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Fixed Points for Weak Contraction Mappings in Complete Generalized Metric Spaces

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Abstract. The aim of this paper is to prove the existence and uniqueness of fixed point for $(\phi - \varphi)$- weak contraction mappings and $(\psi - \varphi)$-weak contraction mappings in a complete and Hausdorff generalized metric space.

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1. Introduction

An element v of a set X is called a periodic point for the mapping $T : X \to X$, if $v = T^p v$ for some $p \in \mathbb{N}$. If equality holds for $p = 1$, then v is called a fixed point of T. So any fixed point is a periodic point but the inverse is not true. A noticeable subject for a mapping $T : X \to X$ is the study of conditions in which a unique fixed point exists.
The fixed point theorem most frequently cited in literature is Banach contraction mapping principle, which asserts that if X is a complete metric space and $T : X \to X$ is a contractive mapping i.e., there exists $\lambda \in [0, 1)$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leqslant \lambda.d(x, y). \tag{1}$$

then T has a unique fixed point. The contractive property (1) implies that T is uniformly continuous. In 1969, Boyd and Wong [4] introduced the notion of

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φ-contraction. A mapping $T : X \to X$ in a metric space is called φ-contraction if there exists an upper semi-continuous function $\varphi : [0, \infty) \to [0, \infty)$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leqslant \varphi(d(x, y)).$$

In 2000, Branciari introduced the notion of a generalized metric space in which the rectangle inequality has been supposed instead of triangle inequality of a metric space. He also extended the Banach contraction principle in such space. After that, many results were established about fixed points in this useful space. For more details about fixed point theory in generalized metric spaces, we refer the reader to Akram and Siddiqui [1], Azam and Arshad [3], Das [7,8], Das and Lahiri [9,10], Fora et al. [12], Mihet [14], Samet [15,16] and Sarma et al. [17]. In 2012, Chen and Chen [6] introduced the notion of $(\phi - \varphi)$ and $(\psi - \varphi)$-weak contraction mapping in a generalized metric space and proved two theorems which assure the existence of a periodic point for these two types of weak contraction.

In this article, we refine these results; in fact we prove the existence and uniqueness of fixed points for these types of functions.

2. Preliminaries

We recall the definition of a generalized metric space as follows.

Difinition 2.1. [5] *Let X be a nonempty set. If the mapping $d : X \times X \to \mathbb{R}$, satisfies:*
(1) $d(x, y) \geqslant 0$, for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(3) $d(x, y) \leqslant d(x, w) + d(w, z) + d(z, y)$ for all $x, y \in X$ and for all distinct points $w, z \in X - \{x, y\}$ (rectangular property).
Then d is called a generalized metric on X and (X, d) is called a generalized metric space (g.m.s.).

Let (X, d) be a generalized metric space, $\{x_n\}$ be a sequence in X and $x \in X$. If for every $\varepsilon > 0$ there is an $n_0 \in \mathbb{N}$ such that $d(x_n, x) < \varepsilon$, for all $n > n_0$ then $\{x_n\}$ is said to be g.m.s. convergent to x. We denote this by $\lim_{n\to\infty} x_n = x$, or $x_n \to x$, as $n \to \infty$. If for every $\varepsilon > 0$ there is an $n_0 \in \mathbb{N}$ such that $d(x_n, x_{n+m}) < \varepsilon$, for all $n > n_0$ then $\{x_n\}$ is called a g.m.s. Cauchy sequence in X. If every g.m.s. Cauchy sequence in X is g.m.s. convergent in X, then X is called a complete generalized metric space.

Now we recall the notion of Meir-Keeler function (see [13]). A function $\varphi : [0, \infty) \to [0, \infty)$ is said to be a Meir-Keeler function if for each $\eta > 0$, there exists $\delta > 0$ such that for $t \in [0, \infty)$ with $\eta \leqslant t < \eta + \delta$, we have $\varphi(t) < \eta$. In

[2,11], the authors proved the existence and uniqueness of fixed points for various Meir-Keeler type contractive functions. In [6] Chen and Chen introduced the below notions of the weaker Meir-Keeler function $\varphi : [0,\infty) \to [0,\infty)$ and stronger Meir-Keeler function $\psi : [0,\infty) \to [0,1)$.

Definition 2.2. [6] *A mapping $\varphi : [0,\infty) \to [0,\infty)$ is called a weaker Meir-Keeler function if for each $\eta > 0$, there exists $\delta > 0$ such that for $t \in [0,\infty)$ with $\eta \leqslant t < \eta + \delta$, there exists $n_0 \in \mathbb{N}$ such that $\varphi^{n_0}(t) < \eta$.*

Definition 2.3. [6] *A mapping $\psi : [0,\infty) \to [0,1)$ is called a stronger Meir-Keeler function if the function ψ satisfies following condition*

$$\forall \eta > 0 \quad \exists \delta > 0 \quad \exists \gamma_\eta \in [0,1) \quad \forall\, t \in [0,\infty)(\eta \leqslant t < \eta + \delta \Rightarrow \psi(t) < \gamma_\eta).$$

In the following we mention some conventions. Throughout the paper we use notations ϕ, φ and ψ, for mappings satisfying the convention.

Conventions

- By ϕ we mean a mapping $\phi : [0,\infty) \to [0,\infty)$ which satisfies:
  (ϕ_1) ϕ is a weaker Meir- Keeler function;
  (ϕ_2) $\phi(t) > 0$ for $t > 0$ and $\phi(0) = 0$;
  (ϕ_3) for all $t \in (0,\infty)$, $\{\phi^n(t)\}$ is decreasing;
  (ϕ_4) for $t_n \subseteq [0,\infty)$, we have
  ($\phi_{4.1}$) if $\lim_{n\to\infty} t_n = r > 0$, then $\lim_{n\to\infty} \phi(t_n) < r$, and
  ($\phi_{4.2}$) if $\lim_{n\to\infty} t_n = 0$, then $\lim_{n\to\infty} \phi(t_n) = 0$.
- Let $\varphi : [0,\infty) \to [0,\infty)$ be a non - decreasing function satisfying:
  (φ_1) $\varphi(t) > 0$ for $t > 0$ and $\varphi(0) = 0$;
  (φ_2) φ is subadditive, i.e. for every $\alpha_1, \alpha_2 \in [0,\infty), \varphi(\alpha_1+\alpha_2) \leqslant \varphi(\alpha_1)+\varphi(\alpha_2)$;
  (φ_3) for all $t_n \in (0,\infty), \lim_{n\to\infty} t_n = 0$ if and only if $\lim_{n\to\infty} \varphi(t_n) = 0$.
- Let the function $\psi : [0,\infty) \to [0,1)$ satisfies the following conditions:
  (ψ_1) $\psi : [0,\infty) \to [0,1)$ is a stronger Meir-Keeler funcion;
  (ψ_2) $\psi(t) > 0$ for $t > 0$ and $\psi(0) = 0$.

3. Fixed Point Theorems

Now we recall the notion of the $(\phi - \varphi)$-weak contraction mapping and then prove existence and uniqueness of a fixed point for the $(\phi - \varphi)$-weak contraction mapping.

Definition 3.1. [6] *Let* (X, d) *be a generalized metric space, and* $T : X \to X$ *be a function satisfying*

$$\varphi(d(Tx, Ty)) \leqslant \phi(\varphi(d(x, y))), \tag{2}$$

for all $x, y \in X$. *Then* T *is said to be a* $(\phi - \varphi)$*-weak contraction mapping.*

Theorem 3.2. *Let* (X, d) *be a Hausdorff and complete generalized metric space. If* $T : X \to X$ *is a* $(\phi - \varphi)$*-weak contraction mapping, then* T *has a unique fixed point in* X.

Proof. Let x_0 be an arbitrary point of X, and define the sequence $\{x_n\}$ inductively by

$$x_{n+1} = Tx_n, \qquad n = 0, 1, 2, \cdots .$$

Since $T : X \to X$ is a $(\phi - \varphi)$-weak contraction mapping, by (2) for each $n, i \in \mathbb{N}$, we observe that

$$\begin{aligned} \varphi(d(x_n, x_{n+i})) &= \varphi(d(Tx_{n-1}, Tx_{n+i-1})) \\ &\leqslant \phi(\varphi(d(x_{n-1}, x_{n+i-1}))) \\ &\leqslant \phi(\phi(\varphi(d(x_{n-2}, x_{n+i-2})))) \\ &= \phi^2(\varphi(d(x_{n-2}, x_{n+i-2}))). \end{aligned}$$

So by induction we have

$$\varphi(d(x_n, x_{n+i})) \leqslant \phi^n(\varphi(d(x_0, x_i))), \quad n, i \in \mathbb{N}.$$

On the other hand according to (ϕ_3), for a fixed $i \in \mathbb{N}$ the sequence $\{\phi^n(\varphi(d(x_0, x_i)))\}$ is decreasing, and so converges to some $\eta \geqslant 0$. In fact, for each $\delta > 0$, there exists $p \in \mathbb{N}$ such that for every $n \geqslant p$ we have

$$\eta \leqslant \phi^n(\varphi(d(x_0, x_i))) < \eta + \delta. \tag{3}$$

We show that $\eta = 0$. Suppose $\eta > 0$. Since ϕ is a weaker Meir Keeler function, corresponding to η, we may choose $\delta > 0$ and $n_0 \in \mathbb{N}$ such that $\phi^{n_0}(t) < \eta$ for every $t \in [\eta, \eta + \delta)$. By (3) with respect to δ, there exists $p_0 \in \mathbb{N}$ such that for all $n \geqslant p_0$

$$\eta \leqslant \phi^n(\varphi(d(x_0, x_i))) < \eta + \delta. \tag{4}$$

Letting $t = \phi^{p_0}(\varphi(d(x_0, x_i)))$, we conclude that

$$\phi^{p_0+n_0}(\varphi(d(x_0, x_i))) < \eta,$$

which contradicts with (4). Therefore, $\lim_{n\to\infty} \phi^n(\varphi(d(x_0, x_i))) = 0$,that is, for each $i \in \mathbb{N}$,

$$\lim_{n\to\infty} \varphi(d(x_n, x_{n+i})) = 0.$$

In particular when $i = 1, 2$, we get

$$\lim_{n\to\infty} \varphi(d(x_n, x_{n+1})) = 0, \tag{5}$$

$$\lim_{n\to\infty} \varphi(d(x_n, x_{n+2})) = 0. \tag{6}$$

Now we show that $\lim_{r,s\to\infty} \varphi(d(x_r, x_s)) = 0$, that is, for every $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $d(x_r, x_s) < \varepsilon$ for $r, s \geqslant n$. If not, there exists $\varepsilon > 0$ such that for any $n \in \mathbb{N}$, there are $r_n, s_n \in \mathbb{N}$ with $r_n > s_n \geqslant n$ satisfying

$$\varphi(d(x_{r_n}, x_{s_n})) \geqslant \varepsilon.$$

Further, corresponding to s_n, we can choose r_n in such a way that it is the smallest integer with $r_n > s_n \geqslant n$ and $\varphi(d(x_{s_n}, x_{r_n})) \geqslant \varepsilon$. Therefore $\varphi(d(x_{s_n}, x_{r_n-1})) < \varepsilon$. By the rectangular inequality and subadditivity of φ, we have

$$\begin{aligned} \varepsilon &\leqslant \varphi(d(x_{r_n}, x_{s_n})) \leqslant \varphi(d(x_{r_n}, x_{r_n-2}) + d(x_{r_n-2}, x_{r_n-1}) + d(x_{r_n-1}, x_{s_n})) \\ &< \varphi(d(x_{r_n}, x_{r_n-2})) + \varphi(d(x_{r_n-2}, x_{r_n-1})) + \varepsilon. \end{aligned}$$

Letting $n \to \infty$, by (5) and (6) we get

$$\lim_{n\to\infty} \varphi(d(x_{r_n}, x_{s_n})) = \varepsilon.$$

On the other hand, we have

$$\begin{aligned} \varphi(d(x_{r_n}, x_{s_n})) &\leqslant \varphi(d(x_{r_n}, x_{r_n-1}) + d(x_{r_n-1}, x_{s_n-1}) + d(x_{s_n-1}, x_{s_n})) \\ &\leqslant \varphi(d(x_{r_n}, x_{r_n-1})) + \varphi(d(x_{r_n-1}, x_{s_n-1})) + \varphi(d(x_{s_n-1}, x_{s_n})), \end{aligned}$$

which shows that

$$\liminf_{n\to\infty} \varphi(d(x_{r_n-1}, x_{s_n-1})) \geqslant \varepsilon,$$

and

$$\begin{aligned} \varphi(d(x_{r_n-1}, x_{s_n-1})) &\leqslant \varphi(d(x_{r_n-1}, x_{r_n}) + d(x_{r_n}, x_{s_n}) + d(x_{s_n}, x_{s_n-1})) \\ &\leqslant \varphi(d(x_{r_n-1}, x_{r_n})) + \varphi(d(x_{r_n}, x_{s_n})) + \varphi(d(x_{s_n}, x_{s_n-1})). \end{aligned}$$

It means that

$$\limsup_{n\to\infty} \varphi(d(x_{r_n-1}, x_{s_n-1})) \leqslant \varepsilon.$$

So we obtain

$$\lim_{n\to\infty} \varphi(d(x_{r_n-1}, x_{s_n-1})) = \varepsilon.$$

Using the inequality (2), then

$$\begin{aligned} \varphi(d(x_{r_n}, x_{s_n})) &= \varphi(d(Tx_{r_n-1}, Tx_{s_n-1})) \\ &\leqslant \phi(\varphi(d(x_{r_n-1}, x_{s_n-1}))). \end{aligned}$$

Letting $n \to \infty$, by the condition (ϕ_4), we have

$$\varepsilon \leqslant \lim_{n\to\infty} \phi(\varphi(d(x_{r_n-1}, x_{s_n-1}))) < \varepsilon.$$

So we get a contradiction. Hence, for every $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $d(x_r, x_s) < \varepsilon$ for $r, s \geqslant n$, that means $\lim_{r,s\to\infty} \varphi(d(x_r, x_s)) = 0$. Now letting $t_n = \sup_{r,s\geqslant n} d(x_r, x_s)$, we see that $\lim_{n\to\infty} \varphi(t_n) = 0$, and then $\lim_{n\to\infty} t_n = 0$ by condition (φ_3); which implies that $\lim_{r,s\to\infty} d(x_r, x_s)) = 0$. Thus $\{x_n\}$ is a g.m.s. Cauchy sequence in complete generalized metric space X, and so it is g.m.s. convergent to some $v \in X$.
In this situation we show that v is a fixed point for T.
In a particular case if there exist $n, m \in \mathbb{N}$ with $n < m$ such that $x_n = x_m$, then, we observe that

$$\{x_n, x_{n+1}, \cdots\} = \{x_n, x_{n+1}, \cdots, x_{m-1}\}. \tag{7}$$

Since $\{x_n\}$ is g.m.s. convergent to v, then for every $\varepsilon > 0$, we have $d(x_r, v) < \varepsilon$ for enough large numbers r. The equality (7) implies that for each $\varepsilon > 0$ and $n \leqslant r \leqslant m-1$, $d(x_r, v) < \varepsilon$. Summing up we have $x_n = x_{n+1} = \cdots = x_m = \cdots = v$. Moreover $v = x_n = Tx_n$ is a fixed point of T.
In the general case by the inequality (2), we obtain

$$\varphi(d(Tx_n, Tv)) \leqslant \phi(\varphi(d(x_n, v))).$$

Therefore, by (φ_3) and (ϕ_4) we get

$$\lim_{n\to\infty} \varphi(d(Tx_n, Tv)) = 0.$$

Put $t_n = d(Tx_n, Tv)$ and use the condition (φ_3) to see that

$$\lim_{n\to\infty} d(Tx_n, Tv) = 0.$$

Since (X, d) is Hausdorff we conclude that

$$Tv = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = v.$$

So v is a fixed point of T.
Finally we show that the fixed point v to T is unique.Suppose that v_1 and v_2 are two distinct fixed points of T.
Putting $x = v_1$ and $y = v_2$ in (2), we have

$$\varphi(d(Tv_1, Tv_2)) \leqslant \phi(\varphi(d(v_1, v_2))).$$

On the other words

$$\varphi(d(v_1, v_2)) \leqslant \phi(\varphi(d(v_1, v_2))). \tag{8}$$

Let $t_n = \varphi(d(v_1, v_2))$. Then we have $\lim_{n\to\infty} t_n = r > 0$ and by (ϕ_4), $\lim_{n\to\infty} \phi(t_n) < r$, which contradicts with inequality (8). Therefore,

$$\varphi(d(v_1, v_2)) = \lim_{n\to\infty} t_n = 0.$$

So by (φ_1), we have $d(v_1, v_2) = 0$, that is, $v_1 = v_2$. □

Remembering the functions ψ and φ, we next define the notion of the $(\psi - \varphi)$-weak contraction mapping and then prove the fixed point theorem for the $(\psi - \varphi)$-weak contraction mappings.

Definition 3.3. [6] *Let (X, d) be a generalized metric space, and let $T : X \to X$ be a function satisfying*

$$\varphi(d(Tx, Ty)) \leqslant \psi(\varphi(d(x, y))).\varphi(d(x, y)), \tag{9}$$

for all $x, y \in X$. Then T is said to be a $(\psi - \varphi)$-weak contraction mapping.

Theorem 3.4. *Let (X, d) be a Hausdorff and complete generalized metric space. If $T : X \to X$ is a $(\psi - \varphi)$-weak contraction mapping, then T has a unique fixed point v in X.*

Proof. Let x_0 be an arbitrary point of X, and the sequence $\{x_n\}$ is defined inductively by

$$x_{n+1} = Tx_n, \qquad (n = 0, 1, 2, \cdots).$$

Since T is a $(\psi - \varphi)$-weak contraction mapping, we have for each $n \in \mathbb{N}$,

$$\begin{aligned} \varphi(d(x_n, x_{n+1})) &= \varphi(d(Tx_{n-1}, Tx_n)) \\ &\leqslant \psi(\varphi(d(x_{n-1}, x_n))).\varphi(d(x_{n-1}, x_n)) \\ &< \varphi(d(x_{n-1}, x_n)). \end{aligned}$$

Thus the bounded below sequence $\{\varphi(d(x_n, x_{n+1}))\}$ is decreasing and hence it is convergent to some $\eta \geqslant 0$. Suppose $\lim_{n\to\infty} \varphi(d(x_n, x_{n+1})) = \eta > 0$. Then for each $\delta > 0$ there exists $n_\delta \in \mathbb{N}$ such that for all $n \in \mathbb{N}$ with $n \geqslant n_\delta$

$$\eta \leqslant \varphi(d(x_n, x_{n+1})) < \eta + \delta. \tag{10}$$

Further, corresponding to η, there exists $\gamma_\eta \in [0, 1)$ such that for all $n \geqslant n_\delta$,

$$\psi(\varphi(d(x_n, x_{n+1}))) < \gamma_\eta.$$

Therefore, it can be deduced that for each $n \in \mathbb{N}$ with $n \geqslant n_\delta + 1$

$$\begin{array}{rcl} \varphi(d(x_n, x_{n+1})) & = & \varphi(d(Tx_{n-1}, Tx_n)) \\ & \leqslant & \psi(\varphi(d(x_{n-1}, x_n))).\varphi(d(x_{n-1}, x_n)) \\ & < & \gamma_\eta.\varphi(d(x_{n-1}, x_n)) \\ & & \vdots \\ & \leqslant & \gamma_\eta^{n-n_\delta}.\varphi(d(x_{n_\delta}, x_{n_\delta+1})). \end{array}$$

Since $\gamma_\eta \in [0,1)$, so we get a contradiction. Therefore $\eta = 0$ and

$$\lim_{n\to\infty} \varphi(d(x_n, x_{n+1})) = 0. \tag{11}$$

A similar process also shows that

$$\lim_{n\to\infty} \varphi(d(x_n, x_{n+2})) = 0. \tag{12}$$

By a similar argument to proof of Theorem 3.2. we find that $\{x_n\}$ is a g.m.s. convergent sequence i.e. $\lim_{n\to\infty} x_n = v$ for some $v \in X$. Now we show that v is the unique fixed point of T.

By using the inequality (9), we obtain

$$\varphi(d(Tx_n, Tv)) \leqslant \psi(\varphi(d(x_n, v))).\varphi(d(x_n, v)).$$

Therefore,

$$\lim_{n\to\infty} \varphi(d(Tx_n, Tv)) = 0.$$

By putting $t_n = d(Tx_n, Tv)$ and using the condition (φ_3), we have

$$\lim_{n\to\infty} d(Tx_n, Tv) = 0.$$

Since (X, d) is Hausdorff,

$$Tv = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = v;$$

which means that v is a fixed point of T. Suppose that v_1 and v_2 are two distinct fixed points of T.

Putting $x = v_1$ and $y = v_2$ in (9), we have

$$\varphi(d(Tv_1, Tv_2)) \leqslant \psi(\varphi(d(v_1, v_2))).\varphi(d(v_1, v_2)).$$

That is,

$$\varphi(d(v_1, v_2)) \leqslant \psi(\varphi(d(v_1, v_2))).\varphi(d(v_1, v_2)). \tag{13}$$

If $t = \varphi(d(v_1, v_2)) > 0$, then by condition (ψ_2), we obtain $\psi(t) > 0$ and since ψ is a stronger Meir-Keeler function then $\psi(t) < 1$, which contradicts with the inequality (13). Therefore, $d(v_1, v_2) = \psi(t) = 0$. That is, $v_1 = v_2$. $\square$

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