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Solving the Integro-Differential Equations Using the Modified Laplace Adomian Decomposition Method

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Abstract. In this work, the modified Laplace Adomian decomposition method (LADM) is applied to solve the integro-differential equations. In addition, examples that illustrate the pertinent features of this method are presented, and the results of the study are discussed. We prove the convergence of LADM applied to the integro-differential equations. Also, the results show that the introduced method is a powerful tool for solving the integro-differential equations.

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1. Introduction

In 1980 George Adomian introduced a new method to solve nonlinear functional equations. This method has since been termed the Adomian decomposition method (ADM) and has been the subject of many investigations ([1, 3, 19]). The ADM involves separating the equation under investigation into linear and nonlinear portions. This method generates a solution in the form of a series whose terms are determined by a recursive relation using the Adomian polynomials. Some fundamental works on various aspects of modifications of the Adomian's decomposition method are given by Andrianov ([2]), Venkatarangan ([20, 21]) and Wazwaz ([22]). The modified form of Laplace decomposition method has been introduced by Khuri ([12, 13]). Agadjanov ([25]) solved the Duffing equation by this method. This numerical technique basically illustrates how the Laplace transform may be used to approximate the

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solutions of the nonlinear partial differential equations by manipulating the decomposition method. Elgasery ([8]) applied the Laplace decomposition method for the solution of Falkner–Skan equation. Hussain and Khan in [10] the modified Laplace decomposition method have applied for solving some PDEs. Recently, the authors have used several methods for the numerical or the analytical solution of linear and nonlinear Fredholm and Volterra integral and integro-differential equations of the second kind ([11]). The nonlinear Fredholm integro-differential equations are given by

$$u^{(n)}(x) = f(x) + \int_a^x K(x,t)[Ru(t) + Nu(t)]dt, \quad u^{(k)}(x) = \alpha_k, \quad 0 \leqslant k \leqslant (n-1), \quad n \geqslant 0 \tag{1}$$

and the nonlinear Volterra integro-differential equations are given by

$$u^{(n)}(x) = f(x) + \int_a^b K(x,t)[Ru(t) + Nu(t)]dt, \quad u^{(k)}(x) = \alpha_k, \quad 0 \leqslant k \leqslant (n-1), \quad n \geqslant 0. \tag{2}$$

where $u^{(n)}(x)$ is the nth derivative of the unknown function $u(x)$ that will be determined, $K(x,t)$ is the kernel of the integral equation, $f(x)$ is an analytic function, $R(u)$ and $N(u)$ are linear and nonlinear functions of u, respectively. For $n = 0$, Eqs. (1) and (2) are the nonlinear Fredholm integral equations and the nonlinear Volterra integral equations, respectively. The Taylor polynomial solution of integro-differential equations has been studied in [14]. The use of Lagrange interpolation in solving integro–differential equations was investigated by Rashed ([17]). Wazwaz ([23]) used the modified decomposition method and the traditional methods for solving nonlinear integral equations. A variety of powerful methods has been presented, such as the homotopy analysis method ([4, 5]), homotopy perturbation method ([6]), the Exp-function method ([15]), variational iteration method ([7]) and the Adomian decomposition method ([7]). By using the LADM we obtain analytical solutions for the integro-differential equations. Our aim in this paper is to obtain the analytical solutions by using the modified Laplace Adomian decomposition method. The remainder of the paper is organized as follows: In Section 2, a brief discussions for the modified Laplace Adomian decomposition method is presented and exact solutions for some examples are obtained. In Section 3, applications of this method and numerical results are considered for the integro-differential equations by LADM. Section 4 ends this paper with a brief conclusion.

2. Modified Laplace Adomian Decomposition Method

The purpose of this section is to discuss the use of modified Laplace decomposition algorithm for the integro-differential equations. We consider the general form of second order nonlinear partial differential equations with initial conditions in the form

$$\mathrm{Lu(x,t) + Ru(x,t) + Nu(x,t) = h(x,t), \quad u(x,0) = f(x), \quad u_t(x,0) = g(x),} \quad (3)$$

where L is the second order differential operator $\mathrm{L_{xx}} = \frac{\partial^n}{\partial x^n}$, R is the remaining linear operator, N represents a general non-linear differential operator and $\mathrm{h(x,t)}$ is the source term. Applying Laplace transform (denoted by $\mathcal{L}$) on both sides of Eq. (3) we have

$$\mathcal{L}[\mathrm{Lu(x,t)}] + \mathcal{L}[\mathrm{Ru(x,t)}] + \mathcal{L}[\mathrm{Nu(x,t)}] = \mathcal{L}[\mathrm{h(x,t)}],$$

and by using the differentiation property of Laplace transform we obtain:

$$\mathrm{s}^2\mathcal{L}[\mathrm{u(x,t)}] - \mathrm{sf(x)} - \mathrm{g(x)} + \mathcal{L}[\mathrm{Ru(x,t)}] + \mathcal{L}[\mathrm{Nu(x,t)}] = \mathcal{L}[\mathrm{h(x,t)}],$$

and so:

$$\mathcal{L}[\mathrm{u(x,t)}] = \frac{\mathrm{f(x)}}{\mathrm{s}} + \frac{\mathrm{g(x)}}{\mathrm{s}^2} - \frac{1}{\mathrm{s}^2}\mathcal{L}[\mathrm{Ru(x,t)}] - \frac{1}{\mathrm{s}^2}\mathcal{L}[\mathrm{Nu(x,t)}] + \frac{1}{\mathrm{s}^2}\mathcal{L}[\mathrm{h(x,t)}]. \quad (4)$$

The next step in Laplace decomposition method is representing the solution as an infinite series given below

$$\mathrm{u(x,t)} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{u_n(x,t)}. \quad (5)$$

The nonlinear operator is decomposed as

$$\mathrm{Nu(x,t)} = \sum_{\mathrm{n}=0}^{\infty} \mathrm{A_n(x,t)}, \quad (6)$$

where for every $\mathrm{n} \in \mathrm{N}$ $\mathrm{A_n}$ is the Adomian polynomial given below

$$\mathrm{A_n} = \frac{1}{\mathrm{n}!}\frac{\mathrm{d^n}}{\mathrm{d}\lambda^{\mathrm{n}}}\left[\mathrm{N}\left(\sum_{\mathrm{i}=0}^{\infty}\lambda^{\mathrm{i}}\mathrm{u_i}\right)\right]_{\lambda=0}.$$

Using (4), (5) and (6) we get

$$\sum_{n=0}^{\infty}\mathcal{L}[u_n(x,t)] = \frac{f(x)}{s} + \frac{g(x)}{s^2} - \frac{1}{s^2}\mathcal{L}[Ru(x,t)] - \frac{1}{s^2}\mathcal{L}\left[\sum_{n=0}^{\infty}A_n(x,t)\right] + \frac{1}{s^2}\mathcal{L}[h(x,t)]. \tag{7}$$

Comparing both sides of (7) we have

$$\mathcal{L}[u_0(x,t)] = k_1(x,s), \tag{8}$$

$$\mathcal{L}[u_1(x,t)] = k_2(x,s) - \frac{1}{s^2}\mathcal{L}[R_0u(x,t)] - \frac{1}{s^2}\mathcal{L}\left[A_0(x,t)\right], \tag{9}$$

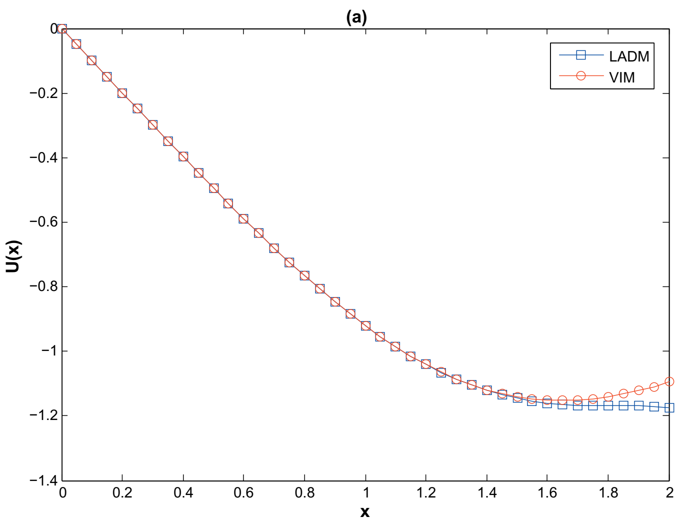
$$\mathcal{L}[u_{n+1}(x,t)] = -\frac{1}{s^2}\mathcal{L}[R_nu(x,t)] - \frac{1}{s^2}\mathcal{L}\left[A_n(x,t)\right], \quad n \geqslant 1, \tag{10}$$

where $k_1(x,s)$ and $k_2(x,s)$ are Laplace transform of $k_1(x,t)$ and $k_2(x,t)$ respectively. Applying the inverse Laplace transform to Eqs. (8)–(10) gives our required recursive relation as follows

$$u_0(x,t) = k_1(x,t), \tag{11}$$

$$u_1(x,t) = k_2(x,t) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}[R_0u(x,t)] - \frac{1}{s^2}\mathcal{L}\left[A_0(x,t)\right]\right], \tag{12}$$

$$u_{n+1}(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}[R_nu(x,t)] - \frac{1}{s^2}\mathcal{L}\left[A_n(x,t)\right]\right], \quad n \geqslant 1. \tag{13}$$



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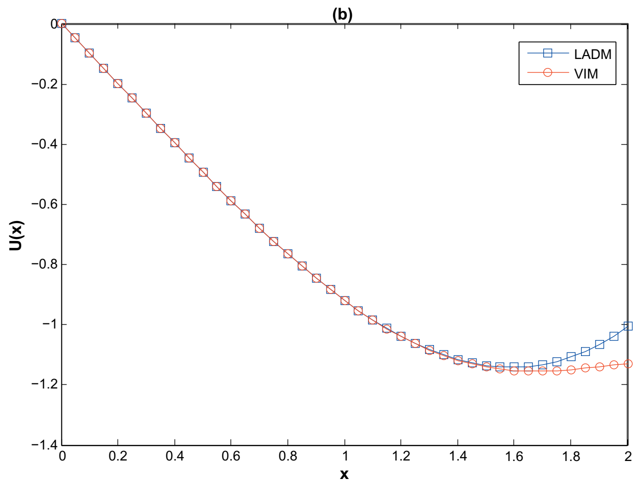

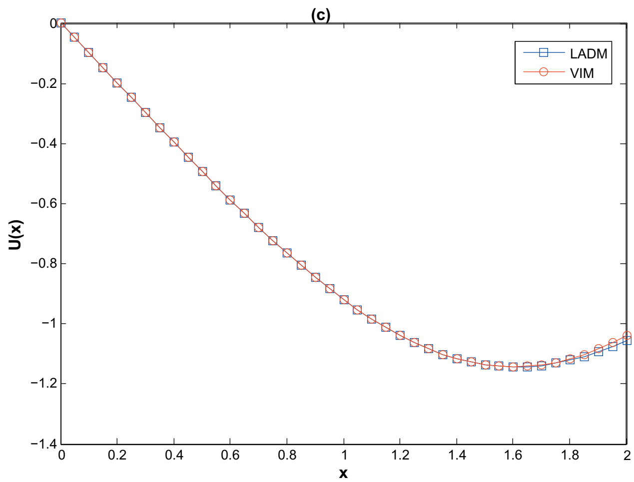

Figure 1: A comparison between
(**a**) 2-iterate LADM solutions and VIM solutions
(**b**) 3-iterate LADM solutions and VIM solutions
(**c**) 4-iterate LADM solutions and VIM solutions for example 1.

The solution through the modified Adomian decomposition method highly depends upon the choice of $k_0(x,t)$ and $k_1(x,t)$, where $k_0(x,t)$ and $k_1(x,t)$ represent the terms arising from the source term and prescribed initial conditions.

3. Application of the Modified Adomian Decomposition Method

In this section we give four examples to illustrate this method for the integro-differential equations.

Example 1. (See [9].) Let us first consider the nonlinear integro-differential equation

$$u'(x) = -1 + \int_0^x u^2(t)dt, \quad u(0) = 0.$$

Applying the Laplace transform and by using the initial condition we have

$$sU(s) = -\frac{1}{s} + \mathcal{L}\left[\int_0^x u^2(t)dt\right],$$

or

$$U(s) = -\frac{1}{s^2} + \frac{1}{s}\mathcal{L}\left(\int_0^x u^2(t)dt\right).$$

Applying the inverse Laplace transform we get

$$u(x) = -x + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(\int_0^x u^2(t)dt\right)\right]. \tag{14}$$

We decompose the solution as an infinite sum given below

$$u(x) = \sum_{n=0}^{\infty} u_n(x). \tag{15}$$

Using (15) on (14) we get

$$\sum_{n=0}^{\infty} u_n(x) = -x + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(\int_0^x \sum_{n=0}^{\infty} A_n(t)dt\right)\right],$$

in which $A_n = \sum_{j=0}^{n} u_j u_{n-j}$. The recursive relation is given below

$$u_0(x) = -x,$$

$$u_1(x) = \frac{x^4}{12},$$

$$u_2(x) = -\frac{x^7}{252}.$$

By repeating above procedure for $n \geqslant 3$, we get to approximate solution as follows

Table 1: The comparison of the values obtained by u_{LADM} and u_{VIM} for example 1.

	second-order approximation		third-order approximation		fourth-order approximation	
x	u_{VIM}	u_{LADM}	u_{VIM}	u_{LADM}	u_{VIM}	u_{LADM}
0.0000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000
0.0938	-0.09379355	-0.09379355	-0.09379355	-0.09379355	-0.09379355	-0.09379355
0.2188	-0.2186091	-0.2186091	-0.2186091	-0.2186091	-0.2186091	-0.2186091
0.3125	-0.3117065	-0.3117065	-0.3117065	-0.3117065	-0.3117065	-0.3117065
0.4062	-0.4039385	-0.4039385	-0.4039385	-0.4039385	-0.4039385	-0.4039385
0.5000	-0.4948226	-0.4948227	-0.4948226	-0.4948225	-0.4948225	-0.4948225
0.6250	-0.6124314	-0.6124321	-0.6124314	-0.6124306	-0.6124306	-0.6124306
0.7188	-0.6969446	-0.6969474	-0.6969447	-0.6969413	-0.6969414	-0.6969414
0.8125	-0.7771007	-0.7771104	-0.7771011	-0.7770897	-0.7770901	-0.7770901
0.9062	-0.8519654	-0.8519942	-0.8519669	-0.8519325	-0.8519340	-0.8519343
1.0000	-0.9205578	-0.9206350	-0.9205629	-0.9204697	-0.9204747	-0.9204761

Example 2. (See [23]) Consider the second–order nonlinear integro-differential equation

$$u''(x) = \sinh(x) + x - \int_0^1 x(\cosh^2(t) - u^2(t))dt, \quad u(0) = 0, \quad u'(0) = 1.$$

Applying the Laplace transform and by using the initial conditions we obtain

$$s^2U(s) - 1 = \frac{1}{s^2-1} + \frac{1}{s^2} - \mathcal{L}\left[\int_0^1 x(\cosh^2(t) - u^2(t))dt\right],$$

or

$$U(s) = \frac{1}{s^2} + \frac{1}{s^2(s^2-1)} + \frac{1}{s^4} - \frac{1}{s^2}\mathcal{L}\left(\int_0^1 x(\cosh^2(t) - u^2(t))dt\right).$$

Applying the inverse Laplace transform we get

$$u(x) = \sinh(x) + \frac{x^3}{6} - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left(x\int_0^1 (\cosh^2(t) - u^2(t))dt\right)\right].$$

Applying the same procedure as in the previous example we arrive the modified recursive relation given below

$$u_0(x) = \sinh(x),$$

$$u_1(x) = \frac{x^3}{6} - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left(x\int_0^1 (\cosh^2(t) - u_0^2(t))dt\right)\right] = 0,$$

$$u_{n+1}(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left(x\int_0^1 (\cosh^2(t) - A_n(t))dt\right)\right] = 0, \quad n \geqslant 1,$$

in which $A_n = \sum_{j=0}^{n} u_j u_{n-j}$, where for every $n \geqslant 1$, $A_n = 0$. Thus, the exact solution is

$$u(x) = \sinh(x).$$

Example 3. (See [11].) Consider the third-order linear integro-differential equation

$$u'''(x) = \sin(x) - x - \int_0^{\frac{\pi}{2}} xtu'(t)dt, \quad u(0) = 1, \quad u'(0) = 0, \quad u''(0) = -1.$$

First, we apply the Laplace transform and by using the initial conditions we have

$$s^3U(s) - s^2 + 1 = \frac{1}{s^2+1} - \frac{1}{s^2} - \mathcal{L}\left[\int_0^{\frac{\pi}{2}} xtu'(t)dt\right],$$

or

$$U(s) = \frac{1}{s} - \frac{1}{s^3} + \frac{1}{s^3(s^2+1)} - \frac{1}{s^5} - \frac{1}{s^3}\mathcal{L}\left(\int_0^{\frac{\pi}{2}} xtu'(t)dt\right).$$

Now applying the inverse Laplace transform we get

$$u(x) = \cos(x) - \frac{1}{24x^4} - \mathcal{L}^{-1}\left[\frac{1}{s^3}\mathcal{L}\left(\int_0^{\frac{\pi}{2}} xtu'(t)dt\right)\right].$$

Applying the same procedure as in the previous example we arrive the modified recursive relation given below

$$u_0(x) = \cos(x),$$

$$u_1(x) = -\frac{1}{24x^4} - \mathcal{L}^{-1}\left[\frac{1}{s^3}\mathcal{L}\left(x\int_0^{\frac{\pi}{2}} tu_0'(t)dt\right)\right] = 0,$$

$$u_{n+1}(x) = -\mathcal{L}^{-1}\left[\frac{1}{s^3}\mathcal{L}\left(\int_0^{\frac{\pi}{2}} xtu_n'(t)dt\right)\right] = 0, \qquad n \geqslant 1,$$

and so the total solution of the above problem is given as

$$u(x) = \cos(x).$$

Example 4. (See [14, 24, 18].) Consider the second-order linear integro-differential equation

$$u''(x) + xu'(x) - xu(x) = e^x - 2\sin(x) + \int_{-1}^{1} \sin(x)e^{-t}u(t)dt, \; u(0) = 1, \; u'(0) = 1.$$

We apply the Laplace transform and by using the initial conditions we have

$$(s^2-1)U(s) - (s-1)\frac{d}{ds}U(s) - s - 1 = \frac{1}{s-1} - \frac{2}{s^2+1} + \mathcal{L}\left[\int_{-1}^{1} \sin(x)e^{-t}u(t)dt\right],$$

or

$$U(s) = \frac{1}{s+1}\frac{d}{ds}U(s) + \frac{1}{s-1} + \frac{1}{(s-1)^2(s+1)} - \frac{2}{(s^2-1)(s^2+1)} - \frac{1}{s^2-1}\mathcal{L}\left(\int_{-1}^{1} \sin(x)e^{-t}u(t)dt\right).$$

Applying the inverse Laplace transform we get

$$u(x) = e^x + \mathcal{L}^{-1}\left[\frac{1}{s+1}\frac{d}{ds}U(s) + \frac{1}{(s-1)^2(s+1)} - \frac{2}{(s^2-1)(s^2+1)} - \frac{1}{(s^2-1)(s^2+1)}\left(\int_{-1}^{1} e^{-t}u(t)dt\right)\right].$$

Applying the same procedure as in the previous example we arrive the modified recursive relation given as

$$u_0(x) = e^x,$$

$$u_1(x) = \mathcal{L}^{-1}\left[\frac{1}{s+1}\frac{d}{ds}\left(\frac{1}{s-1}\right) + \frac{1}{(s-1)^2(s+1)} - \frac{2}{(s^2-1)(s^2+1)} - \frac{1}{(s^2-1)(s^2+1)}\left(\int_{-1}^{1} e^{-t}e^{t}dt\right)\right] = 0,$$

$$u_{n+1}(x) = 0, \quad n \geqslant 1.$$

Thus, the exact solution is

$$u(x) = e^x.$$

4. Convergence Analysis

Here, we will study the convergence analysis as same manner in [16] of the LADM applied to the nonlinear integro-differential equations. Let us consider the Hilbert space H which may define by $H = L^2((\alpha,\beta)X[0,T])$ the set of applications:

$$u : (\alpha,\beta)X[0,T] \to R \quad \text{with} \quad \int_{(\alpha,\beta)x[0,T]} u^2(x,s)dsd\tau < +\infty.$$

Now we consider the nonlinear integro-differential equations in the light of above assumptions and let us denote

$$L(u) = \frac{\partial^n}{\partial x^n}u,$$

then the nonlinear Fredholm integro-differential equations become in a operator form

$$L(u) = f(x) + \int_a^x K(x,t)[Ru(t) + Nu(t)]dt,$$

and the nonlinear Volterra integro-differential equations are given by

$$L(u) = f(x) + \int_a^b K(x,t)[Ru(t) + Nu(t)]dt,$$

The LADM is convergence if the following two hypotheses are satisfied:
(H1) $(\mathrm{L(u)-L(v), u-v}) \geqslant \mathrm{k} \parallel \mathrm{u-v} \parallel^2; \mathrm{k}>0, \forall \mathrm{u,v} \in \mathrm{H}$
(H2) whatever may be $\mathrm{M}>0$, there exist a constant $\mathrm{C(M)}>0$ such that for $\mathrm{u,v} \in \mathrm{H}$ with $\parallel \mathrm{u} \parallel \leqslant \mathrm{M}, \parallel \mathrm{v} \parallel \leqslant \mathrm{M}$ we have:

$$(\mathrm{L(u)-L(v), u-v}) \leqslant \mathrm{C(M)} \parallel \mathrm{u-v} \parallel \parallel \mathrm{w} \parallel$$

for every $\mathrm{w} \in \mathrm{H}$. (see, [16] and the references therein).

Theorem 1. *(Sufficient condition of convergence for example 1). The Laplace Adomian method applied to the nonlinear Voltera integro-differential equation as follows*

$$\mathrm{L(u)} = \frac{\partial}{\partial \mathrm{x}}\mathrm{u} = -1 + \int_0^{\mathrm{x}} \mathrm{u}^2(\mathrm{t})\mathrm{dt},$$

without initial condition, converges towards a particular solution.

Proof. Now, we will verify the conditions (H1) and (H2) of convergence. We will start to verify the convergence hypotheses (H1) for the operator L(u): i.e., $\exists \mathrm{k}>0, \forall \mathrm{u,v} \in \mathrm{H}$, we have:

$$\mathrm{L(u)-L(v)} = \int_0^{\mathrm{x}} (\mathrm{u}^2(\mathrm{t}) - \mathrm{v}^2(\mathrm{t}))\mathrm{dt}.$$

Then we get

$$(\mathrm{L(u)-L(v), u-v}) = \left(\int_0^{\mathrm{x}} (\mathrm{u}^2-\mathrm{v}^2)\mathrm{dt}, \mathrm{u-v}\right).$$

According the Schwartz inequality, we get

$$\left(\int_0^{\mathrm{x}} (\mathrm{u}^2-\mathrm{v}^2)\mathrm{dt}, \mathrm{u-v}\right) \leqslant \mathrm{k}_1 \parallel \mathrm{u}^2-\mathrm{v}^2 \parallel \parallel \mathrm{u-v} \parallel .$$

Now we use the mean value theorem, then we have

$$\left(\int_0^{\mathrm{x}} (\mathrm{u}^2-\mathrm{v}^2)\mathrm{dt}, \mathrm{u-v}\right) \leqslant \mathrm{k}_1 \parallel \mathrm{u}^2-\mathrm{v}^2 \parallel \parallel \mathrm{u-v} \parallel = \frac{1}{3}\mathrm{k}_1\eta^3 \parallel \mathrm{u-v} \parallel^2$$
$$\leqslant \frac{1}{3}k_1M^3 \parallel u-v \parallel^2,$$
$$\left(-\int_0^{\mathrm{x}} (\mathrm{u}^2-\mathrm{v}^2)\mathrm{dt}, \mathrm{u-v}\right) \geqslant \frac{1}{3}\mathrm{k}_1\mathrm{M}^3 \parallel \mathrm{u-v} \parallel^2,$$

where $u < \eta < v$ and $\| u \| \leqslant M, \| v \| \leqslant M$. Therefore:

$$(L(u) - L(v), u - v) \geqslant k \| u - v \|^2,$$

where $k = \frac{1}{3}k_1M^3$. Hence, we find the hypothesis (H1). Now we verify the convergence hypotheses (H2) for the operator L(u) which is for every $M > 0$, there exist a constant $C(M) > 0$ such that for $u, v \in H$ with $\| u \| \leqslant M, \| v \| \leqslant M$ we have $(L(u) - L(v), u - v) \leqslant C(M) \| u - v \| \| w \|$ for every $w \in H$. For that we have:

$$(L(u) - L(v), w) = \left(\int_0^x (u^2 - v^2)dt, w\right)$$
$$\leqslant M^3 \| u - v \| \| w \| = C(M) \| u - v \| \| w \|,$$

where $C(M) = M^3$ and therefore (H2) is hold. The proof is complete.

5. Conclusion

The main idea of this work was to give a simple method for solving the integro-differential equations (IDEs). We carefully applied a reliable modification of Laplace decomposition method for IDEs. The main advantage of this method is the fact that it gives the analytical solution. Also, the solutions which obtained by the Adomian decomposition method and homotopy perturbation method are the same. In the above examples we observed that the LADM with the initial approximation obtained from initial conditions yield a good approximation to the exact solution only in a few iterations. It is also worth noting that the advantage of the decomposition methodology displays a fast convergence of the solutions. The illustrations show the dependence of the rapid convergence depend on the character and behavior of the solutions just as in a closed form solutions. It is shown that this method is a promising tool for some of the linear and nonlinear integro-differential equations.

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