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Contribution of Fuzzy Minimal Cost Flow Problem by Possibility Programming

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Abstract. Using the concept of possibility proposed by zadeh, luhandjula ([4,8]) and buckley ([1]) have proposed the possibility programming. The formulation of buckley results in nonlinear programming problems. Negi [6]re-formulated the approach of Buckley by the use of trapezoidal fuzzy numbers and reduced the problem into fuzzy linear programming problem. Shih and Lee ([7]) used the Negi approach to solve a minimum cost flow problem, whit fuzzy costs and the upper and lower bound. In this paper we shall consider the general form of this problem where all of the parameters and variables are fuzzy and also a model for solving is proposed.

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1. Introduction

Ranking or comparison of fuzzy numbers is one of the basic problems in fuzzy decision making. Since fuzzy numbers are sets and have no linear order, ranking fuzzy numbers is not simple. Many different comparison methods have been proposed in the literature; however, in the present

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paper only the comparison method proposed by Dubois and Prade [2] will be used.
A Trapezoidal Fuzzy number (TrFN) of the form $\{a_1, a_2, a_3, a_4\}$ can also be used, where the value of the membership function is 1 in the interval $[a_2, a_3]$ and zero at a_1 and a_4. For example, the engineer may say that the project can usually be completed in 6 to 8 hours. But, due to uncontrollable factors, the project may take as many as 10 hours or as few as 5 hours. This approximation problem can be represented by the TrFN = [5, 6, 8, 10]. In this paper, two TrFNs $\tilde{B}$ and $\tilde{R}$ are considered which are represented by $\{r_1, r_2, r_3, r_4\}$ and $\{b_1, b_2, b_3, b_4\}$ respectively, the strict exceedance possibility is:

$$poss[\tilde{B} \geqslant \tilde{R}] = \begin{cases} 1 & if \quad b_3 \geqslant r_4 \\ \delta_1 & if \quad b_3 \leqslant r_4, b_4 \geqslant r_3 \\ 0 & if \quad b_4 \leqslant r_3 \end{cases} \tag{1}$$

$$\delta_1 = \frac{(b_4 - r_3)}{(b_4 - b_3) + (r_4 - r_3)} \tag{2}$$

Now, consider the maximization problem with the following format. (Interested readers are recommended to read [2] for more information).

$$\begin{aligned} Max \quad & \tilde{C}X \\ st. \quad & \\ & \tilde{A}X \leqslant \tilde{b} \\ & X \geqslant 0 \end{aligned} \tag{3}$$

Where $\tilde{C}$ and x are n-dimensional vectors, $\tilde{b}$ is m-dimensional, and $\tilde{A}$ is a $(m \times n)$ matrix. Parameters with ˜ are fuzzy numbers which can be TrFN. Since the addition or subtraction of TrFNs results in a TrFN [3], the inequality of constraints in equation (3) can be treated as comparison of fuzzy numbers. Thus, according to equations (1) and (2), the possibility of x to be satisfied by constraint i (F_i) is as follows:

$$poss[x \in F_i] = \begin{cases} 1 & if \quad b_{i3} \geqslant r_{i4} \quad . \quad . \quad . & (4-a) \\ \delta_1 & if \quad b_{i3} \leqslant r_{i4}, b_{i4} \geqslant r_{i3} & (4-b) \\ 0 & if \quad b_{i4} \leqslant r_{i3} \quad . \quad . \quad . & (4-c) \end{cases}$$

$$\delta_{i1} = \frac{(b_{i4} - r_{i3})}{(b_{i4} - b_{i3}) + (r_{i4} - r_{i3})} \tag{5}$$

$$r_{i1} = \sum_{j=1}^{n} a_{ij1} x_j \tag{6-a}$$

$$r_{i2} = \sum_{j=1}^{n} a_{ij2} x_j \tag{6-b}$$

$$r_{i3} = \sum_{j=1}^{n} a_{ij3} x_j \tag{6-c}$$

$$r_{i4} = \sum_{j=1}^{n} a_{ij4} x_j \tag{6-d}$$

Since we are generally not interested in a zero possibility, the decision space is in equation (4-a), (4-b). Thus, the only constraint that is needed to restrict the results in this decision space is:

$$\forall i \quad b_{i4} \geqslant r_{i3}. \tag{7}$$

In order to satisfy all the constraints, we must have:

$$\begin{aligned} poss[x \in F] &= \min\{poss[x \in F_1],\ poss[x \in F_2], ...,\ poss[x \in F_m]\} \\ &= \min\{\delta_1, \delta_2, ..., \delta_m\} \end{aligned} \tag{8}$$

The conditional possibility for the objective function Z, given that the Xs satisfy the strict exceedance possibility for the constraints, are:

$$poss[Z = z|x] = \begin{cases} \theta_1 & for \quad \sum_{j=1}^{n} c_{j1}x_j \leqslant z \leqslant \sum_{j=1}^{n} c_{j2}x_j \quad (9-a) \\ 1 & for \quad \sum_{j=1}^{n} c_{j2}x_j \leqslant z \leqslant \sum_{j=1}^{n} c_{j3}x_j \quad (9-b) \\ \theta_2 & for \quad \sum_{j=1}^{n} c_{j3}x_j \leqslant z \leqslant \sum_{j=1}^{n} c_{j4}x_j \quad (9-c) \end{cases}$$

$$\theta_1 = \frac{z - \sum_{j=1}^{n} c_{j1}x_j}{\sum_{j=1}^{n} c_{j2}x_j - \sum_{j=1}^{n} c_{j1}x_j} \quad , \quad \theta_2 = \frac{\sum_{j=1}^{n} c_{j4}x_j - z}{\sum_{j=1}^{n} c_{j4}x_j - \sum_{j=1}^{n} c_{j3}x_j}.$$

Since we wish to maximize the objective function, the decision space we are interested in is θ_2 , or equation (9-c). In order to keep the results in this decision space, the problem must satisfy the constraint below:

$$\sum_{j=1}^{n} c_{j3}x_j \leqslant z \leqslant \sum_{j=1}^{n} c_{j4}x_j. \tag{10}$$

The unconditional possibility distribution for the objective function can be represented by:

$$poss[Z = z] = \min\{\min\{\delta_1, \delta_2, ..., \delta_m)\}, \theta_2\} = \min\{\delta_1, \delta_2, ..., \delta_m, \theta_2\}. \tag{11}$$

If we have interaction of the decision maker and the decision maker supplied a desired possibility level, say, a level of α , then our problem becomes:

$$\begin{aligned} Max \quad & Z \\ st. \quad & \\ & \delta_1, \delta_2, ..., \delta_m, \theta_2 \geqslant \alpha \\ & b_{i4} \geqslant \sum_{j=1}^{n} a_{ij3}x_j \quad \forall i \\ & x_1, x_2, ..., x_n \geqslant 0. \end{aligned} \tag{12}$$

2. Fuzzy Concepts

Fuzzy numbers described in the first sections are defined by Zadeh [8]. For two TrFNs, $\tilde{A} = [a_1, a_2, a_3, a_4]$ and $\tilde{B} = [b_1, b_2, b_3, b_4]$ the exact formula for extended addition or subtraction may be defined as follows:

$$\tilde{R} - \tilde{B} = (r_1 - b_2, r_2 - b_1, r_3 + b_4, r_4 + b_3) \tag{13}$$

$$\tilde{R} + \tilde{B} = (r_1 + b_1, r_2 + b_2, r_3 + b_3, r_4 + b_4) \tag{14}$$

Moreover for solving the problem in this paper we need the following definitions and operators:

Definition 2.1. *Trapezoidal Fuzzy number (TrFN)* $\tilde{A}$ *of the form* $[a_1, a_2, a_3, a_4]$ *is a positive trapezoidal fuzzy number if* $a_1 > 0$ *, then* $\tilde{A}$ *can be represented by* $\tilde{A} > 0$ *or* $\tilde{A}^+$ *.*

Definition 2.2. *Trapezoidal Fuzzy number* $\tilde{B}$ *of the form* $[b_1, b_2, b_3, b_4]$ *is a negative trapezoidal fuzzy number if* $b_4 < 0$ *, then* $\tilde{B}$ *can be represented by* $\tilde{B} < 0$ *or* $\tilde{B}^-$ *.*

Definition 2.3. *Trapezoidal Fuzzy number* $\tilde{c}$ *of the form* $[c_1, c_2, c_3, c_4]$ *is a semi positive trapezoidal fuzzy number if* $c_1 < 0$ *and* $\exists i, i = 2, 3, 4$ *subject to* $c_i > 0$*. The formula for the extended multiplication of TrFN* $\tilde{A}$ *and* $\tilde{B}$ *and* $\tilde{c}$ *has 3 cases and is presented as follows:*

i. If $\tilde{A} > 0$ *and* $\tilde{B} > 0$

$$\tilde{A}.\tilde{B} = [a_1, a_2, a_3, a_4].[b_1, b_2, b_3, b_4] = [a_1b_1, a_2b_2, a_1b_3+a_3b_1, a_2b_4+a_4b_2] \tag{15}$$

ii. If $\tilde{A} > 0$ *and* $\tilde{B} < 0$

$$\begin{aligned} \tilde{A}.\tilde{B} &= -[a_1, a_2, a_3, a_4].[-b_4, -b_3, -b_2, -b_1] \\ &= [a_2b_1, a_1b_2, a_2b_3 - a_4b_1, a_1b_4 - a_3b_2] \end{aligned} \tag{16}$$

iii. If $\tilde{A} < 0$ *and* $\tilde{B} < 0$

$$\begin{aligned} \tilde{A}.\tilde{B} &= [-a_4, -a_3, -a_2, -a_1].[-b_4, -b_3, -b_2, -b_1] \\ &= [a_2b_2, a_1b_1, -a_2b_4 - a_4b_2, -a_1b_3 - a_3b_1] \end{aligned} \tag{17}$$

3. Minimum Cost Flow Problem with Fuzzy Variable and Parameters

Let G(N, A) be a directed network with a cost c_{ij}. We also let each node $i \in N$ possess a number of resources b_i, which indicates its supply, demand, or transient node depending on whether $b_i > 0$, $b_i < 0$ or $b_i = 0$, respectively. The minimum cost flow (MCF) problem can be formulated as follows:

$$Min \quad f(x) = \sum_{(i,j)\in A} c_{ij}x_{ij}$$

$$st.$$

$$\forall i \in N \qquad \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b_i, \tag{18-a}$$

$$\forall (i,j) \in A \quad x_{ij} \geqslant 0 \quad and \quad integer \tag{18-b}$$

The objective function is to minimize the total cost. Constraint (18-a) represents the conservation of flows. In general, the MCF problems have some additional assumptions such as: (a) supplies, demands, and capacities must be integer, (b) the network is directed, (c) the supply, demand at each node satisfies the condition $\sum b_i = 0$, and (d) the MCF problem has a feasible solution. Although fairly much attention has been paid to theoretical applications of Nahmias Fuzzy variables [5], in this paper we can be considered MCF problem whit fuzzy variable and parameters:

$$Min \quad f(x) = \sum_{(i,j)\in A} \tilde{c_{ij}}\tilde{x_{ij}}$$

$$st.$$

$$\forall (i,j) \in N \qquad \sum_{\{j:(i,j)\in A\}} \tilde{x_{ij}} - \sum_{\{j:(j,i)\in A\}} \tilde{x_{ji}} = \tilde{b_i}, \quad \forall i \in N \tag{19-a}$$

$$\tilde{x_{ij}} \geqslant 0 \tag{19-b}$$

Where $\tilde{c_{ij}} > 0$, $\tilde{x_{ij}}$ and $\tilde{b_i}$ represent the fuzzy variables and parameters. Trapezoidal fuzzy numbers will be used in this paper where

$$c_{ij} = [c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}] \quad b_i = [b_{i1}, b_{i2}, b_{i3}, b_{i4}],$$

$$x_{ij} = [x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}].$$

Thus above problem can be represented as follows:

$$Minf(x) = \sum_{(i,j)\in A} [c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}][x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}] \tag{20-a}$$

$st.$

$$\sum_{\{j:(i,j)\in A\}} [x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}] - \sum_{\{j:(j,i)\in A\}} [x_{ji1}, x_{ji2}, x_{ji3}, x_{ji4}] = [b_{i1}, b_{i2}, b_{i3}, b_{i4}] \tag{20-b}$$

$$x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4} \geqslant 0 \tag{20-c}$$

If $\tilde{B} = \tilde{R}$, (21)
Then we have:

$$b_1 = r_1, b_2 = r_2, b_3 = r_3, b_4 = r_4. \tag{22}$$

Thus, for constraint (20-b) which represents the conservation of flow, and constraint (13) we have:

$$[(\sum_{j=1}^{m} x_{ij1} - \sum_{k=1}^{m} x_{ki2}), (\sum_{j=1}^{m} x_{ij2} - \sum_{k=1}^{m} x_{ki1}), (\sum_{j=1}^{m} x_{ij3} + \sum_{k=1}^{m} x_{ki4}), (\sum_{j=1}^{m} x_{ij4} + \sum_{k=1}^{m} x_{ki3})] = [b_{i1}, b_{i2}, b_{i3}, b_{i4}] \tag{23}$$

Whit constraint (22), the above constraint will be changed to the following constraint:

$$\sum_{j=1}^{m} x_{ij1} - \sum_{k=1}^{m} x_{ki2} = b_{i1}, \sum_{j=1}^{m} x_{ij2} - \sum_{k=1}^{m} x_{ki1} = b_{i2}$$
$$\sum_{j=1}^{m} x_{ij3} + \sum_{k=1}^{m} x_{ki4} = b_{i3}, \sum_{j=1}^{m} x_{ij4} + \sum_{k=1}^{m} x_{ki3} = b_{i4} \tag{24}$$

Since we suppose that $\tilde{x}_{ij} \geqslant 0$ and $\tilde{c}_{ij} \geqslant 0$ the constraint (20-a) using constraint (15) will be changed to the following equation:

$$\sum_{i=1}^{m}\sum_{j=1}^{m}[c_{ij1},c_{ij2},c_{ij3},c_{ij4}].[x_{ij1},x_{ij2},x_{ij3},x_{ij4}] =$$
$$[\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij1},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij2},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij3}+c_{ij3}x_{ij1},$$
$$\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij4}+c_{ij4}x_{ij2}]. \tag{25}$$

Using the possibility theory introduced in section (1) if $\min Z = z$ then:

$$poss[Z=z|x] = \begin{cases} \theta_1 \quad for \quad \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij1} \leqslant z \\ \qquad\qquad \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij2} & (26-a) \\ 1 \quad for \quad \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij2} \leqslant z \\ \qquad\qquad \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij3}+c_{ij3}x_{ij1} & (26-b) \\ \theta_2 \quad for \quad \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij3}+c_{ij3}x_{ij1} \leqslant z \\ \qquad\qquad \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij4}+c_{ij4}x_{ij2}] & (26-c) \end{cases}$$

$$\theta_1 = \frac{z-\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij1}}{\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij2}x_{ij2}-\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij1}x_{ij1}} \tag{27}$$

$$\theta_2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m}(c_{ij2}x_{ij4}+c_{ij4}x_{ij2})-z}{\sum_{i=1}^{m}\sum_{j=1}^{m}(c_{ij2}x_{ij4}+c_{ij4}x_{ij2})-\sum_{i=1}^{m}\sum_{j=1}^{m}(c_{ij1}x_{ij3}+c_{ij3}x_{ij1})} \tag{28}$$

Thus, assuming that the decision maker has decided a cut-off value for α, the general fuzzy minimum cost flow problem can be represented by:

$$Min \quad Z$$
$$st.$$
$$\theta_1 \geqslant \alpha$$
$$\sum_{j=1}^{m}x_{ij1}-\sum_{k=1}^{m}x_{ki2}=b_{i1} \quad for \quad i=1,..,m$$

$$\begin{aligned}
&\sum_{j=1}^{m} x_{ij2} - \sum_{k=1}^{m} x_{ki1} = b_{i2} \quad for \quad i = 1,..,m \\
&\sum_{j=1}^{m} x_{ij3} + \sum_{k=1}^{m} x_{ki4} = b_{i3} \quad for \quad i = 1,..,m \\
&\sum_{j=1}^{m} x_{ij4} + \sum_{k=1}^{m} x_{ki3} = b_{i4} \quad for \quad i = 1,..,m \\
&\sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij1}x_{ij1} \leqslant z \leqslant \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij2}x_{ij2} \\
&\alpha \in [0,1] \\
&(x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}) \geqslant 0.
\end{aligned} \tag{29}$$

4. Fuzzy Minimum Cost Flow Problem with Bounded Variables

In general case, minimum cost flow problem with bounded and fuzzy variables and fuzzy parameters can be represented by following model:

$$Min \quad f(x) = \sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{c_{ij}}\tilde{x_{ij}} \tag{30-a}$$

$$st$$

$$\sum_{i=1}^{m} \tilde{x_{ij}} - \sum_{k=1}^{m} \tilde{x_{ki}} = \tilde{b_i}, \tag{30-b}$$

$$\tilde{l_{ij}} \leqslant \tilde{x_{ij}} \leqslant \tilde{u_{ij}} \tag{30-c}$$

Where $\tilde{x_{ij}}, \tilde{c_{ij}}, \tilde{u_{ij}}, \tilde{l_{ij}}$ are the fuzzy positive or fuzzy negative trapezoidal fuzzy numbers. This assumption is weaker than the assumption $c_{ij} \geqslant 0$ and $x_{ij} \geqslant 0$ of the model (19). Using previous discussion, for constraint (30-c) we have:

$$\tilde{x_{ij}} \leqslant \tilde{u_{ij}}$$

$$poss[\tilde{x_{ij}} \geqslant \tilde{u_{ij}}] = \begin{cases} 1 & if \quad u_{ij3} \geqslant x_{ij4} \\ \delta_{ij1} & if \quad u_{ij3} \leqslant x_{ij4}, u_{ij4} \geqslant x_{ij3} \\ 0 & if \quad u_{ij4} \leqslant x_{ij3} \end{cases} \qquad (31-a)$$

$$\delta_{ij1} = \frac{(u_{ij4} - x_{ij3})}{(u_{ij4} - u_{ij3}) + (x_{ij4} - x_{ij3})} \qquad (31-b)$$

and $\tilde{x_{ij}} \geqslant \tilde{l_{ij}}$

$$poss[\tilde{l_{ij}} \geqslant \tilde{x_{ij}}] = \begin{cases} 1 & if \quad x_{ij3} \geqslant l_{ij4} \\ \delta_{ij2} & if \quad x_{ij3} \leqslant l_{ij4}, x_{ij4} \geqslant l_{ij3} \\ 0 & if \quad x_{ij4} \leqslant l_{ij3} \end{cases} \qquad (32-a)$$

$$\delta_{ij2} = \frac{(x_{ij4} - l_{ij3})}{(x_{ij4} - x_{ij3}) + (l_{ij4} - l_{ij3})}. \qquad (32-b)$$

Now we will partition the objective function as follows:

$$\sum_{(i,j)\in A} \tilde{c_{ij}}\tilde{x_{ij}} = \sum_{(i,j)\in s_1} \tilde{c_{ij}}\tilde{x_{ij}} + \sum_{(i,j)\in s_2} \tilde{c_{ij}}\tilde{x_{ij}} + \sum_{(i,j)\in s_3} \tilde{c_{ij}}\tilde{x_{ij}} + \sum_{(i,j)\in s_4} \tilde{c_{ij}}\tilde{x_{ij}} \qquad (33)$$

Where $s_1 \cup s_2 \cup s_3 \cup s_4 = A$

$$s_1 = \{(i,j)|\tilde{x_{ij}} > 0, \tilde{c_{ij}} > 0, (i,j) \in A \qquad \text{(35-a)}$$

$$s_2 = \{(i,j)|\tilde{x_{ij}} < 0, \tilde{c_{ij}} < 0, (i,j) \in A \qquad \text{(35-b)}$$

$$s_3 = \{(i,j)|\tilde{x_{ij}} > 0, \tilde{c_{ij}} < 0, (i,j) \in A \qquad \text{(35-c)}$$

$$s_4 = \{(i,j)|\tilde{x_{ij}} < 0, \tilde{c_{ij}} > 0, (i,j) \in A \qquad \text{(35-d)}$$

Using equation (15), (16), (17) the objective function (30-a) can be represented by:

$$\sum_{(i,j)\in A} \tilde{c_{ij}}\tilde{x_{ij}} = \sum_{(i,j)\in s_1} [c_{ij1}x_{ij1}, c_{ij2}x_{ij2}, c_{ij1}x_{ij3} + c_{ij3}x_{ij1}, c_{ij2}x_{ij4} + c_{ij4}x_{ij2}] +$$
$$\sum_{(i,j)\in s_2} [c_{ij2}x_{ij2}, c_{ij1}x_{ij1}, -c_{ij2}x_{ij4-c_{ij4}x_{ij2}}, -c_{ij1}x_{ij3} - c_{ij3}x_{ij1}] +$$
$$\sum_{(i,j)\in s_3} [c_{ij2}x_{ij1}, c_{ij1}x_{ij2}, c_{ij3}x_{ij2} - c_{ij1}x_{ij4}, c_{ij4}x_{ij1} - c_{ij1}x_{ij4}] +$$
$$\sum_{(i,j)\in s_4} [c_{ij2}x_{ij1}, c_{ij1}x_{ij2}, c_{ij2}x_{ij3} - c_{ij4}x_{ij1}, c_{ij1}x_{ij4} - c_{ij3}x_{ij4}] \quad (36)$$

Thus

$$\sum_{(i,j)\in A} \tilde{c_{ij}}\tilde{x_{ij}} = [z_1, z_2, z_3, z_4] \quad (37)$$

where

$$z_1 = \sum_{(i,j)\in s_1} c_{ij1}x_{ij1} + \sum_{(i,j)\in s_2} c_{ij2}x_{ij2} + \sum_{(i,j)\in s_3} c_{ij2}x_{ij1} + \sum_{(i,j)\in s_4} c_{ij2}x_{ij1} \quad \text{(38-a)}$$

$$z_2 = \sum_{(i,j)\in s_1} c_{ij2}x_{ij2} + \sum_{(i,j)\in s_2} c_{ij1}x_{ij1} + \sum_{(i,j)\in s_3} c_{ij1}x_{ij2} + \sum_{(i,j)\in s_4} c_{ij1}x_{ij2} \quad \text{(38-b)}$$

$$z_3 = \sum_{(i,j)\in s_1} c_{ij1}x_{ij3} + c_{ij3}x_{ij1} + \sum_{(i,j)\in s_2} -c_{ij2}x_{ij4} - c_{ij4}x_{ij2} + \sum_{(i,j)\in s_3} c_{ij3}x_{ij2} - c_{ij1}x_{ij4} + \sum_{(i,j)\in s_4} c_{ij2}x_{ij3} - c_{ij4}x_{ij1} \quad \text{(38-c)}$$

$$z_4 = \sum_{(i,j)\in s_1} c_{ij2}x_{ij4} + c_{ij4}x_{ij2} + \sum_{(i,j)\in s_2} -c_{ij1}x_{ij3} - c_{ij3}x_{ij1} + \sum_{(i,j)\in s_3} c_{ij4}x_{ij1} - c_{ij1}x_{ij4} + \sum_{(i,j)\in s_4} c_{ij1}x_{ij4} - c_{ij3}x_{ij4} \quad \text{(38-d)}$$

Using the possibility theory introduced in section (1), if $\min Z = z$ then

$$poss[Z = z|x] = \begin{cases} \theta_1 & for \quad z_1 \leqslant z \leqslant z_2 \\ 1 & for \quad z_2 \leqslant z \leqslant z_3 \\ \theta_2 & for \quad z_3 \leqslant z \leqslant z_4 \end{cases} \tag{39}$$

$$\theta_1 = \frac{z - z_1}{z_2 - z_1} \tag{40-a}$$

$$\theta_2 = \frac{z_4 - z}{z_4 - z_3} \tag{40-b}$$

Thus, assuming that the decision maker has decided a cut-off value for α , $\alpha \in [0, 1]$ the general fuzzy minimum cost flow problem with bounded variable can be represented by:

$$\begin{aligned} & Min \quad Z \\ & \quad st. \\ & \qquad \theta_1 \geqslant \alpha \\ & \qquad z_1 \leqslant z \leqslant z_2 \\ & \qquad \delta_{ij1}, \delta_{ij2} \geqslant \alpha \qquad i, j = 1, .., m \\ & \qquad \sum_{j=1}^{m} x_{ij1} - \sum_{k=1}^{m} x_{ki2} = b_{i1} \qquad i = 1, .., m \\ & \qquad \sum_{j=1}^{m} x_{ij2} - \sum_{k=1}^{m} x_{ki1} = b_{i2} \qquad i = 1, .., m \\ & \qquad \sum_{j=1}^{m} x_{ij3} + \sum_{k=1}^{m} x_{ki4} = b_{i3} \qquad i = 1, .., m \\ & \qquad \sum_{j=1}^{m} x_{ij4} + \sum_{k=1}^{m} x_{ki3} = b_{i4} \qquad i = 1, .., m \\ & \qquad x_{ij3} \leqslant u_{ij4} \quad , \quad u_{ij3} \leqslant x_{ij4} \qquad i, j = 1, .., m \\ & \qquad x_{ij3} \leqslant l_{ij4} \quad , \quad l_{ij3} \leqslant x_{ij4} \qquad i, j = 1, .., m \\ & \qquad \alpha \in [0, 1] \end{aligned} \tag{41}$$

Example. This is a Fuzzy MCF problem with 5 nodes and 7 arcs.

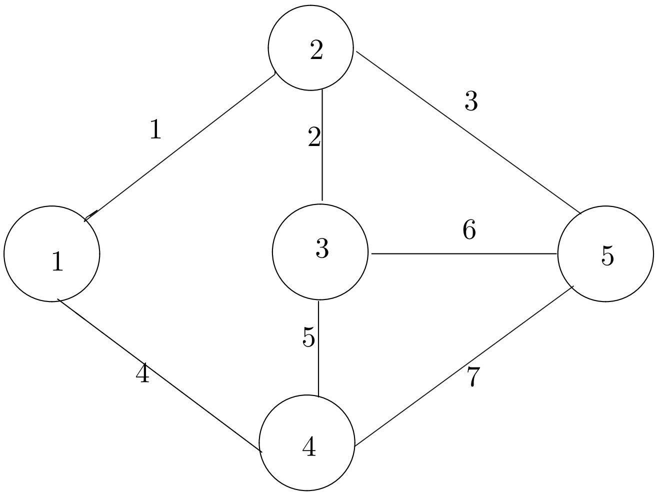

Fig. 1. Network flow

The Fuzzy data together with the structure of the network are summarized in Table 1.

Table 1. Parameters for a Fuzzy MCF problem.

Node no.	supply/demand	Arc no.	Fuzzy costs
1	(1, 2, 3, 4)	$x_{12} = x_1$	(0.5, 1, 1.5, 2)
2	(3, 4, 5, 6)	$x_{23} = x_2$	(0, 0, 0.5, 1)
3	(1, 2, 3, 4)	$x_{25} = x_3$	(5, 6, 7, 8)
4	(0.5, 1, 2, 3)	$x_{14} = x_4$	(1.5, 2, 2.5, 3)
5	(-5.5, -9, -13, -17)	$x_{43} = x_5$	(0.5, 1, 1.5, 2)
		$x_{35} = x_6$	(3, 4, 5, 6)
		$x_{45} = x_7$	(4, 5, 6, 7)

The Fuzzy MCF problem can be formulated as:

$$\theta_1 = \frac{A}{B - C}$$

$A = z - (0.5x_{11} + 5x_{31} + 1.5x_{41} + 0.5x_{51} + 3x_{61} + 4x_{71})$
$B = (x_{12} + 6x_{32} + 2x_{42} + x_{52} + 4x_{62} + 5x_{72})$
$C = (0.5x_{11} + 5x_{31} + 1.5x_{41} + 0.5x_{51} + 3x_{61} + 4x_{71})$

$$
\begin{aligned}
&Min \quad Z \\
&\quad st. \\
&\qquad \theta_1 \geqslant \alpha \\
&\qquad x_{11} + x_{41} = 1 \qquad x_{12} + x_{42} = 2 \\
&\qquad x_{13} + x_{43} = 3 \qquad x_{14} + x_{44} = 4 \\
&\qquad x_{31} + x_{21} - x_{11} = 3 \qquad x_{32} + x_{22} - x_{12} = 4 \\
&\qquad x_{33} + x_{23} - x_{13} = 5 \qquad x_{34} + x_{24} - x_{14} = 6 \\
&\qquad x_{61} - x_{21} - x_{51} = 1 \qquad x_{62} - x_{22} - x_{52} = 2 \\
&\qquad x_{63} - x_{23} - x_{53} = 3 \qquad x_{64} - x_{24} - x_{54} = 4 \\
&\qquad x_{51} + x_{71} - x_{41} = 0.5 \qquad x_{52} + x_{72} - x_{42} = 1 \\
&\qquad x_{53} + x_{73} - x_{43} = 2 \qquad x_{54} + x_{74} - x_{44} = 3 \\
&\qquad - x_{31} - x_{61} - x_{71} = -5.5 \qquad - x_{32} - x_{62} - x_{72} = -9 \\
&\qquad - x_{33} - x_{63} - x_{73} = -13 \qquad - x_{34} - x_{64} - x_{74} = -17 \\
&\qquad x_{ij} \geqslant 0 i, j = 1, .., m \\
&\qquad \alpha \in [0, 1]
\end{aligned}
$$

Notice that for example x_{61} is first component of Fuzzy variable $\tilde{x_6}$. As shown in Table 1, x_6 is equivalent to arc connecting node 3 and 5.

Table 2.Results for a Fuzzy MCF problem.

Arc no.	Fuzzy Results
$x_{12} = x_1$	(1, 2,3,4)
$x_{23} = x_2$	(4, 6, 0, 10)
$x_{25} = x_3$	(0, 0, 8, 0)
$x_{14} = x_4$	(0, 0, 0, 0)
$x_{43} = x_5$	(0.5, 1, 0, 0)
$x_{35} = x_6$	(5.5, 9, 3, 14)
$x_{45} = x_7$	(0, 0, 2, 3)

5. Conclusion

Fuzzy programming is an important area in uncertainty modeling in the field of decision making. In this paper minimal cost flow problem with fuzzy variables and fuzzy parameters has been discussed in general by possibility programming and comparison of fuzzy numbers. Studying the maximum flow problem and transportation problem with fuzzy parameters are left to the next works.

References

[1] J. J. Buckley, Posibilistic linear programming with triangular fuzzy numbers, *Fuzzy Sets and Systems*, 20 (1988), 135-138.

[2] S. Devendra, E. Negi, and S. Lee, Possibility Programming By the Comparision of Fuzzy Numbers, *Computers and Mathematics whit Application*, 9 (1992), 43-50.

[3] D. Dubois and H. Prad, *Possibility theory: An Approach to computerized processing of uncertainty*, Plenum Press, New York, (1988).

[4] A. Kaufmann and M. M. Gupta, *Introduction to fuzzy Arithmetic*, Van Nostrand Refinhold Co., New York, (1985).

[5] M. K. Luhandjula, On Possibilistic Linear Programming, *Fuzzy Sets and Systems*, 18 (1986), 15-30.

[6] S. Nahmias, Fuzzy variables, *Fuzzy Sets and Systems*, 2 (1978), 97-110.

[7] H. shih and L. E. Stanely, Fuzzy Multi-Level Minimum Cost Flow Problems, *Fuzzy Sets and Systems*, 2 (1999), 159-176.

[8] l. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems*, 1 (1978), 3-28.

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