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Evaluation of Financial Ratios in DEA-R Model with Production Trade-Offs and Weight Restrictions

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Abstract. Data envelopment analysis (DEA) is one of the best tools for evaluating units with multiple inputs and multiple outputs. In multiplier models of DEA sometimes data on inputs or outputs is available, and/or some assumptions are imposed to the model that result in some conditions on weights vectors, in addition to non-negative conditions of the weight vectors of u and v. These conditions are called weights restrictions. Applying weights restrictions on the multiplier model creates new variables in its corresponding DEA model. Thus, applying weights restrictions on the multiplier model leads to the development of technology model in the envelopment form. This makes the projection of an inefficient unit that is on the efficiency frontier of the developed technology not to be necessarily producible. Therefore, applying weights restrictions on multiplier models will enjoy this defect. To solve this problem weights restrictions are applied through the trade-off matrix that is a simultaneous change in inputs and outputs. Applying these

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restrictions is mathematically equivalent to applying weights restrictions in conventional methods such as assurance region and the difference is in the technological interpretation of its equivalent envelopment model, which resolves the above-mentioned defect. Applying weights restrictions in DEA model helps us keep under control the significance of one output to other output or the significance of one input to other input and/or the significance of one output to one input. DEA-R model is a model for evaluating the performance of decision-making units (DMUs) that can analyse the role of each input in relation to each output. The specific feature of this model is to evaluate DMUs without imposing redundant weights restrictions that lead to the unreasonable decrease of the efficiency value. In this article, the tradeoff matrix has been utilized in order to impose weight restrictions on the weights of DEA-R models and using the proposed model an efficient method for evaluating business enterprises have been provided based on financial ratios.

AMS Subject Classification: 47N10
Keywords and Phrases: DEA, efficiency, DEA-R, weights restrictions, trade-off

1. Introduction

Data envelopment analysis (DEA) is a non-parametric linear programming designed to evaluate a set of units that utilizes multiple inputs in order to produce multiple outputs. CCR model presented by (Charnes, Cooper, & Rhodes, 1978) is a model which enjoy flexibility in selecting weights in order to evaluate DMUs. In other words, each DMU freely selects weights in order to raise their efficiencies, which leads to the zero weights in the evaluation of units. In some cases, management perspectives or prior data on inputs and outputs can lead to the imposition of some restrictions on weights. In other words, imposition of weights restrictions on DEA models can be considered as a solution to overcome the problem of weights flexibility in DEA. (Allen, Athanassopoulo, Dyson, Thanassoulis 1997) and (Thanassoulis, Portela, & Despi c, 2008) conducted some studies on the technological interpretation in DEA models with weights restrictions. The recognized weights restrictions in DEA models such as Cone ratio (CR) and Assurance region (AR) are accompanied by prior data on inputs and outputs. (Angulo, & Lins, 2002) showed that the interpretation and judgment of data with the aforementioned method based on opinions can not reflect actual results. Adding weights restrictions to the multiplier models of DEA causes adding variables to the corresponding envelopment model. The interpretation of the expansion of technological model using weights restrictions was presented by (Charnes, Cooper, Wei, & Huang, 1989) and (Roll, Cook, & Golany, 1991). (Podinovski, 2004) showed that this expansion can be interpreted as the production trade-off, which means the simultaneous change in inputs and outputs if they are done by each DMU. (Allen, Athanassopoulo, Dyson, Thanassouli 1997) showed that the most obvious problem that arises when using weights restrictions is that utilizing weights restrictions in multiplier models implicitly change the model of production technology in the envelopment form. Specifically, weights restrictions generally shift the efficient frontier to the higher dimensions with altering the model of technology. As stated by (Roll, Cook, & Golany, 1991), the obvious problem with changing the efficient frontier due to weights restrictions is that the efficient projections of inefficient DMUs located in the developed frontier may not be producible. Furthermore, the meaning of efficiency and improvement factor when the efficiency value is considered as the result of the analysis generally become unsubstantiated (Forsond, 2013). Dual relationship of weights restrictions and production trade-off can act as a basis for making weights restrictions, such that developed technology and its related efficient frontier are technologically meaningful and the projection of inefficient DMU on the developed frontier is producible and meaningful. (Podinovski, 2007) utilized imposing weights restrictions with production trade-off to be used for evaluation of higher education. (Amado, & Dyson 2009) utilized this method as an application of DEA method to evaluate health care provision. (Santos , Amado, & Rosado, 2011) applied production trade-off and its related weights restrictions in using DEA for making electricity distribution efficient. (Atici, & Podinovski 2015) presented another application of this type of weights restrictions for agricultural farms. In traditional DEA models including CCR model in order to evaluate input-oriented DMUs the ratio of the weighted sum of outputs to the weighted sum of inputs are considered. Combining DEA and Ratio Analysis (RA), (Despic, Despic, & Paradi, 2007) presented DEA-R model for evaluating units in which inputs are considered one and the ratios of output to input are taken into account as indexes for evaluation. DEA-R model enables DEA models to evaluate DMUs with ratio data. In evaluating hospitals in Taiwan (Wei, Chen, Li, & Tsai, 2011) showed that the ratio of weighted outputs to weighted inputs leads to unreasonable weights restrictions in models that decrease the efficiency value of DMU under evaluation. They called this problem as pseudo inefficiency and proved that the input-oriented DEA-R is the most appropriate model in order to avoid the creation of pseudo inefficiency. DEA-R model can consider the ratio of each output and each input as a separate indicator. This is of importance in three ways.

A) The role and effect of each input in producing each output can be analyzed separately. In models of Ratio Analysis, this leads to the each ratio is evaluated separately among all DMUs and causes some DMUs to be strong in some ratios and weak in others. Therefore, a unit can not be considered as a benchmark and target (Thanassolis, Boussofiane, & Dyson, 1996). Meanwhile, DEA-R model can evaluate ratios simultaneously and introduce a unit as a target.

B) If a specific input has no role in producing specific output, with considering zero weight it is possible to omit the related index from the set of indexes and the same time in other indexes the input retains its role in conjunction with the corresponding output. Forming separate indexes makes controllable the use of a specific input in producing a specific output. This process is possible in the conventional DEA models with imposing complex weights restrictions.

C) In many applications data is ratio. Evaluation of units with ratio data causes disturbance of convexity in the conventional DEA models (Emrouznejad, & Amin, 2009). But, DEA-R model can easily evaluates DMUs with ratio data by producing indexes. In this article DEA-R model with production trade-off and weights restrictions are introduced and an application of it is presented for evaluating the financial ratios of business enterprises. The rest of this article is presented as follows: in the Section 2 the structure of DEA-R model is introduced. In the Section 3 production trade-off and weights restrictions in DEA-R model are presented. The Numerical examples in presented in the Section 4 and finally the Section 5 provides conclusions of the article.

2. Structure of DEA-R Model

2.1 Different weighted means

In some cases the relative importance of the data is essential and may be expressed numerically in the form of non-negative real values: $w_1, w_2, \cdots, w_n$ called weights. The weights can be normalized so that $w_1 + w_2 + \cdots + w_n = 1$, therefore, for a given weights vector $w = (w_1, w_2, \cdots, w_n)$ the weighted means are defined as:

A) Weighted arithmetic mean:

$$A_w = A_w(x_1, x_2, \cdots, x_n) = w_1x_1 + w_2x_2 + \cdots + w_nx_n.$$

B) Weighted harmonic mean:

$$H_w = H_w(x_1, x_2, \cdots, x_n) = 1/(w_1/x_1 + w_2/x_2 + \cdots + w_n/x_n).$$

It is clear that

$$H_w \leqslant A_w. \tag{1}$$

Despic et al (2013) define $T = (t_{ij})_(mn)$ and $w = (w_1, w_2, \cdots, w_m)$ such that $w_i \geqslant 0 (i = 1, 2, \cdots, m)$ and $w_1 + w_2 + \cdots + w_m = 1$. A convex linear combination of the row vectors of T is the vector of the form WT, therefore we have:

$$A_w(T) = WT = (A_w(t_{1j}, t_{2j}, \cdots, t_{mj}))_(1 \times n),$$

$$H_w(T) = (H_w(t_{1j}, t_{2j}, \cdots, t_{mj}))_(1n).$$

Note that, in view of inequality (1), the following vector inequality must hold:

$$H_w(T) \leqslant A_w(T).$$

2.2 DEA-R-I model and its validity

Consider n DMUs (j=1,2,..., n). Each DMU uses m inputs $x_{ij}(i = 1, 2, \cdots, m)$ to produce s outputs $y_{rj}(r = 1, 2, \cdots, s)$ where $x_{ij} \geqslant 0, x_{ij} \neq 0$ and $y_{rj} \geqslant 0, y_{rj} \neq 0$. Suppose we want to evaluate the k th DMU. We define the following matrices:

$$X(K) = \begin{pmatrix} x_{11}/x_{1k} & \dots & 1 & \dots & x_{m1}/x_{mk} \\ \vdots & \dots & \vdots & \dots & \vdots \\ x_{m1}/x_{mk} & \dots & 1 & \dots & x_{mn}/x_{mk} \end{pmatrix}_{m\times n}, \tag{2}$$

and

$$Y(K) = \begin{pmatrix} y_{1k}/y_{11} & \dots & 1 & \dots & y_{1k}/y_{1n} \\ \vdots & \dots & \vdots & \dots & \vdots \\ y_{1k}/y_{11} & \dots & 1 & \dots & y_{sk}/y_{sn} \end{pmatrix}_{s\times n}. \tag{3}$$

Note that the k th column of matrix X(K) and the k th column of matrix Y(K) are columns of ones. Consider the value located in the i th row and j th column of matrix X(K): If this value is greater than one, then unit k is doing better than unit j with respect to input i. Similarly, for a value located at the r th row and j th column of matrix Y(K): if the value is greater than one, then unit k is doing better than unit j with respect to output j. Values smaller than one clearly indicate the opposite. According to Despic et al (2007) the CCR efficiency model can be shown as:

$$e_k = \max_{\substack{\sum_{i,v_i=1,v_i\geqslant 0}, \\ \sum_{r,u_r=1,u_r\geqslant 0}}} \min_i \frac{\sum_{i=1}^m v_i \frac{x_{ij}}{x_{ik}}}{\sum_{r=1}^s v_i \frac{y_{rj}}{y_{rk}}}. \tag{4}$$

The above equation can be written as follows:

$$e_k = \max_{\substack{u\in w_s, \\ v\in w_m}} \min[A_v(X(K)) \times H_u(Y(K))].$$

Another measure of efficiency called harmonic efficiency was also introduced as:

$$\hat{e}_k = \max_{\substack{\sum_{i,v_i=1,v_i\geqslant 0},\\ \sum_{r,u_r=1,u_r\geqslant 0}}} \min_j \frac{1}{\sum_{i=1}^m v_i \frac{x_{ij}}{x_{ik}} \times \sum_{r=1}^s v_i \frac{y_{rj}}{y_{rk}}}. \tag{5}$$

(5) can be rewritten as:

$$\hat{e}_k = \max_{\substack{u\in w_s,\\ v\in w_m}} \min[H_v(X(K)) \times H_u(Y(K))].$$

According to (1) , (4), and (5) we have:

$$\hat{e}_k \leqslant e_k. \tag{6}$$

A type of a measure for the efficiency of the k th DMU is called DEA-R efficiency that is applied to a derived set of input-output data. Thus, the new inputs are represented by $1 \times n$ matrix I where all of the entries are ones and the new outputs are represented by the $(s \times m) \times n$ matrix R whose entries are all the possible ratios:

$$D_{(i,r)j} = \frac{y_{rj}}{x_{ij}}, i = 1, \cdots, m, r = 1, \cdots, s, j = 1, \cdots, n.$$

We show R as follows:

$$R = \begin{pmatrix} \frac{y_{11}}{x_{11}} & \frac{y_{12}}{x_{12}} & \cdots & \frac{y_{1n}}{x_{1n}} \\ \frac{y_{21}}{x_{11}} & \frac{y_{22}}{x_{12}} & \cdots & \frac{y_{2n}}{x_{1n}} \\ \vdots & \star & \vdots & \vdots \\ \frac{y_{s1}}{x_{11}} & \frac{y_{s2}}{x_{12}} & \cdots & \frac{y_{sn}}{x_{1n}} \\ \vdots & \star & \vdots & \vdots \\ \frac{y_{11}}{x_{m1}} & \frac{y_{12}}{x_{m2}} & \cdots & \frac{y_{1n}}{x_{mn}} \\ \frac{y_{21}}{x_{m1}} & \frac{y_{22}}{x_{m2}} & \cdots & \frac{y_{2n}}{x_{mn}} \\ \vdots & \star & \cdots & \vdots \\ \frac{y_{s1}}{x_{m1}} & \frac{y_{s2}}{x_{m2}} & \cdots & \frac{y_{sn}}{x_{mn}} \end{pmatrix}_{(s\times m)\times n}.$$

Note that for example $D_i r$ deal with the ratios corresponding with the i th input and r th output and it is the $(i-1)s+r$ th row of R. The resulting formula for the DEA-R-I efficiency model will be the same:

$$\tilde{e}_k = \max_{\substack{\sum_{i=1}^m \sum_{r=1}^s c_{ir}=1,\\ c_{ir}\geqslant 0}} \min_j \frac{1}{\sum_{i=1}^m \sum_{r=1}^s c_{ir} \frac{x_{ij}}{x_{ik}} \frac{y_{rk}}{y_{rj}}}. \tag{7}$$

In view of the notation introduced above, (7) can be called the harmonic ratio efficiency and rewritten as:

$$\tilde{e}_k = \max_{c \in w_{s\times m}} \min[H_c(R(K))].$$

The DEA-R-I model can be represented as:

$$\begin{array}{l} \min \varphi \\ s.t. \sum_i \sum_r c_{ir} \frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{rk}}{x_{ik}}} \leqslant \varphi, \\ \sum_i \sum_r c_{ir} = 1, \\ c_{ir} \geqslant 0, i = 1, ..., m, r = 1, ..., s. \end{array} \tag{8}$$

Despic et al. (2007) show that:

A- DEA-CCR efficiencies are always greater than or equal to the harmonic efficiencies.

B- DEA-R efficiencies are always greater than or equal to the harmonic efficiencies.

C- DEA-R efficiency score are rarely greater than DEA-CCR efficiency score and they are almost always identical to the harmonic efficiencies.

3. Production Trade-Off and Weights Restrictions in DEA-R Model

3.1 Weights restriction

Preference information introduced by decision makers may appear in different forms, therefore requiring various treatments. The number of variables and DMUs used is directly linked with the discriminating power of DEA models and so with a potential number of zero weights. Complete flexibility in the selection of weights is considered an advantage of DEA. Weights restriction is the simple method for including preference information in DEA. The most current weights restriction method in DEA is the assurance regions (ARs), which imposes certain ranges on the ratios of weights.

3.2 weights restriction and trade-off matrix in DEA-R-I

In this section, we recall the weight restrictions of the DEA-CCR model and provide their equivalent in the DEA-R-I model. Suppose k is a constant and v, u, and c are correspond to the weights associated with inputs, outputs, and ratio respectively.

3.3 Output weights restriction

Assume that in order to improve the discrimination power of the model, we wish to incorporate some weights restrictions. Ideally, by specifying certain proportions (or boundaries of such proportions) between the input and output weights, we should be able to make any extreme values, including zero, infeasible. The most commonly used weights restrictions are homogeneous, which in our case can be represented in the general form as follows: (Podinovski 2002).

$$b_1u_1 + b_2u_2 + \cdots + b_su_s - a_1v_1 - a_2v_2 - \cdots - a_mv_m \leqslant 0, \tag{9}$$

Where each of coefficients $b_1, b_2, \cdots, b_s, a_1, a_2, \cdots, a_m$ can be positive, negative or zero. If we set $b_p = 1, b_q = -k$ and all of the other coefficients are equal to zero then we have:

$$u_p - ku_q \leqslant 0. \tag{10}$$

The corresponding trade-off vectors are represented as:

$$(b_1, b_2, \cdots, b_p, b_q \cdots, b_s) = (0, 0, \cdots, 1, -k, 0, , 0), \tag{11}$$

$$(a_1, a_2, \cdots, a_p, a_q \cdots, a_m) = (0, 0, \cdots, 0, 0, 0, \cdots, 0). \tag{12}$$

We define the trade-off matrix for DEA-R-I model as:

$$E = [E_{11}, E_{12}, \cdots, \overbrace{E_{1p}{}^{corespondingtoD_{1p}}}, E_{1q}, \cdots, E_{1s}, \cdots,$$

$$E_{m1}, \cdots, E_{mp}, \overbrace{E_{mq}{}^{orespondingtoD_{mq}}}, \cdots, E_{ms}]_{1\times(s\times m)},$$

where the number of columns of this matrix corresponds to the number of DEA-R-I weights. We present the elements of this matrix byE_{ir}. Each element of the trade-off matrix is corresponding with a row of output matrix R. for example,E_{12} deals with the weight analogous to the second output and the first input and corresponds to the $((2-1)m+1)th$ row of matrix R. Therefore, the trade-off matrix of the DEA-R-I model corresponding to the (11) and (12) can be shown as:

$$E = [0, \cdots, \overbrace{1}^{coresponding\ to\ D_{1p}}, -k, 0, \cdots, 0, \cdots, \overbrace{1}^{oresponding\ to\ D_{mq}}, -k, \cdots, 0]_{1\times(s\times m} \tag{13}$$

We define the weight matrix of DEA-R-I as follows:

$$C = [C_{11}, ..., C_{1p}, C_{1q}, ..., C_{1s}, ..., C_{m1}, ..., C_{mp}, C_{mq}, ..., C_{ms}]_{1\times(s\times m)}.$$

Therefore the effect of the trade-off matrix on the weights of the DEA-R-I model can be shown as:

$$EC = \sum_{i=1}^{m}(c_{ip} - kc_{iq}) \leqslant 0. \tag{14}$$

3.4 Input weights restriction

With respect to (9), if we set $a_l = -1, a_k = k, a_i = 0, \forall i, i \neq l, k$ and $b_r = 0, r = 1, 2, , s$, then

$$v_l - kv_k \leqslant 0. \tag{15}$$

The corresponding trade-off vectors of (15) can be presented as follows:

$$(b_1, b_2, \cdots, b_p, b_q \cdots, b_s) = (0, 0, \cdots, 0, 0, 0, \cdots, 0), \tag{16}$$

$$(a_1, a_2, \cdots, a_l, a_k \cdots, a_m) = (0, 0, \cdots, 0, -1, k, 0, \cdots, 0). \tag{17}$$

We can show the trade-off matrix analogous to (16) and (17) as:

$$E^{'} = [0, \cdots, 0, \cdots, \overbrace{-1, \cdots, -1}^{corresponding\ to\ lth\ input}, \overbrace{k, \cdots, k}^{corresponding\ to\ kth\ input}, 0, \cdots, 0]_{1\times(s\times m)}. \tag{18}$$

Therefore the weights restrictions of the DEA-R-I model corresponding with the input weights restriction in DEA is obtained as follows:

$$E^{'}C = \sum_{i=1}^{m}(c_{lr} - kc_{kr}) \leqslant 0. \tag{19}$$

3.5 Weight restriction applied to virtual inputs and virtual outputs

In this case, the ratio of the virtual weight of input (outputs) to the virtual weights of another inputs (output) must be greater than or equal to the lower bound be less than or equal to the upper bound; in other words, it must be between the lower bound and the upper bound. We can consider the weights restrictions for virtual outputs as:

$$u_p y_{pj} - ku_q y_{qj} \leqslant 0 \quad (j = 1, 2, , n). \tag{20}$$

These restrictions depend on the value of the p th and value of the q th output. Therefore, considering the output matrix (18) in DEA-R-I model, we have:

$$\sum_{i=1}^{m} c_{ip}\frac{y_{pj}}{x_{ij}} - k\sum_{i=1}^{m} c_{ip}\frac{y_{qj}}{x_{ij}} \leqslant 0 \quad (j = 1, 2, \cdots, n). \tag{21}$$

We can show the weights restrictions for virtual inputs as:

$$v_l x_{lj} - kv_k x_{kj} \leqslant 0 \quad (j = 1, 2, \cdots, n). \tag{22}$$

Similarly to (21), the corresponding restrictions for the DEA-R-I model can be shown as follows:

$$k\sum_{r=1}^{s} c_{pr}\frac{y_{rj}}{x_{lj}} - k\sum_{r=1}^{s} c_{qr}\frac{y_{rj}}{x_{kj}} \leqslant 0 \quad (j = 1, 2, \cdots, n). \tag{23}$$

Considering the above expressions we propose the DEA-R-I model with weight restrictions (WDEA-R-I) as follows:

$$\begin{array}{c} min\ \varphi \\ s.t. \sum_{i=1}^{m}\sum_{r=1}^{s} c_{ir}(\frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{rk}}{x_{ik}}}) \leqslant \varphi, \\ \sum_{i=1}^{m}\sum_{r=1}^{s} c_{ir} = 1, \\ \sum_{i=1}^{m}(c_{ip} - kc_{iq}) \leqslant 0, \\ \sum_{i=1}^{m}(c_{lr} - kc_{kr}) \leqslant 0, \\ c_{ir} \geqslant 0, i = 1, ..., m, r = 1, ..., s. \end{array} \tag{24}$$

The computations should generally be carried out on the dual side since the envelopment model (DWDEA-R-I) is usually easier to solve and interpret.

$$\begin{array}{l} \max\theta_k \\ s.t. \sum_{j=1}^{n}\lambda_j(\frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{rk}}{x_{ik}}}) + m(1-k)\tau - s(1-k)\pi - \theta_k \geqslant 0, \quad (25-1) \\ \sum_{j=1}^{n}\lambda_j = 1, \quad (25-2) \\ \lambda_j \geqslant 0, \tau \geqslant 0, \pi \geqslant 0, \quad j = 1, ..., n. \end{array} \tag{25}$$

Where τ and Π correspond to the first and second inequalities of weight restrictions respectively.

Theorem 3.1. *The DWDEA-R-I model is always feasible and finite.*

Proof. For feasibility it is enough to set $\lambda_k = 1, \lambda_j = 0, j = 1, \cdots, n, j$, $\theta_k = 1$ and set $\pi, \tau = 0$. For finiteness, if we put $\theta_k = 0$ then considering (25-1), we must have $\lambda_j = 0, j = 1, \cdots, n$ and $\pi = \tau = 0$. This contradicts (25-2) since we suppose $X \geqslant 0, Y \geqslant 0$. Therefore $\theta_k > 0$. □

Corollary 3.2. *The WDEA-R-I model is feasible and finite.*

Proof. The DWDEA-R-I model is finite and feasible, thus it is clear by the duality theorem in LP that its dual is also feasible and finite. □

4. Numerical Example

In this section, a practical application of the proposed models in the article is presented. It should be noted that in the above-mentioned examples there are some cases the conventional DEA model cannot be utilized to evaluate their units with this type of data. In the same way, data in this article is ratio type and the ratio of each output to input in the Example 1 introduces a new concept. In the other words, the financial ratios are analyzed. In the present time, in comparing financial indices for evaluation of units, each index is separately considered among units under analysis, which as stated by Thanassoulis (1996) does not lead to the creation of a benchmark. Therefore, the presented DEA-R models in this article provide a new structure for evaluating industrial units and financial enterprises based on the financial viewpoints. In the evaluation of units with traditional input-oriented DEA models the weighted sum of outputs to the weighted sum of inputs are evaluated. This means that the DEA can not evaluate the weighted ratio of a specific output to a specific input. Another point that shows the advantage of the presented models in this article comparing to traditional DEA models is that in the presented models the company director can control the relative importance of some of the financial ratios with applying methods of controlling weights mentioned in this article. This, as will be shown in the example, leads to interesting results in the evaluation. Before presenting the example, it is necessary to explain some of the concepts utilized in this example.

1. Current Ratio= current assets/current liabilities. This ratio reflects the ability of the company to respond obligations in the short term (one year). This ratio shows that how in the short term the company can respond to the current liabilities by converting current assets into the cash. If the ratio is equal to and/or more than 1, it shows company's financial health in this regard.

2. Quick Ratio = quick assets/current liabilities. This ratio represents the ability of the company to pay its current liabilities immediately. In calculating this ratio, the cost of inventory and orders is deducted from current assets. Because it is generally felt that in an emergency, if the company has to pay its current liabilities immediately, selling inventories is not possible. If the ratio is equal to and/or more 1, it displays that the company has no problem in paying current liabilities in an emergency.

3. Inventory Turnover= the cost of goods sold or net sales/the ending inventory. Here the high ratio shows the company' ability in immediate selling of manufactured goods. The low ratio indicates that the company's difficulties in selling their goods, or more than required amount of goods is kept.

4. Return on Total Assets=net profit/total assets This ratio shows that how total assets held by the company have effectively been utilized in order to generate earnings. The higher ratio indicates the company's success in using assets.

5. Total Assets Turnover=revenue/total assets The higher ratio indicates that the company was successful in making use of its total assets and facilities. In Table 1, a selection of financial information of eight enterprises in Iran is provided. The data includes:

Inputs: Current Liabilities (I_1), Ending Inventory (I_2) and Total Assets (I_3).

Outputs: Current Assets (O_1), Quick Assets (O_2), Total Cost (O_3), Revenue (O_4) and Net Profit (O_5).

Table 1: Financial data as inputs and outputs

DMUs	inputs			outputs				
Businesses	I_1	I_2	I_3	O_1	O_2	O_3	O_4	O_5
Business1	616	5671	1327	5466	491	1895	2101	48
Business2	821	862	1980	5851	705	2241	2450	45
Business3	574	495	1245	6028	542	1742	1890	43
Business4	839	623	1964	5923	793	2353	2475	38
Business5	676	591	1479	5392	611	1945	2125	41
Business6	805	847	1836	6690	787	2123	2269	36
Business7	903	914	1985	6584	781	1981	2945	45
Business8	617	887	1432	7368	596	1894	2096	46

Table 2 summarizes the results of using model (8) for the above-mentioned enterprises. It is worth mentioning that the traditional DEA models cannot evaluate data as above since dividing outputs to the input creates new financial concepts. As the focus in the evaluation of these enterprises is on the evaluation of ratios stated above, i.e. current ratio, quick ratio, inventory turnover ratio, total asset turnover ratio and efficiency ratio of total assets, the weight of the other ratios is considered zero, so that the model in the evaluation considers only the stated ratios. It should be noted that this is also the advantage of DEA-R model.

Table 2: Efficiency measures and optimum weights measures using DEA-R-O model

Businesses	efficiency	c_{11}	c_{12}	c_{23}	c_{34}	c_{35}
Business1	1.00	0.000	0.000	0.000	0.000	0.803
Business2	1.12	0.000	0.970	0.035	0.080	0.000
Business3	1.00	0.000	0.876	0.042	0.098	0.000
Business4	1.00	0.000	0.907	0.093	0.000	0.000
Business5	1.05	0.000	0.861	0.041	0.097	0.000
Business6	1.00	0.000	0.890	0.030	0.080	0.000
Business7	1.00	0.000	0.524	0.000	0.486	0.000
Business8	1.00	1.000	0.000	0.000	0.000	0.000

In Table (2) C_{11} represents weights of ratios made by the first output to the first input, i.e. the weight of the current ratio. Similarly, C_{12} represents the weight of ratios made by the second output to the first input, i.e. the weights of quick ratios. C_{23} shows weights of ending inventory turnover ratios, C_{34} represents the weights of total asset turnover ratios and C_{35} shows the weights of the return on total assets. The second column of the above table shows the efficiency values of each enterprise. Based on DEA-RO model, enterprise 2 and 5 did not give good performance, while enterprise 1, 3, 4, 6, 7 and 8 were efficient. Now considering that companies are to give priorities for some inputs, outputs or ratios over other inputs, outputs or ratios, the weight restricted DEA-R-O model is utilized. Suppose that for an enterprise the ratio of the ability to sell goods immediately to the ability to pay liabilities immediately is maximum 3 to 2, i.e. the ratio of inventory turnover to the quick ratio is maximum 3 to 2. In addition, relative importance of the ratio of return on assets to the ability to meet the obligations of the company is maximum 4 to 3, i.e. the relative importance of the ratio of return on total assets to the current ratio is maximum 4 to 3. According to the model (24), these two weight restrictions can be shown as follows:

$$2C_{23} - 3C_{12} \leqslant 0,$$

$$3C_{35} - 4C_{11} \leqslant 0.$$

Moreover, suppose that the relative importance of the current liability is maximum twice the relative importance of ending inventory. This restriction established between two inputs according to (19) can be represented as follows.

$$\sum_{r=1}^{5}(c_{1r} - 2c_{2r}) \leqslant 0,$$

Also, if it is assumed that the relative importance of current assets is at least 2/3 to relative importance of revenue. Based on (14) this restriction can be shown as follows:

$$\sum_{i=1}^{3}(c_{i4} - \frac{2}{3}c_{i1}) \geqslant 0.$$

With applying extra weight restrictions and a re-evaluation of companies, the results of Table (3) were obtained.

Table 3: Efficiency measures and optimum weights at presence weights restrictions

Businesses	efficiency	c_{11}	c_{12}	c_{23}	c_{34}	c_{35}
Business1	1.05	0.235	0.118	0.176	0.157	0.314
Business2	1.20	0.023	0.617	0.319	0.014	0.028
Business3	1.00	0.021	0.612	0.318	0.015	0.031
Business4	1.00	0.014	0.635	0.324	0.009	0.018
Business5	1.06	0.020	0.620	0.320	0.013	0.027
Business6	1.13	0.060	0.579	0.320	0.040	0.000
Business7	1.21	0.026	0.607	0.316	0.017	0.034
Business8	1.11	1.235	0.118	0.176	0.157	0.314

By imposing the above-mentioned weight restrictions, the efficiency values of enterprises change. As indicated in the table (3) after imposing weight restrictions only enterprise 3 and 4 are efficient and others are inefficient. The efficiency of these enterprises indicate that considering the relative importance mentioned above in the example, the performance of efficient enterprises in ratios that enjoy greater relative importance than other ratios was more favorable. As an instance, according to Table 4, the interpretation of one row of the model is provided. Considering enterprise 7, the efficiency value is 1.21. Now it should be analyzed what this number shows in connection with other ratios in the example. Note that the multiplication of weight in the related ratio provides the virtual weights of each ratio. The following table shows the data related to enterprises 7.

Table 4: Ratio, optimum weights and virtual weights of Business 7

Ratios	*Current Ratio*	*Quick Ratio*	*Inventory Turnover*	*TotalAssets Turnover*	*Returnon TotalAssets*
Or/Ii	6584/903	781/903	1981/914	2945/1985	45/1985
Optimum Weights	0.026	0.607	0.316	0.017	0.034
Virtual Weights	0.189573	0.524991	0.684897	0.025222	0.000077

Table 5: Target setting for financial ratio of Business 7

Ratios	*Current Ratio*	*Quick Ratio*	*Inventory Turnover*	*TotalAssets Turnover*	Return on
Total Assets					
Current measures	7.291251	0.864895	2.167396	1.483627	0.02267
Target measure	8.749501	1.046523	2.622549	1.795189	0.027431

The obtained efficiency values show that the unit under analysis is not efficient. For being efficient, each ratio should be multiplied in the efficiency value. For example, the value of the current ratio equals to 6584/903=7.291251. If enterprise7 wants to be efficient, this value should increase to $7.291251 \times 1.2 = 8.749501$. In order to reach to this point, the enterprise should increase its current revenue or reduce its current liability. For each model, the ratio can be determined. The following table value for each pattern determines the ratio of the enterprise 7.

Another point of the table (3) is that since the sum of optimal weights of each ratio in each enterprise equals to 1, we can consider the weight of each ratio as the percentage share of that ratio in the efficiency value of the enterprise. For example, the optimal weight related to the inventory turnover in the enterprise 7 is 0.316, i.e. the percentage share (or percentage of participation) of inventory turnover, the inefficiency of the enterprise 7 is 31.6 percent However, given the efficiency values is 1.21, it can be observed that only (1 / 1.21), i.e. 82.7 percent of this value, i.e. 29.86 percent of the value 31.6 percent has been achieved. In other words, enterprise 7 with the amount of (31.6-29.8) percent failed in the immediate sales of goods or kept surplus goods in its storage.

5. Conclusions

DEA is a practical method in the evaluation of manufacturing and industrial units, bank branches etc. that could evaluate units with multiple inputs and outputs. This method has been developed in many fields such as supply chains and resource allocation problems. Thanassoulis (1996) in an article compared ratio analysis and DEA analysis for 183 medical centers in England. His study showed that the ratio analysis and DEA can not be introduced interchangeably because each of them has its own advantages. For instance, fractional analysis can analyze the ratios formed by inputs and outputs and draw conclusions for each ratio, but it can not provide a general benchmark for units. Using the efficiency values, DEA, however, provides a benchmark of efficiency. On the other hand, DEA has no plan for efficient units, while in fractional analysis there is also a method for improving the efficient ratios. Utilizing ratio data in DEA due to violation of convexity assumption led to a different approach in utilizing ratio data in DEA model, i.e. DEA-R model. This model enjoys several advantages over DEA model:

1. In traditional DEA models, in order to calculate the weighted sum of outputs is divide into the weighted sum of inputs. This causes unnecessary and unreasonable weight restrictions in the evaluation. The reason behind is that in many applications, all inputs do not play a role in producing each of outputs. These restrictions reduce the efficiency values, referred to as the pseudo inefficiency by Wei et al. (2011). In DEA-R model, as the ratio of each output is analyzed against each input, if an output is not related to a specific input, it is possible to remove the related ratio from the set of ratios by assigning zero values for the weight of that ratio. Meanwhile, the input and output in this ratio will have their roles in other related ratios.

2. Due to the violation of convexity condition, using ratio data in DEA without imposing special restrictions is not possible, while DEA-R model easily makes use of ratio data. In case that dividing outputs into inputs makes a new concept and the creation of specific ratios becomes the focus, DEA-R model conducts this evaluation with ease. Performing this analysis in DEA is very difficult and requires the use of complicated restrictions.

3. In DEA if you want to analyze a specific input weight against the weight of one or more specific outputs, it requires the use of weight restrictions that increases the complexity of the model. However, in DEA-R model, since each output makes a discrete ratio with each input, applying these weight restrictions is simply done.

4. In the evaluation with the financial ratios that dividing outputs into inputs makes a new concept, it is needed to calculate the efficiency of the entire system and to analyze each of created ratios separately. In traditional DEA models since the weighted sum of outputs is divided into the weighted sum of inputs, it is not possible to consider and analyze each output against each input. Whilst, this analysis is simply done in DEA-R model.

In the present article, the evaluation of business units using their financial data is considered with practical and analytical approaches. The advantage of this article is that considering results of the evaluation based on the financial ratios, one can analyze the overall financial conditions of the company and identify the reasons behind strong or weak financial performance. This analysis can be utilized by those who are to buy the company's shares in the stock markets.

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