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Inadmissibility of Usual and Mixed Estimators of Two Ordered Gamma Scale Parameters Under Reflected Gamma Loss Function

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Abstract. Let $X_{i1}, \cdots, X_{in_i}$ be a random sample from a gamma distribution with known shape parameter $\nu_i > 0$ and unknown scale parameter $\beta_i > 0$, $i = 1, 2$, satisfying $0 < \beta_1 \leqslant \beta_2$. We consider the class of mixed estimators for estimation of β_1 and β_2 under reflected gamma loss function. It has been shown that the minimum risk equivariant estimator of β_i, $i = 1, 2$, which is admissible when no information on the ordering of parameters are given, is inadmissible and dominated by a class of mixed estimators when it is known that the parameters are ordered. Also, the inadmissible estimators in the class of mixed estimators are derived. Finally the results are extended to some subclass of exponential family.

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1. Introduction

Estimation of ordered parameters when it is known a priori that they are subject to certain order restrictions arises in various practical problems. For example, there are situations where one is interested in estimation the yield output of two machine, of which one is an improvement of the other, it is naturally to assume the improved machine have a yield output more than that of original machine.

The problem of estimation of ordered parameters have been of considerable interest and studied in the literature. Most of the work related to statistical inference under order restriction which appeared in the literature up to 1988 is reviewed by Robertson et al. [17] and up to 2006 is classified and extensively reviewed by van Eeden [20]. The estimation problem with order restriction leads us to some theoretical interesting questions such as admissibility of ordinary unrestricted estimator under the restriction. Some authors address this question for estimation of ordered parameters of some distributions. For example see Katz [4] and Kumar and Sharma [8] for estimation of ordered normal means, Kushary and Cohen [9] for estimation of ordered poisson parameters, Vijayasree and Singh [22, 23], Kaur and Singh [5], Kumar and Kumar [6, 7] and Misra and Singh [13] for estimation of ordered Exponential means and Vijayasree et al. [21], Chang and Shinozaki [1], Misra et al. [11] and Meghnatisi and Nematollahi [10] for estimation of ordered scale parameters of gamma distributions.

Now, suppose $X_{i1}, \cdots, X_{in_i}$, $i = 1, 2$ be two independent random samples from gamma distribution with known shape parameter $\nu_i > 0$ and unknown scale parameters $\beta_i > 0$, $i = 1, 2$, with density

$$f_{X_{ij}}(x) = \frac{1}{\beta_i^{\nu_i}\Gamma(\nu_i)} x^{\nu_i - 1} e^{-x/\beta_i}, x > 0, \nu_i > 0, \beta_i > 0, j = 1, ..., n_i, i = 1, 2. \tag{1}$$

We assume $0 < \beta_1 \leqslant \beta_2$ and want to estimate β_1 and β_2 component-wise. In the literature, estimation of ordered parameters is usually considered under the Squared Error Loss (SEL) or scale-invariant SEL (see van Eeden [20]), which are convex and symmetric, or in some special cases under the LINEX loss (see Misra et al. [12]) or entropy loss (see Parsian and Nematollahi [15], Chang and Shinozaki [2] and Nematollahi and Meghnatisi [14]), which are convex and asymmetric. A major criticism of the SEL, LINEX and entropy loss functions is that these functions continue to increase as the deviation from target increases, rather than attaining an upper limit. In response to the criticism of SEL, Spiring [18] introduced reflected normal loss function, which is appropriate for estimation of location parameter, and in response to the criticism of entropy loss function, Towhidi and Behboodian [19] employed the Reflected Gamma Loss (RGL) function which is defined as

$$L(\beta_i, \delta_i) = k \left\{ 1 - e^{-\gamma^2 \left(\frac{\delta_i}{\beta_i} - \ln \frac{\delta_i}{\beta_i} - 1 \right)} \right\}, \qquad i = 1, 2, \tag{2}$$

where $\gamma > 0$ is a shape parameter and $k > 0$ is the maximum loss parameter. The loss function (2) is appropriate for estimation of scale parameter which is asymmetric and bounded. Without loss of generality, we shall take $k = 1$ in the rest of the paper.

In this paper we study the inadmissibility of usual and mixed estimators of β_1 and β_2 under the model (1) with restriction $0 < \beta_1 \leqslant \beta_2$ and under the RGL function (2). To this end, in Section 2, a subclass of mixed estimators is obtained that beats the Minimum Risk Equivariant (MRE) and admissible estimator of β_1 and β_2, when they are not ordered, and the inadmissible estimators in the class of mixed estimators are derived. Also, the results are extended to a subclass of the scale parameter exponential family and also the family of transformed chi-square distributions introduced by Rahman and Gupta [16].

2. Inadmissibility Results

Let $X_{ij},\ j = 1, \cdots,\ n_i,\ i = 1, 2$, be two independent random samples from Gamma (ν_i, β_i)-distribution, $i = 1, 2$, with density given by (1) where $0 < \beta_1 \leqslant \beta_2$ and $\nu_1,\ \nu_2$ are known positive real valued shape parameters. Let $m_i = n_i \nu_i$ and $\delta_i = \sum_{j=1}^{n_i} X_{ij}/m_i = \overline{X_i}/\nu_i,\ i = 1, 2$. Towhidi and Behboodian [19] showed that under the RGL function (2), δ_i is MRE and admissible (and also maximum likelihood) estimator of β_i, $i = 1, 2$, when β_1 and β_2 are unrestricted. Define the mixed estimator of β_1 and β_2 as

$$\delta_{1\alpha} = \min(\delta_1, \alpha\delta_1 + (1 - \alpha)\delta_2), \qquad 0 \leqslant \alpha < 1, \tag{3}$$

and

$$\delta_{2\alpha} = \max(\delta_2, \alpha\delta_2 + (1 - \alpha)\delta_1), \qquad 0 \leqslant \alpha < 1, \tag{4}$$

respectively. It can be shown that when $\alpha = \frac{m_1}{m_1+m_2}$, $\delta_{1\alpha}$ is the MLE of β_1 and if $\alpha = \frac{m_2}{m_1+m_2}$, then $\delta_{2\alpha}$ is the MLE of β_2 (see Robertson et al. [17] and Chang and Shinozaki [1]).

The risk functions of $\delta_{i\alpha}$ and δ_i with respect to loss (2) are given by

$$R(\boldsymbol{\beta}, \delta_{i\alpha}) = E\left[1 - e^{-\gamma^2(\frac{\delta_{i\alpha}}{\beta_i} - \ln\frac{\delta_{i\alpha}}{\beta_i} - 1)}\right], \qquad i = 1, 2$$

and

$$R(\boldsymbol{\beta}, \delta_i) = E\left[1 - e^{-\gamma^2(\frac{\delta_i}{\beta_i} - \ln\frac{\delta_i}{\beta_i} - 1)}\right], \qquad i = 1, 2$$

respectively. In this section, we find values of α such that $\delta_{i\alpha}$ is inadmissible among the class of mixed estimators of β_i and $\delta_{i\alpha}$ dominates the usual estimator δ_i of β_i, $i = 1, 2$. Let $y_1 = \beta_2/\beta_1$, $y_2 = \beta_1/\beta_2$ and $z = m_1y_1/(m_1y_1 + m_2)$. Since $0 < \beta_1 \leqslant \beta_2$, we have $y_1 \geqslant 1$, $0 < y_2 \leqslant 1$ and $0 < z < 1$.

Theorem 2.1. *Let* $\alpha_1 = \frac{m_1+\gamma^2}{m_1+m_2+\gamma^2}$, *then under the loss function (2), for* $\alpha \in (\alpha_1, 1)$ *and* $0 < \beta_1 \leqslant \beta_2$,

$$R(\boldsymbol{\beta}, \delta_{1\alpha_1}) < R(\boldsymbol{\beta}, \delta_{1\alpha}) < R(\boldsymbol{\beta}, \delta_1).$$

Proof. Let $T_1 = \frac{m_2\delta_2}{m_1y_1\delta_1+m_2\delta_2}$ and $T_2 = \frac{m_1\delta_1}{\beta_1} + \frac{m_2\delta_2}{\beta_2}$. Then $\delta_1 = \frac{\beta_1T_2(1-T_1)}{m_1}$, $\delta_2 = \frac{\beta_2T_1T_2}{m_2}$ and T_1 and T_2 are independent with $T_1 \sim Beta(m_2, m_1)$ and $T_2 \sim Gamma(m_1 + m_2, 1)$. Let $\Delta_1 = R(\boldsymbol{\beta}, \delta_1) - R(\boldsymbol{\beta}, \delta_{1\alpha})$, then using the fact that

$$e^b - e^a \geqslant e^a(b - a), \tag{5}$$

we have

$$
\begin{aligned}
\Delta_1 &= E\left[e^{-\gamma^2(\frac{\delta_{1\alpha}}{\beta_1}-\ln\frac{\delta_{1\alpha}}{\beta_1}-1)} - e^{-\gamma^2(\frac{\delta_1}{\beta_1}-\ln\frac{\delta_1}{\beta_1}-1)}\right] \\
&\geqslant E\left[\gamma^2\left(\frac{\delta_1}{\beta_1}-\ln\frac{\delta_1}{\beta_1}-\frac{\delta_{1\alpha}}{\beta_1}+\ln\frac{\delta_{1\alpha}}{\beta_1}\right)e^{-\gamma^2(\frac{\delta_1}{\beta_1}-\ln\frac{\delta_1}{\beta_1}-1)}\right] \\
&= \gamma^2 E\left[\left\{(1-\alpha)\left(\frac{\delta_1-\delta_2}{\beta_1}\right)+\ln\left(\alpha+\frac{(1-\alpha)\delta_2}{\delta_1}\right)\right\}\right. \\
&\qquad \left. e^{-\gamma^2(\frac{\delta_1}{\beta_1}-\ln\frac{\delta_1}{\beta_1}-1)} I_{[0,\infty]}(\delta_1-\delta_2)\right] \\
&= \gamma^2 e^{\gamma^2} E\left[\left\{\frac{(1-\alpha)(m_2-(m_1y_1+m_2)T_1)T_2}{m_1m_2}\right.\right. \\
&\qquad \left. +\ln\left(\alpha+\frac{(1-\alpha)m_1y_1T_1}{m_2(1-T_1)}\right)\right\} \\
&\qquad \left.\times\left(\frac{1-T_1}{m_1}\right)^{\gamma^2} T_2^{\gamma^2} e^{-\gamma^2(\frac{T_2(1-T_1)}{m_1})} I_{[0,1-z]}(T_1)\right].
\end{aligned}
$$

Now if $A(T_1)$ is a function of T_1, then

$$
E\left[A(T_1)T_2^{\gamma^2+a}e^{-T_2(\frac{\gamma^2(1-T_1)}{m_1})}\,\middle|\, T_1=t_1\right]
$$

$$
= \quad A(t_1)\frac{\Gamma(m_1+m_2+\gamma^2+a)}{\Gamma(m_1+m_2)}\left(\frac{m_1}{m_1+\gamma^2(1-t_1)}\right)^{m_1+m_2+\gamma^2+a}
$$

and hence

$$
E\left[A(T_1)T_2^{\gamma^2+a}e^{-T_2(\frac{\gamma^2(1-T_1)}{m_1})}\right]
$$

$$
= E\left[A(T_1)\frac{\Gamma(m_1+m_2+\gamma^2+a)}{\Gamma(m_1+m_2)}\left(\frac{m_1}{m_1+\gamma^2(1-T_1)}\right)^{m_1+m_2+\gamma^2+a}\right]. \qquad (6)
$$

So,

$$\begin{aligned}\Delta_1 \;\geqslant\; & \frac{\gamma^2 e^{\gamma^2}\Gamma(m_1+m_2+\gamma^2)m_1^{m_1+m_2}}{\Gamma(m_1+m_2)} \\ & \times E\left[f_1(T_1)\frac{(1-T_1)^{\gamma^2}}{(m_1+\gamma^2(1-T_1))^{m_1+m_2+\gamma^2+1}}I_{[0,1-z]}(T_1)\right], \quad (7)\end{aligned}$$

where

$$\begin{aligned}f_1(x) \;=\; & \frac{1-\alpha}{m_2}\Big(m_2-(m_1y_1+m_2)x\Big)(m_1+m_2+\gamma^2) \\ & +\ln\left(\alpha+\frac{(1-\alpha)m_1y_1x}{m_2(1-x)}\right)(m_1+\gamma^2(1-x)). \qquad (8)\end{aligned}$$

Now using the fact that $\ln x \geqslant 1-\frac{1}{x}$ for $x>0$, we have

$$\begin{aligned}f_1(x) \;\geqslant\; & \frac{1}{m_2}\Big\{(1-\alpha)(m_1+m_2+\gamma^2)\Big(m_2-(m_1y_1+m_2)x\Big) \\ & + m_2[m_1+\gamma^2(1-x)]\left[\frac{(1-\alpha)[(m_1y_1+m_2)x-m_2]}{\alpha m_2(1-x)+(1-\alpha)m_1y_1x}\right]\Big\} \\ = \; & \frac{1-\alpha}{m_2[\alpha m_2(1-x)+(1-\alpha)m_1y_1x]}g_1(x), \qquad (9)\end{aligned}$$

where

$$g_1(x) = A_1(y_1,\alpha)x^2 + B_1(y_1,\alpha)x + C_1(y_1,\alpha), \qquad (10)$$

and

$$\begin{aligned}A_1(y_1,\alpha) &= -(m_1y_1+m_2)\Big[(m_1+m_2+\gamma^2)\{(1-\alpha)m_1y_1-\alpha m_2\}+m_2\gamma^2\Big], \\ B_1(y_1,\alpha) &= m_2\Big\{(m_1y_1+m_2)\Big[m_1+\gamma^2-\alpha(m_1+m_2+\gamma^2)\Big] \\ &\quad + (m_1+m_2+\gamma^2)\Big[(1-\alpha)m_1y_1-\alpha m_2)\Big]+m_2\gamma^2\Big\}, \\ C_1(y_1,\alpha) &= m_2^2\Big[\alpha(m_1+m_2+\gamma^2)-m_1-\gamma^2\Big].\end{aligned}$$

Note that $C_1(y_1,\alpha)>0$ for $y_1 \geqslant 1$ and $\alpha>\alpha_1$. When $A_1(y_1,\alpha)\neq 0$, the quadratic form (10) has the roots

$$x_1 = 1-z \qquad \text{and} \qquad x_2 = 1-z+\frac{m_1m_2^2(y_1-1)}{A_1(y_1,\alpha)}.$$

If $A_1(y_1,\alpha) > 0$, then $x_1 = 1 - z$ is the smaller positive root and if $A_1(y_1,\alpha) < 0$, then $x_1 = 1-z$ is the only positive root when $\alpha \in (\alpha_1, 1)$. For the case $A_1(y_1,\alpha) = 0$, $x_1 = 1 - z$ is the only root. So, from (9) $f_1(x) > 0$ for $x \in [0, 1-z]$, and hence $\Delta_1 > 0$ for all $0 < \beta_1 \leqslant \beta_2$

when

$$\alpha \in (\alpha_1, 1), i.e., R(\delta_{1\alpha}, \boldsymbol{\beta}) < R(\delta_1, \boldsymbol{\beta})$$

for $\alpha \in (\alpha_1, 1)$.

Now if $\Delta_1^* = R(\delta_{1\alpha}, \boldsymbol{\beta}) - R(\delta_{1\alpha_1}, \boldsymbol{\beta})$, then by a similar argument that

leads to (7), we have

$$\begin{aligned}
\Delta_1^* \quad \geqslant \quad & \frac{\gamma^2 e^{\gamma^2}\Gamma(m_1+m_2+\gamma^2)(m_1m_2)^{m_1+m_2}}{\Gamma(m_1+m_2)} \\
\times \quad & E\left[f_1^*(T_1)\frac{\Big[m_1y_1(1-\alpha)T_1 + m_2\alpha(1-T_1)\Big]^{\gamma^2}}{\Big[\gamma^2\Big(m_1y_1(1-\alpha)T_1 + m_2\alpha(1-T_1)\Big) + m_1m_2\Big]^{m_1+m_2+\gamma^2+1}}\right. \\
& \left. I_{[0,1-z]}(T_1)\right], \qquad (11)
\end{aligned}$$

where

$$\begin{aligned}
f_1^*(x) \quad = \quad & (\alpha-\alpha_1)\Big[m_2 - (m_1y_1+m_2)x\Big](m_1+m_2+\gamma^2) \\
& + \ln\left(\frac{m_1y_1(1-\alpha_1)x + m_2\alpha_1(1-x)}{m_1y_1(1-\alpha)x + m_2\alpha(1-x)}\right) \\
& \Big\{\gamma^2[m_1y_1(1-\alpha)x + m_2\alpha(1-x)] + m_1m_2\Big\} \\
\geqslant \quad & \frac{(\alpha-\alpha_1)[m_2-(m_1y_1+m_2)x]}{[m_1y_1(1-\alpha_1)x + m_2\alpha_1(1-x)]} \\
& \Big\{(m_1+m_2+\gamma^2)[m_1y_1(1-\alpha_1)x + m_2\alpha_1(1-x)] \\
& - \gamma^2[m_1y_1(1-\alpha)x + m_2\alpha(1-x)] - m_1m_2\Big\} \\
= \quad & \frac{\alpha-\alpha_1}{[m_1y_1(1-\alpha_1)x + m_2\alpha_1(1-x)]}g_1^*(x) \qquad (12)
\end{aligned}$$

and

$$\begin{aligned} g_1^*(x) &= A_1^*(y_1,\alpha)x^2 + B_1^*(y_1,\alpha)x + C_1^*(y_1,\alpha), \qquad (13)\\ A_1^*(y_1,\alpha) &= (m_1y_1+m_2)\Big[(1-\alpha)\gamma^2(m_1y_1+m_2) - m_1m_2(y_1-1)\Big],\\ B_1^*(y_1,\alpha) &= m_2\Big\{m_1m_2(y_1-1) - 2(1-\alpha)\gamma^2(m_1y_1+m_2)\Big\},\\ C_1^*(y_1,\alpha) &= m_2^2\gamma^2(1-\alpha). \end{aligned}$$

Since $C_1^*(y_1,\alpha) > 0$ for $\alpha < 1$, when $A_1^*(y_1,\alpha) \neq 0$ the quadratic function (13) has the roots

$$x_1 = 1-z \qquad \text{and} \qquad x_2 = 1-z+\frac{m_1m_2^2(y_1-1)}{A_1^*(y_1,\alpha)},$$

which is similar to (10) with replacing $A_1^*(y_1,\alpha)$ by $A_1(y_1,\alpha)$. So by a similar argument, $f_1^*(x) > 0$ for $x \in [0,1-z]$, and hence $\Delta_1^* > 0$ for all $0 < \beta_1 \leqslant \beta_2$ when $\alpha \in (\alpha_1,1)$, i.e., $R(\delta_{1\alpha_1},\boldsymbol{\beta}) < R(\delta_{1\alpha},\boldsymbol{\beta})$ for $\alpha \in (\alpha_1,1)$. which completes the proof. □

Theorem 2.2. *Let* $\alpha_2 = \frac{m_2+\gamma^2}{m_1+m_2+\gamma^2}$*, then under the loss function (2), for* $\alpha \in (\alpha_2,1)$ *and* $0 < \beta_1 \leqslant \beta_2$,

$$R(\boldsymbol{\beta},\delta_{2\alpha_2}) < R(\boldsymbol{\beta},\delta_{2\alpha}) < R(\boldsymbol{\beta},\delta_2).$$

The proof of Theorem 2.2. is completely similar to the proof of Theorem 2.1 and hence is omitted.

Remark 2.1. Theorems 2.1. and 2.2. show that the mixed estimators $\delta_{1\alpha}$ and $\delta_{2\alpha}$ are inadmissible whenever $\alpha > \alpha_1$ and $\alpha > \alpha_2$, respectively. In the literature for finding admissible estimators of ordered parameters $0 < \beta_1 \leqslant \beta_2$ in the class of mixed estimators (3) and (4), the values of α that minimizes $R(\boldsymbol{\beta},\delta_{1\alpha})$ and $R(\boldsymbol{\beta},\delta_{2\alpha})$ are obtained by differentiating $-\Delta_1$ and $-\Delta_2$ with respect to α. Because of the complexity of RGL function (2), we cannot find the minimizing value of α explicitly. So, the admissibility of estimators $\delta_{i\alpha}$ for $0 \leqslant \alpha \leqslant \alpha_i$, $i=1,2$ remained unsolved.

Remark 2.2. The results of this section can be extended to a subclass of exponential family as follow. Let $\mathbf{X}_i = (X_{i1}, X_{i2}, \cdots, X_{in_i})$, $i=1,2$

has the joint probability density function

$$f(\mathbf{x}_i, \theta_i) = C(\mathbf{x}_i, n_i)\theta_i^{-\gamma_i} e^{-T_i(\mathbf{x}_i)/\theta_i}, \quad i = 1, 2, \tag{14}$$

where $\mathbf{x}_i = (x_{i1}, \cdots, x_{in_i})$, $C(\mathbf{x}_i, n_i)$ is a function of $\mathbf{x}_i$ and n_i, $\theta_i = \tau_i^r$ for some $r > 0, \gamma_i$ is a function of n_i and $T_i(\mathbf{x}_i)$ is a complete sufficient statistic for θ_i with $Gamma(\gamma_i, \theta_i)$- distribution. For example $Exponential(\beta_i)$ with $\theta_i = \beta_i$, $Gamma(\nu_i, \beta_i)$ with $\theta_i = \beta_i$ and known v_i, $Inverse\ Gaussian(\infty, \lambda_i)$ with $\theta_i = \frac{1}{\lambda_i}$, $Normal(0, \sigma_i^2)$ with $\theta_i = \sigma_i^2$, $Weibull(\eta_i, \beta_i)$ with $\theta_i = \eta_i^{\beta_i}$ and known β_i, $Rayleigh(\beta_i)$ with $\theta_i = \beta_i^2$, $Generalized\ \ Gamma(\alpha_i, \lambda_i, p_i)$ with $\theta_i = \lambda_i^{p_i}$ and known p_i and α_i, $Generalized\ \ laplace(\lambda_i, k_i)$ with $\theta_i = \lambda_i^{k_i}$ and known k_i belong to the family of distributions (14). An admissible linear estimator of $\theta_i = \tau_i^r$ in this family under the reflected gamma loss function (2) can be found in Towhidi and Behboodian [19].

Since $T_i = T_i(\mathbf{X}_i), i = 1, 2$, has a $Gamma(\gamma_i, \theta_i)-$ distribution, therefore we can extend the results of this section to the subclass of exponential family (14) by replacing $m_i = n_i\nu_i$, β_i and $\sum_{j=1}^{n_i} X_{ij} = m_i\delta_i$ by γ_i, θ_i and $T_i(\mathbf{X}_i)$, respectively.

Remark 2.3. The results of this section can also be extended to the family of transformed chi-square distributions which is introduced by Rahman and Gupta [16] and contain pareto and beta distributions. For details see Jafari Jozani et al. [3].

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