**Multi-Period DEA-R Efficiency for Decision Making Units Using Network Structure**

**Maghsoud Ahmad Khanlou Gharakhanlou1, Gasem Thohidi\*2, Nima Azar MirShotorbani3, Shabnam Razavyan4, Roohollah Abbasi Shureshjani5**

1 PhD Student, Department of Applied Mathematics, Islamic Azad University, Tabriz Branch.Tabriz-Iran. Email:kasra.khanlo@gmail.com

2 Associate Professor, Department of Mathematics, Islamic Azad University, Tehran Branch. Tehran- Iran. Email:gas.tobhidi@iauctb.ac.ir

3 Assistant Professor, Department of Applied Mathematics, Islamic Azad University, Tabriz Branch. Tabriz. Email:azarmir\_nim@yahoo.com

4 Associate Professor, Department of Mathematics, Islamic Azad University, South Tehran Branch. Tehran- Iran. Email:sh\_razavyan@azad.ac.ir

5 Assistant Professor, Department of Mathematics, Qom University. Qom- Iran. Email:roohollah31@gmail.com

**Abstract**

**Purpose:** In this paper, P tage network that contains relative data is studied in different time periods and the aim is to evaluate the technical efficiency of such a structure in each time period and overall efficiency after several desired time periods. Also, the efficiency of each stage of such a structure is evaluated in each time period and after several time periods.

**Method:** Considering a subsystem corresponding to each time period, we have a network system with a parallel structure of *T* subsystems, in which each subsystem consists of P processes connected in series. Using the proposed multi-period DEA-R model and also the mathematical relationships in overall efficiencies of the overall system, subsystems and sub- processes, overall efficiency and the multi-period sub-processes efficiency of the system network are measured after several time periods and in any time period.

**Results:** Out of 10 decision units, units D and H were efficient after three time periods, unit D was efficient in the first and second time periods, and unit H was efficient in the third period.

**Conclusion:** It has been shown that overall efficiency scores and the efficiency of each process obtained from this model after several desired time periods are higher than or equal to overall efficiency scores and the efficiency of each process in each time period. Also, a unit becomes efficient after several periods of time if it is efficient in at least one period of time

**Keywords**: data envelopment analysis (DEA), relative data envelopment analysis (DEA-R), network, multi-period, efficiency

**1. Introduction**

DEA is one of the proper and efficient tools in evaluation of the decision making units and it is non-parametric method. The primary method in DEA was proposed by Charnes, Cooper and Rhodes in 1978. They added mathematical programming to the Farrell non-parametric perspective that was proposed in 1957 for evaluation of the decision making units' efficiency with two inputs and one output that was known as CCR model that was capable to measure efficiency with some inputs and outputs (Safari&Azar, 2004). In 1984, Banker, Charnes and Cooper proposed a new model with (BCC) by making shifts in the CCR model (Jahanshahlu et al., 2009). Network data envelopment analysis (NDEA) is a developed model of the data analysis envelopment that tries to consider the internal structures of the decision making units in data development analysis models. Fare and Grosskopf (1996) research is the first studies in this scope. Since then, many researchers studied network data envelopment analysis and these models have been used in various practical usages. Standard DEA evaluates the efficiency of a set of units in a time period only according to inputs and outputs variables. However, it is obvious that the efficiency in a time period *t* not only depends on the inputs of this period but also on inputs of one or some previous periods. This group of the models is called multi-period DEA (Jablonsky et al., 2018). Gazari Neishaburi and his colleagues (2019) proposed a dynamic data envelopment analysis model that measures the process efficiency in a real business. Chen and his colleagues (2010) used a DEA model with network structure for determination of the efficiency of the sub-processes. Kao and Hwang (2013) proposed a model for measuring the network efficiency that in their model, the weight of the inputs and outputs are obtained so that the network efficiency is maximized under the condition that the efficiency of stages should not be more than one. However, Kao and Hwang (2013) model is incapable of determination of the efficiency boundary and a pattern for inefficient units. Seiford & Zhu (1999) and Luo (2003) used two-stage structure for evaluation of banks. Cook and his colleagues (2010) studied the general issue of multi-stage network. Tone and Tsutsui(2014) proposed network slacks-based measure (NSBM) for continuous network structures. Using this model, they could measure the process efficiency with overall efficiency. Tone and Tsutsui(2014) introduced a new structure of multi-period data envelopment analysis under the title of multi-period data envelopment analysis with network structure.

In evaluating a decision unit, it is essential to apply decision makers' constraints on input and output weights to obtain correct results. However, the use of weight constrains in the DEA causes problems. One of these problems is the use of the non-Archimedean constant ε for input and output weights in DEA models, which causes that the resulting weights are never zero. While some inputs may not play a role in producing an output, thus the achieved efficiency will not be accurate. To solve this problem, a model is employed that uses input-to-output ratios instead of inputs and outputs or vice versa, so if an input does not play a role in producing an output, this ratio is omitted from the model. For this purpose, the relative data envelopment analysis (DEA-R) models in the evaluations were introduced by Despic and Paradi (2007). We can used most of the weight constrains of DEA model by producing their equivalents in DEA-R model and produce equivalent results. Nazari and his colleagues (2011) studied efficiency in DEA and DEA-R with weight constraints. Wei et al. (2011) discussed the efficiency estimation in CCR models. Due to the assumption of limiting irrational and unnecessary weight, they used DEA-R models instead of traditional DEA models. Wei et al. (2011) also compared the optimal weights in DEA and DEA-R and they presented input-based DEA-R models and indicated that the efficiency calculated by their proposed model is greater than or equal to the efficiency obtained by the CCR model. Gerami and Mozzaffari (2012) presented a DEA-R model for evaluating network efficiency. They showed that the efficiencies of each stages of the network and overall efficiency of the network resulting from their model are greater than or equal to the corresponding values ​​obtained from previous models. Also, in their proposed model, overall efficiency is obtained as the weighted average of the efficiencies of each stage. Mozzaffari et al. (2017) have proposed a model for calculating overall amount of efficiency in a two-stage network in DEA-R using a linear multi-objective programming structure.

Having access to ratio data on 10 bank branches, Mozzaffari et al. (2020) calculated the efficient hyper planes for these branches using the method proposed in their paper. Overall, their evaluations revealed that the Royal Bank of Scotland (RBS) was not located on any hyper planes.

Given the advantage of DEA-R models over CCR-based models and the fact that time has not been considered in previous DEA-R models, in this paper a multi-period DEA-R model has been proposed for evaluation of the network system efficiency.

 In this paper, P stage network that contains relative data is studied in different time periods and the aim is to evaluate the technical efficiency of such a structure in each time period and overall efficiency after several desired time periods. Also, the efficiency of each stage of such a structure is evaluated in each time period and after several time periods. Considering a subsystem corresponding to each time period, we have a network system with a parallel structure of *T* subsystems, in which each subsystem consists of P processes connected in series. Using the proposed multi-period DEA-R model and also the mathematical relationships in overall efficiencies of the overall system, subsystems and sub- processes, overall efficiency and the multi-period sub-processes efficiency of the system network are measured after several time periods and in any time period. This paper has been organized as follows: section 2 offers an introduction or overview of the necessary concepts. Section 3 introduces the proposed model for determining the efficiency of a multi-period multistage network using the relative data envelopment analysis (DEA-R) model. In section 4, the proposed model is solved with a numerical example and the results are analyzed. The last section of the paper includes conclusion.

**2. Preliminaries**

**2.1 Network production systems**

P-stage networks are evaluated via DEA models and internal relationships and intermediate products are considered. Networks are classified into various structures such as two-stage, series, parallel and a combination of series and parallel. In this section, DEA-R model for two-stage network structure has been proposed by Mozzaffari et al. (2017) and then DEA-R model for multi-stage network structure in series has been introduced by Gerami and Mozzaffari (2012).

**2.1.1. Two-stage network DEA-R models**

The evaluation of DMUs that have relative data such as and requires models that firstly have possibility of production and secondly, they are capable of calculation of the units' efficiency value. In this section, at first, the two-stage DEA-R models and then possibility of production in each stage have been suggested by Mozzaffari and et al.(2017)(Fig.1)

Stage 2

Stage 1

**Fig.1** Two-stage network

**2.1.2 Efficiency in two-stage DEA-R**

We assume that are the data of where and are available ratios. The aim is evaluation of DMUs in a two-stage network using the defined ratios. CCR based output model for the first stage was proposed by Mozzaffari et al. (2017) as follows:

|  |  |
| --- | --- |
|   | (1) |

Output based DEA-R envelop model for evaluation of is as follows:

|  |  |
| --- | --- |
|   | (2) |

In the second stage, output based DEA-R model is as follows (Mozzaffari et al., 2017):

|  |  |
| --- | --- |
|  | (3) |

Output based envelopment model under CCR in the second stage for evaluation of is written as follows (Mozzaffari et al., 2017):

|  |  |
| --- | --- |
|  | (4) |

**2.1.3.Two-stage network DEA-R based on multi-objective linear programming (MOLP)**

Mozzaffari et al. (2017) suggested a two-objective linear programming model for measuring overall efficiency of DMUo with the relative data defined as and (CCR and BCC) as follows. Then, by combining constrains of models (1) and (3), they introduced a two-objective linear model (5) for measuring overall efficiency of the DEA-R two-stage network.

|  |  |
| --- | --- |
|  | (5) |

An envelopment model for evaluation of overall efficiency of a two-stage network DEA-R was proposed by Mozzaffari et al. (2017) as follows:

|  |  |
| --- | --- |
|   | (6) |

Model (6) is a linear programming problem that and are parameters in which determine overall efficiency of two-stage network. Variables and correspond to stage 1 and 2 respectively. If then only stage 1 of the network is considered. Similarly, if then only stage 2 is considered. If and where and then the optimal Pareto answer (5) defines overall efficiency of from two stage models with relative data.

**2.1.4. Measuring the network structure efficiency using DEA-R models**

In this section, the efficiency of the network systems DEA-R is calculated in series as proposed by Gerami and Mozzaffari (2012). Consider a P stage process shown in Figure 2. Overall efficiency is denoted by and the efficiency of each process is denoted by. In the last phase P, all the outputs leave the system and we denote them by .

Stage 2

Stage 1

Stage 3

**Fig.2** Network as series

**Table.1** Model variables

|  |  |
| --- | --- |
|  | Input variable ratio weight to output variable of  |
|  | Input variable ratio weight to output variable of  |
|  | Input variable ratio weight to output variable of  |
|  | Input variable ratio weight to output variable of  |
|  | Input variable ratio weight to output variable of  |
|  | Input variable ratio weight to output variable of  |
|  | Input vector of the first unit stage |
|  | The output vector is for which exits the first step and exits the system and does not enter the next step as input. |
|  | The output vector is for which exits the first step and enters the second step as a part of input. |
|  | The input vector for is in the stage that enters the process at the beginning of this stage.  |
|  | is the output vector for that exits the stage and exits the process and does not enter as input in the stage P+*1* |
|  | is the output vector for that exits the stage and enters as a part of input in the stage P+*1* |
| *،* |  is the input vector of for in the stage P that enters the process at the end of the stage P-1 |

Gerami and Mozzaffari (2012) presented the DEA-R model for calculating the efficiency of system with P network structure in series as follows:

|  |  |
| --- | --- |
|   | (7) |

Model (7) first calculates the relative efficiency score for each weight vector and calculates the smallest score as the efficiency score of this set of weights. Then, by adjusting the weights, the maximum efficiency score is considered as the overall efficiency score. Given that the objective function is the weighted mean of the network sub-phases efficiency, this model maximizes the values ​​of for a selected set of weights to maximize overall efficiency. A change in causes a change in and. Therefore, the overall efficiency and efficiencies of each process depend on the weights of the objective function.

**3. Evaluating the efficiency of production systems with a relative multi-period network structure**

 Most of the multi-period DEA models proposed so far have considered production systems as a black box. These models do not consider the internal relationships within the system as well as the efficiency of each process. DEA models are also unable to calculate efficiency when the prices of inputs and outputs are unknown and the ratio of inputs to outputs or vice versa is known. Also, previous models for the DEA-R two-stage network have been suggested without considering time. Therefore, in this section we introduce a model for determining the efficiency of multi-period decision units DEA-R (in L time period) with multi-network structure.

 Considering a subsystem corresponding to each period, we have a network system with a parallel structure of L subsystems, each system consisting of P stages that are connected in series (Fig. 3).

q

1

P

q

1

P

q

1

P

**Fig.3** Parallel multi-period production network

**3.1. Proposed model for determining the efficiency of DEA-R of decision-making units with a multi-period network structure**

In this section, we consider a set of ,, which is observed in the L time period. Each *DMU* consists of P stages. The internal relations of the processes are the same for all *DMUs*. In a network structure, each process works together that the entire decision-making unit under evaluation to achieve optimal efficiency. A multi-stage P network production model with external inputs , and and final outputs and and the intermediate dimensions of (first stage outputs and second stage inputs), (second stage outputs and third stage inputs), and (P stage outputs) in the period *t* has been proposed under the following assumptions (Fig. 4).

A-The proposed model is the CCR input-oriented DEA-R.

B- The proposed model is a parametric linear model in P step network process. The purpose of this model is to reduce the inputs at each stage in order to evaluate the units with relative data. In all network processes, we consider the constrain corresponding to each step t, provided that Thus, since if then for every ,. In general, for parameters we consider the following states:

1-If and, then the proposed model can calculate the efficiency of each step.

2- If and, then the proposed model can calculate the overall efficiency of the network.

 C- Suppose and are sets of input indices in each P stage. Similarly, and are sets of intermediate size indices and final outputs.

D- In the proposed model, the parameters correspond to the variables respectively. Now since P is the network step corresponding to so the parametersplay a very important role in calculating process efficiencies and overall efficiency.

E- The variables correspond to processes 1, 2 and P respectively.

F-Since the P stage network process is an input-driven, the purpose of the proposed model is to reduce input-to-output data that is reduced radially. The variables are used to reduce the inputs in process 1, 2 and P of the network with relative data respectively.

Our proposed DEA-R model for calculating the overall efficiency of the system with network structure after period L and also the time period t (actually subsystem efficiency) is as follows:

**3.2. Proposed DEA-R model for overall efficiency of the system after L time period**

|  |  |
| --- | --- |
|  | (8) |

In model (8), depict the first stage efficiency and indicates the efficiency of and shows the overall efficiency of the system after L time period.

**3.2. Proposed DEA-R model for overall efficiency of the system in time period t**

|  |  |
| --- | --- |
|  | (9) |

In model (9), depict the first stage efficiency and indicates the efficiency of and shows the overall efficiency of the system in time period t.

q

1

P

**Fig.4** P stage network in time period t

Theorem 1: models (8) and (9) are feasible.

Proof: Assuming that is the optimal value of objective function (8), then according to and it can be said that and is a feasible response of the stage one and according to and it can be said that and is a feasible answer to the stage 2. So, based on and , it can be mentioned that and is a feasible response of P stage. If for each, we assume then according to it can be said that for each is a feasible response to the total network in time period t. In addition, according to feasible response, it is concluded that the optimal value does not exceed from one and it is always higher than zero.

**4. Numerical example**

This example is taken from the paper of Tohinia and Tohidi (2019). In this example we consider 10 DMUs. Each DMU has two stages, stages I and II that are observed in three time periods. Each uses two external inputs and to produce two intermediate products and and an external output , and in the second stage uses two intermediate products and one external input to produce two output products and in time period t Figure (5). The data set is shown in three time periods (t = 1, 2, 3) in Tables 1, 2 and 3.

I

II

I

II

I

II

**Fig.5** Multi-period network production system with T=3 and P=2

| **Table.1** Mean and standard deviation of the data in the first time period |
| --- |
|  | A | B | C | D | E | F | G | H | I | J |
| Mean | 7.2500 | 6.6250 | 8.0000 | 8.7500 | 7.3750 | 7.7500 | 7.3750 | 6.8750 | 5.5000 | 7.7500 |
| Std.Deviation | 4.83292 | 3.37797 | 5.12696 | 5.33854 | 3.73927 | 4.74342 | 3.37797 | 2.85044 | 3.16228 | 4.33425 |

| **Table.2** Mean and standard deviation of the data in the second time period |
| --- |
|  | A | B | C | D | E | F | G | H | I | J |
| Mean | 8.3375 | 9.0000 | 11.0375 | 12.0125 | 9.6500 | 9.6375 | 9.4375 | 8.9125 | 7.5125 | 10.0500 |
| Std..Deviation | 4.97822 | 4.79434 | 8.03473 | 7.82221 | 5.13893 | 5.65229 | 4.47786 | 3.97076 | 4.61006 | 5.75872 |

| **Table.3** Mean and standard deviation of the data in the third time period |
| --- |
|  | A | B | C | D | E | F | G | H | I | J |
| Mean | 9.9862 | 13.5000 | 16.5562 | 20.3588 | 14.4562 | 12.5938 | 12.5125 | 12.4375 | 10.3562 | 12.4857 |
| Std. Deviation | 5.39055 | 7.19151 | 1.20521 | 1.21523 | 7.24527 | 6.56552 | 5.78111 | 4.58661 | 6.03806 | 7.02197 |

After solving model (80 for each overall efficiency of the system after three time periods has shown in table 4.

**Table.4** Results of solving model (8) for

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Objective** |  |
| 0.500000 | 0.375000 | 0.875000 | A |
| 0.441408 | 0.499980 | 0.941388 | B |
| 0.500000 | 0.377419 | 0.877419 | C |
| 0.500000 | 0.500000 | 1.000000 | D |
| 0.484848 | 0.500000 | .984848 | E |
| 0.500000 | 0.205128 | 0.705128 | F |
| 0.486571 | 0.221939 | 0.708510 | G |
| 0.500000 | 0.500000 | 1.000000 | H |
| 0.363439 | 0.500000 | 0.863439 | I |
| 0.500000 | 0. 488939 | 0.988939 | J |

After solving model (9) for each the overall efficiency for any time period has been shown in tables 5 to 7 separately.

**Table.5** Results of solving model (9) for

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Objective** |  |
| 0.500000 | 0.375000 | 0.875000 | A |
| 0.314910 | 0.475105 | 0.790014 | B |
| 0.500000 | 0.375000 | 0.875000 | C |
| 0.500000 | 0.500000 | 1.000000 | D |
| 0.484848 | 0.500000 | 0.984848 | E |
| 0.500000 | 0.203704 | .703704 | F |
| 0.486571 | 0.221939 | 0.708510 | G |
| 0.438298 | 0.470842 | 0.909140 | H |
| 0.363439 | 0.500000 | 0.863439 | I |
| 0.500000 | 0.488939 | 0.988939 | J |

**Table.6** Results of solving model (9) for

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Objective** |  |
| 0.479125 | 0.375000 | 0.854125 | A |
| 0.335018 | 0.475105 | 0.810123 | B |
| 0.500000 | 0.375000 | 0.875000 | C |
| 0.500000 | 0.500000 | 1.000000 | D |
| 0.438672 | 0.500000 | 0.938672 | E |
| 0.500000 | 0.203704 | 0.703704 | F |
| 0.410683 | 0.221939 | 0.632622 | G |
| 0.420654 | 0.470842 | 0.891495 | H |
| 0.342146 | 0.500000 | .842146 | I |
| 0.500000 | 0.488939 | 0.988939 | J |

**Table.7** Results of solving model (9) for

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Objective** |  |
| 0.500000 | 0.371250 | 0.871250 | A |
| 0.441408 | 0.499980 | 0.941388 | B |
| 0.500000 | 0.377419 | 0.877419 | C |
| 0.313725 | 0.500000 | 0.813725 | D |
| 0.365500 | 0.500000 | .865500 | E |
| 0.500000 | 0.205128 | 0.705128 | F |
| 0.422437 | 0.218593 | 0.641030 | G |
| 0.500000 | 0.500000 | 1.000000 | H |
| 0.344713 | 0.500000 | 0.844713 | I |
| 0.475054 | 0.459541 | 0.934596 | J |

We examine the relationship between overall efficiency of the system and the efficiency of each system process after 3 time periods and also the relationship between overall efficiency of the system and each system process in every time period by models (8) and (9). Table 4 summarizes overall efficiency of the system after three time periods .According to this table, units D and H are efficient. The lowest efficiency score after three time periods is for unit F. Table 5 summarizes overall efficiency of the system in the first time period. Unit D is efficient and Unit F has the lowest efficiency score at this time period. Table 6 shows overall efficiency of the network system in the second time period that unit D is efficient in the network system and unit G has the lowest efficiency score. Table 7 indicates the efficiency of the network system in the third time period. Unit H is efficient and G has the lowest efficiency score among the units of the network system in the third time period. According to Tables 4, 5, 6 and 7, it can be said that the efficiency scores obtained from model (8) are higher than or equal to the efficiency scores obtained from model (9). In fact, adding a time period variable increases the constraints of the secondary problem and it is possible that the optimal solution of the problem to be better and the value of the objective function, which is of the maximization type, is increased and since the primary problem optimal response equals the secondary problem, so the obtained efficiency scores are increased.

**5. Conclusion**

A model is required for measuring the efficiency of the system and its processes by considering time in order to evaluate the efficiency of a network system and its processes. In this paper, a model has been proposed for measuring overall efficiency of the system and its processes over several desired time periods. This model has three advantages: First, when the input and output data are unknown and only a proportion of them is known, we can also use this model to prevent false inefficiency and not use the non-Archimedean number ε. Second, in this model, the internal relationships among processes are considered. Finally, the proposed model focuses on changes over time period. It has been shown that overall efficiency scores and the efficiency of each process obtained from this model after several desired time periods are higher than or equal to overall efficiency scores and the efficiency of each process in each time period. Also, a unit becomes efficient after several periods of time if it is efficient in at least one period of time

 For explaining the capability of the proposed model, the efficiency of 10 DMUs has been calculated that each of them is consisting of two phases. The results achieved by model solving help us to identify network processes and periods that reduce system efficiency. In this way, significant results can be obtained by decomposing of a system into subsystems and sub processes. For future research, DEA-R ranking of decision-making units with a multi-period network structure is suggested.

**References**

 Jahanshahlu, G., Hosseinzadeh Lotfi, F., Nikumram, H.: *Introduction to Data Envelopment Analysis*, Islamic Azad University, Science and Research Branch (2009).

Safari, S., Azar, A.: Evaluating the efficiency of organization based on quality award indicators - DEA approach, *Scientific-Research Monthly*, eleventh Year, No. 8, (2004).

 Salehi Sadeghiani, M., Sohraei, S.: Evaluating the cost-effectiveness, revenue and profit efficiency of organization using data envelopment analysis, *Noor Specialized Journals Database*, No. 21, 229-247, (2015).

Charnes, A, Cooper, W.W., Rhodes, E.: Measuring the efficiency of decision making units. *European Journal of Operational Research* 2:429 – 444, (1978).

Cooper, W.W, Seiford, L.M, Tone, K.: Introduction to data envelopment analysis and its uses. *Springer*, New York, (2006).

Cook, W. D., Zhu, J., Bi, G., Yang, F.: Network DEA: Additive efficiency decomposition. *European Journal of Operational Research*, 207(2): 1122-1129. (2010).

Chen, Y., Cook, W. D., Zhu, J.: Deriving the DEA frontier for twostage processes. *European Journal of Operational Research*, 202, 138-142, (2010).

Chen, Y., Cook, W. D., Li, N., Zhu, J.: Additive efficiency decomposition in two-stage DEA. *European Journal of Operational Research*, 196, 1170-1176, (2009).

Despic, O., Paradi, J.C.: DEA-R: Ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications. *Journal of Productivity Analysis*. 28, (1) , pp 33-44, (2007).

Färe, R., Grosskopf, S.: Productivity and intermediate products: A frontier approach. *Economics letters,* 50(1), 65-70, (1996).

Farrell, M. J.: The measurement of productive efficiency*. Journal of the Royal Statistical Society*, 120, 253-281, (1957).

Gazori, a., Khalili-Damghani, K., Hafezalkotob, A.: Multi-period network data envelopment analysis to measure efficiency of a real business. *Journal of Industrial and Systems Engineering*.Vol. 12, No. 3, 55- 77, (2019).

Gerami, J., Mozaffari,M.R.: Measuring performance of network structure by dea-r model. *The 4th National Conference on Data Envelopment Analysis*. University of Mazandaran, Babolsar, Iran, (2012).

Jablonsky, J., et al.: Multi-period data envelopment analysis and resource allocation: *A case study J.* Phys.: Conf. Ser,1-7. (2018).

Kao, C., Hwang. S. N.: Multi period efficiency and Malmquist productivity index in two stage production systems. *European Journal of Operational Research*, 232: 512-521, (2013).

Luo, X. M.: Evaluating the profitability and marketability efficiency of large banks: An application of data envelopment analysis. *Journal of Business Research*, 56, 627-635, (2003).

Lozano, S. Scale and cost efficiency analysis of networks of processes. *Expert Syst Appl* 38:6612–6661, (2011).

Mozaffari, M.R., Saneib, M., Jablonsky, J.: Efficiency Analysis in Multi-Stage Network DEA-R Models, *Int. J. Data Envelopment Analysis* , Vol.5, No.2: 1553-1572, (2017).

Mozaffari. M.R, Dadkhah.F, Jablonsky .J, Fernandes Wanke. P.: Finding efficient surfaces in DEA-R models*, Applied Mathematics and Computation* 386,125497, 1-14, (2020).

Seiford, L. M., & Zhu, J.: Profitability and marketability of the top 55 US commercial banks. *Management Science*, 45, 1270-1288, (1999).

Tone, K., Tsutsui, M.: Dynamic DEA with network structure: A slacks-based measure approach, *Omega*. 42, 124-131, (2014).

Tohidnia , Tohidi,G.Measuring productivity change in DEA-R: A ratio-based profit efficiency model. *Journal of the operational research society, 70*,NO.9,1511-1521, (2018).

Tohidnia, S., Tohidi, G.: Estimating multi-period global cost efficiency and productivity change of systems with network structures. *Journal of Industrial Engineering International*, 15, 171-179, (2019).

Tone, K.: A strange case of the cost and allocative efficiency in DEA. *J Oper Res Soc* 53:1225–1231, (2002).

Wei, C.K., Chen, L.C., Li, R.K., Tsai, C.H.: Using the DEA-R model in the hospital industry to study the pseudoinefficiency problem. *Expert Systems with Applications.38*, 2172-2176, (2011).

Wei, C.K., Chen, L.C., Li, R.K., Tsai, C.H.: A study of developing an inputoriented ratio-based comparative efficiency model. *Expert Systems with Applications. 38*, 2473-2477, (2011).

Wei, C. K., Chen, L. C., Li, R. K., Tsai, C. H.: exploration of efficiency underestimation of CCR model: Based on medical sectors with DEA-R model, *Expert Systems with Applications*, Vol. 38, 3155-3160, (2011).