Developing new robust DEA models to identify the returns to damage under undesirable congestion and damages to return under desirable congestion measured by DEA environmental assessment

Mahsa Nassiria, Mohsen Rostamy-Malkhalifihb[[1]](#footnote-1), Hadi Bagherzadeh Valamia

a Department of Mathematics, Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University, Tehran, Iran

b Department of Mathematics, science and research Branch, Islamic Azad University, Tehran, Iran

**Abstract**

In recent years, the environmental issue has attracted widespread concern from the international community, as gas waste, water waste, and solid wastes generated in the production process of factories. Recent studies on environmental management have forced commercial organizations to re-evaluate their roles and responsibilities for protecting the natural environment. This study focuses on the DEA environmental assessment via the concept of congestion. Recognizing the congestion of units is one of the most attractive issues in the literature of Data Envelopment Analysis (DEA), because the decision maker (DM) can use this concept to decide whether to increase or decrease the size of a Decision Making Unit (DMU). In the DEA literature, congestion is classified into Undesirable Congestion (UC) and Desirable Congestion (DC). In many real-world situations, we cannot determine the exact value for all data, hence, some parameters are inevitably reported as uncertain data, e.g. stochastic data, fuzzy data, interval data and so on. This study focuses on considering Returns to Damage (RTD) under UC and Damages to Return (DTR) under DC in the situation that the input and desirable and undesirable outputs are reported as interval data. For this purpose, some uncertain models under the different production possibility sets (PPS) are formulated and then we use the robust optimization technique to formulate the equivalent certain models. The potential of the proposed methods are illustrated by a numerical example.

**Keyword:** Data Envelopment Analysis; Returns to Damage; Damages to Return; Undesirable Congestion; Desirable Congestion.

1. **Introduction**

Data envelopment Analysis (DEA) is a well-known mathematical programming method to assess the relative efficiency of units. See Charnes et al. (1978), Banker et al. (1984), Fre et al. (1985), Zhu (2002), Cooper et al. (2006) for more studies about the classical DEA models. Evaluating the performance of the DMUs in the presence of undesirable outputs may be difficult because, we should first decide about the way of treatment with these outputs. Thus, many researchers have been attracted to modelling the undesirable outputs in the DEA literature in the last two decades. There are some possible options to handle the undesirable outputs. We can ignore them from the production technology. The undesirable outputs can be treated as the regular inputs or as the normal outputs. Also, we can performe some necessary transformations to take the undesirable outputs into account. Halkos and Petrou (2019) reviewed the existing methods in the DEA literature to handle the undesirable outputs and showed that each method has some benefits and drawbacks which should be taken into account by the researcher. Zhou et al. (2019) proposed an exponential transformation of undesirable outputs into desirable outputs to measure the environmental efficiency by using all kinds of classic models. Toloo and Hančlová (2020) formulated two individual and summative selecting directional distance models and developed a pair of multiplier- and envelopment-based selecting approaches. Shi et al. (2021) proposed a slacks-based measure network data envelopment analysis (SBM-NDEA) model with undesirable outputs to evaluate the performance of production processes that have complex structure containing both series and parallel processes. For more studies about the undesirable outputs, see Yousefi et al. (2018), Zarbakhshnia and Jaghdani (2018), Mo et al. (2020), Pishgar-Komleh et al. (2020), Sun and Huang (2021), Streimikis and Saraji (2021) and Zhao et al. (2022).

The concept of congestion is an important subjects in the DEA literature. It is well-known that, if the decreasing in some inputs of a decision making unit (DMU) results in the increasing in some outputs of that DMU. The concept of congestion can help the decision maker (DM) in order to decide whether to increase or decrease the size of the unit under evaluation. This concept of congestion is called the undesirable congestion (UC). Recognizing UC has attracted the attention of many scholars, for more details, see Fare et al. (1986), Cooper et al. (2001), Tone and Sahoo (2004), Wei and Yan (2004), Sueyoshi and Sekitani (2009), Jahanshahloo and Khodabakhshi (2004), Wu et al. (2015), Wanke et al. (2019) and Khezri et al. (2021). Another type of congestion is called the desirable congestion (DC) which reports an existence of eco-technology innovation and the managerial challenges, used for mitigation of undesirable outputs. It should be noted that, because UC and DC influence the performance of energy firms, hence, the recognizing and separating between UC and DC are crucial from the viewpoint of operating energy sectors. In the environmental assessment literature, the main aim is to overcome global warming and climate change, and so, the DC is more important than UC. For more study, see Sueyoshi and Yuan (2016).

The conventional DEA model deals with precisely known data where inputs and outputs values are deterministic and exactly known. However, in many real-world situations, one may encounter uncertain data due to the different reasons, such as incomplete information, measurement errors or any other source of reason. Robust Optimization (RO) is a technique to model optimization problems with uncertain data which aims to determine an optimal solution which is the best for all or the most possible realizations of the uncertain parameters. For more details, see Ben-Tal and Nemirovski (1998, 1999, 2000) and Bertsimas and Sim (2004). Some scholars incorporated the robust optimization technique into DEA, for more details, see Wang and Wei (2010), Sadjadi and Omrani (2008), Sadjadi and Omrani (2010), Sadjadi et al. (2011), Omrani (2013), Salahi et al. (2016) and Dehnokhalaji et al. (2022).

Given that the importance of UC and DC, this study focuses on proposing new methods to recognize the UC and DC in the situation that the inputs, desirable outputs and undesirable outputs are reported as interval data. For this purpose, we formulate some models under the different PPSs to evaluate the DMUs in the case of interval data and then the robust optimization technique is used to convert the proposed models into the certain models.

The rest of the paper is organized as follows: Section 2 reviews some basic definitions and preliminaries. Section 3 proposes the new method to recognize the undesirable congestion and the desirable congestion. Section 4 uses a data set to show the potentially of the proposed method. Section 5 concludes the paper.

1. **Preliminaries and basic definitions**

Suppose that there are n DMUs, and each unit, e.g. uses m inputs to generate s outputs. It is assumed that, for all and for all are the input and the output, respectively. Assume that is the unit under evaluation.

The following PPS under the variable returns to scale (VRS), namely has been introduced by Banker et al. (1984):

The output oriented BCC model proposed by Banker et al. (1984) for evaluating the efficiency score of units is as follows:

|  |  |
| --- | --- |
|  | (2) |

where is non-Archimedean.

Banker et al. (1984) presented the following definition:

Definition 1. Suppose that is an optimal solution for model (2). If then is called technically efficient. If then is called strongly efficient.

Cooper et al. (2001) and Brocket et al. (2004) presented the classical definition of congestion as follows:

Definition 2. The unit has congestion if the decreases (increases) in some inputs result in the increases (decreases) in some outputs without worsening (improving) other inputs or outputs.

Tone and Sahoo (2004) defined the PPS accepting all assumptions to build except one assumption, strong disposal. They considered weak disposal instead, which was defined as follows:

Definition 3. The PPS satisfies weak disposal assumption if for each belonging to the PPS and vector where and belongs to the PPS.

They proposed the following model to estimate the efficiency score of with respect to

|  |  |
| --- | --- |
|  | (3) |

It is clear that the target unit for located on the strongly efficient frontier of is as follows:

Tone and Sahoo (2004), presented the following definitions for strongly efficient unit, strong congestion and weak congestion, respectively.

Definition 4. The unit is strongly efficient with respect to if

Definition 5. Suppose that is strongly efficient unit with respect to has strong congestion if there is such that and

Definition 6. Suppose that is strongly efficient unit with respect to has weak congestion if there is an activity in that uses less resources in some inputs to produce more products in some outputs.

There are several methods to recognize the congestion of units. The next section develops a method to recognize the desirable congestion (DC) and the undesirable congestion (UC) in the case of interval data.

1. **Our proposed approach to determine the undesirable congestion (UC)**

Consider a system with DMUs, Suppose that each unit, e.g. uses an input vector and produces a vector of desirable outputs and a vector of undesirable outputs . It is assumed that, for all and for all and for all Assume that the lower bound and the upper bound of the intervals have positive values.

* 1. **A possible occurrence of undesirable congestion (UC)**

This section proposes a model to recognize the undesirable congestion of units in the case of interval data. For this purpose, we formulate model (4) as follows:

|  |  |
| --- | --- |
|  | (4) |

Model (4) is an uncertain model which aims to increase the desirable outputs and decrease the undesirable outputs by It is clear that and is efficient if the optimal value of model (4), We use the robust optimization technique to convert model (4) into a certain model. For this purpose, we rewrite model (4) as follows and then the robust optimization technique is used to formulate the equivalent certain model.

|  |  |
| --- | --- |
|  | (5) |

It is clear that at the optimality of model (5). In the following, we prove that at the optimality of model (5). Since, and we have:

1. If then is an inefficient unit. Therefore, and so
2. If then is an efficient unit. Hence,

Hence, model (6) can be obtained as the robust counterpart of model (5) by using the method of Ben-Tal and Nemirovski (2000).

|  |  |
| --- | --- |
|  | (6) |

Model (6) can be converted into the following model:

|  |  |
| --- | --- |
|  | (7) |

In the following, we develop model (7) in order to identify the undesirable congestion of units. For this purpose, model (7) is considered under the weak disposability in the undesirable outputs. Hence, the third constrain of model (7) is considered as the equality constraint.

The data ranges for adjustment are determined by the upper and lower bounds on inputs and those of desirable and undesirable outputs. These upper and lower bounds are specified as follows:

Next, we formulate model (8) to recognize the undesirable congestion of units in the case of interval data:

|  |  |
| --- | --- |
|  | (8) |

Model (8) ignores the slack variables related to the undesirable outputs and so these constraints are considered as the equality constraints. Model (8) maximizes the slack variables related to the input and desirable constraints.

The dual of model (8) can be formulated as follows:

|  |  |
| --- | --- |
|  | (9) |

The efficiency score of is as follows:

|  |  |
| --- | --- |
|  | (10) |

Which shows a possible occurrence of UC. All variables in Eq. (10) are determined at the optimality of models (8) and (9).

After solving model (9), a possible occurrence of UC can be determined as follows:

1. If for at least one then strong UC occurs on
2. If for all then no UC occurs on
3. If for at least one then weak UC occurs on

In the following, we identify the returns to damage (RTD) under UC. Suppose that the dual variables, obtained by model (7), are and and at the optimality of this model. Then, the supporting hyperplane on can be expressed as follows:

|  |  |
| --- | --- |
|  | (11) |

which is characterized by

|  |  |
| --- | --- |
|  | (12) |

where is the reference set for and

Hence, the degree of RTD (DRTD) under UC on is determined as follows:

|  |  |
| --- | --- |
|  | (13) |

The type of RTD is classified as follows:

1. If there is an optimal solution for model (9) which satisfies for all and then has an increasing RTD.
2. If there is an optimal solution for model (9) which satisfies for all and then has the constant RTD.
3. If for each optimal solution for model (9), for all and then has the decreasing RTD.
4. If for each optimal solution for model (9), for at least one then has the negative RTD.
5. For all other cases excluding the cases (a)-(e), has no RTD.

In summary, the type of UC is identified by the sign of dual variables, i.e. The type of UC can be classified into the three categories. Meanwhile, the type of RTD is determined by not only the sign of But also the sign of The type of RTD is classified into the five categorizes.

* 1. **A possible occurrence of desirable congestion (DC)**

This section proposes a model to recognize the desirable congestion of units in the case of interval data. For this purpose, we consider the following PPS and formulate model (15) to evaluate the units.

|  |  |
| --- | --- |
|  | (14) |

|  |  |
| --- | --- |
|  | (15) |

Model (15) is an uncertain model which aims to increase the desirable outputs and decrease the undesirable outputs by It is clear that We use the robust optimization technique to convert model (15) into a certain model. For this purpose, we rewrite model (15) as follows and then the robust optimization technique is used to formulate the equivalent certain model.

|  |  |
| --- | --- |
|  | (16) |

It is clear that at the optimality of model (16). In the following that, we prove that at the optimality of model (16). Since, and we have:

1. If then is an inefficient unit. Therefore, and so
2. If then is an efficient unit. Hence,

Hence, model (17) can be obtained as the robust counterpart of model (16) by using the method of Ben-Tal and Nemirovski (2000).

|  |  |
| --- | --- |
|  | (17) |

Model (17) can be converted into the following model:

|  |  |
| --- | --- |
|  | (18) |

In the following, we develop model (19) in order to identify the desirable congestion of units. For this purpose, model (18) is considered under the weak disposability in the desirable outputs. Hence, the second constrain of model (18) is considered as the equality constraint.

|  |  |
| --- | --- |
|  | (19) |

Model (19) ignores the slack variables related to the desirable outputs such that these constraints are considered as the equality constraints.

The dual of model (19) can be formulated as follows:

|  |  |
| --- | --- |
|  | (20) |

The efficiency score of is as follows:

|  |  |
| --- | --- |
|  | (21) |

Which shows a possible occurrence of DC. All variables in Eq. (21) are determined at the optimality of models (19) and (20).

After solving model (20), a possible occurrence of DC can be determined as follows:

1. If for at least one then has strong DC.
2. If for all then shows no DC.
3. If for at least one then has weak DC.

In the following, we present a method to measure the damage to return (DTR) under DC. Suppose that the dual variables, obtained by model (20), are and and at the optimality of this model. Then, the supporting hyperplane on can be expressed as follows:

|  |  |
| --- | --- |
|  | (22) |

which is characterized by

|  |  |
| --- | --- |
|  | (23) |

where is the reference set for and

Hence, the degree of DTR (DDTR) under DC on is determined as follows:

|  |  |
| --- | --- |
|  | (24) |

The type of DTR is classified as follows:

1. If there is an optimal solution for model (20) which satisfies for all and then has an increasing DTR.
2. If there is an optimal solution for model (20) which satisfies for all and then has the constant DTR.
3. If there is an optimal solution for model (20) which satisfies for all and then has the decreasing DTR.
4. If there is an optimal solution for model (20) which satisfies for at least one then has negative DTR.
5. For all other cases excluding the cases (a)-(i), has no DTR.

In summary, the type of DC is identified by the sign of dual variables, i.e. The type of DC can be classified into the three categories. Meanwhile, the type of DTR is determined by not only the sign of But also the sign of The type of DTR is classified into the five categorizes.

It should be noted that, in this study, the proposed DEA approaches assumes that all unified efficiency measures are uniquely determined on optimality. If the assumption is dropped, then the proposed approach needs to incorporate Strong Complementary Slackness Conditions (SCSCs) into the formulations. See, for example, Sueyoshi and Goto (2017) on how to incorporate SCSCs. Also, see Sueyoshi and Yuan (2016) for more details.

1. **Case study**

This section uses a data set, reported in Khalili-Damghani et al. (2015), to illustrate the potentially of the proposed methods. The data set considers 17 combined cycle power plants in Iran during a six year period, as the DMUs. There are six variables, fossil fuel is considered as input, electricity power is considered as the desirable output and gases such as and are as the undesirable outputs. The data are summarized in Table 1.

Table 1. Interval inputs and outputs of the power plants during the six-year study.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| DMU |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1002243 | 1534381 | 4663820 | 5948123 | 3.6 | 5 | 1.8 | 6.6 | 2338 | 3015 | 0 | 0.1 |
| 2 | 971509 | 1298112 | 4821296 | 5657392 | 3.7 | 4.4 | 2.1 | 4.7 | 2367 | 2727 | 0 | 0.1 |
| 3 | 1331457 | 1831098 | 7220851 | 7699512 | 5.3 | 5.8 | 2.8 | 6.7 | 3478 | 3631 | 0 | 0.1 |
| 4 | 766658 | 1117322 | 3781843 | 4628520 | 2.8 | 3.7 | 1.7 | 4.8 | 1779 | 2250 | 0 | 0.1 |
| 5 | 24213 | 1060942 | 356963 | 3184631 | 0.6 | 3.2 | 0.4 | 2.2 | 318 | 2119 | 0 | 0 |
| 6 | 1045455 | 1283541 | 5339780 | 5975686 | 3.8 | 4.4 | 1.3 | 3 | 2545 | 2806 | 0 | 0 |
| 7 | 412442 | 758142 | 1925856 | 2631210 | 1.7 | 2.3 | 0.1 | 1.1 | 1052 | 1557 | 0 | 0 |
| 8 | 446094 | 1017339 | 1836793 | 4289004 | 1.8 | 3.6 | 1.1 | 4.2 | 1089 | 2229 | 0 | 0.1 |
| 9 | 1244520 | 1820737 | 4222796 | 7935571 | 4.3 | 8 | 0.2 | 12.5 | 2806 | 4788 | 0 | 0.2 |
| 10 | 1056182 | 1410680 | 5126256 | 6213138 | 3.4 | 4.4 | 0.2 | 3 | 2262 | 2802 | 0 | 0 |
| 11 | 311239 | 635257 | 1820209 | 2106015 | 1.6 | 1.9 | 1 | 3.3 | 979 | 1091 | 0 | 0.1 |
| 12 | 204 | 796605 | 515 | 2128410 | 0 | 2.6 | 0 | 5 | 1 | 1595 | 0 | 0.1 |
| 13 | 1234922 | 1303468 | 4500169 | 9886102 | 5.2 | 8.3 | 4.1 | 11.4 | 3209 | 4993 | 0.1 | 0.2 |
| 14 | 422191 | 905874 | 1770332 | 2761553 | 1.9 | 2.9 | 0.4 | 2.4 | 1222 | 1848 | 0 | 0 |
| 15 | 147683 | 2769634 | 5008772 | 1030008 | 5.5 | 8.5 | 3 | 9.2 | 3546 | 5535 | 0 | 0.1 |
| 16 | 161614 | 928637 | 1258570 | 2678996 | 1.9 | 4.5 | 1.4 | 7.7 | 985 | 2661 | 0 | 0.1 |
| 17 | 1298688 | 1961314 | 4785753 | 5898717 | 5.3 | 5.7 | 0.5 | 4 | 3382 | 3828 | 0 | 0.1 |

Now, we solve model (9) to identify the status of units for UC and RTD. The results are summarized in Table 2. In this table, columns 2, 3, 4 and 5 show the optimal value for the weights of the undesirable outputs and column 6 shows the value of Column 7 shows the efficiency score of units, columns 8 and 9 show the UC status and RTD status of DMUs.

Table 2. Type of UC on power plants

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| DMU |  |  |  |  |  | UEN | UC | RTD |
| 1 | -6.0832 | -8.5052 | -5.9656 | -0.4814 | 18.5439 | 1.000 | Strong | Negative |
| 2 | -2.3841 | -1.3779 | 4.9027 | 9.7916 | 6.0633 | 1.000 | Strong | Negative |
| 3 | 0.8787 | 0.2406 | 0.1085 | 0.4814 | 0.2487 | 0.874 | NO | Increasing |
| 4 | 7.1350 | 1.4970 | -1.6050 | 7.2246 | 1.6415 | 1.000 | Strong | Negative |
| 5 | -1.0119 | -9.7433 | -2.1147 | 1.9193 | 2.4554 | 1.000 | Strong | Negative |
| 6 | 0 | 4.3239 | -1.9678 | 2.0056 | 4.3867 | 1.000 | Weak | No |
| 7 | -3.9112 | 7.3420 | -9.3795 | 4.9994 | 0.6483 | 1.000 | Strong | Negative |
| 8 | -1.7631 | 1.0110 | -7.9729 | 1.7415 | 6.1614 | 0.935 | Strong | Negative |
| 9 | 7.2990 | -3.1707 | 0.6483 | 1.1642 | 7.3453 | 0.810 | Strong | Negative |
| 10 | -6.8771 | -3.5683 | 2.1910 | 4.4664 | 3.9753 | 0.759 | Strong | Negative |
| 11 | 0 | 3.3867 | 2.0956 | 1.7415 | 2.5437 | 1.000 | Weak | No |
| 12 | -8.8158 | 0.7800 | -2.9453 | 2.8272 | 8.2234 | 1.000 | Strong | Negative |
| 13 | 4.0265 | -8.0555 | -5.1171 | -4.2690 | -0.4814 | 1.000 | Strong | Negative |
| 14 | -6.8177 | -1.4641 | 8.9462 | 2.2667 | 7.4532 | 1.000 | Strong | Negative |
| 15 | -3.2600 | -1.4009 | 5.0356 | 4.8144 | 12.5450 | 1.000 | Strong | Negative |
| 16 | -9.7301 | -8.1600 | 6.1938 | -1.5242 | 7.9997 | 1.000 | Strong | Negative |
| 17 | 6.7831 | -5.6518 | -2.3287 | -2.9935 | 21.5690 | 1.000 | Strong | Negative |

In the next step, we solve model (20) to identify the status of units for DC and DTR. The results are summarized in Table 3. In this table, column 2 shows the optimal value for the weight of the desirable output and column 3 shows the value of Column 4 shows the efficiency score of units, columns 5 and 9 show the DC status and DTR status of DMUs, respectively.

Table 3. Type of DC on power plants

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| DMU |  |  | UEN | DC | DTR |
| 1 | -8.7401 | 7.4647 | 1.000 | Strong | Negative |
| 2 | -5.0532 | 8.2407 | 0.862 | Strong | Negative |
| 3 | -5.3249 | 4.2157 | 1.000 | Strong | Negative |
| 4 | -6.9406 | -3.6323 | 0.694 | Strong | Negative |
| 5 | 0 | -1.9005 | 1.000 | Weak | No |
| 6 | -1.0745 | 0.1319 | 1.000 | Strong | Negative |
| 7 | -2.2626 | 2.5519 | 1.000 | Strong | Negative |
| 8 | -1.9364 | 6.0038 | 0.794 | Strong | Negative |
| 9 | 8.2157 | 6.9693 | 1.000 | NO | Decreasing |
| 10 | -7.2371 | 2.9668 | 1.000 | Strong | Negative |
| 11 | -2.1954 | 0.8889 | 1.000 | Strong | Negative |
| 12 | -7.1983 | 7.4817 | 1.000 | Strong | Negative |
| 13 | -3.0311 | 2.9416 | 1.000 | Strong | Negative |
| 14 | 6.8480 | -2.3890 | 1.000 | No | Decreasing |
| 15 | 0 | 8.1680 | 1.000 | Weak | No |
| 16 | -4.3400 | -6.4704 | 1.000 | Strong | Negative |
| 17 | 1.4970 | 9.0019 | 1.000 | No | Increasing |

Meanwhile, most combined cycle power plants in Iran, had a large potential to reduce their pollutions with eco-technology development because they had strong DC with negative DTR. However, there were two types of combined cycle power plants that have weak DC with no DTR, indicating the low level of potential for pollution mitigation. This shows that the obtained results by the proposed method can help the DM to decide about the size of the combined cycle power plants. Also, it is strongly hoped that the combined cycle power plants in Iran will be able to change the industrial structure to reduce the environmental pollution.

1. Conclusion

Environmental management is very important in the manufacturing sector due to the unavoidable generation of pollutants during the production process of industrial activities. Regarding the suitability of the DEA models for incorporating the production pollutants, called the undesirable factors, DEA has received great research attention recently. One of the most attractive issues in the literature of Data Envelopment Analysis (DEA) is to recognize the congestion of units, because the decision maker (DM) can use this concept to decide whether to increase or decrease the size of a Decision Making Unit (DMU). In general, congestion can be classified into Undesirable Congestion (UC) and Desirable Congestion (DC). In many real-world applications, the exact value for all data cannot be determined, hence, some parameters may be reported as uncertain data, such as the stochastic data, fuzzy data, interval data and so on. This study focused on considering Returns to Damage (RTD) under UC and Damages to Return (DTR) under DC in the situation that the input and desirable and undesirable outputs were reported as interval data. For this purpose, some uncertain models under the different production possibility sets (PPS) were formulated and then we used the robust optimization technique to formulate the equivalent certain models.

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1. 1Corresponding author’s Email: Mohsen\_rostamy@yahoo.com

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