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An Improvement on STEM Method in Multi-Criteria Analysis

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Abstract. Multi-criteria decision making (MCDM) refers to making decision in the presence of multiple and conflicting criteria. Multiobjective programming method such as multiple objective linear programming (MOLP) are techniques used to solve such multiple criteria decision making (MCDM) problems. One of the first interactive procedures to solve MOLP is step method (STEM).
In this paper we try to improve STEM method by introducing the weight vector of objectives which emphasize that more important objectives be more closer to ideal one. Therefore the presented method try to increase the rate of satisfactoriness of the obtained solution. Finally, a numerical example for illustration of the new method is given to clarify the main results developed in this paper.

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1. Introduction

Optimization can be used only when there is a single objective. The feasible solutions can then be ranked unambiguously according to this objective and the optimal one identified. In the real-world, almost every

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important problem involves more than one objective, which may be related to economic, social and environmental considerations. When there is more than one objective, and the objectives are non-commensurate, which means they cannot be transformed into a single objective, the "optimal" no longer has the same "objective" sense as before. A compromise solution must now be selected on the basis of the decision maker's attitude to achievement of the various objectives. Multi-criteria decision making (MCDM) refers to making decision in the presence of multiple and conflicting criteria. Problems for MCDM may range from our daily life, such as the purchase of a car, to those affecting entire nations, as in the judicious use of money for the preservation of national security.
A MODM problem may not have a single solution that could optimize all objectives simultaneously. The generally accepted solution of a MODM problem is said to be a Pareto solution (or non-dominated solution). A Pareto solution is the one for which any improvement in one objective can only take place if at least one other objective worsens (Keeny et al., [15,19]. Such solutions are referred to as the best compromise. The final solution of a MODM problem should be one of the compromise options that can best satisfy the decision-makers' preferences. There are several approaches to obtaining such solutions. Based on the ways of extracting the decision maker's preference information and using it in decision analysis processes, the MODM methods can be divided into three main categories [11]:

(1) *Priori articulation of preference information*

The most common way of conducting MO is by priori articulation of the DM's preferences. This means that before the actual optimization is conducted, the different objectives are somehow aggregated to one single figure of merit. Weighted sum ([24]), non-linear combination (Andersson, [1,18], utility theory ([15]), fuzzy logic [7,8,27], acceptability functions [17,26], and goal programming [25,24,5,6] methods belong to this category. An obvious drawback of this category is that, in the case of a large number of objective functions, the appropriate weighting is difficult to choose. Weighted-sum is the most popular method under this category because it converts a MO problem to a single objective one and is easy to implement. However, it could get only one or a few Pareto solutions by predetermining the weights using a priori knowledge or simply by a trial-and-error method. It could not generate a set of Pareto solutions. A few attempts have been made to obtain a set of Pareto solutions in one single run using the weighted-sum method. Among them, random weight is the main approach to get a set of Pareto solutions ([21,22,14.24]). The drawback of these methods is that it shows a slow convergence because no search direction exists in the random search.

(2) *Progressive articulation of preference information*
The methods of this category are generally referred to as interactive ones. They rely on progressive information about the DM‘s preferences simultaneously as they search through the solution space. Typically, the optimization program provides an updated set of solutions and lets the DM consider whether or not the weighting of individual objective functions. Tchebycheff method ([24]) and STEM method ([4]) are most common in this category. These methods do not need "a priori" preference information. The DM could give some preference information as the search moves on. Therefore, it is a learning process where the DM gets a better understanding of the problem. However, the solutions are dependent upon how well the DM can articulate his or her preferences and how much effort is required from the DM during the entire search process. In addition, if the preferences are changed, the process has to be restarted.

(3) *Posteriori articulation of preference information*
A number of algorithms can generate a set of Pareto optimal solutions and present them to the DM. The e-constraint method ([9]) normal boundary interaction ([10]), and Generate First-Choose Later (GFCL) approach ([2,3,11]) belong to this category. The main advantage of these methods is that the solution is independent of the DM's preferences.

As discussed above, each MODM method has pros and cons. Generally, none of them is superior to the others.
The paper presented here uses intractive technique for solving MODM problem. The presented method is based on STEM method.
See the following papers that generalized the intractive methods. In [22] an algorithm have designed that combines multiple probing from

the Tchebycheff method, aspiration criterion vectors from the work of Wierzbicki, and reservation vectors from Michalowski and Szapiro. The purpose of that paper was to capture the broad-based powers of these procedures within the context of a single-pattern user interface with a low cognitive burden. In MICA, all that is required at each iteration is to select from a small group of solutions and then specify criterion value intervals for the next iteration, information that is natural and could not be much easier to provide.

Wong et al. ([30]) peoposed a method that establishes the equivalence relationship between the output-orientated DEA dual models and minimax reference point approach of MOLP, showing how a DEA problem can be solved interactively without any prior judgements by transforming it into an MOLP formulation. This provides the basis to apply interactive techniques in MOLP such as STEM method to solve DEA problems and further locate the MPS along the efficient frontier for each DMU.

Taras and Woinaroschy ([28]) propose an interactive optimization framework for sustainable design of chemical and biochemical industrial processes. The proposed framework combines Matlab and SuperPro Designer simulator in order to solve interactive multi-objective optimization problems.

Hosseinzadeh Lotfi et al. ([11]) proposed a method that established an equivalence model between the combined-oriented CCR model and MOLP and showed how a DEA problem can be solved interactively by transforming it into MOLP formulation. This provided the basis to apply interactive techniques in MOLP to solve DEA problems and further locate the MPS along the efficient frontier for each inefficient DMU. We used Zionts-Wallenius (Z-W) method to reflecting the DM's preferences in the process of assessing efficiency.

In this paper we try to improve STEM method by introducing the weight vector of objectives which emphasize that more important objectives be more closer to ideal one.
STEM or Step Method proposed by Benayoun, de Montgolifer, Tergny and Laritchev ([4]) is a reduced feasible region method for solving the

MOLP

$$\begin{array}{ll} \max & (f_1(x), \ldots, f_k(x)) \\ s.t. & \\ & x \in S \end{array} \tag{1}$$

Where all objectives are bounded over S. Each iteration STEM makes a single probe of the efficient set. This is done by computing the point in the iteration's reduced feasible region whose criterion vector is closest to ideal criterion vector. STEM is one first interactive procedure to have impact on the field of multiple objective programming.
Therefore the presented method try to increase the rate of satisfactoriness of the obtained solution.
Rest of the paper is organized as follows:

Section 2. In this section some preliminaries about the following concepts are given:

- MODM Problems.
- Basic definitions.
- STEM Method.

Section 3. In this section, we will focus on the proposed method.

Section 4. In this section, a numerical example is demonstrated.

Section 5. Some conclusions are drawn for the study.

2. Preliminaries

In this section we express the following useful concept, given is [20].

2.1 MODM Problems

Managerial problems are seldom evaluated with a single or simple goal like profit maximization. Today's management systems are much more complex, and managers want to attain simultaneous goals, in which some of them conflict. In the other words, decisions in the real world contexts are often made in the presence of multiple, conflicting, and incommensurate criteria.

Multi-criteria decision making (MCDM) refers to making decision in the presence of multiple and conflicting criteria. Problems for MCDM may range from our daily life, such as the purchase of a car, to those affecting entire nations, as in the judicious use of money for the preservation of national security. However, even with the diversity, all the MCDM problems share the following common characteristics ([13]):

- ***Multiple criteria:*** each problem has multiple criteria, which can be objectives or attributes.
- ***Conflicting among criteria:*** multiple criteria conflict with each other.
- ***Incommensurable unit:*** criteria may have different units of measurement.
- ***Design/selection:*** solutions to an MCDM problem are either to design the best alternative(s) or to select the best one among previously specified finite alternatives.

There are two types of criteria: *objectives* and *attributes.* Therefore, the MCDM problems can be broadly classified into two categories:

- Multi-objective decision making (MODM)
- Multi-attribute decision making (MADM)

The main difference between MODM and MADM is that the former concentrates on continuous decision spaces, primarily on mathematical programming with several objective functions, the latter focuses on problems with discrete decision spaces.

Multi-objective decision making is known as the continuous type of the MCDM. The main characteristics of MODM problems are that decision makers need to achieve multiple objectives while these multiple objectives are non-commensurable and conflict with each other.

An MODM model considers a vector of decision variables, objective functions, and constrains. Decision makers attempt to maximize (or minimize) the objective functions. Since this problem has rarely a unique solution, decision makers are expected to choose a solution from among the set of efficient solutions (as alternatives). Generally, the MODM problem can be formulated as follows:

$$(MODM)\begin{cases} \max & f(x) \\ s.t & \\ & x \in S \quad , \end{cases} \tag{2}$$

where $f(x)$ represents n conflicting objective functions, and x is an n-vector of decision variables, $x \in \mathbb{R}^n$.

Example 2.1. *Example of MODM problem*
For a profit-making company, in addition to earning money, it also wants to develop new products, provide job security to its employees, and serve the community. Managers want to satisfy the shareholders and, at the same time, enjoy high salaries and expense accounts; employees want to increase their take-home pay and benefits. When a decision is to be made, say, about an investment project, some of these goals complement each other while others conflict.

2.2 Basic Definitions

We have the following notion for a complete optimal solution (For more details, see [23]).

Definition 2.2. *x^* is said to be a complete optimal solution, if and only if there exists an $x^* \in X$ such that $f_i(x^*) \geqslant f_i(x)$, $i = 1, \ldots, k$, for all $x \in X$. Also, ideal solution, superior solution, or utopia point are equivalent terms indicating a complete optimal solution.*
In general, such a complete optimal solution that simultaneously maximizes (or minimizes) all objective functions does not always exist when the objective functions conflict with each other. Thus, a concept of Pareto-optimal solution is introduced into MOLP ([23]).

Definition 2.3. *x^* is said to be a Pareto optimal solution, if and only*

if there does not exist another $x \in X$ such that $f_i(x) \geqslant f_i(x^*)$ for all i and $f_j(x) \neq f_j(x^*)$ for at least one j.
The Pareto optimal solution is also named differently by different disciplines: non-dominated solution, non-inferior solution, efficient solution, and non-dominate solution.

Definition 2.4. *(Satisfactory Solution) A satisfactory solution is a reduced subset of the feasible set that exceeds all of the aspiration levels of each attribute. A set of satisfactory solutions is composed of acceptable alternatives. Satisfactory solutions do not need to be non-dominated.*

Definition 2.5. *(Preferred Solution) A preferred solution is a non-dominated solution selected as the final choice through decision maker's involvement in the information processing.*

In the presented method (and in traditional STEM method), in order to measure the distance between two vector we use the following metric:

Definition 2.6. *Consider the weight vector θ where $\sum_{i=1}^{k} \theta_i = 1$ and $\theta_i \geqslant 0$. These weights define the weighted Tchebychev metric:*

$$\|f^* - f(x)\|_\infty^\theta = \max_{i=1,\dots,k} \{\theta_i |f_i^* - f_i(x)|\}. \tag{3}$$

2.3 STEM Method

The STEM-Method or STEP-method was first presented by Benayoun et al [4]. In STEM and related methods, preference information from the DM is used to reduce the solution space successively. The general optimization problem is reformulated as a $\mathfrak{L}_p$-norm problem (min-max formulation), with bounded and weighted objectives.

$$\begin{array}{ll} min & \{\sum_{i=1}^{k} (w_i^h (f_i(x) - f_i^*))^p\}^{\frac{1}{p}} \\ s.t. & \\ & x \in S^h \\ & \sum_{i=1}^{k} w_i^h = 1, \\ & w_i^h > 0 \end{array} \tag{4}$$

h is the iteration counter, and p is the parameter in the $\mathfrak{L}_p$-norm, usually equaling 1 or ∞. The weights are needed in order to solve the min-max formulation and to equalize the magnitude of the different objectives. The weights are not crucial to the outcome of the optimization as the final solution is obtained by means of bounds on the objective rather than variation of weightings. In the literature methods of calculating the weights are given. The problem is solved resulting in an objective vector $\widetilde{f}$. $\widetilde{f}$ is compared with the ideal solution $\mathbf{F}^*$. If some components of $\widetilde{f}$ are acceptable but some are not, the decision-maker must decide on a relaxation on at least on of the objectives. This means that the upper bound for the j:th objective are adjusted to $\widetilde{f_j}+\triangle f_j$. The solution space S^{h+1} is reduced by the new constraint $f_j \geqslant \widetilde{f_j} + \triangle f_j$. The weighting of the j:th objective is set to zero and the optimization problem of is solved again, this time in the reduced solution space. After the second iteration, the decision-maker might be satisfied with the obtained solution, or he/she has to relax the boundaries of another function and start over again. Thus, the algorithm proceeds through progressively reducing the solution space by introducing new constraints on the different objectives.

	f_1	f_2	$\cdots$	f_k	
f_1		f_1^+	f_{12}	$\cdots$	f_{1k}
f_2		f_{21}	f_2^+	$\cdots$	f_{2k}
$\vdots$		$\vdots$	$\vdots$	$\ddots$	$\vdots$
f_k		f_{k1}	f_{k2}	$\cdots$	f_k^+

Table 1: Pay-off table

3. Improved STEM Method

The procedure for improving STEM method has been given as following steps:

Step 1. Identify the weight vector of objectives.
The method requires that the DM gives a vector of weight W relating the objectives. W is generally normalized so that $\sum_{i=1}^{k} W_i = 1$ and the bigger weighting coefficient is associated with the more important

objectives.

Step 2. Construct the pay-off table.
In this step we first maximize each objective function and construct a pay-off table to obtain the positive ideal criterion
vector $f^+ \in \mathbb{R}^k$.
Let $f_j^+ \quad, j = 1, \ldots, k$, be the solutions of the following k problems, namely, positive ideal solution:

$$\begin{array}{lll} f_j^+ = & \max & f_j(x) \\ & s.t. & \\ & & x \in S \quad . \end{array} \tag{5}$$

The pay-off table is of the form Table1.

In Table 1, row j corresponds to the solution vector x^{j+} which maximizes the objective function f_j. A f_{ij} is the value taken by the ith objective f_i when the jth objective function f_j reaches its maximum f_j^+, that is, $f_{ij} = f_i(x^{j+})$.
Then the positive ideal criterion can be define as follows:

$$f^+ = (f_1^+, \ldots, f_k^+) = (f_1(x^{1+}), \ldots, f_k(x^{k+})). \tag{6}$$

And consider that x^+ be the inverse image of f^+. Generally, we know it is may be x^+ not belong to $S^{(h)}$.

Step 3. Calculate the weight factors.
Let f_i^{min} be the minimum value in the ith column of the first pay-off table (Table 1).
Calculate π_i values where:

$$\pi_i = \begin{cases} \frac{f_i^+ - f_i^{min}}{f_i^+} [\sum_{j=1}^n c_{ij}^2]^{-\frac{1}{2}} & , if \ f_i^+ > 0 \\ \\ \frac{f_i^{min} - f_i^+}{f_i^{min}} [\sum_{j=1}^n c_{ij}^2]^{-\frac{1}{2}} & , if \ f_i^+ \leqslant 0. \end{cases} \tag{7}$$

Where c_{ij} are the coefficients of the ith objective.
Then, the weighting factors can be calculated as follows:

$$\lambda_i = \frac{\pi_i}{\sum_{j=1}^n \pi_j} \quad i = 1, \ldots, k. \tag{8}$$

The weighting factors defined as above are normalized, that is they satisfy the following conditions:

$$0 \leqslant \lambda_i \leqslant 1 \quad , i = 1, \ldots, k \quad and \quad \sum_{i=1}^{k} \lambda_i = 1. \tag{9}$$

The weights defined above reflects the impact of the differences of the objective values on decision analysis. If the value $(f_i^+ - f_i^{min})$ is relatively small, then the objective $f_i(x)$ will be relatively insensitive to the changes of solution x. In other words $f_i(x)$ will not play an important role in determining the best compromise solution.

Step 4. Calculation Phase.
The weight factors defined by formula 8 are used to apply the weighted Tchebycheff metric ,Def. 2.2, to obtain a compromise solution, Also, the weight vector of objectives are used to emphasize that more important objectives be more closer to ideal one.
We can obtain a criterion vector which is closest to positive ideal one and emphasize that more important objectives be more closer to ideal one by solve the following model:

$$\begin{array}{ll} \min & \alpha \\ s.t. & \\ & \|W(f^+ - f(x))\|_\infty^\lambda \leqslant \alpha \\ & x \in S^{(h)} \\ & 0 \leqslant \alpha \in \mathbb{R} \end{array} \tag{10}$$

This model can be converted to the following model:

$$\begin{array}{ll} \min & \alpha \\ s.t. & \\ & W_i \lambda_i (f_i^+ - f_i(x)) \leqslant \alpha, \quad 1 \leqslant i \leqslant k \\ & x \in S^{(h)} \\ & 0 \leqslant \alpha \in \mathbb{R} \end{array} \tag{11}$$

We solve the weighted minimax model 11 and obtain the solution $x^{(h)}$. By solving the model 11 we obtain a compromise solution as $x^{(h)}$. In the other words, we obtain a compromise solution $x^{(h)}$ in the reduced feasible region $S^{(h)}$ whose criterion vector is closest to positive ideal criterion vector f^+.

Step 5. (Decision phase)
The compromise solution $x^{(h)}$ is presented to the decision maker, who compares objective vector $f(x^{(h)})$ with the positive ideal criterion vector f^+. This decision phase has the following steps:

- Step 5.1: If all components of $f(x^{(h)})$ are satisfactory, stop with $(x^{(h)}, f(x^{(h)}))$ as the final solution and $x^{(h)}$ is the best compromise solution. Otherwise go to step 5.2.

- Step 5.2: If all component of $f(x^{(h)})$ are not satisfactory, then terminate the interactive process and use other method to search for the best compromise solutions. Otherwise go to step 5.3.

- Step 5.3: If some components of $f(x^{(h)})$ are satisfactory and others are not, the DM must relax a objective $f_j(x)$ to allow an improvement of the unsatisfactory objectives in the next iteration. If the decision maker can not find an objective to sacrifice, then the interactive process will be terminated and other method have to be used for identifying the best compromise solution, otherwise, the DM gives Δf_j as the amount of acceptable relaxation. Δf_j is the maximum amount of $f_j(x)$ we are willing to sacrifice. Now go to step 5.4.

- Step 5.4: Define a new reduced feasible region as:

$$S^{(h+1)} = \left\{ x \in S^{(h)} \;\middle|\; \begin{array}{ll} f_j(x) \geqslant f_j(x^{(h)}) - \Delta f_j & \\ f_i(x) \geqslant f_i(x^{(h)}), & i \neq j, \quad i = 1, \ldots, k \end{array} \right. \tag{12}$$

And the weights π_j are set to zero. set $h = h + 1$ and go to step 3.

4. Numerical Example

Consider a firm that manufactures three products: x_1, x_2 and x_3. The firm's overall objective functions have been estimated as:

$$\begin{aligned} f_1(x) &= -5x_1 + 2x_2 - 3x_3 \\ f_2(x) &= -3x_1 - 4x_2 + 5x_3 \\ f_3(x) &= 2x_1 + 5x_2 - 10x_3 \end{aligned} \tag{13}$$

The following describes the limitations on the firm's operating environment.

$$\begin{aligned} -x_1 + 2x_2 - 2x_3 &\leqslant -5 \\ 2x_1 - x_2 + x_3 &\leqslant 6 \\ 5x_1 - 3x_2 - x_3 &\leqslant 2 \\ -3x_1 + 2x_2 - 2x_3 &\leqslant 8 \\ x_1, x_2, x_3 &\geqslant 0 \end{aligned} \tag{14}$$

Then the MODM problem can be formulated as follows:

$$\begin{aligned} \max \quad & f_1(x) = -5x_1 + 2x_2 - 3x_3 \\ \max \quad & f_2(x) = -3x_1 - 4x_2 + 5x_3 \\ \max \quad & f_3(x) = 2x_1 + 5x_2 - 10x_3 \\ s.t \quad & \\ & -x_1 + 2x_2 - 2x_3 \leqslant -5 \\ & 2x_1 - x_2 + x_3 \leqslant 6 \\ & 5x_1 - 3x_2 - x_3 \leqslant 2 \\ & -3x_1 + 2x_2 - 2x_3 \leqslant 8 \\ & x_1, x_2, x_3 \geqslant 0 \end{aligned} \tag{15}$$

We set $h = 0$ and

$$S^{(0)} = \left\{ x = (x_1, x_2, x_3) \;\middle|\; \begin{array}{ll} -x_1 + 2x_2 - 2x_3 \leqslant -5, & 2x_1 - x_2 + x_3 \leqslant 6, \\ 5x_1 - 3x_2 - x_3 \leqslant 2, & -3x_1 + 2x_2 - 2x_3 \leqslant 8, \end{array} \; x_1, x_2, x_3 \geqslant 0 \right\} \tag{16}$$

In order to find a satisfactory solution we carry out the following steps:

	f_1	f_2	f_2	Solution	Vector	
f_1	$f_1^+ = -7.5$	$f_{12} = 12.5$	$f_{13} = -25$	$x_1^{1+} = 0$	$x_2^{1+} = 0$	$x_3^{1+} = 2.5$
f_2	$f_{21} = -22$	$f_2^+ = 34$	$f_{23} = -80$	$x_1^{2+} = 0$	$x_2^{2+} = 4$	$x_3^{2+} = 10$
f_3	$f_{31} = -10.36$	$f_{32} = 7.99$	$f_3^+ = -19.27$	$x_1^{3+} = 0.82$	$x_2^{3+} = 0$	$x_3^{3+} = 2.09$

Table 2: The pay-off table

- ***Iteration No. 1:***

Step 1. Identification the weight vector of objectives

Assume that decision maker choose the weight vector as $W = (0.6, 0.2, 0.2)$.

Step 2. Construct the pay-off table

The pay-off table of the problem is as shown in Table 3.
Table 3 is constructed using formula 5

Step 3. Calculate the weight factors

Since $f_1^+ = -7.5$ and $f_1^{min} = -22$ and $c_{11} = -5, c_{12} = 2, c_{13} = -3$ then from formula 7 we have:

$$\pi_1 = (\frac{-22-(-7.5)}{-22})\{[(-5)^2 + 2^2 + (-3)^2]\}^{-\frac{1}{2}} = 0.107$$

Similarly, we can get $\pi_2 = 0.108$ and $\pi_3 = 0.067$. From 8 the weight factors are obtained as:

$$\lambda_1 = \tfrac{0.107}{0.107+0.108+0.067} = 0.379 \quad \text{and} \quad \lambda_2 = 0.383 \quad \text{and} \quad \lambda_3 = 0.238$$

Step 4. Calculation Phase

Now we can start the iteration process. We can obtain a criterion vector which is closest to positive ideal one and emphasize that more important objectives be more closer to ideal one by solve model 17 as follows:

$$\begin{array}{ll} \min & \alpha \\ s.t. & \\ & (0.6)(0.605)(-7.5 + 5x_1 - 2x_2 + 3x_3) \leqslant \alpha, \\ & (0.2)(0.383)(34 + 3x_1 + 4x_2 - 5x_3) \leqslant \alpha, \\ & (0.2)(0.238)(-19.27 - 2x_1 - 5x_2 + 10x_3) \leqslant \alpha, \\ & x \in S^{(0)} \\ & 0 \leqslant \alpha \in \mathbb{R} \end{array} \tag{17}$$

The optimal solution of the problem is
$x^{(0)} = (x_1^*, x_2^*, x_3^*) = (0, 0, 4.046)$ with criterion vector

$$f(x^{(0)}) = \{f_1(x^{(0)}), f_2(x^{(0)}), f_3(x^{(0)})\} = \{-12.138, 20.231, -40.461\}$$

Step 5. Decision Phase

The results $x^{(0)} = (x_1^*, x_2^*, x_3^*) = (0, 0, 4.046)$ and $f(x^{(0)}) = \{f_1(x^{(0)}), f_2(x^{(0)}), f_3(x^{(0)})\} = \{-12.138, 20.231, -40.461\}$ are shown to the decision maker. Suppose the solution is not satisfied as $f_1(x^{(0)}) = -12.138$ and $f_3(x^{(0)}) = -40.461$ are too small. Suppose $f_2(x)$ can be sacrificed by 5 units, or $\Delta f_2 = 5$. Then the new search space is given by:

$$S^{(1)} = \left\{ x \in S^{(0)} \;\middle|\; \begin{array}{l} f_2(x) = -3x_1 - 4x_2 + 5x_3 \geqslant 20.231 - \Delta f_2, \\ f_1(x) = -5x_1 + 2x_2 - 3x_3 \geqslant -12.138, \\ f_3(x) = 2x_1 + 5x_2 - 10x_3 \geqslant -40.461 \end{array} \right. \tag{18}$$

We set $\pi_2 = 0$. Therefore we have $\lambda_2 = 0$ and then we begin iteration 2.

- ***Iteration No. 2:*** It is obvious that $\lambda_1 = 0.615$ and $\lambda_3 = 0.385$ and we go to step 4. We can obtain a criterion vector which is closest to positive ideal one and emphasize that more important objectives be more closer to ideal one by solve model 19 as follows:

$$\begin{array}{ll} \min & \alpha \\ s.t. & \\ & (0.6)(0.615)(-7.5 + 5x_1 - 2x_2 + 3x_3) \leqslant \alpha, \\ & (0.385)(0.238)(-19.27 - 2x_1 - 5x_2 + 10x_3) \leqslant \alpha, \\ & -5x_1 + 2x_2 - 3x_3 \geqslant -12.138, \\ & -3x_1 - 4x_2 + 5x_3 \geqslant 15.231, \\ & 2x_1 + 5x_2 - 10x_3 \geqslant -40.461, \\ & x \in S^{(0)} \\ & 0 \leqslant \alpha \in \mathbb{R} \end{array} \tag{19}$$

The optimal solution of the problem is

$$x^{(1)} = (x_1^*, x_2^*, x_3^*) = (0, 0, 3.046),$$

with criterion vector

$$f(x^{(1)}) = \{f_1(x^{(1)}), f_2(x^{(1)}), f_3(x^{(1)})\} = \{-9.138, 15.231, -30.461\}.$$

Note that, $x^{(1)}$ is the point in feasible region whose criterion vector has minimum distance to positive ideal and cause to objective $f_1(x)$ be more closer to ideal one.
According to the behavioral assumptions of the STEM method (discussed in decision phase), in this example the decision maker is satisfied with the solution $x^{(1)}$.

For problems having more than three objectives. In such circumstances, whether the decision maker is satisfied with a solution depends on the range of solutions he has investigated. Also, the sacrifices of multiple objectives should also be investigated in addition to the sacrifice of a single objective at each iteration.

5. Conclusion

The suggested method in this paper improves the STEM method by introducing the weight vector of objectives which emphasizes that more important objectives be more closer to ideal one. Therefore, we find a point in reduced feasible region whose criterion vector is closest to positive ideal criterion vector and also, more important objectives are more closer to ideal one. Therefore, the presented method increases the rate of satisfactoriness of the obtained solution.

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