

An Algorithm for Improving Profit Efficiency

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Abstract. This paper applies the nonparametric Data Envelopment Analysis (DEA) methodology to estimate and improve profit efficiency using an algorithm. The proposed method improves profit efficiency of inefficient units. This method uses the ratio of output prices to input costs in every decision making units. We have explained our method by an example.

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1. Introduction

Profit efficiency of economic and financial institutions have been investigated in the empirical literature to a far lesser extent in contrast with cost and technical efficiency; Among 130 studies, for example, on efficiency of financial institutions reviewed in the extensive survey by Berger and Humphrey (1997)([1]), only nine analysis devoted to profit efficiency. Berger and Mester (1997) showed that profit efficiency is not always positively correlated with cost efficiency, suggesting the possibility that

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cost and revenue inefficiencies may be negatively correlated ([2]). Using the thick frontier approach, Lozano estimated profit efficiency of Spanish savings banks ([7]). Rogers estimated cost, revenue and profit efficiency of US commercial banks by using models with and without nontraditional outputs ([8]). His results suggested that the standard model, which omits nontraditional outputs, understates bank efficiency. Performance evaluation of organizations has important role in making future decisions. For this purpose, efficiency and utilization of organizations should be studied. One of the appropriate and efficient tools in this field, is data envelopment analysis which is used as a nonparametric method for the computing of efficiency of the decision making units. Kuosmanen et al. have explored the nonparametric approach to profit efficiency analysis both at the firm and industry levels when information about production technology and economic prices is unavailable ([6]). Sam Park and Jin-Wan Cho's approach has implied technical efficiency in DEA and has given rise to the upper bound of profit efficiency, referred to as pro-efficiency ([9]). The usual approach to evaluate profit efficiency uses both input and output quantitative indicators and information about unit prices in order to study productivity defined as the ratio of weighted outputs to weighted inputs. Here, market prices of outputs and inputs are the weights. Profit efficiency measured in the current study uses an approach where inputs represented by expenses are minimized and outputs represented by revenues are maximized. In the past three decades, parametric and nonparametric frontier approaches have been developed and increasingly used in applied economics and management science to evaluate the profit efficiency of various types of decision making units (DMUs), including for-profit and non-profit organizations. DEA is an efficient frontier technique that computes a comparative ratio of weighted outputs to weighted inputs for each decision-making unit (DMU) using linear programming. It is a non-parametric data analytic technique that is extensively used by various research communities since its introduction by Charnes et al. ([3]). Great interest has been shown in DEA, with major progress made in both methodological terms and range of application. Tone presented a model for enhancing the ratio of input to cost ([10]). This model is applicable for DMUs to subtract the

budget. Cooper et al. have considered a model for gaining more lost profits and showed that the insufficient amount of profits is paramount's to the total technical and special inefficiency ([5]). This paper uses the nonparametric method of DEA to measure profit efficiency, then by use of a proposed algorithm it is shown how it can be improved.

The paper is organized as follow: Section 2 surveys profit efficiency in DEA. Section 3 describes the proposed algorithm, and finally Section 4 presents the conclusion.

2. Profits Efficiency

Consider a set of n production units, also known as Decision Making Units ($DMUs$), each consuming various amounts of m inputs to produce s outputs. Let $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ represent the input and output vectors respectively for DMU_j $j = 1, \dots, n$. We also employ X to denote the $m \times n$ matrix of inputs and Y to denote the $s \times n$ matrix of outputs.

$$\begin{aligned} &Min \quad \theta \\ &s.t. \\ &\quad \theta x_0 \geq X\lambda, \\ &\quad y_0 \leq Y\lambda, \\ &\quad 1^T \lambda = 1, \\ &\quad \lambda \geq 0. \end{aligned}$$

For a commercial firm, both inputs and outputs are choice variables and the only constraint would be the feasibility of the input-output bundle chosen. For such a firm, the criterion of efficiency is profit maximization. At input cost and output price are shown by C and P , respectively; the actual profit of the firm, producing the output bundle y_o from the input bundle x_o is $\pi_o = p_o y_o - c_o x_o$. The maximum profit feasible for the firm is: $\max p_o y_o - c_o x_o = e\bar{y} - e\bar{x}$ which p_o is the vector of different outputs price and c_o is the vector of different inputs cost in DMU_o : ([4])

$$\begin{aligned} P &= (p_1, \dots, p_n) & C &= (c_1, \dots, c_n) \\ \bar{X} &= (\bar{x}_1, \dots, \bar{x}_n) & \bar{x}_j &= (c_{1j}x_{1j}, \dots, c_{mj}x_{mj}) \\ \bar{Y} &= (\bar{y}_1, \dots, \bar{y}_n) & \bar{y}_j &= (p_{1j}y_{1j}, \dots, p_{sj}y_{sj}) \end{aligned}$$

In any empirical application, we compute maximal profit for each DMU, relative to the technology T , via the linear programme: [10]

$$\begin{aligned}
 e\bar{y}^* - e\bar{x}^* &= \max e\bar{y} - e\bar{x} \\
 s.t. \quad \bar{x} &= \bar{X}\lambda \leq \bar{x}_o \\
 \bar{y} &= \bar{Y}\lambda \geq \bar{y}_o \\
 L &\leq e\lambda \leq U \\
 \lambda &\geq 0,
 \end{aligned} \tag{1}$$

e is a linear vector and all of its components are 1.

If $L=U=1$ considered as variable returns to scale and $U=\infty, L=0$ as fixed returns to scale, the possible production set will be defined as follow:

$$T = \{(\bar{x}, \bar{y}) \mid \bar{x} \geq \bar{X}\lambda, \bar{y} \leq \bar{Y}\lambda, L \leq e\lambda \leq U, \lambda \geq 0\},$$

by assuming that (\bar{x}^*, \bar{y}^*) is an optimal solution, profit efficiency is determined as follow :

$$(\bar{\pi}_o) = \frac{(e\bar{y}_o - e\bar{x}_o)}{(e\bar{y}_o^* - e\bar{x}_o^*)}. \tag{2}$$

This measure also bounded between 0 and 1 except in the case where the actual profit is negative while the maximum profit is positive. In that case $\bar{\pi}_o$ is less than 0.

Definition 2.1. *DMU_o is profit efficient if and only if $\bar{\pi}_o = 1$.*

3. An Algorithm for Improving Profite Efficiency

Our goal is accounting the ratio of output price to input cost in a way that could change the inefficient profit unit to efficient profit unit. Therefore, weight $(d_1, d_2) \geq 0$ which is a description of decisional, will to reduce price and increase cost and parameters β, δ , which are the rate of price growth and cost reduction, approach to goal function and condition of profit efficiency model this model obtained through rewriting the

previous model proposed by Wei et al. ([11]):

$$\begin{aligned}
 & \max d_2\beta - d_1\delta \\
 & s.t. \quad \bar{X}\lambda \leq \delta\bar{x}_o \\
 & \quad \bar{Y}\lambda \geq \beta\bar{y}_o \\
 & \quad \lambda = 1 \\
 & \quad \delta \leq 1 \quad \beta \geq 1 \quad \lambda \geq 0.
 \end{aligned} \tag{3}$$

Therefore, for different weight (d_1, d_2) , different price and cost is possible through this algorithm:

Algorithm

Step 1:

(i) Let $d_1 = 1$, $d_2 = 1$, and solve

$$\begin{aligned}
 & \max (\beta - \delta) \\
 (P_1) \quad & s.t. \quad (\delta\bar{x}_o, \beta\bar{y}_o) \in T \\
 & \quad \delta \leq 1, \quad \beta = 1,
 \end{aligned}$$

to obtain the optimal solution $(\delta_1, \beta_1) = (\delta_1^o, 1)$.

(ii) Let $d_1 = 1$, $d_2 = 1$, and solve

$$\begin{aligned}
 & \max (\beta - \delta) \\
 (P_2) \quad & s.t. \quad (\delta\bar{x}_o, \beta\bar{y}_o) \in T \\
 & \quad \delta = 1, \quad \beta \geq 1
 \end{aligned}$$

to obtain the optimal solution $(\delta_2, \beta_2) = (1, \beta_2^o)$. Obviously,

$$\delta_1^o = \delta_1 \leq \delta_2 = 1; \quad \beta_1 = 1 \leq \beta_2 = \beta_2^o.$$

and $\ell := 2$.

Step 2: denote

$$d_1^i = \beta_{i+1} - \beta_i, \quad d_2^i = \delta_{i+1} - \delta_i, \quad \rho_i = \frac{d_1^i}{d_2^i}, \quad i = 1, \dots, \ell - 1$$

When $\ell \geq 3$, we have

$$\rho_1 > \rho_2 > \dots > \rho_{\ell-1}.$$

For $i = 1, \dots, \ell - 1$, solve

$$\begin{aligned} & \max (d_2^i \beta - d_1^i \delta) \\ (\text{P } (d_1^i, d_2^i)) \text{ s.t. } & (\delta \bar{x}_o, \beta \bar{y}_o) \in T \\ & \beta \geq 1, \quad \delta \leq 1. \end{aligned}$$

to obtain its optimal solution (δ'_i, β'_i) . if

$$\frac{\delta'_i - \delta_i}{\delta_{i+1} - \delta_i} = \frac{\beta'_i - \beta_i}{\beta_{i+1} - \beta_i}, \quad (4)$$

then (δ'_i, β'_i) is abandoned. Assume that there exists $\bar{\ell}$ points not satisfying (4). if $\bar{\ell} = 0$ then go to step 3, else continue.

Sort these newly solved $\bar{\ell}$ optimal solutions (δ'_i, β'_i) , $i = 1, 2, \dots, \bar{\ell}$, and original ℓ points (δ_i, β_i) $i = 1, 2, \dots, \ell$, into a non-decreasing order:

$$\begin{aligned} \delta_1^o &= \delta_1 < \delta_2 < \dots < \delta_\ell = 1, \\ 1 &= \beta_1 < \beta_2 < \dots < \beta_\ell = \beta_1^o. \end{aligned}$$

Let $\ell := \ell + \bar{\ell}$ and go back to step 2.

Step 3: At this time, we obtain

$$\begin{aligned} \delta_1^o &= \delta_1 < \delta_2 < \dots < \delta_{\ell+\bar{\ell}} = 1, \\ 1 &= \beta_1 < \beta_2 < \dots < \beta_{\ell+\bar{\ell}} = \beta_2^o, \end{aligned}$$

and

$$d_1^i = \beta_{i+1} - \beta_i, \quad d_2^i = \delta_{i+1} - \delta_i, \quad \rho_i = \frac{d_1^i}{d_2^i}, \quad i = 1, \dots, \ell - 1.$$

For convenience, denote

$$\begin{aligned} d_1^o &= +\infty, \quad d_2^o = 1, \quad \rho_o = \frac{d_1^o}{d_2^o}, \\ d_1^\ell &= 0, \quad d_2^\ell = 1, \quad \rho_\ell = \frac{d_1^\ell}{d_2^\ell}. \end{aligned}$$

Then, we have

$$\infty = \rho_o > \rho_1 > \rho_2 > \dots > \rho_{\ell-1} > \rho_\ell = 0.$$

We have

(i) If $\frac{d_1}{d_2} \in (\rho_{i+1}, \rho_i) = \left(\frac{d_1^{i+1}}{d_2^{i+1}}, \frac{d_1^i}{d_2^i}\right)$, $0 \leq i \leq \ell - 1$, then $(\delta_{i+1}, \beta_{i+1})$ is an optimal solution of $(P(d_1^i, d_2^i))$,

$$(P(d_1, d_2)) \text{ s.t. } \begin{aligned} & \max (d_2\beta - d_1\delta) \\ & (\delta\bar{x}_o, \beta\bar{y}_o) \in T \\ & \beta \geq 1, \quad \delta \leq 1. \end{aligned}$$

Denote

$$s\left(\bar{x}_o, \bar{y}_o, \frac{d_1}{d_2}\right) = \{(\delta_{i+1}\bar{x}_o, \beta_{i+1}\bar{y}_o)\};$$

(ii) If $\frac{d_1}{d_2} = \rho_i$, $1 \leq i \leq \ell - 1$ then the optimal solution of problem $(P(d_1^i, d_2^i))$ is a set

$$\{(\delta, \beta) \mid \beta = \rho_i(\delta - \delta_{i+1}) + \beta_{i+1}, \delta_i \leq \delta \leq \delta_{i+1}\}.$$

Denote

$$s\left(\bar{x}_o, \bar{y}_o, \frac{d_1}{d_2}\right) = \{(\delta\bar{x}_o, \beta\bar{y}_o) \mid \beta = \rho_i(\delta - \delta_{i+1}) + \beta_{i+1}, \delta_i \leq \delta \leq \delta_{i+1}\}.$$

The above algorithm is always convergent. As we were introduced in the step 2, the changes of the ρ during the steps of algorithm are strictly descending, furthermore, according to the step 3 the changes of the δ and β are strictly ascending. For the δ and β , upper and under bound are 1 and 1 respectively. So the process of decreasing of input and increasing of output that created in algorithm leads to appropriate and convergent result for the problem.

Example 3.1. Consider the four DMUs with single input and single output; we will account the projection of profit efficiency.

Table 1: profits efficiency

DMU	x	C(x)	y	P(y)	$\bar{\pi}$
A	1	2	1	4	1
B	3	3	3	6	1
C	6	4	4	7	1
D	5	2	2	5	0

By considering (3) and the algorithm we will have:

(i) When the ratio of cost and price weights, $\frac{d_1}{d_2}$, is in $[0, 0.667)$, then $\bar{x}_D = 10$, $\bar{y}_D = 18.667$.

(ii) When the ratio of cost and price weights, $\frac{d_1}{d_2} = 0.667$, then price and cost vector would be such a set:

$$\{(\bar{x}, \bar{y}) \mid \bar{y} = 0.667\bar{x} + 11.997, 9 \leq \bar{x} \leq 10\}$$

(iii) When the ratio of cost and price weights, $\frac{d_1}{d_2}$, is in $(0.667, 2)$, then $\bar{x}_D = 9$, $\bar{y}_D = 18$.

(iv) When the ratio of cost and price weights, $\frac{d_1}{d_2} = 2$, then price and cost vector would be such a set:

$$\{(\bar{x}, \bar{y}) \mid \bar{y} = 2\bar{x}, 5 \leq \bar{x} \leq 9\}$$

(v) When the ratio of cost and price weights, $\frac{d_1}{d_2}$, is in $(2, \infty)$, then $\bar{x}_D = 5$, $\bar{y}_D = 10$.

4. Conclusion

In econometric applications, one specifies some explicit form of the production, cost, or profit function to represent the benchmark technology for efficiency measurement. The validity of the derived measures of efficiency does critically depend on the appropriateness of the functional specification. In the nonparametric alternative, one makes a number of fairly general assumptions about the technology but leaves the functional form unspecified. Typically, it is assumed that the production possibility set is convex and inputs and outputs are both freely disposable. In

any given context, the correct measure of efficiency can be obtained only if the choice variables of the firm are correctly identified. DEA not only can evaluate the application in nonfinancial systems but also have an extremely capability in evaluation of financial cases. We can put some of them into the note like the models efficiency of cost, price and revenue. We have marked the models of profits efficiency for the inefficient segments by presenting a proposed algorithm and by the ratio of output price to input cost so that the segments of decision marker being considered by the efficiency of profits. Models in the proposed algorithm are feasible, and this algorithm is convergent for the all of the steps.

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