**Evaluation and ranking of rail freight and passenger transportation in same Asian countries with new method in Data Envelopment Analysis**

**A.R. Salehi, F. Hosseinzadeh Lotfi, M.Rostamy-Malkhlifeh**

**Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran**

**Abstract**

Rail transportation plays an important role in one country's economic development. Therefore, many researchers have focused attention on the measurement of efficiency and ranking in the rail transportation industry. Evaluating organizational performance can produce significant effects on the organization’s activities. Data Envelopment Analysis (DEA) is a mathematical programming method, which measures the relative efficiency of the organizational units, which have multiple inputs and outputs. In this paper, we proposed a method for ranking all the Decision Making Units (DMUs) that is based on strong and weak supporting hyperplanes. As an applied project, we evaluate rail freight and passenger transportation in some Asian countries. The ranking is based on data from the International Union of Railways (UIC) in 2016.

**Keywords:** Data Envelopment Analysis (DEA), ranking, Rail transportation, Decision Making Units (DMUs), strong supporting hyperplanes.

1. **Introduction**

It is universally recognized that transport is crucial for sustained economic growth and modernization of a nation. Rail transport plays an important role in the economic development of a country. High safety, reliability, low pollution, low energy consumption, capacity and other advantages of rail than road transportation caused world countries using rail to transport goods, also Being a safe, energy efficient and sustainable mode of transportation, railways play a significant role in moving passengers worldwide. Undoubtedly, every organization has to measure and evaluate its performance in order to apply its limited resources more optimally and efficiently. Considering that rail, transportation plays an important role in one country's economic development. Consequently, many researchers have focused attention on the measurement of efficiency in the rail transportation industry. For example, Yu and Lin [1], Oum and Yu [2] found that railway systems highly depend on public subsidies that are significantly less efficient, and that systems with high degree of managerial autonomy achieve higher levels of efficiency. Gathon et al. [3] discovered that in the pre-liberalization period (1961- 1988), technical efficiency of European railways was negatively related to the degree of government influence. In addition, Oum et al. [4], published a complete overview of productivity and efficiency in rail transport in which it was clear that the results of these estimates were very sensitive to outputs specification. Cantos et al. [5], obtained efficiency indicators using non-parametric approaches, Cowie and Riddington [6], used alternative methodologies. Evaluating organizational performance can produce significant effects on the organization’s activities. Moreover, it is one of the challenging areas in management. There have been many ways to measure efficiency in the relevant research, though, among them data envelopment analysis (DEA) has been accepted more widely by academic circles and industries.

 DEA is a technique that based on mathematical programming and it is applied to determine the efficiency of a set of the decision-making units that are homogeneous. DEA was first established in the article known as CCR in 1978 by of Charnes, Cooper and Rhodes [7]. They generalized Farrell's initial analysis that was introduced in mode of single input and output to several inputs and outputs. Then, Charnes- Cooper and Banker in 1984 were able to establish the BCC model by recognizing the return to scale method and modifying the CCR model [8]. DEA efficiency studies in railways were surveyed by Merkert, Smith, and Nash (2010) [11]. DEA continues to be widely used to evaluate the efficiency and performance of railways in Iran (Movahedi, Abtahi, & Motamedi, 2011[12]; Rayeni & Saljooghi, 2014)[13], Korea (Kim et al., 2011)[14], China (Li & Hu, 2010[15]; Song, Zhang, Zeng, Liu, & Fang, 2015[16]; Teng, Feng, & Zhang, 2010[17]), Japan (Jitsuzumi & Nakamura, 2010[18]; Oum, Pathomsiri, & Yoshida, 2013[19]; Sekiguchi, Terada, & Terada, 2010[20]), India (Deshpande & Weisskopf, 2014[21]; Ranjan, Chatterjee, & Chakraborty, 2016[22]) and US (Mallikarjun, Lewis, & Sexton, 2014) [23].

One of the important issues discussed in DEA literature is ranking efficient units since the efficient units obtained in the efficiency score of one cannot be compared with each other based on this criterion any more. Ranking models were formed following the development of performance evaluation models and the increase in the need for managers to distinguish efficient units. Many papers presented in the literature for ranking efficient units; one of the first papers is Young and Hamer [24]. One important field in ranking is cross-efficiency; to name a few, consider Sexton et al. [25], Wu et al. [26], In the literature there exist other methods based on finding optimal weights in DEA analysis as Jahanshahloo et al. [27], Wang et al. [28], Hosseinzadeh Lotfi et al [29]. One of the important fields in ranking is super efficiency presented by Andersen and Petersen [30], Mehrabian et al. [31], Tone [32], Li et al. [33], Rezai Balf et al. [34]. Another important field in ranking is benchmarking methods such as to Sueyoshi [35], Jahanshahloo et al. [36] one important field is using statistical tools for ranking units first suggested by Friedman and Sinuany-Stern [37] and Mecit and Alp [38]. One of the significant fields in ranking is unseeing multi criteria decision-making (MCDM) methodologies and DEA analysis. Few, consider Joroet al. [39], Hosseinzadeh Lotfi et al. [40]. Also there exist some other ranking methods not much developed and extended in the literature, Seiford and Zhu [41], Amirteimoori [42], Khodabakhshi and Aryavash [43]. In addition to the theoretical papers presented in ranking literature there exist a variety of papers which used these new model sin applications such as Cook and Kress [44]. In this paper, we proposed a method for ranking all the decision-making units including extreme efficient, non-extreme efficient, weak efficient and inefficient units. For case study we evaluation and ranking of rail freight and passenger transportation in Asian countries. The comparison based on data from the International Union of Railways members in 2016. The units ranked on the assumption that the ranking of the extreme efficient units is higher than the one of non-extreme efficient units, the ranking of non-extreme units is higher than the one of weak efficient units, and the ranking of weak efficient units is higher than the one of inefficient units. The proposed method virtually does not suffer from the common problems in ranking including infeasibility of the model, instability for small data, inability to rank the non-extreme units, and not having false ranking.

1. **Preliminaries and basic ranking methods**

In this section, we describe some of the basic DEA method and main ranking methods with their advantages and drawbacks. Consider a set of **n** DMUs, which is associated with **m** inputs and **s** outputs. Particularly, each  consumes amount  of input **i** and produces amount  of output **r**. The production possibility set  is based on postulate sets, which are presented with a brief explanation. One of the most representative DEA models for evaluate the relative efficiency of a set of DMUs is BCC model, proposed by Banker et al. [8]. The production possibility set (PPS) of BCC model can be defined as follows:



In which $x\_{j}$ and $y\_{j}$ are vectors of input and output of $DMU\_{j}$ , respectively. The input-oriented BCC model, corresponds to $DMU\_{K}$ , $K\in j$ is given by

|  |  |
| --- | --- |
|  | (1) |

Where $ε$ is non-Archimedean small and positive number and  are called slack variables belong to . Note that  represent input excesses and  represent output shortfalls. Also  and  , are real numbers and . The models (1) called envelopment forms (with non-Archimedean number). The dual of models (1) (without $ε$ i.e. $ε=0$), which are called multiplier forms, are as (2) respectively:

|  |  |
| --- | --- |
|  | (2) |

If  is an optimal solution of model (2) then  is equation of supporting hyperplane of the PPS.

**Definition 1.** $DMU\_{k}$ is said to be BCC-efficient (strong efficient) if an optimal solution for model (1) satisfies  and  Otherwise, $DMU\_{k}$ is BCC-inefficient. Also BCC-inefficient $DMU\_{k}$ is said to be weak efficient if  and  (or  and )

**Note:** $DMU\_{k}$ is said to be strong efficient if in (2); and  for some optimal solutions. If and no exits then $DMU\_{k}$ is called weak efficient.

**Definition 2.** The BCC-efficient $DMU\_{k}$ is said to be extreme efficient if model (1) has unique optimal solution with  and . Otherwise, $DMU\_{k}$ is non-extreme efficient.

**Definition 3.** The defining Hyperplane  of PPS is strong defining hyperplane of PPS if only if it is supporting, at least m+s extreme efficient DMUs of PPS lie on H and all of components of its gradient (normal vector) are strictly positive. We call each virtual DMU on each infinite edge of PPS ‘‘weak efficiency virtual DMU”,

**Definition 4.** The defining Hyperplane of PPS is weak defining hyperplane of PPS if and only if it is supporting, at least m‏+s extreme efficient and weak efficient virtual DMUs of PPS lie on H. (In this case at least one components of its gradient (normal vector) is zero).

In the course of improving various abilities of data envelopment analysis (DEA) models, many investigations have been carried out for ranking decision-making units (DMUs). There exist a variety of papers which apply different ranking methods to a real data set. We describe some of the main ranking methods and their advantages and drawbacks.

* 1. **AP model**

Super efficiency models introduced in DEA technique is based upon the idea of leave one out and assessing

this unit trough the remanding units. In this subsection, we are going to review AP ranking model in data envelopment analysis. Andersen and Petersen [30] developed a new procedure for ranking efficient units. The methodology enables an extreme efficient DMUo to achieve an efficiency score greater than or equal to one by removing the O-th constraint in the primal formulation, they omitted the efficient DMU from the PPS, and ran CCR model[7] for other units to rank them. The mathematical formulation of model (3) is as follows:

|  |  |
| --- | --- |
|  | (3) |

The dual formulation of the super-efficient model, as seen in model (4), computes the distance between the Pareto frontier, evaluated without DMUo, and the unit itself i.e. for$\left\{j=1,…,n,j\ne o\right\}$.

|  |  |
| --- | --- |
|  | (4) |

However, The AP method has the following problems:

* First, Andersen and Petersen refer to the DEA objective function value as a rank score for all units, despite the fact that each unit is evaluated according to different weights. This value in fact explains the proportion of the maximum efficiency score that each unit O attained with its chosen weights in relation to a virtual unit closest to it on the frontier. Furthermore, if we assume that the weights reflect prices, then each unit has different prices for the same set of inputs and outputs within the same organization.
* Second, the super-efficient methodology can give ‘‘specialized’’ DMUs an excessively high ranking. To avoid this problem, Sueyoshi [35] suggest a method to avoid this problem.
* The third problem lies with an infeasibility issue, which if it occurs, means that the super-efficient technique cannot provide a complete ranking of all DMUs. Mehrabian et al. [31] suggested a modification to the dual formulation in order to ensure feasibility; we will refer to it later. Notice that, the AP model is feasible when we use this model in output oriented form.
* Fourth, In some cases, small changes in the data may change a lot $θ^{\*}$, of course, this The problem does not occur in output oriented form.
* Fifth, AP model does not have any suggestion for ranking non-extreme efficient units I fact, super efficiency method cannot rank the non-extreme efficient DMUs.
	1. **Cross-efficiency method**

Sexton et al. [25] proposed the cross-efficiency method. In cross-efficiency evaluation, each DMU is self and peer evaluated. Specifically, each unit determines a set of weights in the traditional DEA model, resulting in *n* sets of weights. Then, each DMU is evaluated by the *n* sets of weights obtaining *n* efficiency scores. The cross-efficiency of each unit is the average of the *n* efficiency scores.

In this method, by solving model (2) *n* times for each DMU, we can obtain the optimal solution  for each  Then, the cross efficiency score of corresponding to  can be calculated as the following Eq. (1).



Then, the cross-efficiency score of can be calculated as the average of  in Eq. (2).



There are three principal advantages of cross-efficiency evaluation:

1. This approach provides a unique ordering of the DMUs (Doyle and Green [45]).
2. It eliminates unrealistic weight schemes without incorporating weight restrictions (Anderson et al. [46]).
3. Cross-efficiency method distinguishes good and poor performers among the units (Boussofiane et al. [47]).

Because of these advantages, cross-efficiency evaluation has been extensively applied in various cases ( See Sexton et al. [25], Liang et al. [48], Ma and Li [49] and so on for more details).

A factor that possibly reduces the usefulness of the cross-efficiency evaluation method is that the cross-efficiency scores may not be unique due to the presence of alternative optimal weights. As a result, it is suggested that secondary goals are introduced in cross-efficiency evaluation. Doyle and Green [50] proposed two linear programming problems which are known as the aggressive formulation and benevolent formulation for cross-efficiency evaluation. The aggressive formulation aims to minimize the cross-efficiencies and the benevolent formulation aims to maximize the cross-efficiencies of other DMUs.

Liang et al. [48] extended Doyle and Green’s models by incorporating alternative secondary objective functions based on deviations to its ideal point 1. However, Wang and Chin [51] pointed out that the ideal points in the model of Liang et al. [48] are not realizable for the inefficient units. They improved the models by changing the target efficiency from the ideal point 1 to the CCR efficiency. It could be found that, the traditional benevolent and aggressive models only consider the desirable targets (1 or the original efficiency scores) as the referenced efficiencies for all units. However, Dotoli et al. [52] pointed out that the undesirable targets are also important indicators that the DMUs need to consider. Wu et al. [53], Ma et al. [54], Jahanshahloo et al. [55] incorporated a symmetric technique into DEA cross-efficiency evaluation and gave secondary goal models, which can choose symmetric weights for units. Wu et al. [56] incorporated a target identification model to get reachable targets for all DMUs. They proposed several secondary goal models for weights selection considering both desirable and undesirable cross-efficiency targets for all DMUs. Although secondary goal models were suggested to solve the problem of the cross-efficiency evaluation, but the existing secondary goal models in the literature have some drawbacks. Note, none of the secondary goal models in the literature guarantees that the optimal weights are unique. Hence, the problem of existing alternative optimal solutions does not solve completely. This is the main drawbacks of secondary goal models. On the other hand, most of the existing secondary goal models in literature solve *n(n-1)* model to obtain the rank of units. Therefore, if *n* be a large number, then the number of models that should be solved is very large, so the computational complexity is very high and this is another drawbacks of secondary goal models.

1. **the proposed method**

To determine the unit  ranking, it is first removed from observation inclusion and then a new PPS is produced for the remained units:



There may be two cases as follow:

1. : We want to find a strong supporting hyperplane on a PPS such as , which has the greatest distance from the unit under evaluation. Know the hyperplane H divides the space $R^{m+s}$ into two semi space:





According to the definition of $H^{-}$ :



So the supporting hyperplane H is chosen so that, then 
 , Note that $z>0$ is a multiple of the distance of the unit  to the supporting hyperplane

1. : As , then  and it can be written  in this case 

In both cases, we recommend the following linear model to find a strong supporting hyperplane which has the greatest distance to the unit under evaluation.

|  |  |
| --- | --- |
|  | (5) |

In model (5), then. Therefore, maximizing the value Z means maximizing . Since Z is maximized, thus the large positive number M is added to the problem to achieve positive values U and V if the constraints in the problems are as. This makes the achieved hyperplane a strong hyperplane.

**Theorem 1:** model 5 is always feasible. (**Proof in appendix [1]**)

**Theorem 2:** assume that  is the optimal solution for model 5, then supports hyperplane on .(**Proof in appendix [2]**)

**Theorem 3**: assume that  is the feasible solution for model 5, then  if and only if the unit  is extreme efficient unit. (**Proof in appendix [3]**)

**Theorem 4:** assume that is the optimal solution for model (5); if  and , then the unit  is non-extreme efficient. (**Proof in appendix [4]**)

**Remark 1:** if the unit  is efficient, then.

Consider a case in which  is the optimal solution for model 6 and. In this case, the unit  is either inefficient or weak efficient. In case, to distinguish if the unit  is either inefficient or weak efficient, model 6 is solved as follows

|  |  |
| --- | --- |
|  | (6) |

**Theorem 5:** assume that the unit  is weak efficient; if is the optimal solution for model 5 and is the optimal solution for model 6, then or . **(Proof in appendix [5])**

**Theorem 6:** assume that  is the optimal solution for model 5 and is the optimal solution for model 6. **(Proof in appendix [6])**

1. **An algorithm for ranking all the units**

**STEP 1:** model 5 is solved for all the units. The set  is defined as follows:



Where  is the set of all the extreme efficient units having the ranking and  is the set of all the non-extreme efficient units. If, so will be the ranking of the only member of the set  in case it has just one member. Therefore, the ranking of all the extreme and non-extreme units is achieved and if, the second step is done as follows:

**STEP 2:** put, so the new PPS for the members of the set is formed as follows:

 Model 7 is solved again for members in . Assume 

|  |  |
| --- | --- |
|  | (7) |

By considering  In this case, will be the ranking of the units which are members of. It is obvious that the sectional members of are non-extreme efficient units which ranked like this. Now, if  , then the ranking of all the efficient units including extreme and non-extreme is gained and if  , the set  is defined as follows



And the new PPS for the members of is formed as follows



Continuing this process, model 1 is solved for members of until we gain the ranking of all the non-extreme efficient units

**The th step of the algorithm** : The sets  are defined as follows:



In this case, will be the ranking of the units, which are in the set.  is formed on the members of  as follows  Model 8 is solved for the members of; therefore, if, we have

|  |  |
| --- | --- |
|  | (8) |

Noting the definition of  , it can easily be understood that  Assume that in the repetition we have, then the ranking of all the extreme and non-extreme units is gained.

 The set  are weak efficient or inefficient units.

Assume that means  and in this case  is on the weak frontier and, therefore, it is weak efficient. And if  means, then model (a) –which is mentioned in Appendix- is solved for the unit, and if, the unit  is inefficient and if, the unit  is weak efficient. The ranking criterion for the weak efficient units is in this way that if  then  is the ranking criterion and if  then  will be a criterion for ranking. In addition, we consider this issue that.

* 1. **Numerical example**

To illustrate the algorithm process in this subsection, we describe a small numerical example and then apply the algorithm to real data. Consider 13 DMUs with one input and one output. Table 1 shows 13 decision-making units with one input and one output.

Table 1. data of 13 DMUs

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| DMU | A | B | C | D | E | F | G | H | I | J | K | M | N |
| In put | 1 | 1 | 2 | 3 | 4 | 5 | 7 | 9 | 10 | 13 | 13 | 7 | 9 |
| Out put | 1 | 2 | 4 | 6 | 8 | 9 | 11 | 13 | 13 | 13 | 13 | 3 | 6 |

The optimal solution of model 5 for the DMUs is show in table 2.

Table 2. The optimal solution of model 6

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | A | B | C | D | E | F | G | H | I | J | K | M | N |
|  | 0 | 0.33 | 0 | 0 | 0.08 | 0 | 0 | 0 | 0.11 | -0.1 | -0.3 | -0.7 | -0.1 |
|  | 1 | 1 | 0.5 | 0.33 | 0.25 | 0.2 | 0.167 | 0.143 | 0.11 | 0.1 | 0.07 | 0.143 | 0.111 |
|  | 0 | 0.33 | 0.25 | 0.167 | 0.167 | 0.2 | 0.167 | 0.143 | 0.167 | 0.1 | 0.07 | 0.071 | 0.056 |
|  | 1 | 0.66 | 0 | 0 | -0.25 | -0.8 | -0.66 | -0.57 | -1.1 | -0.4 | -0.3 | 0 | 0 |



The ranking of the units is specified to be  respectively. Therefore, the units  are extreme efficient. The set  is formed as follows:



As, so PPS is formed for the units in the set  as follows. Then, model 5 is solved again for the members of  in PPS. New PPS for members of The optimal solution of model 5 for the members of in the new PPS is explained as follows.

**Table3. The optimal solution of model 5**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | C | D | F | G | H |
|  | 0.1 | 0.066 | 0.05 | 0 | **0.047** |
|  | 0.5 | 0.33 | 0.2 | 0.167 | **0.143** |
|  | 0.2 | 0.2 | 0.15 | 0.167 | **0.19** |
|  | 0.3 | -0.13 | -0.3 | -0.66 | **-1** |



Ranking of the members of  is as follows.



And as, then the ranking of the unit G is also gained,. As a result, the ranking of the extreme and non-extreme units is gained as follows.



**Ranking the weak efficient and inefficient units :** Optimal value of the objective function for members of the set is as follows:



Since, then it is located on a weak hyperplane and it is weak efficient and it is not capable of reducing inputs. Therefore, it has a higher ranking among the members of the set .

Now, to determine the ranking of the units J, K, M, and N; model 6 is solved for the members of . The optimal solution of model 6 for the members of  is as follows:



This shows that the units J and K are weak efficient and the units M and N are inefficient. Since the priority is given to reducing the inputs, ranking of the units in the set is as follows:



Therefore, ranking all the units will be as follows:



1. **Analysis of Indicators of Productivity Evaluation**

In this section, we evaluation and ranking of rail freight and passenger transportation in Asian countries. Statistical community of recent studies includes all railways in the world. Information and statistics of 60 UIC (International Railway Statistics, Union International des Chemins defer) member countries are gathered and used. The data and information have been extracted from the statistical yearbook of the International Union of railways up to 2016. The database includes information on population and area of the country, the length of rail lines, number of freight and passenger wagons, the number of locomotives and the amount of freight and passenger transported in the countries; as an example, the information about the countries in the Asia in 2016 are shown in Table 4. [http://www.uic.org].

**Table 4. Information about the countries in the Asia**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Country** | **Area (1000 km2)** | **Population (million)** | **The Total length of lines (km)** | **Two - lane lines (km)** | **The number of locomotives** | **Passengers tansported (million)** | **Passenger - km (million)** | **Tonnage of goods transported (million)** | **Ton - km (million)** | **Average staff strength (thousands)** |
| **China** | 9,563 | 1,370.84 | 67,092 | 34,777 | 19,988 | 1,544.36 | 723,006 | 2,294.10 | 1,980,061 | 2004 |
| **India** | 3,287 | 1,311.05 | 66,030 | na | 10,730 | 8,224 | 1,147,190 | 1,095.26 | 681,696 | 1326 |
| **Indonesia** | 1,911 | 257.56 | 5,279 | 486 | 357 | 198 | 18,510 | 20.44 | 5,452 | 27 |
| **Pakistan** | 796 | 188.93 | 9,255 | 2,866 | 452 | 52.95 | 20,288 | 3.60 | 3,301 | 32.98 |
| **Bangladesh** | 148 | 161.00 | 2,835 | na | 286 | 65.60 | 7,305 | 2.71 | 710 | 27.97 |
| **Japan** | 378 | 126.82 | 19,204 | 7,593 | 194 | 9,090.74 | 260,192 | 31.00 | 20,255 | 128.89 |
| **Vietnam** | 331 | 91.71 | 2,480 | 0 | 296 | na | 4,233 | na | na | 28.36 |
| **Iran** | 1,745 | 79.11 | 8,576 | 1,900 | 915 | 24.45 | 14,938 | 35.65 | 25,014 | 9.02 |
| **Thailand** | 513 | 67.96 | 5,327 | 346 | 265 | 44 | 7,504 | 10.86 | 2,455 | 26.32 |
| **South Korea** | 100 | 50.63 | 3,944 | 2,342 | 492 | 134.44 | 23,071 | 37.38 | 9,564 | 27.85 |
| **Iraq** | 435 | 35.87 | 2,138 | na | na | na | 99 | 1.00 | 249 | 8.8 |
| **Saudi Arabia** | 2,150 | 31.54 | 1,412 | 0 | na | 0.99 | 297 | 4.03 | 1,852 | 1.59 |
| **Uzbekistan** | 447 | 31.19 | 4,192 | na | na | 17.12 | 3,437 | 82.39 | 22,686 | 20.94 |
| **Malaysia** | 331 | 30.33 | 2,250 | 350 | 92 | 40.20 | 3,293 | 11.83 | 3,071 | 5.45 |
| **China-Taiwan** | 36 | 23.31 | 1,410 | 1,069 | 281 | 276.75 | 19,757 | 10.91 | 634 | 13.18 |
| **Syria** | 185 | 22.35 | 2,139 | 0 | na | 3.59 | 1,857 | 8.51 | 2,206 | 12.57 |
| **Kazakhstan** | 2,725 | 17.51 | 14,758 | 3,759 | 1,892 | 21 | 16,595 | 280.00 | 223,583 | 78.81 |
| **Azerbaijan** | 87 | 9.65 | 2,068 | 803 | 326 | 1.89 | 494 | 17.09 | 6,210 | 16.15 |
| **Tajikistan** | 143 | 8.48 | 621 | na | na | 0.55 | 24 | 8.41 | 554 | 3.06 |
| **Israel** | 22 | 8.35 | 1,340 | na | na | 52.81 | 2,608 | 7.50 | 1,155 | 3.3 |
| **Jordan** | 89 | 6.74 | 509 | na | na | 0.04 | 503 | 2.13 | 344 | 6.5 |
| **Kyrgyzstan** | 200 | 5.93 | 417 | na | na | 0.55 | 75 | 6.91 | 922 | 2.94 |
| **Turkmenistan** | 488 | 5.37 | 3,115 | na | na | 6.47 | 1,811 | 26.84 | 11,992 | 14.39 |
| **Georgia** | 70 | 3.72 | 1,491 | na | 50 | 2.73 | 549 | 16.68 | 4,987 | 9.068 |
| **Armenia** | 30 | 3.02 | 703 | 8 | 61 | 0.84 | 50 | 1.64 | 345 | 3.43 |
| **Mongolia** | 1,564 | 2.96 | 1,810 | na | na | 3.31 | 1,194 | 19.15 | 11,463.00 | 14.68 |

Information about all Asian countries is given in the table above. In this article, we evaluate the performance and ranking of the populous Asian countries. Also, given that the information in some countries was not available in the table above, they are presented with (na). For accurate evaluation, these countries have not been investigated. The inputs and outputs in this study are presented in Table 5.

**Table 5. The inputs and outputs**

|  |  |  |
| --- | --- | --- |
| **ID** | **variables** | **type of variables** |
| 1 | The Total length of lines (km) | input |
| 2 | The number of locomotives | input |
| 3 | Passenger - km (million) | output |
| 4 | Ton - km (million) | output |

The countries evaluated in this paper, along with their inputs and outputs, are presented in Table 6:

**Table 6. The inputs and outputs for 7 countries**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Country** | **The Total length of lines (km)** | **The number of locomotives** | **Passenger - km (million)** | **Ton - km (million)** |
| **China** | 67,092 | 19,988 | 723,006 | 1,980,061 |
| **Indonesia** | 5,279 | 357 | 18,510 | 5,452 |
| **Pakistan** | 9,255 | 452 | 20,288 | 3,301 |
| **Japan** | 19,204 | 194 | 260,192 | 20,255 |
| **Iran** | 8,576 | 915 | 14,938 | 25,014 |
| **Thailand** | 5,327 | 265 | 7,504 | 2,455 |
| **South Korea** | 3,944 | 492 | 23,071 | 9,564 |

The minimum values and the maximum mean variance and the deviations to the criteria for inputs and outputs are summarized in Table 7.

**Table 7. Statistical indexes**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **The Total length of lines (km)** | **The number of locomotives** | **Passenger - km (million)** | **Ton - km (million)** |
| **Max data** | 67,092 | 19,988 | 723,006 | 1,980,061 |
| **Min data** | 3,944 | 194 | 7,504 | 2,455 |
| **Average** | 16,954 | 3,238 | 152,501 | 292,300 |
| **Variance**  | 514849980.5 | 54610994.29 | 71534500972 | 5.53957E+11 |
| **Standard deviation** | 22690.30587 | 7389.925188 | 267459.3445 | 744282.5906 |

The optimal sum of the model 5 with the rank of each country is shown in Table 8.

**Table 8.** The optimal sum of the model 6 with the rank

|  |  |  |
| --- | --- | --- |
| **Country** | $$z^{\*}$$ | rank |
| **China** | 0.00114 | 2 |
| **Indonesia** | -0.0003 | 5 |
| **Pakistan** | -0.372 | 6 |
| **Japan** | 0.002 | 1 |
| **Iran** | -0.38 | 7 |
| **Thailand** | 0.00058 | 4 |
| **South Korea** | 0.00091 | 3 |

The assessment results show that East Asian countries have a particular interest in rail transport and the optimal use of this industry. Other countries, especially developing countries in Asia, can develop rail industry by simulating East Asian countries and transferring advanced technologies in the coming years. The development of the rail industry can play an important role in improving environmental conditions and reducing pollution. Given the safety of rail travel, the use of this industry can play an important role in reducing mortality in all countries. An indicator is a variable which is used to measure the status and efficiency of the system and should be comparable. In this study, several indicators are defined whereby the status and efficiency of rail freight transport could be compared between various countries. Most of the variables of the database cannot be considered as an indicator, but significant and comparable indices could be created with their no dimensionality and application of algebraic relationships between several variables. Definition of 7 indicators made by the database variables and how to calculate them are shown in Table 9. In this Table, the indicators are divided into four general parts including the development of railways, navigation development, exploitation of rail lines, and exploitation of rail fleet.

**Table 9. Sample of information about countries in the Asia**

|  |  |  |
| --- | --- | --- |
| **Types of indicators** | **indicators** | **Symbols and formulas** |
| `Railroads development indicators | Density of the length of lines in the area |    |
|
| The length of rail road per capita |  |
|
| Locomotive number | the number of locomotive to the length of rail lines |  |
|
| Efficiency of lines | Annual ton-Km per a rail road-Km |  |
|
| Annual Passenger-Km per a rail road-Km |  |
|
| Efficiency of locomotive | Annual Passenger-Km per The number of locomotives |  |
|
| Annual ton-Km per The number of locomotives |  |
|

**Figure 1. The status countries of the Asia by comparing indicators of development and fleet**

In general, China and Japan have the highest values based on the indexes. In Japan, in particular, rail infrastructure has a high density and spread.  Also, in China, the fleet used has a significant difference with other countries. So two countries rank first and second in the annual passenger and freight turnover.   After that Korea ranked third with average fleet.

**Conclusion:**

Inability in ranking the efficient units is one of the major weaknesses of traditional methods of data envelopment analysis. Researchers have provided various methods for ranking efficient units. In this paper, we suggested a method for ranking all the DMUs, which based on strong and weak supporting hyperplanes. In addition, decision-making units ranked on the assumption that the ranking of the extreme efficient units is higher than the one of non-extreme efficient units, the ranking of non-extreme units is higher than the one of weak efficient units, and the ranking of weak efficient units is higher than the one of inefficient units. The proposed method virtually does not suffer from the common problems of ranking including infeasibility of the model, instability for small data, inability to rank the non-extreme units, and false ranking. As a case study, we implement the presented model on railway transportation data of some Asian countries. By introduction of performance indicators, we conclude that in East Asian countries, the efficiency of rail transportation is more than the other ones. As we know in mentioned countries which get the high ranks-Japan, China, South Korea- rail is a major means of transport so that their rail networks are among the busiest in the world that has expanded with the new high speed, conventional and commuter lines during last decades and have a long-term plan to expand the networks in future. Sometimes the privatized networks are highly efficient, requiring few subsidies and running extremely punctually. Therefore, the efficiency of rail transportation in East Asia is more considerable.

**Reference**

[1]. Yu, M. and Lin, E. (2008)., "Efficiency and effectiveness in railway performance using a multi-activity network DEA model", Omega, Vol. 36, pp. 1005–1017.

[2]. Oum, T. H., Yu, C., (1994). Economic efficiency of railways and implications for public policy: A

comparative study of the OECD countries’ railways, Journal of transport economics and policy, 28, 121-138.

[3]. Gathon, H. J., Pestieau, P., (1995). Decomposing Efficiency into its Managerial and its Regulatory

Components: The Case of European Railways. European Journal of Operational Research, 80, 500-507.

[4]. Oum, T. H., Waters II, W. G., Yu, C., (1999). A survey of productivity and efficiency measurement in rail transport. Journal of transport economics and policy, 33(1), 9-44.

[5]. Cantos, P., Pastor, J. M., Serrano, L., (1999). Productivity, Efficiency and Technical Change in the

European Railways, a Non-Parametric Approach. Transportation, 26(4), 337-357.

[6]. Cowie, J., Riddington, G., (1996). Measuring the Efficiency of European Railways. Applied Economics, 28,1027-1035.

[7]. Charnes, A., Cooper, W.W., Rhodes, E., 1978. Measuring the efficiency of decision-making units. European Journal of Operational Research 2 (6), 429–444.

[8]. Banker, R. Charnes, and W. W. Cooper, 1984, Some Methods for estimating Technical and Scale Inefficiencies in Data Envelopment Analysis, Management Science.

 [11]. Merkert, R., Smith, A. S. J., & Nash, C. A. (2010). Benchmarking of train operating firms—a transaction cost efficiency analysis. Transportation Planning and Technology, 33,35–53.

[12]. Movahedi, M., Abtahi, S., & Motamedi, M. (2011). Iran railway efficiency analysis, using DEA: An international comparison. International Journal of Applied Operational Research, 1, 1–6.

[13]. Rayeni, M. M., & Saljooghi, F. H. (2014). Ranking and measuring efficiency using secondary goals of cross-efficiency evaluation—a study of railway efficiency in Iran. International Journal of Services and Operations Management, 17, 1–16.

[14]. Kim, H. -G., Choi, C. -Y.,Woo, J. -W., Choi, Y., Kim, K., &Wu, D. D. (2011). Efficiency of themodal shift and environmental policy on the Korean railroad. Stochastic Environmental Research and Risk Assessment, 25, 305–322.

[15]. Li, L. -B., & Hu, J. -L. (2010). Efficiency analysis of the regional railway in China: An applicationof DEA-Tobit approach. Journal of Information and Optimization Sciences, 31,1071–1085.

[16]. Song, M., Zhang, G., Zeng, W., Liu, J., & Fang, K. (2015). Railway transportation and environmental efficiency in China. Transportation Research Part D: Transport and Environment.

[17]. Teng, Y., Feng, F. -L., & Zhang, J. (2010). Assessment of China railway logistics performance based on DEA. Journal of Railway Science and Engineering, 5, 023.

[18]. Jitsuzumi, T., & Nakamura, A. (2010). Causes of inefficiency in Japanese railways: Applicationof DEA for managers and policymakers. Socio-Economic Planning Sciences, 44,161–173.

[19]. Oum, T. H., Pathomsiri, S., & Yoshida, Y. (2013). Limitations of DEA-based approach and alternative methods in the measurement and comparison of social efficiency across firms in different transport modes: An empirical study in Japan. Transportation Research Part E: Logistics and Transportation Review, 57, 16–26.

[20]. Sekiguchi, Y., Terada, K., & Terada, H. (2010). A study on the third-sector urban railway efficiency in Japan. Journal of the Eastern Asia Society for Transportation Studies, 8,1275–1287.

[21]. Deshpande, A., & Weisskopf, T. E. (2014). Does affirmative action reduce productivity? A case study of the Indian railways. World Development, 64, 169–180.

[22]. Ranjan, R., Chatterjee, P., & Chakraborty, S. (2016). Performance evaluation of Indian railway zones using DEMATEL and VIKOR methods. Benchmarking: An International Journal, 23, 78–95.

[23]. Mallikarjun, S., Lewis, H. F., & Sexton, T. R. (2014). Operational performance of U.S. public rail transit and implications for public policy. Socio-Economic Planning Sciences, 48,74–88.

[24]. F. W. Young and R. M. Hamer, Multi-dimensional Scaling, History, Theory and Applications, Lawrence Erlbaum, London, UK,1987.

[25]. T.R. Sexton, R.H. Silkman, andA.J. Hogan, “Data envelopment analysis: critique and extensions,” in Measuring Efficiency: An Assessment of Data Envelopment Analysis, R. H. Silkman, Ed., pp.73–105, Jossey-Bass, SanFrancisco, Calif, USA,1986.

 [26]. J. Wu, J. Sun, L. Liang, and Y. Zha, “Determination of weights for ultimate cross efficiency using Shannon entropy,” Expert Systems with Applications, vol.38, no.5, pp.5162–5165,2011.

[27]. G. R. Jahanshahloo, A. Memariani, F. H. Lotfi, and H. Z. Rezai, “A note on some of DEA models and finding efficiency and complete ranking using common set of weights,” Applied Mathematics and Computation, vol. 166, no. 2, pp. 265–281, 2005.

[28]. Y.-M. Wang, Y. Luo, and Z. Hua, “Aggregating preference rankings using OWA operator weights,” Information Sciences, vol.177, no.16, pp.3356–3363,2007.

[29]. F. Hosseinzadeh Lotfi, A. A. Noora, G. R. Jahanshahloo, and M. Reshadi, “One DEA ranking method based on applying aggregate units,” Expert Systems with Applications, vol. 38, no. 10, pp.13468–13471,2011.

[30]. P. Andersen and N. C. Petersen, “A procedure for ranking efficient units in data envelopment analysis,” Management Science, vol.39, no.10, pp.1261–1294,1993.

[31]. S. Mehrabian, M. R. Alirezaee, and G. R. Jahanshahloo, “A complete efficiency ranking of decision making units in data envelopment analysis,” Computational Optimization and Applications, vol.14, no.2, pp.261–266,1999.

[32]. K. Tone, “A slacks-based measure of super-efficiency in data envelopment analysis,” European Journal of Operational Research, vol.143, no.1, pp.32–41,2002.

[33]. S. Li, G. R. Jahanshahloo, and M. Khodabakhshi, “A super efficiency model for ranking efficient units in data envelopment analysis,” Applied Mathematics and Computation, vol.184, no.2, pp.638–648,2007.

[34]. F. Rezai Balf, H. Zhiani Rezai, G. R. Jahanshahloo, and F. Hosseinzadeh Lotfi, “Ranking efficient DMUs using the Tchebycheff norm,” Applied Mathematical Modelling, vol. 36, no. 1, pp. 46– 56,2012.

[35]. T. Sueyoshi, “DEA non-paramedic ranking test and index measurement: slack-adjusted DEA and an application to Japanese agriculture cooperatives,” Omega, vol. 27, no. 3, pp. 315–326, 1999.

[36]. G. R. Jahanshahloo, H. V. Junior, F. H. Lotfi, and D. Akbarian, “A new DEA ranking system based on changing the reference set,” European Journal of Operational Research, vol. 181, no. 1, pp.331–337,2007.

[37]. L. Friedman and Z. Sinuany-Stern, “Scaling units via the canonical correlation analysis in the DEA context,” European Journal of Operational Research, vol.100, no.3, pp.629–637,1997.

[38]. E. D. Mecit and I. Alp, “A new proposed model of restricted data envelopment analysis by correlation coefficients,” Applied Mathematical Modeling, vol.3, no.5, pp.3407–3425,2013.

[39]. T. Joro, P. Korhonen, and J. Wallenius,“ Structural comparison of data envelopment analysis and multiple objective linear programming,” Management Science,vol.44,no.7,pp.962–970, 1998.

[40]. F. Hosseinzadeh Lotfi, M. Rostamy-Malkhalifeh, N. Aghayi, Z. Ghelej Beigi, and K. Gholami, “An improved method for ranking alternatives in multiple criteria decision analysis,” Applied Mathematical Modelling, vol.37, no.1-2, pp.25–33,2013.

[41]. L. M. Seiford and J. Zhu, “Context-dependent data envelopment analysis: measuring attractive ness and progress,” Omega, vol.31, no.5, pp.397–408,2003.

 [42]. A. Amirteimoori, “DEA efficiency analysis: efficient and anti-efficient frontier,” Applied Mathematics and Computation, vol. 186, no.1, pp.10–16,2007.

[43]. M. Khodabakhshi and K. Aryavash, “Ranking all units in data envelopment analysis,” Applied Mathematics Letters, vol.25, no. 12, pp.2066–2070,2012. 20 Journal of Applied Mathematics.

[44]. W. D. Cook and M. Kress, “ Anmth generation model for weak rang in go players in a tournament,” Journal of the Operational ResearchSociety,vol.41,no.12,pp.1111–1119,1990.

[45]. Doyle, J. R., Green, R. H, Cross-evaluation in DEA: Improving discrimination among DMUs. INFOR, (1995) 33(3), 205–222.

[46]. Anderson, T. R., Hollingsworth, K. B., Inman, L. B, The fixed weighting nature of a cross-evaluation model. Journal of Productivity Analysis, (2002) 18(1), 249–255.

[47]. Boussofiane, A., Dyson, R. G., Thanassoulis, E., Applied data envelopment analysis. European Journal of Operational Research, (1991) 52(1), 1–15.

[48]. Liang, L., Wu, J., Cook, W. D., Zhu, J., Alternative secondary goals in DEA cross efficiency evaluation, International Journal of Production Economics, (2008) 113, 1025–1030.

[49] Ma, Z. X., Li, P. (2011). The cross-efficiency evaluation method for energy enterprise. Energy Procedia, 5, 37–41.

[50]. Doyle, J. R., Green, R. H., Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. Journal of the Operations Research Society, (1994) 45(5), 567–578.

[51]. Wang, Y. M., Chin, K. S., Some alternative models for DEA cross-efficiency evaluation. International Journal of Production Economics, (2010)128, 332–338.

[52]. Dotoli, M., Epicoco, N., Falagario, M., Sciancalepore, F., A cross-efficiency fuzzy data envelopment analysis technique for performance evaluation of decision making units under uncertainty. Computers and Industrial Engineering, (2015)79, 103-114.

[53]. Wu, J., Liang, L., Chen, Y., DEA game cross-efficiency approach to Olympic rankings. Omega, (2009)37, 909–918.

[54]. Ma, R., Yao, L., Jin, M., Ren, P., The DEA game cross-efficiency model for supplier selection problem under competition. Applied Mathematics, (2014)8(2), 811–818.

[55]. Jahanshahloo, G. R., Hosseinzadeh Lofti, F., Yafari, Y., Maddahi, R., Selecting symmetric weights as a secondary goal in DEA cross-efficiency evaluation, Applied Mathematical Modelling, (2011)35, 544–549.

[56]. Wu, J., Chu, J., Sun, J., Zhu, Q., Liang, L., Extended secondary goal models for weights selection in DEA cross-efficiency evaluation, Computer and Industrial Engineering. (2015).

**Appendix**

[1] Proof Theorem 1:

Let  ,  ,  and for   and  for otherwise. where , then  is the feasible solution for model (5).

[2] Proof Theorem 2:

Assume that, we show that 

Then we have 

Now, we show that ,  Proof by contradiction, Assume that for each index such as j  .  is defined as and the value of is found in a way that  is feasible. To reach this goal:





As

so  However, it can be  Putting, as M is a very large positive number, so. This shows that  is a feasible solution for model (5). However,  And as, so it can be understood that  which in contradiction to the optimality of for model (5). Therefore, the contradiction proposition is nullified and the assertion is proved.

[3] Proof Theorem 3:

Assume that  as  is the feasible solution for model 5.

 This shows that  and on the other hand , thus. it is implied that the unit  is extreme efficient unit.Sufficient condition: now, we assume that the unit  is extreme efficient we show that .the dual of model 5 is as follows:

|  |  |
| --- | --- |
|  | (a) |

Assume that  is the optimal solution for model (a). First, we show that. Assume that, then we have  by the equation, it is understood that; also, by the in equation it is understood that. Which is in contradiction to the equation. Therefore, the contradiction proposition is nullified and the assertion is proved now, we show that. Assume that  because, then. However, we have:



As, it can be expressed that:



put, we have:

 

In addition, this shows that  and this is contradiction to the unit  being extreme efficient. Therefore, the contradiction proposition is nullified and it is understood that 

[4] Proof Theorem 4:

Let  ,  This shows that the unit  is on the supporting hyperplane  and based on the proposition, which is, then the unit  is efficient. However, the unit  is non-extreme efficient because if we consider this unit as extreme efficient, based on the theorem 3  which is in contradiction to the proposition.

[5] Proof Theorem 5:

based on the proposition of the problem, the unit is weak efficient. Therefore, it is not possible to improve all its elements. Thus, we have the two following cases:

1) it is not possible to improve all the elements on the vector. In this case, is the optimal solution for model 1 and is the optimal solution for model 2, therefore



Choosing  and regarding  , it is understood that is the feasible solution for model 5. As  is the optimal value for model 5, then. On the other hand, ; otherwise, if , based on the theorem 8 it is understood that the unit  is extreme efficient which is in contradiction to the proposition of the theorem

2) It is not possible to improve all the elements on the vector. (Proof like the part (1))

[6] Proof Theorem 6:

* If, then the unit  is inefficient.

**Proof.** By contradiction, Assume the unit  is not inefficient; subsequently, one of the following cases occurs:

1) The unit  is efficient, subsequently it is concluded that since (based on the result 1) and this is in contradiction to the problem proposition.

2) The unit  is weak efficient, subsequently based on the theorem 10  or which is in contradiction to the problem proposition. Thus, the contradiction proposition is nullified and the result is achieved.

* If  or, then the unit  is weak efficient.

**Proof.**

1) If the case  occurs  This shows that the unit  is on the supporting hyper plane and as, based on the proposition , ; then the unit is not efficient and this unit weak efficient.

2) If the case  occurs  This shows that the unit  is on the supporting hyperplane and as, based on the proposition, ; then the unit is not efficient. As the frontier  includes efficient or weak efficient points, then it is understood that the unit  is weak efficient.