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Super-Efficiency and Sensitivity Analysis Based on Input-Oriented DEA-R

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Abstract. This paper suggests a method of finding super-efficiency scores and modification of input-oriented models for sensitivity analysis of decision making units. First, by using DEA-R (ratio-based DEA) models in the input orientation, the models of super-efficiency and also models of super-efficiency modification are suggested. Second, the worst-case scenarios are considered where the efficiency of the test DMU is deteriorating while the efficiencies of the other DMUs are improving. Then, by combining these two ideas, a model is suggested which increases the super-efficiency score and modifies the change ranges in order to preserve the performance class. In the end, the super-efficiency and change interval of efficient decision making units for 23 branches of Zone 1 of the Islamic Azad University are calculated.

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1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. ([2]), is a nonparametric methodology for assessing the performances of a group

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of decision making units (DMUs) which use multiple inputs to produce multiple outputs. During recent years, the issue of sensitivity and stability of DEA results has been extensively studied. The first DEA sensitivity analysis paper by Charnes et al. ([3]) examined change in a single output. In recent years, many studies have been performed on sensitivity analysis of inputs and outputs of DMUs. Among these, sensitivity analysis using super-efficiency models has received much attention. Zhu ([10]) used the worst-case scenario where the efficiency of the test DMU is worsening while the efficiencies of the other DMUs are improving. He thus determined the necessary and sufficient conditions for preserving the efficiency classification of a DMU when various data changes are applied to all DMUs. Moreover, Zhu presented the necessary and sufficient conditions for the infeasibility of the super-efficiency model. Despic et al. ([7]) proposed a new mathematical model for sensitivity analysis, which combines the DEA methodology with the idea of ratio analysis. Similar to Andersen and Petersen's ([1]) idea, Wei et al. ([5]) studied efficiency and super-efficiency using DEA-R and also compared the optimal weights of DEA and DEA-R. In DEA-R models, the efficiency scale is greater than or equal to that in DEA models and the classification and modification of DMUs depend on the super-efficiency score of the unit. Thus, using DEA-R models in the input orientation in order to find the super-efficiency with greater or equal amounts can adjust the change ranges of DMUs. The present paper is an extension of Zhu's works and addresses super-efficiency and sensitivity analysis. The paper is organized as follows. In section 2, we review Zhu's modified models. Section 3 contains an introduction of DEA-R. Super-efficiency and sensitivity analysis are discussed in section 4. An application of our proposed model to the data of 23 branches of Zone 1 of the Islamic Azad University is given in section 5 together with the camparisan of the results with those of Zhu's modified models. Section 6 provides the conclusion.

2. Data Envelopment Analysis

Consider n $DMUs$ with m inputs and s outputs. The input and output vectors of DMU_j $(j = 1, \ldots, n)$ are $X_j = (x_{1j}, \ldots, x_{mj})^t, Y_j = (y_{1j}, \ldots, y_{sj})^t$ where $X_j > 0, \quad Y_j > 0$.
By using the constant returns to scale, convexity, and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

$$T_c = \left\{ (X, Y) : X \geqslant \sum_{j=1}^{n} \lambda_j X_j, Y \leqslant \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geqslant 0, j = 1 \ldots, n \right\}.$$

Let I and O denote, respectively, the input and output subsets in which we are interested. That is, we consider the data changes in set I and set O. The input-oriented super–efficiency model for DMU_p using the constant returns to scale (CRS) assumption is as follows.

$$\begin{aligned} \theta_{(p)}^{*super} = \ & Min \quad \theta_{(p)}^{Super} \\ s.t \ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij} \leqslant \theta_{(p)}^{Super} x_{ip}, \qquad i = 1, \ldots, m \\ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j y_{rj} \geqslant y_{rp}, \qquad r = 1, \ldots, s \qquad (1) \\ & \lambda_j \geqslant 0, \qquad j = 1, \ldots, n, j \neq p. \end{aligned}$$

By modifying Model (1) and separating the inputs and by using the CRS assumption, Seiford and Zhu ([8]) proposed the following model to obtain the stability region of DMU_p.

$$\begin{aligned} \theta_{I-(p)}^{*} = \ & Min \quad \theta_{I-(p)} \\ s.t \ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij} \leqslant \theta_{I-(p)} x_{ip}, \qquad i \in I \\ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij} \leqslant x_{ip}, \qquad i \notin I \qquad (2) \\ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j y_{rj} \geqslant y_{rp}, \qquad r = 1, \ldots, s \\ & \lambda_j \geqslant 0, \qquad j = 1, \ldots, n, j \neq p. \end{aligned}$$

3. Ratio-Based DEA

Despic et al. ([6]) proposed a new mathematical model for sensitivity analysis, which combines the DEA methodology with the idea of ratio analysis. Similar to Andersen and Petersen's ([1]) idea, Wei et al. ([4,5,6]) studied efficiency and super-efficiency using DEA-R and also compared the optimal weights of DEA and DEA-R. The input-oriented DEA-R model for DMU_p using the constant returns to scale (CRS) assumption is as follows.

$$\begin{array}{ll} & Max \quad \Delta \\ s.t. & \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}}\right) \geqslant \Delta, \qquad j=1,\ldots,n \\ & \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}=1, \\ & w_{ir}\geqslant 0, \quad \Delta\geqslant 0 \qquad i=1,\ldots,m, \quad r=1,\ldots,s. \end{array} \tag{3}$$

Model (3) is a linear programming problem in which dual of Model (3) by considering dual variables λ_j for all j and θ_R respectively corresponding to input and output constraints and the convex combination constraint is as follows.

$$\begin{array}{ll} & Min \quad \theta_R \\ s.t. & \sum_{j=1}^{n}\lambda_j\left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}}\right) \leqslant \theta_R, \qquad i=1,\ldots,m \quad r=1,\ldots,s \\ & \sum_{j=1}^{n}\lambda_j \geqslant 1, \\ & \lambda_j \geqslant 0, \qquad j=1,\ldots,n. \end{array} \tag{4}$$

Definition 3.1. *DMU_p is R-CCR-I-efficient (input-oriented CCR-R-efficient) if and only if $\theta_R^*=1$.*
Model (3) has the following properties.

1.The efficiency and super-efficiency scores obtained by this model are greater than or equal to those of the CCR model.

2. The efficiency scores of the model in the input and output orientations are not necessarily equal.

3. In a situation involving no weight restrictions, the input-target *improvement strategy given by DEA-R-I is always better than the CCR-I model.*

4. When DEA-R-I weights are concentrated on one output, the CCR-I efficiency and DEA-R-I efficiency are the same.

4. Super-Efficiency Based on DEA-R

In this section, we discuss sensitivity analysis and obtaining the super-efficiency score based on DEA-R. Considering set I for inputs and by modifying the inputs based on Zhu's idea, we have the following relations:

$$\begin{array}{llll} \overline{x}_{ip} = \delta_i x_{ip} \quad \delta_i \geqslant 1, & i \in I, & \overline{x}_{ij} = \frac{x_{ij}}{\overline{\delta}_i} \quad \overline{\delta}_i \geqslant 1, & i \in I, \\ \overline{x}_{ip} = x_{ip} \quad i \notin I, & & \overline{x}_{ij} = x_{ij} \quad i \notin I. & \end{array}$$

The DMUs are divided into those lying on the frontier and those that are not on the frontier. Also, extreme efficient, non-extrme efficient, and weak efficient DMUs are denoted by E, É , and F, respectively.

Lemma 4.1. *Assume that $DMU_p \in F$ with non-zero input/output slack values associated with set I/ set O. Then DMU_p with inputs $\overline{x}_{ip}$and outputs $\overline{y}_{rp}$ as defined above still belongs to set F when other DMUs are fixed.*

Proof. By the complementary slackness theorem for Models (3) and (4), we have $w^*_{ir}s^*_{ir} = 0$. Since $s^*_{ir} \neq 0 \quad for \quad i \in I, r \in O,$ we have $w^*_{ir} = 0 \quad for \quad i \in I, r \in O$. Therefore, $w^*_{ir}$is a feasible solution to (3) for DMU_p with inputs $\overline{x}_{ip}$ and outputs $\overline{y}_{rp}$. Therefore, the DMU_p still belongs to set F.

We consider the super-efficiency model based on DEA-R-I as follows.

$$\begin{array}{ll} & \theta^*_{R-I(p)^{Super}} = Min \quad \theta_{R-I(p)^{Super}} \\ s.t. & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}} \right) \leqslant \theta^{Super}_{R-I(p)}, \qquad i = 1, \ldots, m \quad r = 1, \ldots, s \\ & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \geqslant 1, \\ & \lambda_j \geqslant 0, \qquad j = 1, \ldots, n, \quad j \neq p. \end{array} \tag{5}$$

We consider the modified super-efficiency model based on DEA-R-I as follows.

$$
\begin{aligned}
\theta^*_{R-I(p)} = Min \quad & \theta_{R-I(p)} \\
s.t. \quad & \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}} \right) \leqslant \theta_{R-I(p)}, \qquad i \in I, \quad r = 1, \ldots, s \\
& \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}} \right) \leqslant 1, \qquad i \notin I, \quad r = 1, \ldots, s \\
& \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j \geqslant 1, \\
& \lambda_j \geqslant 0, \qquad j = 1, \ldots, n, \quad j \neq p.
\end{aligned} \tag{6}
$$

Example 4.1. (Taken from Zhu [10]) The results of Models (1), (2), (5) and (6) for four DMUs with two inputs and one output are presented in Table 1, below.

Table 1: Result Models (1), (2), (5) and (6).

A	2	5	1	1.1538	1.25	1.4000
B	3	3	1	1.2381	1.5	1.4167
C	6	2	1	1.5000	Inf	1.5000
D	2	7	1	1.0000	1	0.7143

With regard to the models presented based on DEA-R, we present the following theorems and lemmas for the sensitivity analysis of data.

Lemma 4.2. *If* $\theta^{*super}_{R-I(p)} = 1, \quad then \quad \theta^*_{R-I(p)} \leqslant 1.$

Proof. The proof is obvious from the fact that $(\lambda^{*super}, \theta^*_{R-I(p)}) = (\lambda^*, 1)$ is a optimal solution to (5), so $(\lambda, \theta_{R-I(p)}) = (\lambda^*, \theta^{*super}_{R-I(p)})$ is a feasible solution to (6). □

Lemma 4.3. *If* $\theta^{*super}_{R-I(p)} = 1, \quad and \quad \theta^*_{R-I(p)} < 1, \; then \quad DMU_p \in F.$

Proof. $\theta^{*super}_{R-I(p)} = 1$ indicates that $DMU_p \in E' \bigcup F$. Moreover, $\theta^*_{R-I(p)} <$

1 indicates that there are non-zero slack values in $x_{ip} \quad for \quad i \in I$. Thus, $DMU_p \in F$. $\square$

In Table (1), DMU D with $\theta^{*super}_{R-I(p)} = 1$ and $\theta^{*super}_{R-I(p)} = 1$ belongs to F.

Theorem 4.1. *If $\theta^{*super}_{R-I(p)} = 1$ and $\theta^{*}_{R-I(p)} < 1$ then for any $\delta_i \geqslant 1$ and $\overline{\delta}_i \geqslant 1$ for $(i \in I)$ DMU_p remains set F.*

Proof. From Lemma 4.3, we know that $DMU_p \in F$with non-zero slack values in$x_{ip} \quad for \quad i \in I$. By Lemma 4.1 and its proof of Lemma 4.1, we know that for any $\delta_i \geqslant 1 \quad and \quad \overline{\delta}_i \geqslant 1$ and , with an objective function value of 1, w^*_{ir} is a feasible solution to (3) in which inputs are replaced by $\overline{x}_{ip}$ for $i \in I$ and x_{ij} for $i \notin I$. Thus, DMU_p remains in set F after input data changes set I in all DMUs. $\square$

Corollary 4.1. *Infeasibility of Model (6) can only be associated with extreme-efficient DMUs in set E.*

Proof. Lemma 4.2 implies that Model (6) are always feasible for DMUs in set E or set F. Also, Model (6) is always feasible for non frontier DMUs. Therefore, infeasibility of Model (6) may only occur for extreme-efficient DMUs in set E. $\square$

Theorem 4.2. *A specific super-efficiency DEA model associated with set I is infeasible if and only if for any $\overline{\delta}_i \geqslant 1$ $(i \in I), DMU_p$ remains extreme-efficient. (See [8]).*

Lemma 4.3. *If model (6) is feasible and $\theta^{*super}_{R-I(p)} > 1$ then $\theta^{*}_{R-I(p)} > 1$.*

Proof. Suppose$\theta^{*}_{R-I(p)} \leqslant 1$. Then the input constraints of (6) turn into

$$\sum^{n}_{\substack{j=1 \\ j \neq p}} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}} \right) \leqslant \theta^{*}_{R-I(p)} \leqslant 1, \qquad i \in I, \quad r = 1, \ldots, s,$$

$$\sum^{n}_{\substack{j=1 \\ j \neq p}} \lambda_j \left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{ip}}{y_{rp}}} \right) \leqslant 1, \qquad i \notin I, \quad r = 1, \ldots, s \ ,$$

which indicates that $(\lambda^*, \theta^*_{R-I(p)^{Super}}) = (\lambda^*, 1)$ is a optimal solution to

(5) , so $(\lambda, \theta_{R-I(p)}) = (\lambda^*, \theta^*_{R-I(p)^{Super}})$ is a feasible solution to (6) . Therefore, $\theta^*_{R-I(p)^{Super}} = 1$, which is a contradiction. $\square$

Theorem 4.4. *Suppose* $\theta^{*super}_{R-I(p)} > 1$ *then if* $1 \leqslant \delta_{i(p)}\overline{\delta}_{i(p)} < \theta^*_{R-I(p)}$ *for* $(i \in I)$, *then* DMU_p *remains extreme-efficient. Furthermore, if equality holds for* $\delta_{i(p)}\overline{\delta}_{i(p)} = \theta^*_{R-I(p)}$, *that is,* $1 \leqslant \delta_{i(p)}\overline{\delta}_{i(p)} = \theta^*_{R-I}$, *then* DMU_p *remains on the frontier where*$\theta^*_{R-I(p)}$ *is the optimal value to (6).*

Theorem 4.5. *Suppose* $\theta^{*super}_{R-I(p)} > 1$ *then If* $\delta_i\overline{\delta}_i > \theta^*_{R-I(p)}$ *for* $(i \in I)$, *then* DMU_p *will not be extreme-efficient, where* $\theta^*_{R-I(p)}$ *is the optimal value to (6).*

5. An Application and Discussion

In this section, we consider the data of 23 branches of Zone 1 of the Islamic Azad University, with the inputs: number of scholarship receivers (I1), number of staff (I2), number of faculty members (I3), and number of students (I4), and the outputs: income (O1) and score of the branch (O2), as follows. Then, we discuss the results of the proposed models.
In this section, by considering the result in Table 3, we compare the super-efficiency scores obtained by the input-oriented CCR and DEA-R models. One can see that the super-efficiency scores by the input-oriented DEA-R model are greater than or equal to those by the input-oriented CCR model, which has also been pointed out in [5]. Consider columns 2 and 3 of Table 3. DMUs 1, 3, 5, 7, 12, 16, and 23 are efficient and the scores obtained by the super-efficiency DEA-R model for these DMUs are not less than those obtained by the super-efficiency CCR model. So,

$$E = \{DMU_1, DMU_3, DMU_5, DMU_7, DMU_{12}, DMU_{16}, DMU_{23}\}.$$

With regard to the theorems presented in the paper, we use the super-efficiency scores to classify the efficient units. By Theorem 4.1, DMU_{23} is extreme efficient and the ranges of perturbation in the first input of this DMU for the CCR and DER-R super-efficiency models are 1.0951 and 1.1455, respectively. Both the input-oriented CCR and DEA-R super-efficiency models are infeasible for the second input, i.e., the second input of DMU_7 can increase and those of other DMUs decrease by any amount while the efficiency classification of the DMU is preserved. On the other hand, for the second input have $\theta^*_{I=\{1\}} = 1.6000$ and $\theta^*_{R-I=\{1\}} = 1.9614$. That is, for CCR-I super efficiency model for the first input we have $0 \leqslant \delta_1 \overline{\delta}_1 \leqslant 1.6000$ and for the DEA-R super-efficiency model for the second input we have $0 \leqslant \delta_1 \overline{\delta}_1 \leqslant 1.9614$ as the range of perturbations. The DEA-R model yields a broader range for perturbations in the second input.

6. Conclusion

In this paper, we dealt with super-efficiency and sensitivity analysis in DEA, using DEA-R models. As the DEA-R efficiency score is greater than or equal to the DEA efficiency score, it is greater than or equal to the DEA super-efficiency score, as well. Furthermore, since the variation interval of the DMUs depends on the super-efficiency score, the interval expands as the super-efficiency score increases, which is a very important point. Calculation of super-efficiency scores by using the super-efficiency SBM model in DEA-R is suggested for future studies.

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Table 2. Inputs and Outputs.

DMU	I1	I2	I3	I4	O1	O2
01	8	32	51	2569	1984200	415.39
02	17	128	134	4687	3543100	545.95
03	9	108	85	3200	2416500	596.88
04	2	70	29	2170	1671000	193.18
05	1	31	22	2540	1994000	161.66
06	6	157	105	4360	3301500	522
07	2	68	61	2918	2238400	339.29
08	5	28	15	1465	1140500	106.68
09	2	44	28	3200	2509000	200.46
10	6	53	36	2550	1974500	219.25
11	14	47	69	3650	2820500	197.37
12	1	80	24	3000	2335500	100
13	57	336	274	20561	15978300	1826.40
14	2	21	25	1372	1061100	117.05
15	64	111	142	8500	6570500	1000
16	8	94	75	4227	3255600	743.4
17	28	142	131	5947	4541600	907.84
18	23	17900	174	8300	6365000	1347.96
19	5	112	106	4093	3109900	475.46
20	5	42	46	2235	1718500	224.37
21	46	206	210	13842	10737600	1598.12
22	3	47	25	2134	1657200	174.20
23	8	74	58	4256	3305800	598.51

Table 3: Results of Models (1), (2), (5), and (6).

DMU	θ_I^{*super}	θ_{R-I}^{*super}	$\theta_{\{1\}}^*$	$\theta_{R-\{1\}}^*$	$\theta_{\{2\}}^*$	$\theta_{R-\{2\}}^*$	$\theta_{\{3\}}^*$	$\theta_{R-\{3\}}^*$
01	1.5515	1.5515	Inf	Inf	1.6232	1.6496	Inf	Inf
02	0.9705	0.9718	0.1901	0.1901	0.4395	0.4427	0.4175	0.4344
03	1.0606	1.0606	Inf	Inf	Inf	Inf	Inf	Inf
04	0.9845	0.9970	0.6469	0.9838	0.4116	0.5118	0.7653	0.9899
05	1.4584	1.5213	Inf	Inf	Inf	Inf	Inf	Inf
06	0.9733	0.9784	0.5488	0.5648	0.4364	0.5027	0.5188	0.6150
07	1.3602	1.3738	1.6000	1.9614	Inf	Inf	Inf	Inf
08	0.9930	0.9942	0.1387	0.1995	0.6355	0.6736	0.9006	0.9528
09	0.9988	0.9988	0.6263	0.6275	0.8865	0.8865	0.9866	0.9875
10	0.9895	0.9915	0.2199	0.3156	0.5842	0.5885	0.7122	0.7746
11	0.9843	0.9843	0.1002	0.1002	0.9330	0.9330	0.4492	0.4492
12	1.1713	1.1713	Inf	Inf	Inf	Inf	Inf	Inf
13	0.9934	0.9954	0.2196	0.3483	0.7465	0.7854	0.7696	0.8401
14	0.9882	0.9902	0.4172	0.6643	0.7922	0.8218	0.5491	0.5965
15	0.9933	0.9964	0.2350	0.3071	0.9393	0.9755	0.8503	0.9796
16	1.1968	1.1968	Inf	Inf	Inf	Inf	Inf	Inf
17	0.9865	0.9870	0.3021	0.3233	0.5144	0.5152	0.6824	0.6831
18	0.9927	0.9932	0.6057	0.6390	0.6780	0.6833	0.7714	0.7762
19	0.9765	0.9829	0.5605	0.5605	0.5866	0.6905	0.4830	0.6088
20	0.9846	0.9866	0.3004	0.4200	0.6453	0.6506	0.5326	0.5748
21	0.9956	0.9981	0.4235	0.5764	0.8263	0.9468	0.8291	0.9443
22	0.9918	0.9936	0.3924	0.6164	0.5520	0.5979	0.8352	0.9051
23	1.0951	1.1455	Inf	Inf	Inf	Inf	Inf	Inf

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