# A novel algorithm for solving bi-objective fractional transportation problems with fuzzy numbers

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**Abstract:** A new method is proposed for finding a set of efficient solutions to bi-objective fractional transportation problems with fuzzy numbers using ranking function. This method is an important tool for the decision makers to obtain efficient solutions and select the preferred optimal solution from the satisfaction level. The procedure allows the user to identify next efficient solution to the problem from the current efficient solution. This new approach enables the decision makers to evaluate the economic activities and make satisfactory managerial decisions when they are handling a variety of logistic problems involving two objectives. An illustrative example is presented to clarify the idea of the proposed approach.
**Key words:** Bi-objective fractional transportation problems, Bi-objective fuzzy fractional transportation problems, Linear fractional programming problems, Level of satisfaction, trapezoid fuzzy number, ranking function, α-cut set

## 1. Introduction

Transportation problem nourishes economic and social activity and is cardinal to operations research and management science. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost multi-objective transportation problem and linear fractional programming problem, which has attracted the attention of many researchers in the past. In general, the real life problems are modeled with multi-objectives, which are measured in different scales and at the same time are in conflict. In actual classical transportation problems, the multi-objective functions are generally considered, which include average delivery time of the commodities, reliability of transportation, product deterioration and so on. A number of optimization problems are actually multi-objective optimization problems (MOPs), where the objectives are conflicting. As a result, there is usually no single solution, which optimizes all objectives simultaneously. A number of techniques have been developed to find a compromise solution to MOPs. Miettinen [1] refers the reader to the recent book about the theory and algorithms for MOPs. Fractional programming problems (FPPs) arise from many applied areas such as portfolio selection, stock cutting, game theory, and numerous decision problems in management science. Many approaches for FPPs have been exploited in considerable details. See, for example, Avriel et al. [2], Schaible[3] and Stancu-Minasian [4]. Multi-objective Linear fractional programming problems are useful targets in production and financial planning and return on investment. There are several ways to solve the linear fractional programming (LFP) and multi-objective linear fractional programming (MOLFP) problems [5, 6,7]. Tantawy (2007) proposed a new method for solving linear fractional programming problems [8]. Singh, Sharma and Dangwal proposed a solution concept to MOLFP problem using the Taylor polynomial series at optimal point of each linear fractional function in feasible region [9]. Sulaiman and Abulrahim used transformation technique for solving multi-objective linear fractional programming problems to single objective linear fractional programming problem through a new method using mean and median and then solved the problem by modified simplex method [10, 11]. Bodkhe et al., (2010) used the fuzzy programming technique with hyperbolic membership function to solve a bi-objective TP as vector minimum problem [12]. in 2005, an algorithm was proposed by Omar and Yunes for solving multi –objective 1transportation problems using fuzzy factors [13]. In 2011, Pendian presented a new method for solving two objective transportation problems [14]. Pandian and Natarajan, (2010) have introduced a new method for finding an optimal solution for transportation problems [15]. Amit and Pushpinder (2010) have introduced ranking of generalized trapezoidal fuzzy numbers based on rank [16].

In this paper, we propose a new method namely; dripping method for finding the set of efficient solutions to bi-objective transportation fractional problem with fuzzy numbers using ranking function and percent level of satisfaction of a solution for transportation problem with the proposed model is introduced. In the proposed method, one can identify next solution to the problem from the current solution, which differs from utility function method, goal programming approach, fuzzy programming technique, genetic approach and evolutionary approach. The percentage level of satisfaction of a solution of the bi-objective transportation fractional problem is then introduced. The dripping method is illustrated with the help of a numerical example. This new approach enables the decision makers to evaluate the economic activities and make self- satisfied managerial decisions when they are handling a variety of logistic problems involving two objectives.

## 2. The Fractional Transportation Simplex Method [17]

As in the case of a general linear fractional programming problem, the solution process of a linear fractional transportation problem (LFTP) consists of two phases:

1) Finding an initial basic feasible solution (BFS);
2) Improving the current basic feasible solution until the optimality criterion is satisfied. Since the process of finding initial BFS for LFPT is the same as in the linear problem (LP) case, we will focus mainly on the second stage.

Consider the following LFPT problem:

$$\text{(LFTP)} \qquad Q = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij} + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} + d_0}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i = 1,2,\dots,m \qquad (2.1)$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,m \qquad (2.2)$$

$$x_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n. \qquad (2.3)$$

Here and in what follows we suppose that $D(x) > 0, \forall x = (x_{ij}) \in S$, where **S** denotes a feasible set defined by constraints (2.1)-(2.3). In addition, we assume that

$$a_i > 0, \qquad b_j > 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n$$

and that total demand equals to total supply, i.e.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \qquad (2.4)$$

We now show how the simplex method may be adapted to the case when an LFPT problem must be solved. First, we have to introduce special simplex multipliers $u_i', v_j'$ and $u_i'', v_j''$ associated with numerator P(X) and denominator D(x), respectively. Elements $u_i'$ and $u_i''$, $i = 1,2,\dots,m$, correspond to m supply constraints and elements $v_j'$ $and$ $v_j''$, $j = 1,2,\dots,n$, correspond to n demand constraints. We calculate these variables from the following systems of linear equations

$$u_i' + v_j' = p_{ij} \quad \text{and} \quad u_i'' + v_j'' = d_{ij}, \quad (ij)\epsilon J_B \qquad (2.5)$$

Then, using the variables $u_i^{'}, v_j^{'}, u_i^{"}$ $and$ $v_j^{"}$ we define the following 'reduced costs' $\Delta_{ij}^{'}$ $and$ $\Delta_{ij}^{"}$

$$\left.\begin{array}{l}\Delta_{ij}^{\prime}= u_i^{\prime} + v_j^{\prime} - p_{ij}\\ \Delta_{ij}^{"}= u_i^{"} + v_j^{"} - d_{ij}\end{array}\right\} for\ \ i = 1,2,\dots,m, j = 1,2,\dots,n \tag{2.6}$$

It is easy to show that the latter may also be expressed as follows

$$\Delta_{ij}(x) = \Delta_{ij}^{'} - Q(x)\Delta_{ij}^{"}, i = 1,2,\dots,m, j = 1,2,\dots,n \tag{2.7}$$

**Theorem 2.1**: [18] Basic feasible solution $\mathrm{x} = (\mathrm{x_{ij}})$ of LFPT problem is optimal if

$$\Delta_{ij}(x) \geq 0, i = 1,2,\dots,m, j = 1,2,\dots,n \tag{2.8}$$

## 3. Bi-objective Fractional Transportation Problem

Consider the following Bi-objective Fractional Transportation Problems (BFTP):

$$\text{(BFTP)}\quad \text{Maximize}\quad \mathrm{Q}_1 = \frac{\mathrm{P}_1(\mathrm{x})}{\mathrm{D}_1(\mathrm{x})} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^1 x_{ij} + p_0^1}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 x_{ij} + d_0^1}$$

$$\text{Maximize}\quad \mathrm{Q}_2 = \frac{\mathrm{P}_2(\mathrm{x})}{\mathrm{D}_2(\mathrm{x})} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^2 x_{ij} + p_0^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2 x_{ij} + d_0^2}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i = 1,2,\dots,m \tag{3.1}$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,n \tag{3.2}$$

$$x_{ij} \geq 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n \tag{3.3}$$

Here and in what follows we suppose that

$$\mathrm{D}_1(\mathrm{x}) > 0, \mathrm{D}_2(\mathrm{x}) > 0, \forall \mathrm{x} = (\mathrm{x_{ij}}) \in \mathrm{S},$$

where **S** denotes a feasible set defined by constraints (1.1) to (1.3). Further, we assume that

$$a_i > 0, \qquad b_j > 0,\ \ i = 1,2,\dots,m; j = 1,2,\dots,n$$

and that total demand equals to total supply, *i.e.*

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

**Definition 3.1: [17]** A set $X^0 = \{x_{ij}^0, i = 1,2,\dots,m; j = 1,2,\dots,n\}$ is said to be feasible to the problem (FP) if $X^0$ satisfies the conditions (1.1) to (1.3).

**Definition 3.2: [7]** A feasible solution $X^0$is said to be an efficient solution to the problem (P) if there exists no other feasible X of BTP such that $Q_1(X) \geq Q_1(X^0)$ and $Q_2(X) > Q_2(X^0)$ or $Q_2(X) \geq Q_2(X^0)$ and $Q_1(X) > Q_1(X^0)$. Otherwise, it is called non-efficient solution to the problem (P).

## 4. Bi-objective Fuzzy Fractional Transportation Problem

Consider the following Bi-objective Transportation Problem of linear fractional programming problems

$$\text{(FP)}\qquad \widetilde{\mathrm{Q}}_1(\mathrm{x}) = \frac{\widetilde{\mathrm{P}}_1(\mathrm{x})}{\widetilde{\mathrm{D}}_1(\mathrm{x})} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{p}_{ij}^1 x_{ij} + \tilde{p}_0^1}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{d}_{ij}^1 x_{ij} + \tilde{d}_0^1}$$

$$\widetilde{\mathrm{Q}}_2(\mathrm{x}) = \frac{\widetilde{\mathrm{P}}_2(\mathrm{x})}{\widetilde{\mathrm{D}}_2(\mathrm{x})} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{p}_{ij}^2 x_{ij} + \tilde{p}_0^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{d}_{ij}^2 x_{ij} + \tilde{d}_0^2}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i = 1,2,\dots,m \tag{1.1}$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,n \tag{1.2}$$

$$x_{ij} \geq 0, \;\; i = 1,2, \dots, m; j = 1,2, \dots, n. \qquad (1.3)$$

Here and in what follows we suppose that

$$D_1(x) > 0 , D_2(x) > 0, \forall x = (x_{ij}) \in S,$$

where **S** denotes a feasible set defined by constraints (1.1) to (1.3). Further, we assume

$$a_i > 0, \; b_j > 0, \;\; i = 1,2, \dots, m; j = 1,2, \dots, n$$

and that total demand equals to total supply, *i.e.*

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

**Definition 4.1: [16]** The percentage level of satisfaction of the objective of the transportation problem for the solution U of the transportation problem, $L(Z_t; U)$ is defined as follows:

$$L(Z_t, U) = \begin{cases} \left(\dfrac{Z_t(U)}{Z_t(X_t^0)}\right) \times 100 & \text{if the problem is maximization type} \\ \left(\dfrac{2Z_t(X_t^0) - Z_t(U)}{Z_t(X_t^0)}\right) & \text{if the problem is minimization type} \end{cases}$$

Where $Z_t(U)$ is the objective value at the solution U and $Z_t(X_t^0)$ is the optimal objective value of the transportation problem.

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**Definition 4.2: [7]** Let R be real numbers set, $\tilde{a}$ fuzzy number is a map with below conditions:

1) $\mu_{\tilde{a}}$ is continuous.
2) $\mu_{\tilde{a}}$ on $[a_1, a_2]$ is ascending and continuous.
3) $\mu_{\tilde{a}}$ on $[a_3, a_4]$ is descending and continuous.

That $a_1, a_2, a_3$ and $a_4$ are real numbers and fuzzy numbers is shown as $\tilde{a} = [a_1, a_2, a_3, a_4]$ and it is called trapezoidal fuzzy number.

**Definition 4.3: [7]** If $\tilde{a}$ is trapezoid fuzzy number, membership function is as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & a_3 \leq x \leq a_4 \end{cases}$$

**Definition 4.4: [16]** $\alpha$-cut set of a fuzzy number $\tilde{a}$ is shown by $A_\alpha$ and is defined as below:

$$A_\alpha = \{x | \mu_{\tilde{a}}(x) \geq \alpha\} = [a_\alpha^l, a_\alpha^u]$$

**Definition 4.5:[15]** Robust ranking technique which satisfy compensation, linearity, and additively properties and provides results which consist human intuition. If $\tilde{a}$ is a fuzzy number then the Robust Ranking is defined by

$$R(\tilde{a}) = \frac{1}{2} \int_0^1 (a_\alpha^l + a_\alpha^u) d\alpha$$

Where $[a_\alpha^l, a_\alpha^u]$ is the $\alpha -$ cut of the fuzzy number $\tilde{a}$.

In this paper, we use this method for ranking the objective values. The robust ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$ .

**Definition 4.6: [15]** let $\tilde{a}$ and $\tilde{b}$ are two fuzzy numbers then

1) $\tilde{a} \geq \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$
2) $\tilde{a} = \tilde{b}$ if and only if $R(\tilde{a}) = R(\tilde{b})$

3) $\tilde{a} \leq \tilde{b}$ if and only if $R(\tilde{a}) \leq R(\tilde{b})$

**Definition 4.7: [15]** Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{a} \ominus \tilde{b} = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

(iii) $k\tilde{a} = k(a_1, a_2, a_3, a_4) = \begin{cases} (ka_1, ka_2, ka_3, ka_4) & for\ k \geq 0 \\ (ka_4, ka_3, ka_2, ka_1) & for\ k < 0 \end{cases}$

(iv) $\tilde{a} \otimes \tilde{b} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (t_1, t_2, t_3, t_4)$

where $t_1 = \text{minimum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$;
$t_2 = \text{minimum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$;
$t_3 = \text{maximum}\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$ and
$t_4 = \text{maximum}\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$

(v) $\frac{\tilde{a}}{\tilde{b}} = \frac{(a_1, a_2, a_3, a_4)}{(b_1, b_2, b_3, b_4)} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

Now, we need the following theorem, which is used in the proposed method.

**Theorem 2.1: [14]** Let $X^0 = \{x_{ij}^0, i = 1,2, \dots, m: j = 1,2, \dots, n\}$ be an optimal solution to (P1) where

$$(P_1)\ \text{minimize}\ Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ i = 1,2, \dots, m$$
$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ j = 1,2, \dots, m$$
$$x_{ij} \geq 0,\ \ i = 1,2, \dots, m; j = 1,2, \dots, n.$$

In addition, $Y^0 = \{y_{ij}^0, i = 1,2, \dots, m: j = 1,2, \dots, n\}$ be an optimal solution to (P2) where

$$(P_2)\ \text{minimize}\ Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ i = 1,2, \dots, m$$
$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ j = 1,2, \dots, m$$
$$x_{ij} \geq 0,\ \ i = 1,2, \dots, m; j = 1,2, \dots, n.$$

Then, $U^0 = \{u_{ij}^0, i = 1,2, \dots, m: j = 1,2, \dots, n\}$ which is obtained from$X^0 = \{x_{ij}^0, i = 1,2, \dots, m: j = 1,2, \dots, n\}$ or $Y^0 = \{y_{ij}^0, i = 1,2, \dots, m: j = 1,2, \dots, n\}$, is an efficient / non efficient solution to the problem (P).

## 5. Proposed Method:

We now propose a new method namely, dripping method for finding all the solutions to the bi-objective transportation problem of fractional programming problems (P).The dripping method proceeds as follows:

**Step 1:**

Convert fuzzy bi-objective transportation problems into bi-objective transportation problems with classic number by using Robust Ranking Technique

**Step 2:**

Construct two individual problems of the given BTFP namely, first objective transportation problem of fractional programming problems (FOTFP) and second objective transportation problem of fractional programming problems (SOTFP).

**Step 3:**
Obtain an optimal solution to the problems FOTFP and SOTFP using transportation algorithm of fractional programming problems (in order to solve fuzzy single objective problem, obtain solution with maximum profit method and then optimize it with modified distribution method).
**Step 4:**
Start with an optimal solution of FOTFP in the SOTFP as a feasible solution, which is an efficient solution to BTFP.
**Step 5:**
Select the allocated cell ($t$, $r$) with the minimize penalty in the SOTFP. Then, construct a rectangular loop\that starts and ends at the allocated cell ($t$, $r$) and connect some of the unallocated and allocated cells.
**Step 6:**
Add and subtract $\lambda$ to and from the transition cells of the loop in such a way that the rim requirements remain satisfied and assign a sequence of values to $\lambda$ one by one in such a way that the allocated cell remains non-negative. Then, obtain a feasible solution to SOTFP for each value of $\lambda$ , which is also an efficient / a non-efficient solution to BTFP by the Theorem 2.1.
**Step 7:**
Check whether the feasible solution to SOTFP obtained from the step 5. It is the optimum solution. If not, repeat the Steps 4 and 5 until an optimum solution to SOTFP is found. If so, the process can be stopped and movement to the next step can be made.
**Step 8:**
Start with an optimal solution of the SOTFP in the FOTFP as a feasible solution which is an efficient/ non-efficient solution to BTFP.
**Step 9:**
Repeat the steps 4, 5 and 6 for the FOTFP.
**Step 10:**
Combine all solutions (efficient / non efficient) of BTFP obtained using the optimal solutions of FOTFP and SOTFP. From this, a set of efficient solutions and a set of non-efficient solutions to the BTFP can be obtained.

**6. Numerical Example:**
The proposed method for solving a BFFTP is illustrated by the following example.
**Example:** Assume there are two objectives under consideration: The first objective function is the maximization of the ratio of the total delivery speed to total waste along the shipping route and the second objective function is the maximization of ratio of total profit to total cost. The ratio of the total delivery speed to total waste along the shipping route and the second objective function is the maximization of ratio of total profit to total cost are given in the following tables:

(FOTFP)

| Destination→<br>source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|
| 1 | (2,3,5,6)<br>(0,1,1,2) | (3,5,8,10)<br>(1,2,3,6) | (2,3,6,9)<br>(1,2,5,8) | (0,1,3,4)<br>(5,5,7,7) | 15 |
| 2 | (1,1,2,4)<br>(2,4,5,5) | (3,5,6,6)<br>(1,2,3,6) | (0,0,2,2)<br>(4,4,7,8) | (2,3,5,6)<br>(0,1,3,4) | 25 |
| 3 | (1,1,2,4)<br>(3,4,6,7) | (0,0,1,3)<br>(1,3,4,4) | (2,2,5,7)<br>(2,2,3,5) | (2,2,4,4)<br>(1,2,2,3) | 20 |
| demand | 14 | 18 | 12 | 16 | |

(SOTFP)

| Destination→<br>source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| 1 | (9,10,10,11)<br>(12,14,17,17) | (13,13,14,16)<br>(11,11,13,13) | (6,7,9,10)<br>(14,14,17,19) | (10,10,13,15)<br>(5,7,9,11) | 15 |
| 2 | (7,8,8,9)<br>(8,9,11,12) | (9,11,13,15)<br>(5,5,7,7) | (10,13,16,17)<br>(10,11,14,17) | (6,6,9,11)<br>(9,11,13,15) | 25 |
| 3 | (7,8,10,11)<br>(10,13,14,15) | (4,5,6,9)<br>(13,14,16,17) | (12,15,16,17)<br>(9,11,12,16) | (7,9,10,10)<br>(7,8,11,14) | 20 |
| demand | 14 | 18 | 12 | 16 | |

Convert fuzzy bi-objective transportation problems into bi-objective transportation problems with classic number by using Robust Ranking Technique to obtain a problem with below characteristics:

(FOTFP):

| Destination→<br>source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|
| 1 | 4<br>1 | 6<br>3 | 5<br>4 | 2<br>6 | 15 |
| 2 | 2<br>4 | 5<br>3 | 1<br>6 | 4<br>2 | 25 |
| 3 | 2<br>5 | 1<br>3 | 4<br>3 | 3<br>2 | 20 |
| demand | 14 | 18 | 12 | 16 | |

(SOTFP):

| Destination→<br>source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|
| 1 | 10<br>15 | 14<br>12 | 8<br>16 | 12<br>8 | 15 |
| 2 | 8<br>10 | 12<br>6 | 14<br>13 | 8<br>12 | 25 |
| 3 | 9<br>13 | 6<br>15 | 15<br>12 | 9<br>10 | 20 |
| demand | 14 | 18 | 12 | 16 | |

Obtain fuzzy solution with maximum profit method and then optimize it with modified distribution method, then FFOTP optimal solution is:

$x_{11} = 14\,, x_{12} = 1\,, x_{21} = 17\,, x_{24} = 8\,, x_{33} = 12\,, x_{34} = 8$

Obtain fuzzy solution with maximum profit method and then optimize it with modified distribution method then FFOTP optimal solution is:

$x_{14} = 15, x_{21} = 7\,, x_{22} = 18\,, x_{31} = 7\,, x_{33} = 12\,, x_{34} = 1$

Now, as in Step 3, we consider the optimal solution of the FOTFP in the SOTFP as a feasible solution in the following table:

| Destination→ source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|
| 1 | 10<br>15 **14** | 14<br>12 **1** | 8<br>16 | 12<br>8 | 15 |
| 2 | 8<br>10 | 12<br>6 **17** | 14<br>13 | 8<br>12 **8** | 25 |
| 3 | 9<br>13 | 6<br>15 | 15<br>12 **12** | 9<br>10 **8** | 20 |
| demand | 14 | 18 | 12 | 16 | |

Thus, $(\frac{251}{136}, \frac{704}{543})$ is the bi-objective value of BTP for the feasible solution $x_{11} = 14, x_{12} = 1, x_{21} = 17, x_{24} = 8, x_{33} = 12, x_{34} = 8$

According to Step 4, we construct a rectangular loop (2,4) - (2,2) - (1,2) - (1,4) - (2,4) . By using the Step 5, we have the following reduced table.

| Destination→ source↓ | 1 | 2 | 3 | 4 | supply |
|---|---|---|---|---|---|
| 1 | 10<br>15 **14** | 14<br>12 $\mathbf{1-\lambda}$ | 8<br>16 | 12<br>8 $\boldsymbol{\lambda}$ | 15 |
| 2 | 8<br>10 | 12<br>6 $\mathbf{17+\lambda}$ | 14<br>13 | 8<br>12 $\mathbf{8-\lambda}$ | 25 |
| 3 | 9<br>13 | 6<br>15 | 15<br>12 **12** | 9<br>10 **8** | 20 |
| demand | 14 | 18 | 12 | 16 | |

Now, the current solution to SOTFP is not the optimum solution. Repetition of Steps 5 and 6 results in the following feasible solution which is better than the prior feasible solution of SOTFP.

Thus, by using Steps 5 and 6, we obtain the set of all-efficient / non-efficient solution from FOTFP to SOTFP as given below:

| Iteration | $\lambda$ | Bi-objective value |
|---|---|---|
| 1 | {0,1} | $(\frac{(94,145+\lambda,245+\lambda,312+2\lambda)}{(153,193-\lambda,274+2\lambda,360+3\lambda)}, \frac{(561-\lambda,649-3\lambda,757+\lambda,849+3\lambda)}{(481+4\lambda,534+7\lambda,651+3\lambda,746+4\lambda)})$ |
| 2 | {1,2,…,6} | $(\frac{(94-2\lambda,146+4\lambda,246+5\lambda,314+4\lambda)}{(153-7\lambda,192-7\lambda,276-8\lambda,363-6\lambda)}, \frac{(560-2\lambda,646-2\lambda,758-2\lambda,852-2\lambda)}{(410+8\lambda,496+9\lambda,596+10\lambda,690+9\lambda)})$ |
| 3 | {1,2,…,9} | $(\frac{(112+2\lambda,170+2\lambda,276+4\lambda,338+6\lambda)}{(111-5\lambda,150-4\lambda,228-4\lambda,347-3\lambda)}, \frac{(548,634-2\lambda,746-3\lambda,840-3\lambda)}{(458+10\lambda,550+10\lambda,656+10\lambda,744+10\lambda)})$ |
| 4 | {1,2,…,8} | $(\frac{(130-2\lambda,188-3\lambda,312-4\lambda,392-5\lambda)}{(66-3\lambda,114-2\lambda,192-4\lambda,300-7\lambda)}, \frac{(548-7\lambda,616-6\lambda,749-8\lambda,813-7\lambda)}{(548+4\lambda,640+4\lambda,746+6\lambda,834+6\lambda)})$ |
| 5 | {1,2,…,8} | $(\frac{(114+3\lambda,164+4\lambda,280+4\lambda,352+\lambda)}{(42+\lambda,98,160-2\lambda,244+\lambda)}, \frac{(492+6\lambda,588+9\lambda,823+8\lambda,757+5\lambda)}{(580-10\lambda,672-11\lambda,794-11\lambda,882-11\lambda)})$ |

Similarly, by using Steps 8 and 9, we obtain the set of all solutions $S_2$ from SOTFP to FOTFP is given below:

| Iteration | $\lambda$ | Bi-objective value |
|---|---|---|
| 1 | {0,1} | $(\frac{(138-2\lambda,196-2\lambda,312-4\lambda,360-6\lambda)}{(50+5\lambda,98+4\lambda,144+4\lambda,252+3\lambda)}, \frac{(540,640+2\lambda,718+2\lambda,797+3\lambda)}{(500-10\lambda,576-10\lambda,693-10\lambda,794-10\lambda)})$ |
| 2 | {1,2,…,7} | $(\frac{(136-2\lambda,194-3\lambda,308-6\lambda,354-7\lambda)}{(55+3\lambda,102+3\lambda,148+3\lambda,255+2\lambda)}, \frac{(556-\lambda,642+\lambda,722+\lambda,800-\lambda)}{(490-\lambda,566-3\lambda,696,784)})$ |
| 3 | {1,2,…,14} | $(\frac{(122-3\lambda,173-3\lambda,280-4\lambda,305-2\lambda)}{(76+7\lambda,123+6\lambda,169+3\lambda,269+9\lambda)}, \frac{(533+\lambda,649-\lambda,699+3\lambda,793+5\lambda)}{(483-4\lambda,545-2\lambda,696-5\lambda,784-5\lambda)})$ |
| 4 | {1,2,…,7} | $(\frac{(80+2\lambda,131+2\lambda,224+3\lambda,277+5\lambda)}{(174-3\lambda,207-2\lambda,309-5\lambda,395-5\lambda)}, \frac{(547+2\lambda,635+\lambda,771-2\lambda,863-2\lambda)}{(427-3\lambda,517-4\lambda,626-5\lambda,714-4\lambda)})$ |

Now the set of all solutions S of the BTP obtained from FOTP to SOTP and from SOTP to FOTP is given below:

| number | $\widetilde{Q}_1(x)$ | $\widetilde{Q}_2(x)$ | $(R(\widetilde{Q}_1),R(\widetilde{Q}_1))$ | Level Satisfaction |
|---|---|---|---|---|
| 1 | $(\frac{92}{357},\frac{151}{268},\frac{251}{185},\frac{318}{146})$ | $(\frac{558}{699},\frac{644}{606},\frac{756}{505},\frac{850}{418})$ | (1.0889,1.3478) | (34.21,97.25) |
| 2 | $(\frac{90}{351},\frac{156}{268},\frac{256}{178},\frac{322}{139})$ | $(\frac{556}{708},\frac{151}{616},\frac{754}{185},\frac{848}{426})$ | (1.1527,1.3212) | (36.22,95.34) |
| 3 | $(\frac{88}{345},\frac{161}{268},\frac{261}{171},\frac{326}{132})$ | $(\frac{554}{717},\frac{151}{626},\frac{752}{185},\frac{846}{434})$ | (1.2224,1.2955) | (38.41,93.48) |
| 4 | $(\frac{86}{339},\frac{166}{268},\frac{266}{164},\frac{330}{125})$ | $(\frac{552}{726},\frac{151}{636},\frac{750}{185},\frac{844}{442})$ | (1.2989,1.2706) | (40.81,91.69) |
| 5 | $(\frac{84}{333},\frac{171}{268},\frac{271}{157},\frac{334}{118})$ | $(\frac{550}{735},\frac{151}{646},\frac{748}{185},\frac{842}{450})$ | (1.3833,1.2466) | (43.47,89.95) |
| 6 | $(\frac{82}{327},\frac{176}{268},\frac{276}{150},\frac{338}{111})$ | $(\frac{548}{744},\frac{151}{656},\frac{746}{185},\frac{840}{458})$ | (1.4769,1.2241) | (46.41,88.33) |
| 7 | $(\frac{114}{350},\frac{172}{224},\frac{280}{146},\frac{344}{106})$ | $(\frac{548}{754},\frac{632}{666},\frac{743}{560},\frac{837}{648})$ | (1.5641,1.1977) | (49.15,86.42) |
| 8 | $(\frac{116}{353},\frac{174}{220},\frac{284}{142},\frac{350}{101})$ | $(\frac{548}{764},\frac{630}{676},\frac{740}{570},\frac{834}{478})$ | (1.6462,1.1730) | (51.72,84.64) |
| 9 | $(\frac{118}{356},\frac{177}{216},\frac{288}{138},\frac{356}{96})$ | $(\frac{548}{774},\frac{628}{686},\frac{737}{580},\frac{831}{488})$ | (1.7353,1.1492) | (54.52,82.92) |
| 10 | $(\frac{120}{359},\frac{180}{212},\frac{292}{134},\frac{362}{91})$ | $(\frac{548}{784},\frac{626}{696},\frac{734}{590},\frac{828}{498})$ | (1.8327,1.1262) | (57.59,81.26) |
| 11 | $(\frac{122}{362},\frac{183}{208},\frac{296}{130},\frac{368}{86})$ | $(\frac{548}{794},\frac{624}{706},\frac{731}{600},\frac{825}{508})$ | (1.9395,1.1040) | (60.94,79.67) |
| 12 | $(\frac{124}{365},\frac{186}{204},\frac{300}{126},\frac{374}{81})$ | $(\frac{548}{804},\frac{622}{716},\frac{728}{610},\frac{822}{518})$ | (2.0575,1.0826) | (64.65,78.12) |
| 13 | $(\frac{126}{368},\frac{189}{200},\frac{304}{122},\frac{380}{76})$ | $(\frac{548}{814},\frac{620}{726},\frac{725}{620},\frac{819}{528})$ | (2.1885,1.0619) | (68.77,76.63) |
| 14 | $(\frac{128}{371},\frac{192}{196},\frac{308}{118},\frac{386}{71})$ | $(\frac{548}{824},\frac{618}{736},\frac{722}{630},\frac{816}{538})$ | (2.3351,1.0418) | (73.37,75.18) |
| 15 | $(\frac{130}{374},\frac{195}{192},\frac{312}{114},\frac{392}{66})$ | $(\frac{548}{834},\frac{616}{746},\frac{719}{640},\frac{813}{548})$ | (2.5007,1.0224) | (78.57,73.78) |
| 16 | $(\frac{126}{286},\frac{182}{184},\frac{304}{110},\frac{382}{60})$ | $(\frac{534}{846},\frac{604}{758},\frac{765}{648},\frac{799}{556})$ | (2.6399,1.0114) | (82.95,72.98) |
| 17 | $(\frac{124}{279},\frac{179}{180},\frac{300}{108},\frac{377}{57})$ | $(\frac{527}{852},\frac{598}{764},\frac{773}{652},\frac{792}{560})$ | (2.7076,1.0003) | (85.08,72.18) |
| 18 | $(\frac{122}{272},\frac{176}{176},\frac{296}{106},\frac{372}{54})$ | $(\frac{520}{858},\frac{592}{770},\frac{781}{656},\frac{785}{564})$ | (2.7824,0.9893) | (87.42,71.38) |
| 19 | $(\frac{120}{265},\frac{173}{172},\frac{292}{104},\frac{367}{51})$ | $(\frac{513}{864},\frac{586}{776},\frac{789}{660},\frac{778}{568})$ | (2.8656,0.9785) | (90.04,70.61) |
| 20 | $(\frac{118}{258},\frac{170}{168},\frac{288}{102},\frac{362}{48})$ | $(\frac{506}{870},\frac{580}{782},\frac{797}{664},\frac{771}{572})$ | (2.9586,0.9678) | (92.96,69.84) |
| 21 | $(\frac{114}{244},\frac{164}{160},\frac{280}{98},\frac{352}{42})$ | $(\frac{492}{882},\frac{568}{794},\frac{813}{672},\frac{757}{580})$ | (3.1825,0.9470) | (100,68.33) |
| 22 | $(\frac{117}{245},\frac{168}{158},\frac{284}{98},\frac{353}{43})$ | $(\frac{498}{871},\frac{577}{783},\frac{663}{660},\frac{762}{570})$ | (3.1620,0.9125) | (99.35,65.84) |
| 23 | $(\frac{120}{246},\frac{172}{156},\frac{288}{98},\frac{354}{44})$ | $(\frac{504}{860},\frac{586}{772},\frac{671}{648},\frac{767}{560})$ | (3.1436,0.9376) | (98.78,67.65) |
| 24 | $(\frac{123}{247},\frac{176}{154},\frac{292}{98},\frac{355}{45})$ | $(\frac{510}{849},\frac{595}{761},\frac{679}{636},\frac{772}{550})$ | (3.1273,0.9635) | (98.26,69.52) |
| 25 | $(\frac{126}{248},\frac{180}{152},\frac{296}{98},\frac{356}{46})$ | $(\frac{516}{838},\frac{604}{750},\frac{687}{624},\frac{777}{540})$ | (3.1129,0.9902) | (97.81,71.45) |
| 26 | $(\frac{129}{249},\frac{184}{150},\frac{300}{98},\frac{357}{47})$ | $(\frac{522}{827},\frac{613}{739},\frac{695}{612},\frac{782}{530})$ | (3.1004,1.0179) | (97.42,73.45) |
| 27 | $(\frac{132}{250},\frac{188}{148},\frac{304}{98},\frac{358}{48})$ | $(\frac{528}{816},\frac{622}{728},\frac{703}{600},\frac{787}{520})$ | (3.0896,1.0418) | (97.08,75.17) |
| 28 | $(\frac{135}{251},\frac{192}{146},\frac{308}{98},\frac{359}{49})$ | $(\frac{534}{805},\frac{631}{717},\frac{711}{588},\frac{792}{510})$ | (3.0807,1.0764) | (96.81,77.67) |
| 29 | $(\frac{138}{252},\frac{196}{144},\frac{312}{98},\frac{360}{50})$ | $(\frac{540}{794},\frac{640}{706},\frac{719}{576},\frac{797}{500})$ | (3.0731,1.1072) | (96.56,79.90) |
| 30 | $(\frac{126}{265},\frac{179}{163},\frac{278}{117},\frac{319}{70})$ | $(\frac{550}{784},\frac{647}{696},\frac{727}{551},\frac{795}{485})$ | (2.1267,1.1477) | (66.82,82.82) |

| | | | | |
|---|---|---|---|---|
| 31 | $(\frac{124}{267},\frac{176}{169},\frac{272}{120},\frac{312}{73})$ | $(\frac{549}{784},\frac{648}{696},\frac{728}{548},\frac{794}{484})$ | (2.0116,1.1504) | (63.21,83.01) |
| 32 | $(\frac{122}{269},\frac{173}{172},\frac{266}{123},\frac{305}{76})$ | $(\frac{548}{784},\frac{649}{696},\frac{729}{545},\frac{793}{483})$ | (1.9088,1.1530) | (59.97,83.20) |
| 33 | $(\frac{110}{305},\frac{161}{181},\frac{264}{147},\frac{297}{104})$ | $(\frac{537}{764},\frac{645}{676},\frac{711}{537},\frac{813}{467})$ | (1.4755,1.1805) | (46.36,85.19) |
| 34 | $(\frac{107}{314},\frac{158}{184},\frac{260}{153},\frac{295}{111})$ | $(\frac{538}{759},\frac{644}{671},\frac{714}{535},\frac{818}{463})$ | (1.3891,1.1925) | (43.65,86.05) |
| 35 | $(\frac{104}{323},\frac{155}{187},\frac{256}{159},\frac{293}{118})$ | $(\frac{539}{754},\frac{643}{666},\frac{717}{533},\frac{823}{459})$ | (1.3109,1.2046) | (41.19,86.92) |
| 36 | $(\frac{101}{332},\frac{152}{190},\frac{252}{165},\frac{291}{125})$ | $(\frac{540}{749},\frac{642}{661},\frac{720}{531},\frac{828}{455})$ | (1.2399,1.2170) | (38.96,87.82) |
| 37 | $(\frac{98}{341},\frac{149}{193},\frac{248}{171},\frac{289}{132})$ | $(\frac{541}{744},\frac{641}{656},\frac{723}{529},\frac{833}{451})$ | (1.1691,1.2295) | (36.73,88.72) |
| 38 | $(\frac{95}{350},\frac{146}{196},\frac{244}{177},\frac{287}{139})$ | $(\frac{542}{739},\frac{640}{651},\frac{726}{527},\frac{838}{447})$ | (1.1149,1.2422) | (35.03,89.64) |
| 39 | $(\frac{92}{359},\frac{143}{199},\frac{240}{183},\frac{285}{146})$ | $(\frac{543}{734},\frac{639}{646},\frac{729}{525},\frac{843}{443})$ | (1.0596,1.2551) | (33.29,90.59) |
| 40 | $(\frac{86}{377},\frac{137}{205},\frac{232}{195},\frac{281}{160})$ | $(\frac{545}{724},\frac{637}{636},\frac{735}{521},\frac{853}{435})$ | (0.9606,1.2815) | (30.18,92.47) |
| 41 | $(\frac{80}{395},\frac{131}{211},\frac{224}{207},\frac{277}{174})$ | $(\frac{547}{714},\frac{635}{626},\frac{741}{517},\frac{863}{427})$ | (0.8744,1.3187) | (27.47,95.16) |
| 42 | $(\frac{86}{380},\frac{137}{294},\frac{233}{201},\frac{292}{165})$ | $(\frac{553}{702},\frac{638}{611},\frac{765}{505},\frac{857}{418})$ | (0.8553,1.3493) | (26.87,97.36) |
| 43 | $(\frac{92}{365},\frac{143}{279},\frac{242}{195},\frac{307}{156})$ | $(\frac{559}{690},\frac{641}{596},\frac{759}{493},\frac{851}{409})$ | (0.7434,1.3765) | (23.35,99.32) |
| 44 | $(\frac{94}{360},\frac{145}{274},\frac{245}{193},\frac{312}{153})$ | $(\frac{561}{686},\frac{642}{591},\frac{757}{489},\frac{849}{406})$ | (0.7247,1.3858) | (22.77,100) |

The above satisfaction level table is very much useful for the decision makers to select the appropriate efficient solutions to bi-objective fuzzy fractional transportation problems according to their level of satisfaction of objectives.

## 7. Conclusion:

In this paper, the proposed method provides the set of efficient solutions for bi-objective fractional transportation problems with fuzzy numbers using ranking function and percent of function one solution is introduced for transportation problem. This method is new method that has been used for solving multi -objective fractional transportation problems with fuzzy numbers in which decision maker can determine the preferred solution from efficient solution using it. Here the two objectives inherently take care of at each iteration and the pairs recorded at any step identify the next efficient pair, thus providing a direction of movement without making use of any utility function. This method enables the decision makers to select an appropriate solution, depending on their financial position and their level of satisfaction of objectives.

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