Journal of Mathematical Extension
Vol. 14, No. 4, (2020), 21-51
ISSN: 1735-8299
URL: http://www.ijmex.com
Original Research Paper

Value Efficiency Analysis in Different Technologies

J. Gerami
Shiraz Branch, Islamic Azad University

Abstract. One way to apply decision-making preference information in the efficiency evaluation process in data envelopment analysis is to use the value efficiency approach. In this paper, we first describe the concept of value efficiency and use the directional distance function to calculate the efficiency. By choosing different directions, we calculate the value efficiency scores in different orientations, and obtain value efficiency scores in constant and variable return to scale and (Free Disposal Hull) FDH technologies by developing the directional distance function model. We apply the approach presented in the paper to the bank dataset in the following and show that we can obtain value efficiency scores for other banks by using bank management opinion in choosing the most valid and efficient branches as Must Prefer Solution (MPS) units. Finally, we present the results of the research.

AMS Subject Classification: 65K99
Keywords and Phrases: Data envelopment analysis, value efficiency, efficiency analysis, directional distance function

1. Introduction

Data Envelopment Analysis (DEA) by Charnes et al. [5], is a linear programming method in measuring the relative efficiency of a set of comparable decision making units. One of the strengths of the DEA is that there is no need for preference information in the performance analysis process, and we call the traditional DEA models value-free. But the weights obtained by the traditional DEA models are not real and this causes the efficiency scores corresponding to

Received: July 2018; Accepted: December 2019

the decision making units to be incorrectly calculated (Bal et al. [3]). The traditional DEA models do not incorporate Decision Maker's (DM) priori knowledge into the efficiency evaluation process and if it is necessary to apply DM preference information in the efficiency evaluation process and to include the DM point in the process of evaluating the efficiency of decision-making units, we need to adjust the DEA models. In this regard, different models have been proposed for applying DM preference information. The concept of value judgment was first introduced by Athanassopoulis [1]. Other methods of applying preference information to the DEA include target setting techniques in the articles by Galony's [10] and Athanassopoulis [2]. Weight restrictions is the of the methods for applying DM preference information in data envelopment analysis [6,21]. Thanassoulis and Allen [21] used weight restriction technique on inputs and outputs to apply preference information in DEA. In the weight restrictions method, the input and output components take precedence over each other; the confidence-area method in the article by Thompson et al. [22] and relative weights are mentioned in Wong and Beasley [23] articles and the cone ratio method in Charnes and Cooper [6] among these methods. In the weight restrictions method, problems such as infeasibility and high computation rate were noted. In the following, Zhu [26] explores data envelopment analysis with preference information structure. Other ways to apply DM preference in the process of evaluating the efficiency include interactive methods that employ both DEA and MOLP approaches. These methods use DM information in the process of problem solving. Galony's [10] presented an interactive model that combines both DEA and MOLP approaches. Researchers examine the relationship between DEA and MOLP issues in many articles and use MOLP interactive methods to solve multi-objective DEA models and they also use problem solving from the manager's point of view to prioritize objective functions and choose the optimal solution. Following articles can be mentioned in this regard (see, e.g., Golany [10], Joro, Korhonen and Wallenius [16], Wong, Luque and Yang [24], Yang et al. [25], Malekmohammadi, Hosseinzadeh Lotfi and Jaafar [17], Hosseinzadeh Lotfi et al. [14, 15], Ebrahimnejad and Tavana [7]). High computations and approximate solutions can be mentioned as problems of interactive methods in solving multi-objective DEA models. One of the best ways to apply DM 's priori knowledge in the process of evaluating the efficiency is value efficiency analysis (see, Halme et al. [12]).

Value Efficiency Analysis (VEA) is based on the assumption that the DM compares decision maker's units using an unknown value function. This function is assumed to be pseudo-concave and pseudo-convex based on the outputs and inputs, respectively. This function reaches its maximum at a point called the MPS unit. The MPS point can be any existing or virtual unit on the efficient frontier. The purpose of value efficiency analysis is to obtain the rate of increase in outputs and decrease the inputs to reach the frontier of the value function passing through the MPS point. By attention to the value function is an unknown function and its frontier is not precisely specified, we specify a region containing of input and output vectors that are less or equally preferred to the MPS. Value efficiency analysis uses a linear approximation of the frontier of the unknown value function and calculates the rate of the improvement of input and output scores to reach the frontier of the linear approximation. The value efficiency score is measured technically and preference information is taken into account. Note that if the MPS unit is one of the existing units that we call it the Most Preferred Unit (MPU).

First, the administrator in this method selects units as MPU units; these units are the ones that perform best in terms of management. These units are selected in two ways:
1. from the units in the reference set corresponding to the unit under evaluation. And,
2. according to the manager's opinion, including units that have a specific feature.

It should be noted that the units selected as MPS are on the efficient frontier and can be part of the existing or virtual units. Given that the value function is an unknown function, the frontier of this function is not precisely specified. For this reason, we specify a region consisting of input and output vectors that are less or equally preferred to MPS units. Value efficiency is calculated in terms of improvement ratio in input and output scores to reach the frontier of linear approximation and the score of efficiency is calculated in terms of technical efficiency and preference information. Further work on value efficiency was done by Halme et al. [11] who addressed the problem of applying preference information by imposing weight restrictions on inputs and outputs. They calculated the value efficiency using the input and output price information. They improved the accuracy of calculating value efficiency. However, previous problems involving high computation and computational complexity persisted in the presence of large numbers of DMUs. In addition, works done on value efficiency analysis to apply DM preference information in the process of value efficiency calculation, Joro et al. [16] presented an interactive approach to improve value efficiency estimation. However, their approach provided an improvement in the method of calculating value efficiency. Zohrehbandian [27] presented a MOLP model which its objective functions are input/output variables subject to the defining constraints of Production Possibility Set (PPS) in data envelopment analysis models. He used the Zoint-wallenius method [28] to solve the proposed model. The method proposed by Zohrehbandian [27] were based on the simplex method, however, it was more accurate than the method proposed by Halme et al. [12]; however, the method was inadequate despite the large number of DMUs.

Eskelinen, Halme and Kallio [8] used value efficiency analysis to evaluate the bank branch sales in Finland. They used their joint project with the Bank of Finland to evaluate the sales performance of the bank and they used the ability of the bank branches to generate profits and the absence of technical efficiency considerations. Halme et al. [13] developed non-convex value efficiency analysis to calculate value efficiency in FDH technology. They explained that when decision-makers apply preference information to existing efficient units over virtual efficient units, we can develop value efficiency analysis. They showed that the concept of the original value efficiency can no longer be used to calculate value efficiency and developed the concept of value efficiency in terms of the vertex and polar cones concepts to define a new approximate frontier and presented a DMs priori knowledge of the DEA models in the absence of a hypothetical convexity in the structure of the FDH models. Soleimani-damaneh et.al [20] described the properties of a value function and considered the non-smooth condition for this function and showed that continuity is necessary in the case of true value efficiency. Gerami [9] introduced a (Multiple objective Linear Programming) MOLP model for calculating efficiency, which was a non-radial model for calculating efficiency. He developed the proposed model to calculate the component value efficiency and used the interactive STEM method to calculate the component value efficiency. The method proposed by Gerami [9] was computationally better than previous methods, and the calculated component value efficiency scores depended on the superiority of the DM in the interactive process.

The directional distance model was introduced by Chamber et al. [4] at first. This model was presented to evaluate the efficiency and ranking of decision-making units in the presence of negative inputs and outputs. Silva Portela and Thanassoulis [19] then used the directional distance function approach to measure the technical efficiency of a set of decision units in the presence of the negative input and output components. They used the (Range Direction Model) RDM model to evaluate the efficiency of bank branches. Pourmahmoud et al. [18] used a directional distance function model to evaluate the efficiency of decision-making units in the presence of negative data and developed the previous models to deal with the problem of inefficiency.

The remainder of this article is organized as follows. In the second section, we describe the concepts of value efficiency and the model of directional distance function. In the third section, we present the properties of the directional distance function model and we use the directional distance function model to calculate value efficiency in different technologies. At first, we develop the above model to calculate value efficiency in constant and variable return to scale technologies and then, we develop the above model to calculate value efficiency in FDH technology by presenting the concept of polar cone and vertex. In section four, we analyze the value efficiency in different technologies for the set of bank branches and finally, we present the conclusions of the research.

2. Background and Motivation

In this section, we first describe the concept of value efficiency geometrically based on technical efficiency concept and we obtain value efficiency scores through traditional DEA models. In the following, we describe the directional distance function model and its properties.

2.1 Value efficiency analysis

The purpose of value efficiency analysis is to evaluate the efficiency of each decision maker unit according to the value function contour passing through the MPU unit. Function $g : R^{m+p} \to R$ is called a value function if it has the following property. Suppose x, $x^* \in R^m$, y, $y^* \in R^p$, $g(x^*, y^*) > g(x, y)$ if (x^*, y^*) dominate vector (x, y). It should be noted that the function g is strictly increasing and decreasing based on the outputs and inputs. Assume that the value function is a pseudo-convex and pseudoconcave function based on outputs and inputs, respectively. For the sake of simplicity, define the function f as follows.

$f(-x, y) = g(x, y)$. Thus, the function f will be a strictly pseudoconcave. The function f is called a pseudo-convex if the function f has the following property. $\nabla f(z_1)^T(z_2 - z_1) \geqslant 0 \to f(z_2) \geqslant f(z_1)$ for all $z_1, z_2 \in R^{m+p}$. ∇f denote gradient of the function f. The function f. is called a pseudo-concave if the function $-f$ is pseudo-convex. Without any ambiguity, call function f a value function and consider it a pseudo-concave function. The purpose of value efficiency is to calculate and evaluate the efficiency of each decision maker with respect to the indifference contour derivative of the value function that crosses the MPS point, but since the value function is an unknown function, we cannot accurately specify the derivative curve and the value function at the point MPS is determined by all possible linear functions that reach their optimal value at this point and these functions represent all possible tangents of an unknown value function. Since the value function is an unknown function, we must approximate the corresponding contour. The value function at this point is approximated by all possible linear functions that reach the optimal value at MPU point. These functions represent all possible tangents of the unknown value function. We now describe the concept of value efficiency geometrically. Consider the data set in Table 1 containing 7 decision making units with two outputs and one input. All units have the same input.

Table 1: The data set of a numerical example

Unit	Input	Output1	Output2
A	1	1	10
B	1	2	9
C	1	3	7
D	1	3.4	4
E	1	4.6	2
F	1	2	6.5
G	1	2.6	4.5

The production possibility set in the output space is shown in figure 1. As it can be seen, units A, B, C, E are efficient units and other units are inefficient. Suppose that we want to obtain the value efficiency score corresponding to unit G. As it is shown in figure 1, the contour corresponding to the value function is specified as a dot and at the point MPU reaches its maximum. The technical efficiency score corresponding to unit G is $|OG/OG^1| = 0.77$ which is the line segment that originates from the point G^2 on the efficiency contour and crosses the point G that is specified.

In order to calculate value efficiency, we consider B and C units as MPS units. We approximate the contour from all possible tangents of a pseudo-concave function reaching its maximum at the MPS and is uniquely defined and coincides with the line segment B-C. We obtain the value efficiency score corresponding to unit G based on the radial distance of unit G to point G^2. This radial distance is determined by using a line that passes from the origin to point G^2 on the approximate contour and passes through point G. As it is clear in figure 1, the score of the true value efficiency is about $|OG/OG^3| = 0.65$.

Given that the value function and its boundary are unknown, we approximate the contour in all possible tangents to a pseudo-concave function that reaches its maximum at the point MPS. In this state the tangent is uniquely defined. It is represented by the line segment B-C, which is the supporting hyperplane to the production possibility set that passes through B and C points. We measure the radial distance from unit G to point G^2 used to determine the value efficiency score of unit G and this approximation of the value efficiency score is equals to $|OG/OG^2| = 0.65$, which is optimistic to the correct score. Now, we describe how to obtain value efficiency using traditional DEA models. To calculate value efficiency through DEA models, we first solved the DEA model and we get the optimal solution as corresponding to the unit under evaluation and then selected MPS units from the existing units in the reference set corresponding to the unit under evaluation. Of course, these units can be selected according to the manager's opinion. In order to calculate value efficiency, we first solve DEA model in envelopment format. We identify the MPS units. For example, suppose, we obtain MPS units using BCC model in variable return to the scale technology. MPS unit is a combination of the existing units, supposing that $(\lambda_1^*, \lambda_2^*, , \lambda_n^*)$ is optimal solution of BCC model. We obtain optimal solutions in BCC model corresponding to DMU under evaluation. MPS unit is presented as follows.

$y_k^* = \sum_{j=1}^{n} \lambda_j^* y_{kj}, \quad k = 1, \cdots, p,$
$x_i^* = \sum_{j=1}^{n} \lambda_j^* x_{ij}, \quad i = 1, \cdots, m,$
$\sum_{j=1}^{n} \lambda_j^* = 1,$

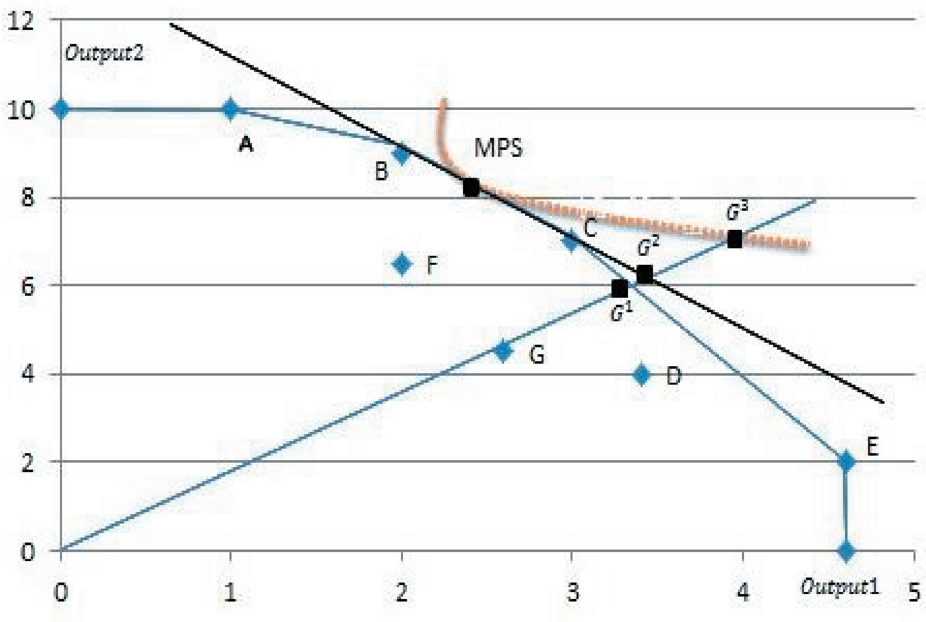

Figure 1. An illustration of value efficiency analysis of unit G.

Suppose that $(\lambda_1^*, \lambda_2^*, , \lambda_n^*)$ is optimal solution of traditional DEA models in envelopment format corresponding to DMU under evaluation. Note that if the MPS unit is one of the existing units that we call it the MPU. These MPU units are the units with the multiples of $\lambda_j^* > 0$ in optimal solution of traditional DEA models and belong to the reference set corresponding to the DMU under evaluation. Of course, we can only consider the managers choice of units as MPUs. After determining MPU units and in the second stage, we solve DEA model in envelopment format by paying attention to MPU units. We consider λ_j being free in sign, in accordance with the MPU units and λ_j being greater or equal than zero in sign in accordance with other units.

2.2 Directional distance function

Consider n decision-making units (x_j, y_j), $j = 1, \cdots, n$ that consume input vector $x_j = (x_{1j}, \cdots, x_{mj}) \in R_+^m$ in order to produce output vector $y_j = (y_{1j}, \cdots, y_{pj}) \in R_+^p$. x_{ij}, $i = 1, \cdots, m$ and y_{kj}, $k = 1, \cdots, p$, represent the i-th and r-th components of the input and output vector corresponding to DMU_j, respectively. Suppose we denote the unit under evaluation with $DMU_o = (x_o, y_o)$. λ_j, $j = 1, \cdots, n$, are intensity variables. The directional distance function (DDF) is introduced as follows (Chambers et al. [4]).

$$
\begin{aligned}
\beta_o^* = \max \quad & \beta_o \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{io} - \beta_o g_{io}^-, \qquad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{kj} \geqslant y_{ko} + \beta_o g_{ko}^+, \qquad k = 1, \cdots, p, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n.
\end{aligned} \tag{1}
$$

Adding slack and extra variables, the above model is presented as follows.

$$
\begin{aligned}
\beta_o^* = \max \quad & \beta_o + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{k=1}^{p} s_k^+) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} - \beta_o g_{io}^-, \qquad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{kj} - s_k^+ = y_{ko} + \beta_o g_{ko}^+, \qquad k = 1, \cdots, p, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n.
\end{aligned} \tag{2}
$$

The directional distance function [4,18,19] with a directional vector is defined as $g = (-g^-, g^+) \neq 0_{m+p}$, $g^+ \in R^p_+$ and $g^- \in R^m_+$, $g^+ = (g^+_1, \cdots, g^+_p)$, $g^- = (g^-_1, \cdots, g^-_m)$.

We denote the direction vector corresponding to the unit under evaluation, namely $DMU_o = (x_o, y_o)$ with (g^-_o, g^+_o). In order to measure the inefficiency of a unit under evaluation of the dataset, the directional distance function model depicts the unit under evaluation on the weak efficiency boundary of the production possibility set along with a positive radius defined by the vector (g^-_o, g^+_o). The vector g has the same dimension as the vector (x_o, y_o) and its unit of measure is the same as the unit of input and output. To solve the directional distance model, we first have to choose the vector g. Vector g can be constant. In the special case, if we put $g_o = (x_o, 0)$, $g_o = (0, y_o)$, $g_o = (x_o, y_o)$ then model (2) is converted to the input and output and mix orientation models, respectively, and depict the unit under evaluation in the above directions on efficiency boundary of the production possibility set. We can put vector g as $g_o = ((x_o - \underline{x}), (\bar{y} - y_o))$ where $\underline{x} = (\underline{x}_1, \cdots, \underline{x}_m)$, $\bar{y} = (\bar{y}_1, \cdots, \bar{y}_p)$, $\underline{x_i} = \min\{x_{ij} \mid j = 1, \cdots, n\}$, $i = 1, \cdots, m$, $\bar{y}_k = \max\{y_{kj} \mid j = 1, \cdots, n\}$, $k = 1, \cdots, p$. In this case, it calls the RDM model. The vector (g^-_o, g^+_o) is nonnegative, and is called the possible improvement rate of DMU_o. And vectors are the ideal direction of the input and output levels. Both inputs and outputs decrease and increase at the same time with the β^*_o ratio, respectively. And β^*_o indicates technical inefficiency. Note that if $\beta^*_o = 0$, then the units under evaluation are efficient and if $\beta^*_o > 0$, so the units under evaluation are inefficient. There should at least be one inefficient unit in assessing the efficiency with the directional distance function [19].

Theorem 2.2.1. *The efficiency score resulting of model (2) in input-oriented (output) put in the [0,1] interval.*

Proof. To get the result, consider model (2) in the input-oriented state. So model (2) is transformed as follows:

$$\beta^*_o = \max \quad \beta_o + \epsilon(\sum_{i=1}^{m} s^-_i + \sum_{k=1}^{p} s^+_k)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s^-_i = (1 - \beta_o) x_{io}, \qquad i = 1, \cdots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{kj} - s^+_k = y_{ko}, \qquad k = 1, \cdots, p,$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n.$$

$s_i^- \geqslant 0, \quad s_k^+ \geqslant 0, \qquad i = 1, \cdots, m, \qquad k = 1, \cdots, p.$

In model (2), we put $1 - \beta_o = \theta_o$. Therefore, the first part of the objective function in model (2) is transformed as follows from the optimization point of view.
$\max\,(1 - \theta_o) = 1 + \max\,(-\theta_o) = 1 - \min\ \theta_o.$

So the first part of the objective function in model (2) is equal to $\min \theta_o$. As consider $\theta_o = 1$, $\lambda_j = 0$, $j \neq o$, $j = 1, \cdots, n$, $\lambda_o = 1$, $s_i^- = s_k^+ = 0$, $i = 1, \cdots, m$, $k = 1, \cdots, p$ is a feasible solution to the above problem. Since the above problem is a minimization problem, so $0 \leqslant \theta_o^* \leqslant 1$. If we show the optimal scores of θ_o and β_o with θ_o^* and β_o^* respectively. By defining the efficiency score in model (2) as $1 - \beta_o^*$ so $0 \leqslant 1 - \beta_o^* \leqslant 1$ and the proof are complete.

We now show similarly that the efficiency score of model (2) in output- oriented state is in the interval of [0,1]. If we put $g_o = (0, y_o)$ in model (2) then the output oriented model obtains. So model (2) is transformed as follows:

$$\beta_o^* = \max \quad \beta_o + \epsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{k=1}^{p} s_k^+\right)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i = 1, \cdots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{kj} - s_k^+ = (1 + \beta_o) y_{ko}, \qquad k = 1, \cdots, p,$$

$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n.$$

$$s_i^- \geqslant 0, \quad s_k^+ \geqslant 0, \qquad i = 1, \cdots, m, \qquad k = 1, \cdots, p.$$

If we put $1 + \beta_o = \gamma_o$ in model (2) so the first part of the objective function in model (2) is transformed as follows from the optimization point:
$\max\,(-1 + \gamma_o) = -1 + \max\,(\gamma_o).$

So the first part of the objective function in model (2) is equal to $\max\ \gamma_o$. Given that in this state: $\gamma_o = 1$, $\lambda_j = 0$, $j \neq o$, $j = 1, \cdots, n$, $\lambda_o = 1$, $s_i^- = s_k^+ = 0$, $i = 1, \cdots, m$, $k = 1, \cdots, p$ is a feasible solution to the above problem. Since the above problem is a maximization problem, so $1 \leqslant \gamma_o^*$. If we show the optimal scores of γ_o and β_o with γ_o^* and β_o^* respectively, by defining the efficiency score in model (2) as $1/(1 + \beta_o^*)$, so $0 \leqslant 1/\gamma_o^* = 1/(1 + \beta_o^*) \leqslant 1$ and the proof are complete. □

The distance function model has the property of translation invariant and unit invariance. Let's describe the translation invariant property. Suppose β_o^*, β_o^{t*}

are optimal solution scores of model (2) to evaluate vectors (x_o, y_o) and $(x_o + t^-, y_o + t^+)$ respectively. We showed $(g_o^-, g_o^+) = (g_o^{t-}, g_o^{t+})$ where (g_o^{t-}, g_o^{t+}) is distance vector after transferring all the data as $(x_j + t^-, y_j + t^+)$, $(t^-, t^+) \neq 0_{m+p}$. The distance function model has the property of translation invariant, if

1) The optimal solutions score of model (2) remained unchanged namely $\beta_o^* = \beta_o^{t*}$.

2) The image point of vector $(x_o + t^-, y_o + t^+)$ that is as $(x_o + t^-, y_o + t^+) + \beta_o^*(g_o^{t-}, g_o^{t+})$ and image point of vector (x_o, y_o) as $(x_o, y_o) + \beta_o^*(g_o^-, g_o^+) + (t^-, t^+)$ are the same.

In other words, vector should not be dependent on sample of units. Directional distance function for constant scores has translation invariant specifications. In other words, direction vector should not be dependent on sample of units. DDF for constant values has translation invariant specifications. The top priority of using the directional distance function model over other methods can be summarized as follows.

A) The model is always feasible and it has translation invariant and unit invariance properties.

B) Using directional distance function model, we can calculate the technical and value efficiency scores for different orientations and technologies.

C) One of the important issues in calculating the value efficiency is choosing the right direction for depicting the unit under evaluation on the approximate boundary of the value function, which has the capability of the directional distance function model and can be easily selected.

2.3 FDH model

Considering the observed DMUs, FDH model [13] is introduced in order to calculate the efficiency in the absence of convex situation in production possibility set. Now, we present the FDH model for computing efficiency score of the unit under evaluation based on the directional distance function as follows.

$$
\begin{aligned}
\beta_o^* = \max \quad & \beta_o + \epsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{k=1}^{p} s_k^+\right) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} - \beta_o g_{io}^-, \qquad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{kj} - s_k^+ = y_{ko} + \beta_o g_{ko}^+, \qquad k = 1, \cdots, p, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \in \{0, 1\}, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n.
\end{aligned}
\tag{3}
$$

The difference between traditional DEA models and FDH models is the convexity condition. There is no convexity condition in the production possibility set dependent to the FDH model. If the number of DMUs is greater than the sum of the number of inputs and outputs, the number of efficient units derived from FDH models is higher than the number of efficient units derived from traditional models such as BCC and CCR, and one unit may be efficient in evaluating with the FDH model, however, it is ineffective in evaluating BCC and CCR models. In the case of $\beta_o^* = 0$ we call the DMU_o unit FDH efficient otherwise it is inefficient.

3. Calculation of Value Efficiency Based on Directional Distance Function

In this section, we obtain value efficiency scores in different technologies based on the directional distance function. We first develop the directional distance function model in constant and variable return to scale technologies to calculate value efficiency.

3.1 Value efficiency in variable return to scale model

To calculate the value efficiency scores in constant and variable return to scale technologies, we first need to specify the MPU units corresponding to the unit under evaluation. MPUs are the units that perform best from a management standpoint and have a certain property. These units are efficient units on the efficiency boundary. To select MPU units, we first solve model (2). To solve model (2) in constant and variable return to scale technologies, we need to choose suitable directions. If we choose $g_o = (x_o, 0)$, $g_o = (0, y_o)$, $g_o = (x_o, y_o)$ in model (2), we obtain the models in the input, output, and mix oriented respectively and the unit under evaluation is depicted in the above directions on the efficiency boundary. However, we can choose the direction vector as $g_o = ((x_o - \underline{x}), (\bar{y} - y_o))$, which yields the RDM model [19]. If we omit $\sum_{j=1}^{n} \lambda_j = 1$ from model (2), we will obtain distance function model in constant return to scale technology. To calculate value efficiency, we first determine MPU units. After solving model (2), we specify the reference set corresponding to the unit under evaluation. These units are the units on the efficient frontier. Suppose $(\lambda_1^*, \lambda_2^*, \cdots, \lambda_n^*)$ is an optimal solution of model (2) corresponding to the unit under evaluation, namely DMU_o. We define the reference set corresponding DMU_o as follows.

Reference set $=\{DMU_j \mid \lambda_j^* > 0\}$.

We choose MPU units in one of the following two ways.

1) MPU units can include all or a subset of the units in the reference set corresponding to the unit under evaluation.

2) MPU units can be selected according to the manager. These units can be the ones with a specific property that the manager is looking for. After solving model(2) and determining the corresponding solution with each unit under evaluation, we specify the MPU units. In the following, we solve model (4) by paying attention to obtain vector $(\lambda_1^*, \lambda_2^*, \cdots, \lambda_n^*)$ from model (2). Suppose $(\lambda_1^*, \lambda_2^*, \cdots, \lambda_n^*)$ is optimal solution of model (2) corresponding to DMU under evaluation. MPU units are the units with the multiples $\lambda_j^* > 0$ in optimal solution of model (2) and belong to the reference set corresponding to the DMU under evaluation. Of course, we can only consider the managers choice of units as MPUs and consider the λ_j multiples sign free in sign corresponding to MPU units in model (4). Then we consider λ_j being free in sign, in accordance with the MPU units and λ_j being greater or equal than zero in sign in accordance with the other units. We place $\lambda_j > 0$ only for a subset of the set $N - \{j \mid \lambda_j^* > 0\}$ and limit MPU selection and determine the amount of value efficiency. The following model is presented in order to calculate the value efficiency.

$$\begin{aligned}
\beta_o^* = \max \quad & \beta_o + \epsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{k=1}^{p} s_k^+\Big) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} - \beta_o g_{io}^-, \qquad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{kj} - s_k^+ = y_{ko} + \beta_o g_{ko}^+, \qquad k = 1, \cdots, p, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geqslant 0, \qquad j = 1, \cdots, n, \\
& \lambda_j \geqslant 0 \quad if \quad \lambda_j^* = 0, \quad j = 1, \cdots, n, \\
& \lambda_j \quad is \quad free \quad if \quad \lambda_j^* > 0, \quad j = 1, \cdots, n, \quad \epsilon > 0.
\end{aligned} \tag{4}$$

The value efficiency scores resulting from model (4) in the input and output oriented model are $(1 - \beta_o^*)$ and $1/(1 + \beta_o^*)$ respectively. In model (4), we select the λ_j sign corresponding to the MPU units free in the sign and select theλ_j sign corresponding to the other units greater than or equal to zero. In model (4), we select the sign λ_j corresponding to all units that in optimal solution $\lambda^* = (\lambda_1^*, \lambda_2^*, \cdots, \lambda_n^*)$ corresponding to the unit under evaluation, have a sign greater than zero, that is $\lambda_j > 0$, the free in the sign. Of course, we can select the λ_j sign only for some units in the reference set corresponding to the unit under evaluation free in the sign, which are a subset of the units in the corresponding reference set to the unit under evaluation. It should be noted that by releasing the λ_j sign corresponding to the MPU units, the unit under evaluation is depicted on the approximate boundary of the value function. This boundary includes all hyperplanes passing through MPU units. The interpretation is that we use the five-fold principle in constructing a production possibility set. One of these is the principle of convexity and we will have convex composition of decision-making units in the production possibility set. Then we add constraint $\sum_{j=1}^{n} \lambda_j = 1,\ \ \lambda_j \geqslant 0,\ \ j = 1, \cdots, n$, to the set of constraints and obtain the production possibility set in variable return to scale state. If we select the λ_j sign corresponding to the MPU units free in sign, in this case, the convexity condition for these units to not be met and the efficient frontier changes. In these locations supporting hyperplane on the production possibility set that passing through these units continues and there will be a new contour for the production possibility set than before. The hyperplane that passing through MPU units is considered as a linear approximation of the contour of the unknown value function and these hyperplanes create a new contour. The condition of convexity is no longer established and these planes continue. The λ_j scores corresponding to the MPU units can be negative in model (4). As shown in Fig. 1, In the case of $\lambda_B > 0$, $\lambda_C > 0$ a line segment that passes from units B, C is obtained and if the sign of λ_B, λ_C is considered free in sign, the line that passes from units B, C is obtained as shown by the dash in Fig. 1.

3.2 Value efficiency in FDH technology

Production possibility set in FDH technology is not convex. The value efficiency analysis used in traditional DEA models can no longer be used if we want to do value efficiency analysis only for the existing units. Production possibility set in FDH Technology is a subset of the production possibility set in variable return to scale technology. The scores are never more pessimistic in FDH models. If the number of units is lower than the number of inputs and outputs, the efficiency analysis leads to the number of more efficient units in the FDH models. Now, we calculate the value efficiency in FDH technology. If the production possibility set is non-convex, the linear approximation used in the original VEA does not work well. Firstly, because these units are dominated by convex combined with other efficient units for efficient units called dominated convex. The linear approximation of the value function cannot reach its optimal value at the dominated convex point and the linear approximation cannot be applied. Second, even if the unit under evaluation is not dominated convex, the linear approximation may not work well, and the linear approximation may be pessimistic, and the true value efficiency is a score between the approximation score of value efficiency and the efficiency score resulting from FDH model. As we know, the value efficiency approximation is always optimistic when we use the original value efficiency. But in FDH technology, using the original value approximation may be more optimistic or more pessimistic than the true value efficiency. For example, considering the data of Table (1), production possibility set regarding the equal inputs in the output space is as Fig. 2.

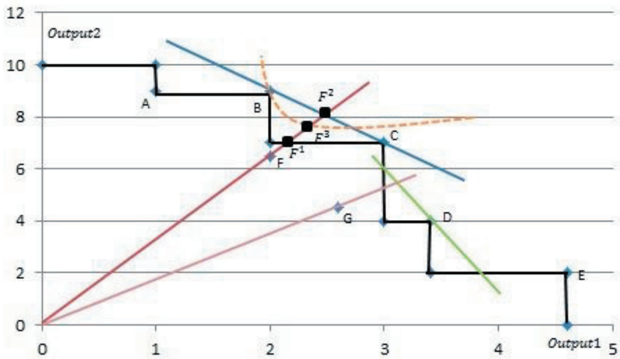

Figure 2. The linear value function and the FDH model.

As you can see, A, B, C, D and E units are efficient units in FDH model while only units A, B, C and E are efficient in the BCC model. Because D unit is the combination of E and C units, the efficiency value of D unit in BCC model equals $0.89 < 1$. Note that FDH model produces more or equally optimistic efficiency scores in comparison with DEA traditional models which are greater or equal to this credible issue. Because FDH production possibility set is a subset of production possibility set of traditional models and the values are never worse. While the number of observed units is less comparing to the number of inputs and outputs. Efficiency analysis leads to more efficiency units in FDH models compared with traditional models. At this stage, we intend to introduce the superior information in FDH models. As previously discussed, there is one assumption to calculate VEA, which is the production possibility set is convex to be able to have an unknown value function frontier. Convex production possibility set is a convex to enable us to approximate value unknown function frontier through a linear frontier and the approximate obtained VEA is always greater than technical efficiency and the optimistic model occurs. If the production possibility set is not convex, the previously presented linear approximation cannot be used as the value function approximation. VEA calculation method should be corrected because the value function in this point is not at its optimistic value in D point. Another reason why linear approximation cannot work in this mode except in the case that the unit under assessment is dominant by convex combining of other units, is that linear approximation might be pessimistic. Considering the value function in figure (2), this function achieves its maximum value in the points of B or C. Assume this point is B. The linear approximation method for the value function with a linear function is not right because the strength is very pessimistic. Consider F unit. The true value efficiency score is between its approximate score and FDH efficiency score, i.e., $0.81 < 0.83 < 0.93$. In this mode, it is possible that the approximation score is either very optimistic or very pessimistic. In the following, use the convex cones theorem based on the assumption that the value function is a quasi-concave function. Function f is said to be quasi-convex if $f(\mu z_1 + (1-\mu)z_2) \leqslant \max\{f(z_1), f(z_2)\}$ for all $z_1, z_2 \in R^{m+p}$ and $\mu \in [0,1]$. Function f is said to be quasi-concave if -f is quasi-convex. This hypothesis is relatively general and is still robust in taking advantage of superior information. Principal Value Efficiency (VEA) applies the assumption that the value function is pseudo-concave, note that the pseudo-concave value functions are also quasi-concave. VEA employed the assumption if a pseudo-concave value function which is more restrictive that pseudo-concave functions are quasi-concave. Assuming the value function allows us to approximate the boundary of this function using convex cones and obtain lower bounds for the true scores of value efficiency. In olden DEA models, the value function contour is approximated using hyperplane, whereas in FDH models we use convex cones. The accuracy of the approximation is completely dependent on the DM information. To address the above problems, Halme et al. [13] used convex cones to approximate the value function boundary. They assumed that the value function was a quasi-concave function. Assume that $f : R^{m+p} \to R$ is a quasi-concave function. Consider r point $z_1, \cdots, z_r \in R^{m+p}$ while $z_l \geqslant z_q,\ l = 1, \cdots, r,\ l \neq q$. Suppose $\geqslant$ denotes that z_1 dominates over z_q. Cone $C(z_1, \cdots, z_r; z_q)$ with vertex z_q is defined as follows.

$$\begin{aligned} C(z_1, \cdots, z_r; z_q) = \{z \mid z = z_q + \sum_{l=1,\ l\neq q}^{r} (z_q - z_l)\mu_l, \\ \mu_l \geqslant 0,\ \ l = 1, \cdots, r,\ \ l \neq q\ \ \}. \end{aligned} \tag{5}$$

A cone can include one or two points or r points, assuming that $f : R^{m+p} \to R$ function is quasi-concave and $f(z_q) = \min\{f(z_l) \mid l = 1, \cdots, r\}$. Based on the quasi-concavity assumption [13] of the value function f, we have $\forall z \in C(z_1, \cdots, z_r; z_q)$ we have $f(z_l) \geqslant f(z_q) \geqslant f(z),\ z \in R^{m+p},\ l = 1, \cdots, r,\ l \neq q$. For identifying whether $z \in R^{m+p}$ is dominated by points cone (5), Halme et al. [13] presented the following model.

$$
\begin{aligned}
\max \quad & \beta + \epsilon 1^T s \\
s.t. \quad & z_q + \sum_{l=1, l\neq q}^{r} (z_q - z_l)\mu_l - \beta w - s = z, \\
& \mu_l \geqslant 0,\ \ w \in R^{m+p},\ \ w \geqslant 0,\ \ w \neq 0,\ \ s \geqslant 0,\ \ l = 1, \cdots, r, \\
& s \in R^{m+p},\ \ 1 = [1, \cdots, 1]^T,\ \ \epsilon > 0,\ \ (Non - Archimedean).
\end{aligned} \tag{6}
$$

They showed that if $\beta^* > 0$ for some $w \geqslant 0$ and $w \neq 0$ then z is dominated by Cone C. In model (6) if $\beta^* > 0$, then the point z is dominated by the cone $C(z_1, \cdots, z_r; z_q)$. Otherwise the point z is not dominated by the cone $C(z_1, \cdots, z_r; z_q)$. If the amount of ϵ is unbounded in model(6) and for some $w \neq 0$ and $w \in R^{m+p}$, $w \geqslant 0$, then superiority information of $z_l \geqslant z_q$, $l = 1, \cdots, r$, $l \neq q$ would not be compatible (see [13]).

The model (6) is a typical formulation that used in DEA. We can convert the model (6) to DEA model structure with slightly change.

We assume the utility function is quasi-concave. Now consider the concept of convex cones if DM is only intended to evaluate the existing units. Since the value function is quasi-convex and quasi-concave in terms of inputs and outputs respectively, so assuming that the value function must be quasi-concave, we substitute the input values with their negatives and consider vector $z = (-x, y)$. For $q \in \{1, \cdots, n\}$ consider r distinct points $z_1, \cdots, z_r \in R^{m+p}$ while $z_l \geqslant z_q$, $l = 1, \cdots, r$, $l \neq q$, $r < n$. And denote the unit under evaluation with z_o. z_o can be an element of the set $\{z_1, \cdots, z_r\}$. The goal is to calculate the relative distance z_o from the approximate derivative boundary that passes through the MPU units.
If we select
$w = g^o = (g_1^{-o}, \cdots, g_m^{-o}, g_1^{+o}, \cdots, g_p^{+o})$, $z = z_o = (-x_o, y_o)$, $z_l = (-x_l, y_l)$, $l = 1, \cdots, r$, $r < n$, $z_q = (-x_q, y_q)$, $s = (s_1^-, \cdots, s_m^-, s_1^+, \cdots, s_p^+)$,
we can write model (6) as follows.

$$
\begin{aligned}
\max \quad & \beta + \epsilon 1^T s \\
s.t. \quad & z_q + \sum_{l=1, l\neq q}^{r} (z_q - z_l)\mu_l - s = \beta g^o + z_o, \\
& \mu_l \geqslant 0, \;\; g^o \in R^{m+p}, \;\; g^o \geqslant 0, \;\; g \neq 0, \;\; s \geqslant 0, \;\; l = 1, \cdots, r, \\
& r < n, \;\; s \in R^{m+p}, \;\; 1 = [1, \cdots, 1]^T, \;\; \epsilon > 0.
\end{aligned} \tag{7}
$$

Model (7) was developed to calculate the value efficiency score of unit $z_o = (-x_o, y_o)$ with respect to cone $C(z_1, \cdots, z_r; z_q)$ as follows:

$$
\begin{aligned}
\max \quad & \beta + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{k=1}^{p} s_k^+) \\
s.t. \quad & x_{iq} + \sum_{l=1}^{r}(x_{iq} - x_{il})\mu_l + s_i^- = x_{io} - \beta g_i^{-o}, \;\; i = 1, \cdots, m, \\
& y_{kq} + \sum_{l=1}^{r}(y_{kq} - y_{kl})\mu_l - s_k^+ = y_{ko} + \beta g_k^{+o}, \;\; k = 1, \cdots, p, \\
& \mu_l \geqslant 0, \;\; s_i^-, s_k^+ \geqslant 0, \;\; i = 1, \cdots, m, \;\; k = 1, \cdots, p, \\
& r < n, \;\; l = 1, \cdots, r, \;\; \epsilon > 0, \;\; (Non - Archimedean).
\end{aligned} \tag{8}
$$

By selecting the vector $g^o = (0^T, y_o)$ in the model (8), we can extend model (8) to calculate value efficiency in output oriented model. The value efficiency score is defined by the cone C in the output oriented as $1/(1+\beta^*)$ where β^* is the optimal score of obtained from model (7). To get model (8) in the input oriented format, we set $g^o = (-x^o, 0^T)$. The value efficiency score is defined by the cone C in the input oriented as $1 - \beta^*$.

Consider Figure (3) and assume that a unit B is considered as MPU, dominant points like F and G cannot be considered as MPU. We introduce two points' cones $C(B, C; C)$ with vertex C and lines that pass of C trough B. We can make four relevant two-point cones with vertex A, C, D and E. Lines that pass of B trough A, C, D and E units are shown in Figure 3. G, F and D, E units are dominated by cone $C(B, C; C)$ where all points of cones $C(B, E; E)$ and $C(B, D; D)$ are also dominated. Cones $C(B, A; A)$, $C(B, C; C)$ include all the information we require.

If B and D units are considered as MPU units in DM we can produce the three-point cone $C(B, C, D; C)$. Each point of this cone dominated C unit and some of these points also dominate B and D points. The corresponding solution of model (8) is unbounded. Regarding figure (3), there is no quasi-concave function whose optimal solutions are B and D points and thus the unbounded of model (8) shows that superiority information does not exist in accordance with the assumption that value function is pseudo-concave.

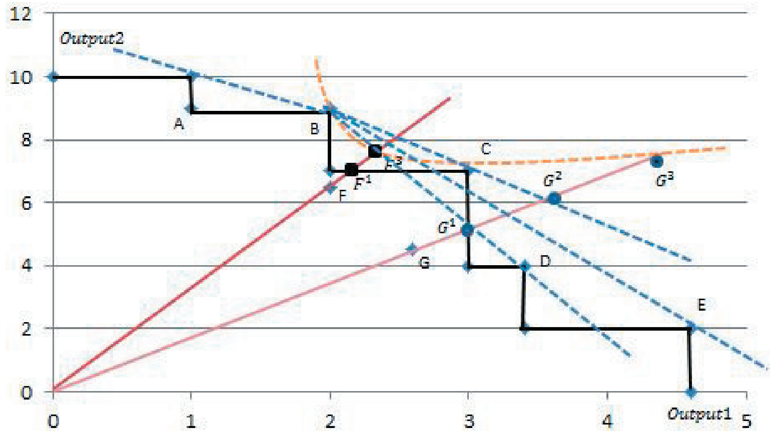

Figure 3. Analysis units when B is the most preferred unit.

Now, we can use convex cones approximation to achieve the lower approximation of quasi-concave value function frontier that passes MPU unit. Considering unit G in figure 3, with regard to the unknown value function, the true efficiency score for this unit that is $|OG/oG^2|$ is approximated as $|OG/oG^3|$ and our approximation is less pessimistic because $|OG/oG^3| = 0.63 < |OG/oG^2| = 0.75$.

4. Value Efficiency Analysis Based on Bank Branches

In this section, we calculated the value efficiency set of bank branches based on the presented approach. In this regard, the presented approaches in this paper were used to calculate the efficiency of branches of Helsinki metropolitan bank [13]. Branches of Helsinki metropolitan bank are owned by OP-Pohjola Group and continue group service in Finland. Banks present financing, investment, daily and insurance services for private customers and small businesses. Note that bank services are important channels for the presentation of sale performance on banks and management seeks opportunities to improve sale performance in the branches. This issue is also discussed in Eskelinen, Halme and Kallio [8] from various aspects. Sale performance of a branch is defined as below and the amount sale of non-produced is by its sale force. Bank management seeks to identify units with weaker performances to improve them. Inputs and outputs are considered as below. Outputs include the transactional sales volumes of the branches. Sales are in two classes. The first class is financial services and daily banking investment services of funds services are excluded from our analyses. Insurance services have entered investments but non-life insurance services are not considered. The weights of importance of factors for the development of output are determined through management. Only one input is considered in our analyses: the work of the sales force. A sale force management does not consider other operational costs due to the fact that they are not controllable. The input quality used is the overall use of work times in sales activity as full-time equivalents. The value efficiency was calculated using CCR, BCC, RDM and FDH models. The value efficiency of bank branches is assessed with the opinion of management of banks. Thus, MPU units are specified and other units should achieve the levels of their activity of MPU units. The input and output data are given in Table (2).

Table 2: Input and outputs.

DMU	O_1	O_2	I_1
1	1090	497	26
2	2633	1111	47.7
3	3320	1477	60.7
4	1147	353	25.2
5	1180	540	21.6
6	3821	1769	75.5
7	1574	716	36.4
8	1171	1004	29.1
9	1174	449	22.5
10	1203	568	27.2
11	928	384	22
12	4393	2210	65.9
13	2642	931	38.8
14	3362	1505	53.1
15	2263	541	26.9
16	3619	1541	70.3
17	4163	1594	73.6
18	3075	805	46.7
19	5757	2601	93
20	1763	496	29
21	3825	1961	83.1
22	2354	792	42.4
23	5289	3160	104
24	1108	332	24.2
25	743	354	22.4

At first, we evaluate the value efficiency of 25 bank branch using CCR and BCC models in the input orientation. Therefore, we put $g = (g_o^-, g_o^+) = (x_o, 0)$ in model (2) and we obtain the efficiency scores of bank branches in the input oriented.

With the removal of restrictions $\sum_{j=1}^{n} \lambda_j = 1$, CCR model is obtained from model (2). First, consider the second and third columns of Table (3). As it can be seen, units 8, 12 and 15 are CCR efficient, regarding units 12 and 15 as branches that have the best performance from management's perspective. The third column of Table (3) shows value efficiency scores, as it can be seen only units of 12 and 15 are the value efficient. The scores of the technical and value efficiency corresponding to CCR model are distinct in the input orientation for units 8, 21 and 23. For example, the technical efficiency score of unit 21 is equal to 0.703, while the value efficiency score is equal to 0.696, which is less than the one in the technical efficiency score. Unit 8 is traditionally efficient, while its value efficiency score is 0.772 and it is not the value efficient. Now, we consider the scores of the technical and value efficiency corresponding to BCC model in input orientation. So to solve models (2), (4), we must consider vector g as $g = (g_o^-, g_o^+) = (x_o, 0)$. By regarding the third and fourth columns of Table (3), we see that units 5, 8, 12, 15, 19 and 23 are BCC efficient units. By regarding units 12 and 15 as the MPU units, the value efficiency scores would come in the last column in Table (3) corresponding to BCC model in input orientation, as it can be seen units 1, 4, 5, 7, 9, 10, 11, 13, 14, 19, 20, 23, 24 and 25 have the distinct scores for the technical and value efficiency. For example, the technical and value efficiency scores corresponding to BCC model in the input orientation of unit 9 are 0.96, 0.879 respectively. Units 5, 19 and 23 are BCC efficient but not the value efficient.

Table (4) shows scores of the technical and value efficiency corresponding to CCR and BCC models in the output orientation. First, consider the technical efficiency scores of the output oriented CCR model. According to the second and third columns of Table (4), units 8, 12 and 15 are CCR efficient. With regarding units 12 and 15 as MPU units, the value efficiency scores are shown in the third column of Table (4). As it can be seen, only units 12 and 15 are value efficient and the amounts of the technical and value efficiency of CCR model in the output-orientation for the units 8, 21, 23 are distinct. For example, the value efficiency scores corresponding to CCR model in the output orientation of unit 21 are 0.703 and 0.696, respectively. The third and fourth columns of Table (4) show the technical and value efficiency scores corresponding to BCC model in the output orientation.

Table 3: The technical and value efficiency scores of the CCR and BCC input orientation models.

DMU	CCR-input -o-efficiency	CCR-input -o-value-efficiency	BCC-input -o-efficiency	BCC-input -o-value-efficiency
DMU01	0.606	0.606	0.831	0.775
DMU02	0.775	0.775	0.805	0.805
DMU03	0.783	0.783	0.792	0.792
DMU04	0.578	0.578	0.857	0.715
DMU05	0.79	0.79	1	0.988
DMU06	0.735	0.735	0.736	0.736
DMU07	0.624	0.624	0.725	0.723
DMU08	1	0.772	1	1
DMU09	0.708	0.708	0.96	0.879
DMU10	0.647	0.647	0.815	0.806
DMU11	0.588	0.588	0.982	0.792
DMU12	1	1	1	1
DMU13	0.9	0.9	0.913	0.913
DMU14	0.908	0.908	0.918	0.918
DMU15	1	1	1	1
DMU16	0.725	0.725	0.736	0.736
DMU17	0.768	0.768	0.838	0.838
DMU18	0.8	0.8	0.894	0.894
DMU19	0.891	0.891	1	0.977
DMU20	0.753	0.753	0.843	0.813
DMU21	0.703	0.696	0.708	0.708
DMU22	0.724	0.724	0.745	0.745
DMU23	0.896	0.82	1	0.832
DMU24	0.577	0.577	0.893	0.722
DMU25	0.487	0.487	0.964	0.713

Units 5, 8, 12, 15, 19 and 23 are BCC efficient units. With regarding units 12 and 15 as MPU units and resolving model (4) with the vector selection of $g = (g_o^-, g_o^+) = (0, y_o)$ we will see that only units 8, 12 and 15 are value efficient. According to the last column of Table (4), the amounts of technical and value efficiency of BCC model in the output orientation for units 1, 4, 5, 6, 9, 10, 11, 16, 17, 19, 21, 23, 24 and 25 are distinct. For example, the technical and value efficiency scores of BCC model in the output orientation for unit 11 are 0.710 and 0.735, respectively. Then the amount of the value efficiency is smaller. Units 5, 19 and 23 are BCC efficient but not value efficient.

Table (5) shows the technical and value efficiency scores corresponding to CCR and BCC models in the mix oriented. To calculate the value efficiency scores, units 12 and 15 are considered as MPU units that have the best performance from management's perspective. First, we will consider CCR model. In this way, we choose the vector of $g = (g_o^-, g_o^+) = (x_o, y_o)$to solve the models (2), (4). As it can be seen in the second column of Table (5), units 8, 12 and 15 are the mix efficient units. Only units 12 and 15 are the mix value efficient units. Amounts of the technical and value efficiency corresponding to CCR model in the mix oriented for units 8, 21 and 23 are distinct according to the second and third columns in Table (5). All value efficiency scores are less than the amounts comparing to the traditional efficiency. For example, amounts of the technical and value efficiency corresponding to CCR model in the mix oriented for unit 21 are 0.825 and 0.821, respectively, and it is clear that the amount of the value efficiency is smaller.

Unit 8 is CCR efficient but not value efficient and its value efficiency score is equal to 0.871. The fourth and fifth columns of Table (5) show the technical and value efficiency scores corresponding to the BCC model in the mix oriented. Units 12 and 15 are included as MPU units. As it is seen, units 8, 12 and 15 are the efficient units of BCC and other units are inefficient. The value efficiency score corresponding to the BCC model in the mix oriented of all units except units 8, 12 and 15 with their corresponding scores show that the traditional efficiency is distinct in accordance to the fourth and fifth columns of Table (5). For example, the amount of the value and traditional efficiency of unit 18 is 0.889 and 0.952, respectively.

The value efficiency scores corresponding to BCC model in the mix oriented of all units are larger than their corresponding scores of the technical efficiency. As it was observed, we can apply the management opinion for value efficiency analysis.

In this section, we evaluate the value efficiency of 25 bank branch using FDH model in the input and output oriented. By considering the second column of Table (6), we will see that 12 of 25 units are FDH efficient units. Other units are inefficient.

Table 4: The technical and value efficiency scores of the CCR and BCC output orientation models.

DMU	CCR-output -o-efficiency	CCR-output -o-value-efficiency	BCC-output -o-efficiency	BCC-output -o-value-efficiency
DMU01	0.606	0.606	0.708	0.705
DMU02	0.775	0.775	0.776	0.776
DMU03	0.783	0.783	0.808	0.808
DMU04	0.578	0.578	0.625	0.622
DMU05	0.79	0.79	1	0.983
DMU06	0.735	0.735	0.784	0.777
DMU07	0.624	0.624	0.666	0.666
DMU08	1	0.772	1	1
DMU09	0.708	0.708	0.861	0.833
DMU10	0.647	0.647	0.75	0.749
DMU11	0.588	0.588	0.735	0.71
DMU12	1	1	1	1
DMU13	0.9	0.9	0.907	0.907
DMU14	0.908	0.908	0.91	0.91
DMU15	1	1	1	1
DMU16	0.725	0.725	0.784	0.781
DMU17	0.768	0.768	0.871	0.865
DMU18	0.8	0.8	0.919	0.919
DMU19	0.891	0.891	1	0.98
DMU20	0.753	0.753	0.762	0.762
DMU21	0.703	0.696	0.766	0.717
DMU22	0.724	0.724	0.757	0.757
DMU23	0.896	0.82	1	0.821
DMU24	0.577	0.577	0.647	0.626
DMU25	0.487	0.487	0.612	0.603

Table 5:The technical and value efficiency scores of the mix orientation models.

DMU	CCR-Mix -o-efficiency	CCR-Mix -o-value-efficiency	BCC-Mix -o-efficiency	BCC-Mix -o-value-efficiency
DMU01	0.754	0.754	0.754	0.854
DMU02	0.873	0.873	0.873	0.883
DMU03	0.878	0.878	0.878	0.881
DMU04	0.733	0.733	0.733	0.806
DMU05	0.883	0.883	0.883	0.993
DMU06	0.847	0.847	0.847	0.862
DMU07	0.769	0.769	0.769	0.822
DMU08	1	0.871	1	1
DMU09	0.829	0.829	0.829	0.924
DMU10	0.786	0.786	0.786	0.877
DMU11	0.741	0.741	0.741	0.862
DMU12	1	1	1	1
DMU13	0.948	0.948	0.948	0.95
DMU14	0.952	0.952	0.952	0.955
DMU15	1	1	1	1
DMU16	0.841	0.841	0.841	0.864
DMU17	0.869	0.869	0.869	0.92
DMU18	0.889	0.889	0.889	0.952
DMU19	0.942	0.942	0.942	0.989
DMU20	0.859	0.859	0.859	0.883
DMU21	0.825	0.821	0.825	0.821
DMU22	0.84	0.84	0.84	0.841
DMU23	0.945	0.901	0.945	0.905
DMU24	0.732	0.732	0.732	0.81
DMU25	0.655	0.655	0.655	0.8

Table 6: The technical and value efficiency scores of the FDH input and output orientation models.

DMU	FDH-input -o-efficiency	FDH-input -o-value-efficiency	FDH-output -o-efficiency	FDH-output -o-value-efficiency
DMU01	0.831	0.831	0.924	0.924
DMU02	1	0.984	1	0.917
DMU03	0.875	0.781	0.988	0.887
DMU04	0.857	0.857	0.972	0.972
DMU05	1	1	1	1
DMU06	0.873	0.825	0.87	0.852
DMU07	1	1	1	1
DMU08	1	1	1	1
DMU09	0.96	0.96	0.995	0.995
DMU10	1	1	1	1
DMU11	0.982	0.982	0.786	0.786
DMU12	1	1	1	1
DMU13	1	1	1	1
DMU14	1	1	1	1
DMU15	1	1	1	1
DMU16	0.937	0.937	0.824	0.824
DMU17	0.895	0.895	0.948	0.948
DMU18	1	1	1	1
DMU19	1	1	1	1
DMU20	0.928	0.928	0.917	0.917
DMU21	0.793	0.677	0.887	0.786
DMU22	0.915	0.915	0.891	0.891
DMU23	1	0.872	1	0.892
DMU24	0.893	0.893	0.939	0.939
DMU25	0.964	0.964	0.656	0.656

The sale network management distinguishes units 12, 13 and 15 as the best branches in terms of the performance and we respect them as MPU units. We obtain the value efficiency scores, accordingly. As we saw, unit 13 isnt efficient based on CCR and BCC models but it is efficient in FDH model.

By considering the convex combination assumed in the collection, this unit is a convex combination of other units. We use model (8) to evaluate the value efficiency of these units. We use four-point cones that all other efficient units except three of them can be used as the vertex of the cones. This means that we use any other efficient unit as the vertex of the cone. We use the three units as the other components of the four-point cone. Three units 12, 13 and 15 are considered as units with having the best performance in terms of management. As it can be seen from Table (6), units 2 and 23 are FDH efficient while they arent value efficient, they obtain levels of 0.984 and 0.872, respectively, as the amount of value efficiency in the input orientation. This indicates that the value efficiency scores will be different with the technical efficiency scores by mentioning the considered decision-making preference information. The fourth and fifth columns of the Table (6) are the representation of the technical and value efficiency scores of FDH in the output orientation, respectively. As it can be seen, units 2 and 5, 7, 8, 10, 12, 13, 14, 15, 18, 19 and 23 are FDH efficient in the output oriented model and other units are inefficient by considering the same cones with the input orientation. The value efficiency scores corresponding to FDH output orientation model are shown in the last column of the Table (6). As it can be seen, units 2 and 23 are FDH efficient but they arent FDH value efficient. The technical and value efficiency scores for units 2 and 3, 6, 21 and 33 are distinct. For example, the technical and value efficiency scores corresponding to unit 6 are 0.87 and 0.852 respectively. The value efficiency score of unit 6 compared to its technical efficiency is smaller in the output oriented. For the technical and value efficiency scores of the input orientation in accordance to the second column of Table (6) we have also similar interpretation. For example, the technical and value efficiency scores corresponding to unit 6 are 0.873 and 0.825, respectively.

The value efficiency score of unit 6 compared to its technical efficiency is smaller in the input oriented. It should be noticed that unit 25, as a new unit, was not considered as MPU unit. Unit 10 also has the same performance with unit 15 and the service amount produces the similar investment and nearly 50 percent of finance services less than unit 15.

5. Conclusions

In this paper, we developed the directional distance function model to calculate value efficiency. We have shown that we can apply the above model to various technologies to calculate value efficiency. We first calculated the scores of value efficiency in constant and variable return to scale technologies and then developed it for FDH technology. The strengths of the approach presented over previous approaches in computing value efficiency is that we can apply it to different technologies. We obtain this model in different orientations including input, output and mixed by choosing the suitable direction. As described in the numerical example section, we can incorporate DM 's priori knowledge into the efficiency evaluation process by applying the presented models. In calculating value efficiency in constant and variable return to scale technologies, we select MPU units from units that are characterized by management and are strong efficient units, and calculate the value efficiency scores of other units in terms of MPU units. It should be noted that the distance function model is suitable in calculating value efficiency and has stable properties in relation to the transfer and modification of the data unit, allowing the administrator to apply the model to different datasets. Due to the problems of the method of calculating the original value efficiency, we developed the model in applying the model to FDH technology and showed that it does not work well in the calculation of the true value efficiency and that we have problems with the so-called convex dominated units and approximation does not obtain the optimal scores for value efficiency, and by developing the directional distance function model to calculate the value efficiency of the existing units against all units, we calculated the value efficiency scores and approximated the value function counter with convex cones. We developed a directional distance function model with DEA structure to calculate value efficiency in FDH technology and incorporated value information into the evaluation process by selecting the appropriate cone. Finally, we applied the presented approach to the bank dataset and showed that using the presented approach we can incorporate bank management's view into the performance appraisal process and use management units of the bank to evaluate the performance of other units. Finally, we can extend the above approach to other data structures such as network structure and inaccuracy.

References

[1] A. Athanassopoulis, Goal programming and data envelopment analysis (GoDEA) for target-based multi-level planning: allocating central grants to the Greek local authorities, *European Journal of Operational Research*, 87 (1995), 535-550.

[2] A. Athanassopoulis, Decision support for target-based resource allocation of public services in multiunit and multilevel systems, *Management Science*, 39 (10) (1998), 1261-1264.

[3] H. Bal, H. H. Orkcu, and S. Celebioglu, Improving the discrimination power and weights dispersion in the data envelopment analysis, *Computers and Operations Research*, 37 (2010), 99-107.

[4] R. G. Chambers, Y. Chung, and R. Fre, Profit, directional distance functions, and Nerlovian efficiency, *Journal of Optimization Theory and Applications*, 98 (2) (1998), 351-364.

[5] A. Charnes, W. W. Cooper, and E. Rhodes, Measuring efficiency of decision making units, *European Journal of Operational Research*, 2 (6) (1978), 429-444.

[6] A. Charnes and W. Cooper, Polyhedral cone-ratio DEA models with an illustrative application to large commercial banking, *Journal of Econometrics*, 46 (1990), 73-79.

[7] A. Ebrahimnejad and M. Tavana, An interactive MOLP method for identifying target units in output-oriented DEA models: The NATO enlargement problem, *Measurement*, 52 (2014), 124-134.

[8] J. Eskelinen, M. Halme, and M. Kallio, Bank branch sales evaluation using extended value efficiency analysis, *European Journal of Operational Research*, 232 (2014), 654-663.

[9] J. Gerami, An interactive procedure to improve estimate of value efficiency in DEA, *Expert Systems with Applications*, 137 (2019), 29-45.

[10] B. Golany, An interactive MOLP procedure for the extension of DEA to effectiveness analysis, *Journal of Operational Research Society*, 39 (1988), 725-734.

[11] M. Halme, T. Joro, and P. Korhonen, Restricting weights in value efficiency analysis, *European Journal of Operational Research*, 126 (2000), 175-188.

[12] M. Halme, T. Joro, P. Korhonen, S. Salo, and J. Wallenius, A value efficiency approach to incorporating preference information in data envelopment analysis, *Management Science*, 45 (1) (1999), 103-15.

[13] M. Halme, P. Korhonen, and j. Eskelinen, Non-convex value efficiency analysis and its application to bank branch sales evaluation, *Omega*, 48 (2014), 10-18.

[14] F. Hosseinzadeh Lotfi, A. A. Noora, G. R. Jahanshahloo, J. Jablonsky, M. R. Mozaffari, and J. Gerami, An MOLP based procedure for finding efficient units in DEA models, *Central European Journal of Operational Research*, 17 (2009), 1-11.

[15] F. Hosseinzadeh Lotfi, G. R. Jahanshahloo, M. Soltanifar, A. Ebrahimnejad, and S. M. Manosourzadeh, Relationship between MOLP and DEA based on output-orientated CCR dual model, *Expert Systems with Applications*, 37 (6) (2010), 4331-4336.

[16] T. Joro, P. Korhonen, and S. zionts, An interactive approach to improve estimates of value efficiency in data envelopments analysis, *European Journal of Operational Research*, 149 (2003), 688-699.

[17] N. Malekmohammadi, F. Hosseinzadeh Lotfi, and A. B. Jaafar, Target setting in data envelopment analysis using MOLP, *Applied Mathematical Modelling*, 35 (2011), 328-338.

[18] J. Pourmahmoud, A. Hatami-Marbini, and E. Babazadeh, A comment on a new super-efficiency model in the presence of negative data, *Journal of the Operational Research Society*, 67 (3) (2016), 530-534.

[19] M. C. A. Silva Portela, E. Thanassoulis, and G. Simpson, A directional distance approach to deal with negative data in DEA: An application to bank branches, *Journal of the Operational Research Society*, 55 (10) (2004), 1111-1121.

[20] M. Soleimani-damaneh, P. Korhonen, and J. Wallenius, On value efficiency, *Optimization*, 63 (4) (2014), 617-631.

[21] E. Thanassoulis and R. Allen, Simulating weights restrictions in data envelopment analysis by means of unobserved DMUs, *Management Science*, 44 (4) (1998), 586-594.

[22] R. Thompson, L. Langemeier, C. Lee,and R. Thrall, The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics*, 46 (1990), 93-108.

[23] H. Wong and J. Beasley, Restricting weight flexibility in data envelopment analysis. *Journal of Operational Research Society*, 41 (9) (1990), 829-835.

[24] B. Y. H. Wong, M. Luque, and J. B. Yang, Using interactive multiobjective methods to solve DEA problems with value judgements, *Computers and Operations Research*, 36 (2009), 623-636.

[25] J. B. Yang, B. Y. H. Wong, Xu. Dong-Ling, and T. J. Stewart, Integrating DEA-oriented performance assessment and target setting using interactive MOLP methods, *European Journal of Operational Research*, 195 (2009), 205-222.

[26] J. Zhu, Data envelopment analysis with preference structure, *Journal of Operational Research Society*, 47 (1996), 136-150.

[27] M. Zohrehbandian, Using ZiontsWallenius method to improve estimate of value efficiency in DEA, *Applied Mathematical Modelling*, 35 (2011), 3769-3776.

[28] S. Zionts and J. Wallenius, An interactive programming method for solving the multiple criteria problem, *Management Science*, 22 (6) (1976), 652-663.

Javad Gerami
Assistant Professor of Mathematics
Department of Mathematics
Shiraz Branch, Islamic Azad University
Shiraz, Iran
E-mail: Geramijavad@gmail.com