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Fixed Point Theorems for F-Contractions in Dislocated S_b-Metric Spaces

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Abstract. In this paper, we introduce the notion of dislocated S_b-metric space and describe some fixed point theorems concerning F-contraction in the setup of such spaces. We provide some examples to verify the effectiveness and applicability of our main results.

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1. Introduction

In the year 1989, Bakhtin introduced the concept of b-metric spaces as a generalization of metric spaces [2]. Later several authors proved so many results on b-metric spaces [5, 6, 7, 8]. Mustafa and Sims defined the notion of a generalized metric space, which is called a G-metric space and established a fixed point theory for various mappings in this new structure [9]. Shoaib et al. [15] obtained some fixed point theorems for

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a contractive dominated self-mapping in an ordered complete dislocated quasi G-metric space. Aghajani, Abbas, and Roshan presented a new type of metric which is called G_b-metric and studied some properties of this metric [1]. Sedghi, Shobe, and Aliouche gave the notion of an S-metric space and obtained some fixed point theorems for a self-mapping on a complete S-metric space [14]. Recently, Sedghi et al. [13] defined S_b-metric spaces, using the concept of S-metric spaces.

On the other hand, Wardowski [16] introduced a new contraction, the so-called F-contraction, and proved some fixed point results for such mappings on a complete metric space. After that, Wardowski and Dung [17] defined the notion of F-weak contractions in metric spaces and generalized the theorem of Wardowski [16]. Dung and Hang [3] studied the notion of a generalized F-contraction and extended a fixed point theorem for such mappings. Piri and Kumam [11] further described a large class of functions by replacing condition $(F3')$ instead of the condition $(F3)$ in the definition of F-contraction.

Motivated by these researches, in this paper we introduce the notion of dislocated S_b-metric spaces and prove some fixed point theorems for F-contractions in complete dislocated S_b-metric spaces. We provide some examples to verify the effectiveness and applicability of our results.
We begin with some definitions and auxiliary facts which will be needed further on.

Throughout this paper, $\mathbb{R}$, $\mathbb{R}_+$, and $\mathbb{N}$ denote the set of all real numbers, the set of all nonnegative real numbers, and the set of all positive integers, respectively.

Definition 1.1. [13] *Let X be a nonempty set. An S-metric on X is a function $S : X^3 \to [0, \infty)$ that satisfies the following conditions for each $x, y, z, a \in X$:*

(S1) $0 < S(x, y, z)$ *for all* $x, y, z \in X$ *with* $x \neq y \neq z \neq x$,

(S2) $S(x, y, z) = 0$ *if and only if* $x = y = z$,

(S3) $S(x, y, z) \leqslant S(x, x, a) + S(y, y, a) + S(z, z, a)$.

Then the pair (X, S) is called an S-metric space.

Lemma 1.2. [14] *Let (X, S) be an S-metric space. Then for each $x, y \in X$ we have $S(x, x, y) = S(y, y, x)$.*

Definition 1.3. [13] *Let (X, S) be an S-metric space, let $\{x_n\}$ be a sequence in X, and let $x \in X$. Then*

• *(i) the sequence $\{x_n\}$ is said to be a Cauchy sequence if, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S(x_n, x_n, x_m) < \varepsilon$ for each $m, n \geqslant n_0$;*

• *(ii) the sequence $\{x_n\}$ is said to be convergent to a point $x \in X$ if, for each $\varepsilon > 0$, a positive integer n_0 exists such that for all $n \geqslant n_0$, $S(x, x, x_n) < \varepsilon$.*

• *(iii) (X, S) is said to be complete if every Cauchy sequence is convergent.*

Remark 1.4. • *Let (X, d) be a metric space. Put $S_d(x, y, z) = d(x, z) + d(y, z)$ for all $x, y, z \in X$; then (X, S_d) is an S-metric space. S_d is called the S-metric generated by d. It can be easily shown that (X, d) is complete if and only if (X, S_d) is complete [10].*
• *Let $X = \mathbb{R}$. Consider the function $S(x, y, z) = |x - z| + |x + z - 2y|$ for all $x, y, z \in \mathbb{R}$. Then (X, S) is an S-metric space and $S \neq S_d$ for all metrics d; see [10].*

Definition 1.5. [13] *Let X be a nonempty set, and let $b \geqslant 1$ be a given real number. Suppose that a mapping $S_b : X^3 \to \mathbb{R}_+$ is a function satisfying the following properties:*

(S_b1) $0 < S_b(x, y, z)$ *for all $x, y, z \in X$ with $x \neq y \neq z \neq x$,*

(S_b2) $S_b(x, y, z) = 0$ *if and only if $x = y = z$,*

(S_b3) $S_b(x, y, z) \leqslant b(S_b(x, x, a) + S_b(y, y, a) + S_b(z, z, a))$ *for all $x, y, z, a \in X$.*

Then the function S_b is called an S_b-metric on X and the pair (X, S_b) is called an S_b-metric space.

Definition 1.6. *Let (X, S_b) be an S_b-metric space, and let $b > 1$. Then S_b is called symmetric if*

$$S_b(x, x, y) = S_b(y, y, x) \tag{1}$$

for all $x, y \in X$.

According to Lemma 1.2, the symmetry condition (1) is automatically satisfied by an S-metric.

Definition 1.7. [13] *If (X, S_b) is an S_b-metric space, then a sequence $\{x_n\}$ in X is said to be:*

(1) Cauchy if, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S_b(x_n, x_n, x_m) < \varepsilon$ for each $m, n \geqslant n_0$.

(2) convergent to a point $x \in X$ if, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S_b(x_n, x_n, x) < \varepsilon$ or $S_b(x, x, x_n) < \varepsilon$ for all $n \geqslant n_0$. In this case, we denote it by $\lim_{n\to\infty} x_n = x$.

Definition 1.8. [13] *An S_b-metric space (X, S_b) is called complete if every Cauchy sequence is convergent in X.*

Example 1.9. [4] Let $X = [0, 1]$. Define $S_b : X^3 \to \mathbb{R}_+$ by $S_b(x, y, z) = (|y + z - 2x| + |y - z|)^2$; then (X, S_b) is a complete S_b-metric space with $b = 4$.

We conclude this section, recalling the following fixed point theorems of Wardowski and Dung [16, 17]. Before this, we quote some definitions.

Definition 1.10. *[16] Let $\mathcal{F}$ be the family of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ such that:*

(F1) F is strictly increasing, that is, for all $\alpha, \beta \in \mathbb{R}_+$ if $\alpha < \beta$, then $F(\alpha) < F(\beta)$;

(F2) for each sequence $\{\alpha_n\}$ of positive numbers, $\lim_{n\to+\infty} \alpha_n = 0$ if and only if $\lim_{n\to+\infty} F(\alpha_n) = -\infty$;

(F3) there exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Recently, Piri and Kumam [11] described a large class of functions by replacing the condition $(F3)$ by the following one:

$(F3')$ F is continuous on $(0, +\infty)$.

They denote by $\mathfrak{F}$ the family of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ which satisfy conditions $(F1)$, $(F2)$, and $(F3')$.

Example 1.11. [11] Let $F_1(\alpha) = -\frac{1}{\alpha}$, $F_2(\alpha) = -\frac{1}{\alpha} + \alpha$, $F_3(\alpha) = \frac{1}{1-e^{\alpha}}$, $F_4(\alpha) = \frac{1}{e^{\alpha}-e^{-\alpha}}$, and $F_5(\alpha) = \ln \alpha$. Then $F_1, F_2, F_3, F_4, F_5 \in \mathfrak{F}$.

Definition 1.12. [16] *Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be an F-contraction on (X, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$,*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \leqslant F\big(d(x, y)\big).$$

Theorem 1.13. [16] *Let (X, d) be a complete metric space, and let $T : X \to X$ be an F-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$, the sequence $\{T^n x\}$ converges to x^*.*

There is an analogue to the theorem above for $F \in \mathfrak{F}$ (see [11]).

Definition 1.14. [17] *Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be an F-weak contraction on (X, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$,*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \leqslant F\big(M(x, y)\big),$$

where

$$M(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\right\}.$$

Theorem 1.15. [17] *Let (X, d) be a complete metric space, and let $T : X \to X$ be an F-weak contraction. If T or F is continuous, then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}$ converges to x^*.*

2. Main Results

In this section, we introduce the concept of dislocated S_b-metric space and then we demonstrate some fixed point results for F-contractions in the setup such spaces. Our results are remarkable for two reasons: first, the dislocated S_b-metric is more general; second, the contractivity condition involves auxiliary functions form a wider class.

Definition 2.1. *Let X be a nonempty set, and let $b \geqslant 1$ be a given real number. A function $S_b : X^3 \to \mathbb{R}_+$ is a dislocated S_b-metric if, for all $x, y, z, a \in X$, the following conditions are satisfied:*

(dS_b1) $S_b(x, y, z) = 0$ *implies* $x = y = z$,

(dS_b2) $S_b(x, y, z) \leqslant b\big(S_b(x, x, a) + S_b(y, y, a) + S_b(z, z, a)\big)$.

A dislocated S_b-metric space is a pair (X, S_b) such that X is a nonempty set and S_b is a dislocated S_b-metric on X. In the case when $b = 1$, S_b is denoted by S and it is called the dislocated S-metric, and the pair (X, S) is called the dislocated S-metric space.

Definition 2.2. *Let (X, S_b) be a dislocated S_b-metric space, let $\{x_n\}$ be any sequence in X, and let $x \in X$. Then:*

(i) *the sequence $\{x_n\}$ is said to be a Cauchy sequence in (X, S_b) if, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $S_b(x_n, x_n, x_m) < \varepsilon$ for each $m, n \geqslant n_0$.*

(ii) *the sequence $\{x_n\}$ is said to be convergent to x if, for each $\varepsilon > 0$, a positive integer n_0 exists such that $S_b(x, x, x_n) < \varepsilon$ for all $n \geqslant n_0$ and we denote it by $\lim_{n\to\infty} x_n = x$.*

(iii) *(X, S_b) is said to be complete if every Cauchy sequence is convergent.*

The following examples show that a dislocated S_b-metric is not necessarily a dislocated S-metric.

Example 2.3. Let $X = \{0, 1, 2\}$. Define $S_b : X^3 \to \mathbb{R}_+$ by

$$S_b(x, y, z) = \begin{cases} \frac{1}{4}, & x = y \neq z \textit{ or } x \neq y \neq z \neq x, \\ \frac{1}{3}, & x = z \neq y, \\ \frac{1}{2}, & y = z \neq x, \\ \frac{1}{5}, & x = y = z = 0, \\ 0, & x = y = z = 1 \text{ or } 2, \end{cases}$$

for all $x, y, z \in X$. It is easy to show that (X, S_b) is a complete dislocated S_b-metric space when $b = 2$. It is not a dislocated S-metric space. For this, we show that (dS_b2) does not hold when $b = 1$. To prove this, let $x = 1$ and $y = z = 2$. Then $\frac{1}{2} = S(1,2,2) \nleq S(1,1,2) + 2S(2,2,2) = \frac{1}{4}$. Note that S_b is symmetric, in the sense of Definition 1.6.

Example 2.4. Let $X = [0,1]$; then the mapping $S_b : X^3 \to \mathbb{R}_+$ defined by $S_b(x,y,z) = x + \frac{y}{2} + 2z$ is a complete dislocated S_b-metric on X with $b = \frac{3}{2}$. Obviously, S_b is not symmetric. Also, it is not a dislocated S-metric space. Indeed, we have

$$2 = S(0,0,1) \nleqslant 2S(0,0,0) + S(1,1,0) = \frac{3}{2}.$$

Example 2.5. Let $X = \mathbb{R}_+$; then the mapping $S : X^3 \to \mathbb{R}_+$, defined by $S(x,y,z) = x+2y+z$, is a complete dislocated S-metric on X, which is not symmetric, since $1 = S(0,0,1) \neq S(1,1,0) = 3$.

Definition 2.6. *Let (X, S_b) be a dislocated S_b-metric space. A mapping $T : X \to X$ is said to be an F-contraction on (X, S_b) if there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that for all $x, y \in X$,*

$$S_b(Tx,Tx,Ty) > 0 \Rightarrow \tau + F\big(b^2 S_b(Tx,Tx,Ty)\big) \leqslant F\big(S_b(x,x,y)\big). \quad (2)$$

Our first main result is the following.

Theorem 2.7. *Let (X, S_b) be a complete dislocated S_b-metric space, and let $T : X \to X$ be an F-contraction. Then T has a unique fixed point in X.*

Proof. Let $x_0 \in X$ be arbitrary and fixed. Let $\{x_n\}$ be the Picard sequence of T based on x_0, that is, $x_{n+1} = Tx_n$ for $n = 0, 1, 2, \ldots$. If $n_0 \in \mathbb{N}$ exists such that $S_b(x_{n_0}, x_{n_0}, x_{n_0+1}) = 0$, then x_{n_0} is a fixed point of T and the existence part is proved. On the contrary case, assume that $S_b(x_n, x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. Applying the contractivity condition (2), we get

$$\tau + F\big(b^2 S_b(Tx_n, Tx_n, T^2 x_n)\big) \leqslant F\big(S_b(x_n, x_n, Tx_n)\big). \quad (3)$$

We will show that

$$S_b(x_{n+1}, x_{n+1}, Tx_{n+1}) < S_b(x_n, x_n, Tx_n), \quad (4)$$

for all $n \in \mathbb{N}$. Suppose, on the contrary, that $S_b(x_{n_0+1}, x_{n_0+1}, Tx_{n_0+1}) \geqslant S_b(x_{n_0}, x_{n_0}, Tx_{n_0})$ for some $n_0 \in \mathbb{N}$. From (3), we have

$$F\big(b^2 S_b(x_{n_0+1}, x_{n_0+1}, Tx_{n_0+1})\big) \leqslant F\big(S_b(x_{n_0}, x_{n_0}, Tx_{n_0})\big) - \tau,$$

which together condition $(F1)$ implies that

$$S_b(x_{n_0+1}, x_{n_0+1}, Tx_{n_0+1}) < S_b(x_{n_0}, x_{n_0}, Tx_{n_0}).$$

It gives us a contradiction. Therefore, (4) holds. So, $\{S_b(x_n, x_n, Tx_n)\}$ is a decreasing positive sequence in $\mathbb{R}_+$ and it converges to some $A \geqslant 0$. We claim that $A = 0$. To prove the claim, let it be untrue, and let $A > 0$. Then, for any $\varepsilon > 0$, it is possible to find a positive integer m such that

$$S_b(x_m, x_m, Tx_m) < A + \varepsilon.$$

By $(F1)$, we get

$$F\big(S_b(x_m, x_m, Tx_m)\big) < F(A + \varepsilon). \tag{5}$$

Since $S_b(x_n, x_n, x_{n+1}) > 0$ for all n, then by repeatedly using (2) and taking (5) into account, we obtain

$$\begin{aligned}
&F\big(b^2 S_b(T^n x_m, T^n x_m, T^{n+1} x_m)\big)\\
&\qquad \leqslant F\big(S_b(T^{n-1} x_m, T^{n-1} x_m, T^n x_m)\big) - \tau\\
&\qquad \leqslant F\big(b^2 S_b(T^{n-1} x_m, T^{n-1} x_m, T^n x_m)\big) - \tau\\
&\qquad \leqslant F\big(S_b(T^{n-2} x_m, T^{n-2} x_m, T^{n-1} x_m)\big) - 2\tau\\
&\qquad \vdots\\
&\qquad \leqslant F\big(S_b(x_m, x_m, Tx_m)\big) - n\tau\\
&\qquad < F(A + \varepsilon) - n\tau.
\end{aligned}$$

Letting $n \to \infty$ in the above inequality, we get

$$\lim_{n \to +\infty} F\big(b^2 S_b(T^n x_m, T^n x_m, T^{n+1} x_m)\big) = -\infty.$$

Hence, from $(F2)$, we derive $\lim_{n\to+\infty} S_b(T^n x_m, T^n x_m, T^{n+1}x_m) = 0$. Then $S_b(x_{m+n}, x_{m+n}, Tx_{m+n}) < A$ for n large enough. It is a contradiction with the definition of A; therefore,

$$\lim_{n\to+\infty} S_b(x_n, x_n, Tx_n) = 0. \tag{6}$$

Now, we prove that $\{x_n\}$ is a Cauchy sequence in (X, S_b). Suppose the contrary. Then $\varepsilon > 0$ exists for which we can find monotonically increasing sequences $\{p(n)\}$ and $\{q(n)\}$ of natural numbers such that

$$p(n) > q(n) > n,$$

$$S_b(x_{q(n)}, x_{q(n)}, x_{p(n)}) \geqslant \varepsilon$$
$$S_b(x_{q(n)}, x_{q(n)}, x_{p(n)-1}) < \varepsilon. \tag{7}$$

Regarding (2) and (7), we can write

$$\begin{aligned} F\big(b^2 S_b(x_{q(n)}, x_{q(n)}, x_{p(n)})\big) &\leqslant F\big(S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1})\big) - \tau \\ &< F\big(S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1})\big), \end{aligned} \tag{8}$$

which together $(F1)$ implies

$$S_b(x_{q(n)}, x_{q(n)}, x_{p(n)}) < S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1}).$$

Using this together with (dS_b2), we get

$$\begin{aligned} \varepsilon &\leqslant S_b(x_{q(n)}, x_{q(n)}, x_{p(n)}) \\ &< S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1}) \\ &\leqslant 2bS_b(x_{q(n)-1}, x_{q(n)-1}, x_{q(n)}) + bS_b(x_{p(n)-1}, x_{p(n)-1}, x_{q(n)}) \\ &\leqslant 2bS_b(x_{q(n)-1}, x_{q(n)-1}, x_{q(n)}) + 2b^2 S_b(x_{p(n)-1}, x_{p(n)-1}, x_{p(n)-1}) \\ &\quad + b^2 S_b(x_{q(n)}, x_{q(n)}, x_{p(n)-1}) \\ &\leqslant 2bS_b(x_{q(n)-1}, x_{q(n)-1}, x_{q(n)}) + 6b^3 S_b(x_{p(n)-1}, x_{p(n)-1}, x_{p(n)}) \\ &\quad + b^2 S_b(x_{q(n)}, x_{q(n)}, x_{p(n)-1}). \end{aligned}$$

By virtue of this fact and in view of (6) and (7), we have

$$\varepsilon \leqslant \limsup_{n\to+\infty} S_b(x_{q(n)}, x_{q(n)}, x_{p(n)}) \leqslant \limsup_{n\to+\infty} S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1}) \leqslant b^2\varepsilon. \tag{9}$$

Using (8), (9), $(F1)$, and $(F3)$, we find that

$$\begin{aligned} F(b^2\varepsilon) &\leqslant F\big(b^2 \limsup_{n\to+\infty} S_b(x_{q(n)}, x_{q(n)}, x_{p(n)})\big) \\ &\leqslant F\big(\limsup_{n\to+\infty} S_b(x_{q(n)-1}, x_{q(n)-1}, x_{p(n)-1})\big) - \tau, \\ &\leqslant F(b^2\varepsilon) - \tau, \end{aligned}$$

which leads to a contradiction with the assumption $\tau > 0$. Therefore $\{x_n\}$ is a Cauchy sequence in the complete dislocated S_b-metric space X. Then, $v \in X$ exists such that $x_n \to v$ as $n \to \infty$, that is, for any $\varepsilon > 0$, there exists $n_1 \in \mathbb{N}$ such that $S_b(v, v, x_n) < \varepsilon$ for all $n \geqslant n_1$. We are going to show that v is a fixed point of T. First note that if $S_b(Tv, Tv, Tx_n) = 0$, for some $n \geqslant n_1$, then, from (dS_b2), we obtain

$$S_b(Tv, Tv, v) \leqslant 2bS_b(Tv, Tv, Tx_n) + bS_b(v, v, Tx_n) < b\varepsilon.$$

On the other hand, if for each $n \geqslant n_1$, $S_b(Tv, Tv, Tx_n) > 0$, then, using (2), we have

$$F\big(b^2 S_b(Tv, Tv, Tx_n)\big) \leqslant F\big(S_b(v, v, x_n)\big) - \tau.$$

It enforces that $S_b(Tv, Tv, Tx_n) < S_b(v, v, x_n) < \varepsilon$ for each $n \geqslant n_1$. From (dS_b2) it follows that

$$\begin{aligned} S_b(Tv, Tv, v) &< 2bS_b(Tv, Tv, Tx_n) + bS_b(v, v, Tx_n) \\ &< 3b\varepsilon \end{aligned}$$

for each $n \geqslant n_1$. Since $\varepsilon > 0$ is arbitrary, in each cases, we deduce $S_b(Tv, Tv, v) = 0$, that is $Tv = v$. Hence, v is a fixed point of T. Next, we study the uniqueness of the fixed point of T. Assume that T has two different fixed points v_1 and v_2. Then $S_b(v_1, v_1, v_2) > 0$, and from condition (2) we get

$$0 < \tau \leqslant F\big(S_b(v_1, v_1, v_2)\big) - F\big(b^2 S_b(Tv_1, Tv_1, Tv_2)\big)$$

$$\leqslant F\big(b^2 S_b(v_1, v_1, v_2)\big) - F\big(b^2 S_b(v_1, v_1, v_2)\big) = 0,$$

which is a contraction. Then $S_b(v_1, v_1, v_2) = 0$, and so $v_1 = v_2$ which is a contradiction. Therefore, the fixed point is unique. $\square$

Remark 2.8. *Let (X,d) be a complete metric space. If we take $S(x,y,z) = \frac{d(x,z)+d(y,z)}{2}$, then (X,S) is a complete S-metric space and $S(x,x,y) = d(x,y)$. So Theorem 1.13 (see [16]) is a special case of Theorem 2.7.*

Now, we present an example illustrating the applicability of our main result.

Example 2.9. Let (X,S_b) be the same as Example 2.4. Define the mapping $T : X \to X$ by

$$T(x) = \begin{cases} \frac{x}{40}, & x \in [0,1), \\ \frac{1}{10}, & x = 1, \end{cases}$$

and take $F(\alpha) = \ln \alpha$; we obtain the result that T is an F-contraction with $0 < \tau \leqslant \ln 3$. To see this, let us consider the following calculations. First observe that

$$\tau + F\big(b^2 S_b(Tx,Tx,Ty)\big) \leqslant F\big(S_b(x,x,y)\big) \Leftrightarrow \ln \frac{S_b(x,x,y)}{b^2 S_b(Tx,Tx,Ty)} \geqslant \tau.$$

We distinguish the following cases:

(i) For $x = 1$ and $0 \leqslant y < 1$, we have $S_b(Tx,Tx,Ty) > 0$ and

$$\ln \frac{\frac{3}{2} + 2y}{\frac{9}{80}y + \frac{27}{80}} \geqslant \tau.$$

(ii) For $0 \leqslant x < 1$, $y = 1$, we have $S_b(Tx,Tx,Ty) > 0$ and

$$\ln \frac{\frac{3}{2}x + 2}{\frac{27}{320}x + \frac{18}{40}} \geqslant \tau.$$

(iii) For $x = y = 1$, we have $S_b(T1,T1,T1) = \frac{7}{20}$ and

$$\ln \frac{40}{9} \geqslant \tau.$$

(iv) For $0 \leqslant x < y < 1$, we have $S_b(Tx,Tx,Ty) > 0$ and

$$\ln \frac{\frac{3}{2}x + 2y}{\frac{27}{320}x + \frac{9}{80}y} \geqslant \tau.$$

(v) For $0 \leqslant y < x < 1$, we have $S_b(Tx,Tx,Ty) > 0$ and

$$\ln \frac{\frac{3}{2}x + 2y}{\frac{27}{160}x + \frac{9}{40}y} \geqslant \tau.$$

(vi) For $0 < x = y < 1$, we have $S_b(Tx, Tx, Ty) > 0$ and

$$\ln \frac{160}{9} \geqslant \tau.$$

From the above cases it may readily be seen that if $0 < \tau \leqslant \ln 3$, then $\ln \frac{S_b(x,x,y)}{b^2 S_b(Tx,Tx,Ty)} \geqslant \tau$. Hence T is an F-contraction. So, all the required hypotheses of Theorem 2.7. are satisfied and T has the unique fixed point 0.

For $F_1(\alpha) = \frac{1}{1-e^{\alpha}}$, $F_2(\alpha) = \ln \alpha + \alpha$, $F_3(\alpha) = \frac{-1}{\alpha} + \alpha$, and $F_4(\alpha) = \frac{-1}{\sqrt{\alpha}}$, we also include the range of changes of τ in Table 1.

Table 1: The range of changes of τ

F_1	$(0, 0.8037]$
F_2	$(0, 1.9016]$
F_3	$(0, 0.421296]$
F_4	$(0, 0.5923]$

On the other hand, $F(\alpha) = \frac{1}{e^{\alpha} - e^{-\alpha}}$ does not satisfy condition (2). To see this, take $x = 1$, $y \in [0, 1)$. Then

$$F\big(S_b(1,1,y)\big) - F\big(b^2 S_b(\frac{1}{10}, \frac{1}{10}, \frac{y}{40})\big) = \frac{1}{e^{\frac{3}{2}+2y} - e^{-\frac{3}{2}-2y}} - \frac{1}{e^{\frac{9y+27}{80}} - e^{-\frac{9y+27}{80}}}. \quad (10)$$

Minimum value of (10) is -7.29579×10^{10}, and it occurs at $y = 5.74918 \times 10^{-11}$. It enforces that $\tau < 0$, which is a contradiction.

Definition 2.10. *Let (X, S_b) be a dislocated S_b-metric space. A mapping $T : X \to X$ is said to be an F-weak contraction on (X, S_b) if there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that for all $x, y \in X$,*

$$S_b(Tx, Tx, Ty) > 0 \Rightarrow \tau + F\big(b^2 S_b(Tx, Tx, Ty)\big) \leqslant F\big(M(x, y)\big), \quad (11)$$

where

$$M(x, y) = \max\big\{S_b(x, x, y), S_b(Tx, Tx, Ty), \frac{S_b(y, y, Tx)}{10b^2}, \frac{S_b(y, y, Ty)}{10b}\big\}.$$

Our second fixed point result is a version of Theorem 1.15, for F-weak contractions in dislocated S_b-metric spaces.

Theorem 2.11. *Let (X, S_b) be a complete dislocated S_b-metric space, and let $T : X \to X$ be an F-weak contraction satisfying the following condition:*

$$\max\Big\{\frac{S_b(y,y,Ty)}{10b}, \frac{S_b(y,y,Ty)}{5b} + \frac{S_b(Tx,Tx,Ty)}{10b}\Big\} \leqslant S_b(Tx,Tx,Ty).$$

Then T has a unique fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary and fixed. Define a sequence $\{x_n\}$ in X such that $x_1 = Tx_0$ and $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. We may suppose that $x_{n+1} \neq x_n$ for all n, otherwise T has obviously a fixed point. Then $S_b(x_n, x_n, x_{n+1}) > 0$ for any $n \in \mathbb{N} \cup \{0\}$ and hence (11) implies that

$$F\big(b^2 S_b(Tx_{n-1}, Tx_{n-1}, Tx_n)\big) \leqslant F\big(M(x_{n-1}, x_n)\big) - \tau. \tag{12}$$

Now, using (dS_b2), we obtain

$$\begin{aligned}
&\max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(Tx_{n-1},Tx_{n-1},Tx_n)\big\}\\
&\qquad\leqslant M(x_{n-1},x_n)\\
&\qquad= \max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(Tx_{n-1},Tx_{n-1},Tx_n),\\
&\qquad\qquad \frac{S_b(x_n,x_n,Tx_{n-1})}{10b^2}, \frac{S_b(x_n,x_n,Tx_n)}{10b}\big\}\\
&\qquad\leqslant \max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(Tx_{n-1},Tx_{n-1},Tx_n),\\
&\qquad\qquad 3\frac{S_b(x_n,x_n,Tx_n)}{10b}, \frac{S_b(x_n,x_n,Tx_n)}{10b}\big\}\\
&\qquad= \max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(x_n,x_n,x_{n+1})\big\},
\end{aligned}$$

then (12) becomes

$$F\big(b^2 S_b(Tx_{n-1},Tx_{n-1},Tx_n)\big) \leqslant F\big(\max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(x_n,x_n,x_{n+1})\big\}\big) - \tau.$$

If we assume that

$$\max\big\{S_b(x_{n-1},x_{n-1},x_n), S_b(x_n,x_n,x_{n+1})\big\} = S_b(x_n,x_n,x_{n+1})$$

for some n, then from (12) we have

$$\begin{aligned}
F\big(b^2 S_b(Tx_{n-1},Tx_{n-1},Tx_n)\big) &\leqslant F\big(S_b(Tx_{n-1},Tx_{n-1},Tx_n)\big) - \tau\\
&< F\big(S_b(Tx_{n-1},Tx_{n-1},Tx_n)\big).
\end{aligned}$$

Using condition $(F1)$, we conclude that $S_b(x_n, x_n, x_{n+1}) < S_b(x_n, x_n, x_{n+1})$, which is a contradiction. Therefore

$$\max\{S_b(x_{n-1}, x_{n-1}, x_n), S_b(x_n, x_n, x_{n+1})\} = S_b(x_{n-1}, x_{n-1}, x_n)$$

for each n. Applying again (12) and $(F1)$, we deduce that

$$S_b(x_n, x_n, x_{n+1}) < S_b(x_{n-1}, x_{n-1}, x_n).$$

That is, $\{S_b(x_n, x_n, x_{n+1})\}$ is a strictly decreasing positive sequence in $\mathbb{R}_+$ and it converges to some $A \geqslant 0$. We declare that $A = 0$. Suppose, it is not true, then $A > 0$. For each $\varepsilon > 0$, let us choose $m \in \mathbb{N}$ such that

$$S_b(x_m, x_m, Tx_m) < A + \varepsilon.$$

From $(F1)$, we have

$$F\big(S_b(x_m, x_m, Tx_m)\big) < F(A + \varepsilon). \tag{13}$$

Since T is an F-weak contraction and taking into account $S_b(Tx_m, Tx_m, T^2x_m) > 0$, we get

$$\tau + F(b^2 S_b(Tx_m, Tx_m, T^2x_m)) \leqslant F\big(M(x_m, Tx_m)\big). \tag{14}$$

Since $M(x_m, Tx_m) = \max\Big\{S_b(x_m, x_m, Tx_m), S_b(Tx_m, Tx_m, T^2x_m)\Big\}$, then from (11) and $(F1)$, we get

$$\max\{S_b(Tx_m, Tx_m, T^2x_m), S_b(x_m, x_m, Tx_m)\} = S_b(x_m, x_m, Tx_m).$$

Hence (14) becomes

$$F\big(b^2 S_b(Tx_m, Tx_m, T^2x_m)\big) \leqslant F\big(S_b(x_m, x_m, Tx_m)\big) - \tau.$$

This yields

$$\begin{aligned} F\big(b^2 S_b(T^2x_m, T^2x_m, T^3x_m)\big) &\leqslant F\big(S_b(Tx_m, Tx_m, T^2x_m)\big) - \tau \\ &\leqslant F\big(b^2 S_b(Tx_m, Tx_m, T^2x_m)\big) - \tau \\ &\leqslant F\big(S_b(x_m, x_m, Tx_m)\big) - 2\tau. \end{aligned}$$

Continuing the above process and using (13), we observe that

$$\begin{aligned}
&F\big(b^2 S_b(T^n x_m, T^n x_m, T^{n+1} x_m)\big)\\
&\qquad\leqslant F\big(S_b(T^{n-1}x_m, T^{n-1}x_m, T^n x_m)\big) - \tau\\
&\qquad\leqslant F\big(b^2 S_b(T^{n-1}x_m, T^{n-1}x_m, T^n x_m)\big) - \tau\\
&\qquad\leqslant F(S_b(T^{n-2}x_m, T^{n-2}x_{m-1}, T^{n-1}x_{m-1})\big) - 2\tau\\
&\qquad\quad\vdots\\
&\qquad\leqslant F\big(S_b(x_m, x_m, Tx_m)\big) - n\tau\\
&\qquad< F(A+\varepsilon) - n\tau.
\end{aligned}$$

Passing to the limit $n \to +\infty$ in the above relation, we obtain

$$\lim_{n\to+\infty} F\big(b^2 S_b(T^n x_m, T^n x_m, T^{n+1} x_m)\big) = -\infty.$$

It follows from $(F2)$ that

$$\lim_{n\to+\infty} S_b(T^n x_m, T^n x_m, T^{n+1} x_m) = 0.$$

So, $S_b(T^n x_m, T^n x_m, T^{n+1} x_m) = S_b(x_{m+n}, x_{m+n}, Tx_{m+n}) < A$ for n sufficiently large, which is a contradiction with the definition of A. Therefore

$$\lim_{n\to+\infty} S_b(x_n, x_n, x_{n+1}) = 0. \tag{15}$$

Next, we intend to show that the sequence $\{x_n\}$ is a Cauchy sequence in (X, S_b). Arguing by contradiction, we assume that $\varepsilon > 0$, and the sequences $\{p(n)\}$ and $\{q(n)\}$ of natural numbers exist such that, for all $n \in \mathbb{N}$,

$$p(n) > q(n) > n,$$

$$S_b(x_{q(n)}, x_{q(n)}, x_{p(n)}) \geqslant \varepsilon,$$

$$S_b(x_{q(n)}, x_{q(n)}, x_{p(n)-1})) < \varepsilon. \tag{16}$$

In the light of (16) and condition (11), we find that

$$F(b^2 S_b(Tx_{q(n)-1}, Tx_{q(n)-1}, Tx_{p(n)-1}) \leqslant F(M(x_{q(n)-1}, x_{p(n)-1})\big) - \tau. \tag{17}$$

Applying (dS_b2) and our hypothesis, we get

$$
\begin{aligned}
\max&\{S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}),S_b(Tx_{q(n)-1},Tx_{q(n)-1},Tx_{p(n)-1})\}\\
&\leqslant M(x_{q(n)-1},x_{p(n)-1})\\
&= \max\Big\{S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}),\\
&\quad S_b(Tx_{q(n)-1},Tx_{q(n)-1},Tx_{p(n)-1}),\frac{S_b(x_{p(n)-1},x_{p(n)-1},Tx_{q(n)-1})}{10b^2},\\
&\quad \frac{S_b(x_{p(n)-1},x_{p(n)-1},Tx_{p(n)-1})}{10b}\Big\}\\
&\leqslant \max\Big\{S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}),S_b(Tx_{q(n)-1},Tx_{q(n)-1},Tx_{p(n)-1}),\\
&\quad \frac{S_b(x_{p(n)-1},x_{p(n)-1},Tx_{p(n)-1})}{5b}+\frac{S_b(Tx_{q(n)-1},Tx_{q(n)-1},Tx_{p(n)-1})}{10b},\\
&\quad \frac{S_b(x_{p(n)-1},x_{p(n)-1},Tx_{p(n)-1})}{10b}\Big\}\\
&\leqslant \max\Big\{S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}),S_b(Tx_{q(n)-1},Tx_{q(n)-1},Tx_{p(n)-1})\Big\}
\end{aligned}
$$

As a consequence of (17) and $(F1)$, we have

$$
\max\Big\{S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}),S_b(x_{q(n)},x_{q(n)},x_{p(n)})\Big\}=S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}).
$$

Accordingly, (17) becomes

$$
F\big(b^2S_b(x_{q(n)},x_{q(n)},x_{p(n)})\big)\leqslant F\big(S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1})\big)-\tau,
$$

and so using $(F1)$, we get

$$
S_b(x_{q(n)},x_{q(n)},x_{p(n)})\big)<S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}). \tag{18}
$$

By (16), (18), and using (dS_b2), we obtain

$$
\begin{aligned}
\varepsilon &\leqslant S_b(x_{q(n)},x_{q(n)},x_{p(n)})\\
&< S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1})\\
&\leqslant 2bS_b(x_{q(n)-1},x_{q(n)-1},x_{q(n)})+bS_b(x_{p(n)-1},x_{p(n)-1},x_{q(n)})\\
&\leqslant 2bS_b(x_{q(n)-1},x_{q(n)-1},x_{q(n)})+2b^2S_b(x_{p(n)-1},x_{p(n)-1},x_{p(n)-1})\\
&\quad +b^2S_b(x_{q(n)},x_{q(n)},x_{p(n)-1})\\
&\leqslant 2bS_b(x_{q(n)-1},x_{q(n)-1},x_{q(n)})+6b^3S_b(x_{p(n)-1},x_{p(n)-1},x_{p(n)})\\
&\quad +b^2S_b(x_{q(n)},x_{q(n)},x_{p(n)-1}).
\end{aligned}
$$

Regarding to (15) and (16), we have

$$\begin{aligned}\varepsilon &\leqslant \limsup_{n\to+\infty} S_b(x_{q(n)},x_{q(n)},x_{p(n)})\\ &\leqslant \limsup_{n\to+\infty} S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1}) \leqslant b^2\varepsilon.\end{aligned} \tag{19}$$

In view of (18) and (19) together with $(F1)$, $(F3')$, we have

$$\begin{aligned}F(b^2\varepsilon) &\leqslant F\big(b^2 \limsup_{n\to+\infty} S_b(x_{q(n)},x_{q(n)},x_{p(n)})\big)\\ &\leqslant F\big(\limsup_{n\to+\infty} S_b(x_{q(n)-1},x_{q(n)-1},x_{p(n)-1})\big)-\tau\\ &\leqslant F(b^2\varepsilon)-\tau.\end{aligned}$$

It is a contradiction with $\tau>0$, and it follows that $\{x_n\}$ is a Cauchy sequence in X. By completeness of (X,S_b), $\{x_n\}$ converges to some point $v\in X$. Therefore, for each $\varepsilon>0$, there exists $n_1\in\mathbb{N}$ such that

$$S_b(v,v,x_n)<\varepsilon, \tag{20}$$

for all $n\geqslant n_1$. We claim that v is a fixed point of T. If $S_b(Tv,Tv,Tx_n)=0$ for some $n\geqslant n_1$, then from (dS_b2) we deduce that

$$S_b(Tv,Tv,v)\leqslant 2S_b(Tv,Tv,Tx_n)+bS_b(v,v,Tx_n)<b\varepsilon.$$

Also, if $S_b(Tv,Tv,Tx_n)>0$ for all $n\geqslant n_1$, then from (11) we have

$$F\big(b^2S_b(Tv,Tv,Tx_n)\big)\leqslant F\big(M(v,x_n)\big)-\tau. \tag{21}$$

In view of (dS_b2) and our assumptions, we obtain

$$\begin{aligned}&\max\{S_b(v,v,x_n),S_b(Tv,Tv,Tx_n)\}\\ &\leqslant M(v,x_n)\\ &\leqslant \max\Big\{S_b(v,v,x_n),S_b(Tv,Tv,Tx_n),\frac{S_b(x_n,x_n,Tx_n)}{5b}\\ &\qquad+\frac{S_b(Tv,Tv,Tx_n)}{10b},\frac{S_b(x_n,x_n,Tx_n)}{10b}\Big\}\\ &\leqslant \max\Big\{S_b(v,v,x_n),S_b(Tv,Tv,Tx_n)\Big\}.\end{aligned}$$

Then (21) turns into

$$F\big(b^2S_b(Tv,Tv,Tx_n)\big)\leqslant F\Big(\max\Big\{S_b(v,v,x_n),S_b(Tv,Tv,Tx_n)\Big\}\Big)-\tau.$$

If $\max\Big\{S_b(v,v,x_n), S_b(Tv,Tv,Tx_n)\Big\} = S_b(Tv,Tv,Tx_n)$, then from (21) and $(F1)$ we lead to a contradiction and consequently

$$\max\Big\{S_b(v,v,x_n), S_b(Tv,Tv,Tx_n)\Big\} = S_b(v,v,x_n)$$

and so

$$F\big(b^2 S_b(Tv,Tv,Tx_n)\big) \leqslant F\big(S_b(v,v,x_n)\big) - \tau. \tag{22}$$

Employing (22) and $(F1)$, we derive that

$$S_b(Tv,Tv,Tx_n) < S_b(v,v,x_n). \tag{23}$$

From (dS_b2), (23), and (20) for each $n \geqslant n_1$, we have

$$S_b(Tv,Tv,v) \leqslant 2S_b(Tv,Tv,Tx_n) + bS_b(v,v,Tx_n) < 3b\varepsilon.$$

Thus, in each cases $S_b(Tv,Tv,v) = 0$ which implies that $Tv = v$. Hence, v is a fixed point of T. Finally, we show that T has at most one fixed point. Indeed, if $v_1, v_2 \in X$ are two fixed points of T such that $v_1 \neq v_2$, then from (11) we obtain

$$F\big(b^2 S_b(Tv_1,Tv_1,Tv_2)\big) \leqslant F\big(M(v_1,v_2)\big) - \tau. \tag{24}$$

Applying (dS_b2) and the assumption of the theorem, it follows that

$$\begin{aligned} S_b(v_1,v_1,v_2) &\leqslant M(v_1,v_2) \\ &\leqslant \max\Big\{S_b(v_1,v_1,v_2), S_b(Tv_1,Tv_1,Tv_2), \\ &\quad \frac{S_b(v_2,v_2,Tv_2)}{5b} + \frac{S_b(Tv_1,Tv_1,Tv_2)}{10b}, \\ &\quad \frac{S_b(v_2,v_2,Tv_2)}{10b}\Big\} \\ &\leqslant \max\Big\{S_b(v_1,v_1,v_2), S_b(Tv_1,Tv_1,Tv_2)\Big\} \\ &= S_b(v_1,v_1,v_2). \end{aligned}$$

Due to this fact and applying (24), we yield $S_b(v_1,v_1,v_2) < S_b(v_1,v_1,v_2)$, which is a contradiction. Hence $v_1 = v_2$. This completes the proof. $\square$

Example 2.12. Let $X = \mathbb{R}$, and define $S_b : \mathbb{R}^3 \to \mathbb{R}_+$ by $S_b(x,y,z) = \frac{|x|}{4} + \frac{|y|}{4} + \frac{3}{2}|z|$. Then (X, S_b) is a complete dislocated S_b-metric space with

$b = 3$. Let $T : X \to X$ be defined by $T(x) = \frac{x}{16}$. Also, take $F(\alpha) = \ln \alpha$ for $\alpha > 0$. Note that $M(x, y) = \frac{|x|+3|y|}{2}$ and if x or y is nonzero, then $S_b(Tx, Tx, Ty) > 0$. In this case, we have

$$\tau + F\big(b^2 S_b(Tx, Tx, Ty)\big) \leqslant F\big(M(x, y)\big) \Leftrightarrow \ln(\frac{16}{9}) \geqslant \tau.$$

Also, we observe that

$$\begin{aligned} \max\Big\{\frac{S_b(y, y, Ty)}{10b}, \frac{S_b(y, y, Ty)}{5b} + \frac{S_b(Tx, Tx, Ty)}{10b}\Big\} &= \max\Big\{\frac{19|y|}{960}, \frac{41|y| + |x|}{960}\Big\} \\ &\leqslant \frac{30|x| + 90|y|}{960} \\ &= S_b(Tx, Tx, Ty) \end{aligned}$$

for all $x, y \in X$. Now, if we assume that $0 < \tau \leqslant \ln(\frac{16}{9})$, then all conditions of Theorem 2.11 hold and 0 is a unique fixed point of T.

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References

[1] A. Aghajani, M. Abbas, and J. R. Roshan, Common fixed point of generalized weak contractive mappings in partially ordered G_b-metric spaces, *Filomat*, 28 (6) (2014), 1087–1101.

[2] I. A. Bakhtin, The contraction mapping principle in quasimetric spaces, *Funct. Anal. Unianowsk Gos. Ped. Inst.*, 30 (1989), 26–37.

[3] N. V. Dung and V. L. Hang, A fixed point theorem for generalized F-contractions on complete metric spaces, *Vietnam J. Math.*, 43 (2015), 743–753.

[4] G. N. V. Kishore, K. P. R. Rao, D. Panthi, R. B. Srinuvasa, and S. Satyanaraya, Some applications via fixed point results in partially ordered S_b-metric spaces, *Fixed Point Theory Appl.*, 10 (2017), 14 pages.

[5] Z. Mustafa, M. M. M. Jaradat, A. H. Ansari, B. Z. Popović, and H. M. Jaradat, C-class functions with new approach on coincidence point results for generalized (ψ, φ)-weakly contractions in ordered b-metric spaces, *Springerplus,* 5 (802) (2016), 18 pages.

[6] Z. Mustafa, M. M. M. Jaradata, and H. M. Jaradat, Some common fixed point results of graphs on b-metric space, *J. Nonlinear Sci. Appl.*, 9 (6) (2016), 4838–4851.

[7] Z. Mustafa, J. R. Roshan, V. Parvaneh, and Z. Kadelburg, Some common fixed point results in ordered partial b-metric space, *J. Inequal. Appl.*, 562 (2013), 26 pages.

[8] Z. Mustafa, J. R. Roshan, V. Parvaneh, and Z. Kadelburg, Fixed point theorems for weakly T-Chatterjea and weakly T-Kannan contractions in b-metric spaces, *J. Inequal. Appl.*, 46 (2014), 14 pages.

[9] Z. Mustafa and B. Sims, A new approach to generalized metric spaces, *J. Nonlinear Convex Anal.*, 7 (2) (2006), 289–297.

[10] N. Y. Özgür and N. Taş, Some new contractive mappings on S-metric spaces and their relationships with the mapping (S25), *Math. Sci.*, 11 (2017), 7–16.

[11] H. Piri and P. Kumam, Some fixed point theorems concerning F-contraction in complete metric spaces, *Fixed Point Theory Appl.*, 210 (2014), 11 pages.

[12] H. Piri and P. Kumam, Fixed point theorems for generalized F-Suzuki-contraction mappings in complete b-metric spaces, *Fixed Point Theory Appl.*, 90 (2016), 13 pages.

[13] S. Sedghi, A. Gholidahneh, T. Došenović, J. Esfahani, and S. Randenović, Common fixed point of four maps in S_b-metric spaces. *J. Linear Topol. Algebra*, 5 (2) (2016), 93–104.

[14] S. Sedghi, N. Shobe, and A. Aliouche, A generalization of fixed point theorem in S-metric spaces, *Mat. Vesn.*, 64 (2012), 258–266.

[15] A. Shoaib, M. Arshad, T. Rasham, and M. Abbas, Unique fixed point results on closed ball for dislocated quasi G-metric spaces, *Trans. A. Razmadze Mathematical Institute,* 171 (2017), 221–230.

[16] V. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, *Fixed Point Theory Appl.*, 94 (2012), 6 pages.

[17] D. Wardowski and N. V. Dung, Fixed points of F-weak contractions on complete metric spaces, *Demonstr. Math.*, 1 (2014), 146–155.

[18] C. Zhu, C. Chen, and X. Zhang, Some results in quasi-b-metric-like spaces, *J. Inequal. Appl.*, 437 (2014), 8 pages.

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