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Finding Targets With Ratio Data Based on Value Efficiency

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Abstract. In many organizations, ratio data are always available and are considered as a criterion for decision-making. In this case, evaluating decision making units (DMUs) with ratio data and finding targets can be of great importance. Therefore, in the present article, a method is proposed for finding the targets of decision making units employing ratio-based DEA (DEA-R) models and using value efficiency. Units with the most preferred solution (MPS) have a crucial role in finding the targets of decision making units. Thus, utilizing value efficiency by choosing MPS according to the management's idea and DEA-R models presents a new target for ratio data. In this paper, two different ideas based on the Max-Min model are studied for 20 tourism companies. In the end, the targets of decision making units based on ratio data are provided.

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1. Introduction

In many organizations, it is imperative to be able to differentiate between efficient and inefficient units and find suitable targets for the inefficient

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units. However, when the volume of input and output parameters is unknown and only a ratio of them is available, it would be difficult to find a suitable target. Furthermore, we would need to use interactive methods in order to find proper targets based on the manager's preferences. Generally, determining the efficiency values, differentiating between efficient and inefficient units and finding suitable targets are the most important applications of DEA. In cases where the volume of inputs and outputs are not available and we only have a ratio of input and output data (e.g. ratio of faculty members to students, assets to liabilities, number of hotel beds in a city to number of tourists, etc.), DEA-R models can solve our problems. Lately, based on the idea presented by Joro and Korhonen [6], DEA models are modified to calculate the value efficiency score based on the manager's preferences. Nowadays, the tourism industry and tourist attraction play a significant role in improvement of economy in any country. Therefore, it is essential to evaluate the tourism companies, differentiate between efficient and inefficient companies and motivate the inefficient ones to reach the conditions of their relative target. Now, the main problem in evaluation of tourism companies is the lack of access to the volume of input and output parameters and the high costs of collecting that information. However, it is possible to evaluate these companies through DEA-R models if we have access to the input/output ratios. What's more, only through an integration of value efficiency, DEA and DEA-R models can we involve the most efficient tourism companies according to the manager (most preferred solutions) in our targeting. The research gap in this article falls into two categories:

1) When the volume of input and output parameters is not available, it won't be possible to evaluate the DMUs using DEA models. In this regard, DEA-R models (using linear programming structure) can easily evaluate the DMUs in such situations, and when we do have access to the volume of input and output parameters, they will perform the evaluations exactly the same as DEA models ([2], [3], [4], [5] and [6]).

2) In DEA and DEA-R models, efficiency measurement and targeting are done without consideration to the manager's preferences and the most efficient units. Therefore, it is quite important to incorporate value

efficiency models into DEA and DEA-R.

In the following, we will present a thematic history of DEA, DEA-R, value efficiency and the tourism industry, and in summary review the current situation of tourism companies in Shiraz, Iran. Data Envelopment Analysis was first suggested by Farrell [7], for evaluating units with multiple inputs and one output. Charnes et al. [8], then presented the CCR model for evaluating units with multiple inputs and multiple outputs, which was afterward introduced as the fundamental part of DEA. Later, Banker et al. [9], presented BCC models with the idea of variable returns to scale. One of the objectives of CCR and BCC models is to find the suitable targets for decision-making units under the following conditions:

- Reality of the target.
- Availability of the target.
- Potential of the target to help the company make progress.

DEA provides significant advantages over other similar methods including ranking DMUs, determining returns to scale, and finding the suitable target for DMUs on the efficiency frontier. In recent years, DEA has made considerable progress and has been utilized in many areas such as performance evaluation of financial and educational organizations, and also providing a suitable target for companies with the goal of achieving success and reaching their ideal conditions. The effective presence of DEA in various fields has brought to the fore the type of input and output data and given rise to the question whether the basic models of DEA can be utilized for ratio data. In real problems, examples of such data include percentages, proportions, rates, and averages. Sometimes, ratios obtained from inputs and outputs have more explicit meaning. In some situations, some and not all of the inputs are involved in the production of one output; a case which can be described with the ratios between inputs and outputs. Ratio-based DEA (DEA-R) is the combination of DEA and ratio data having the following three features:

- Removing pseudo-inefficiency.

- Overcoming the problem of weights restrictions through defining efficiency as the weighted sum of input to output ratio or vice versa.
- Evaluating units with only ratio data. In other words, it can be utilized when the volume input and output data are not available, and there exists only a ratio of them.

In recent years, some studies have been conducted with ratio data (e.g., [10], [12], [12]). By combining DEA and ratio data, Despic et al. [5], presented models for DEA with ratio data known as DEA-R. Yang et al. [13], confirmed the equivalence of the dual output-oriented DEA model and Min-max model in multi-objective programming. The convexity condition is one of the leading conditions in basic DEA models. By considering the problems raised in the convexity condition with ratio data, Emrouzinejad and Amin [14], provided a new convexity assumption as well as enhancements to basic DEA models to tackle this problem. Studying 21 health care centers in Taiwan, Wei et al. [2], showed the presence of pseudo-inefficiency using DEA-R models. Wei et al. [3], presented the advantage of these models over CCR models by studying DEA-R models. By enhancing input-oriented DEA-R models, Wei et al. [4], calculated the efficiency and super-efficiency with CRS assumption of the production technology. Liu et al. [6], studied 15 research institutes in China and proposed DEA models without explicit inputs. Presenting the relationship between DEA and DEA-R, Mozaffari et al.[15], initially provided output-oriented envelopment and multiplier models based on harmonic efficiency. Subsequently, they suggested the enhanced Russell model for proper targeting of units. Utilizing the idea of Despic et al. [5] and MOLP structure, Mozaffari et al. [16], studied cost and revenue efficiency models in DEA-R and used them for 21 health care centers under two conditions: 1- volume data are available, and 2- only ratio data are available. By providing an example, Olesen et al. [17], indicated that the basic DEA models suffer from calculation problems in evaluating ratio data and indeed pointed out the importance of ratio data. They proposed that under constant returns to scale conditions, the ratio data should be categorized according to their responses to the scaling of the volume data. Taking this into account, the axioms in the conventional models were revisited, and ratio data were categorized into six groups. This categorization is performed by considering the worst-case condition for each item of ratio data and its response to the scaling of the volume data. The other idea presented in this paper is that if an item of ratio data acts like a unfavourable input, it is regarded as output, and vice versa. Finally, Olesen et al. [17], presented a model in which some restrictions are stated for various ratio data. Despite the detailed and extensive discussion of the ratio data in Olesen et al. [17], the problem of recognizing the type of each ratio data item may still be a difficult task, which is in some cases even based on a subjective decision with regard to the behaviour of the data. It is worth mentioning that in real conditions, it may be difficult to use this model and recognize the type of data. In the present article, a more straightforward model is provided utilizing the idea of Olesen et al. [17]. In many financial institutes, ratios like quick ratio, defined as the ratio of current assets to the current debts, are of great significance and can be beneficial for evaluating institutes. Overall, the ratio of assets to cost in a financial institute or the ratio of staff members to students in an educational institute are figures obtained without spending much cost and energy and can be utilized in the effective evaluation of decision making units. Thus, in every organization, using the concept of value efficiency that includes introducing MPS units can lead to value efficiency based DEA-R models. In various organizations such as educational institutes, health care centers, banks, tourism companies, etc., real problems have multiple objectives and can be programmed as multi-objective problems for which Pareto-optimal solutions can be calculated. These solutions can be obtained according to the decision maker's priorities. MPS units significantly affect the determination of the efficiency of units, and a manager can practice his viewpoints by introducing MPSs among efficient units and determine suitable targets for inefficient units. In conventional DEA models, units that are theoretically efficient are determined, and the efficiency of the rest of the units is calculated considering the efficient units. Inefficient units try to approach the efficient ones. The advantage of the value efficiency method is that efficient units that are considered by the manager as having particular importance are determined as MPS units and are provided as targets for inefficient units. Korhonen and Laakso [18], solved a multi-objective problem using interactive methods. Following them, Joro et al. [19],compared the DEA method with multi-objective linear programming. Halme et al. [20], can be considered as the first to suggest a new method in DEA by utilizing value efficiency (VE). They showed that if the value function is taken as a quasi-concave function, the value inefficiency of each DMU can be described by using a tangent cone of which the MPS is the apex. Halme and Korhonen [21], presented VE objectives with weights restrictions. A comprehensive study of VE and its applications have been conducted by Korhonen and Syrjanen [22], and Soleimani-damaneh et al. [23]. Later, a new method was presented by Halme and Korhonen [24], for benchmarking with heterogeneous units utilizing value efficiency analysis (VEA). Halme et al. [25], carried out a case study of bank branches using VEA. Joro and Korhonen [6], published a book about the relationship between DEA and VE in 2015. One of the primary objectives of DEA is to introduce suitable targets for inefficient units, according to which each inefficient DMU attempts to be efficient. There are various ideas for presenting such targets including:

- Determining the target based on input/output-oriented radial or non-radial models.
- Determining the target based on grading or ranking of units.
- Determining the target based on available areas and management's viewpoints.
- Determining the target based on directional distance models.
- Determining the target based on the combination of DEA and MOLP.

For a long time, various models have been suggested for determining suitable targets for inefficient DMUs according to the decision maker's priorities; models such as Thanassoulis and Dyson [26], in which weights restrictions are used. In a study by Aparicio et al. [27], a single-stage procedure was proposed in which the nearest targets on the Pareto-efficiency frontier are introduced by utilizing different distance functions and/or different efficiency measures so that each inefficient DMU could transform into an efficient unit with minimum effort. What is of crucial importance in this approach is the characterization of the set of Pareto-efficient points dominating the unit under assessment using a set of linear constraints; these points being corresponding to a mathematical programming problem whose objective is either the distance or the efficiency measure used. In an article by Lozano and Gutirrez [28], the issue of targets was analysed for Seville Airport. Using the min-max reference point, MOLP programming, and target setting, Malekmohammadi et al. [29], improved the output-oriented DEA multiplier models such that it simultaneously led to the improvement of inputs and outputs. It was achieved by solving a single model instead of n models. The proposed model was applied to data from the UK retail bank industry. In the article by Aparicio and Pastor[30], they tried to present a well-defined efficiency measure based on the closest targets in DEA which could be justifiable from both the mathematical theory and economic viewpoints. In the article, the Russel output-oriented model was presented in a new form in which the nearest targets are obtained. Showing that the output-oriented BCC model in the presence of unfavourable outputs is equivalent to the weighted min-max model, Ebrahimnejad and Tavana [31], presented targets so that inefficient DMUs move towards increasing favourable outputs and decreasing unfavourable outputs. In the article, a pilot study was carried out on NATO[1]. In a study by Suzuki et al. [32], an attempt was made to increase the efficiency of renewable energies like solar energy. Conducted to help energy-environment policy making in Japan, the study was carried out with the target of minimizing the Euclidean distance of inefficient DMU from the efficiency frontier. The TO-EDM (Target-Oriented Euclidean Distance Minimization) method was used in the article.

[1]NATO: North Atlantic Treaty Organization

1.1 motivational example

As is known, tourism industry entails substantial benefits to the economic section of the host country. The reason behind this is the amount of money the travellers spend for transportation, accommodation, food, sightseeing, buying souvenirs, and so on. Tourism industry considered as the third most crucial industry following oil and car industries in the world has excellent effects on employment and revenue of the host country. With the decrease in the fossil sources like oil and gas, it sounds vital for states and governments to plan for the substitution of the tourism industry for oil and gas industries. Unlike other industries, tourism industry needs a little amount of investment for creating job opportunities. In the announcement of World Tourism Organization (2004), one can observe the impact of tourism industry on the economic systems of the countries. Tourism industry includes various sections of transportation, hotels, stores, information system, the security system of a country, etc. In the competition among countries in the tourism industry, a country that provides travellers with better facilities with higher efficiency could be prosperous. It is crystal clear that the role of advertisement at the international level is of great significance, which in turn has its costs. From the viewpoint of historical-cultural sites and ecotourism sites, Iran is lying among the top ten and the top five countries in the world, respectively. Lack of sufficient advertisement at the international level and technical work forces familiar with the field of tourism has led to the insignificant share of the tourism industry in the economy of Iran. Therefore, it is necessary to consider the improvement of the industry employing unique viewpoint.

Example 2.1. In Iran, the city of Shiraz enjoys excellent potential for becoming one of the largest tourist centers at the international level owing to its location and historical and cultural sites. In addition, there are essential and substantial medical facilities in this city, which could make it a high potential center for health tourism. With its 124 historical-cultural sites, eight historical-environmental sites, and 18 environmental sites, the city of Shiraz ranks fifth in the number of tourist attractions. Undoubtedly, one of the critical factors in attracting tourist is the number of the high quality hotel in a city. Considering the number of tourists coming to Shiraz annually, according to the statistics, the quantity and quality of the hotels should increase.

Based on the recent year statistics, the number of hotels in Shiraz could be summarized in Table 1.

Table 1: The number of hotels in Shiraz

HOTEL	2013	2014	2015	2016
5 STARS	4	5	5	6
4 STARS	5	5	7	7
3 STARS	5	7	9	11

Based on the statistics published for 2013, 2014, and 2015, the number of tourists coming to Shiraz was 120000, 147000, and 170000 persons, respectively, and it is estimated that this number reaches 200000 persons for 2016. According to these statistics, if we ignore the differences between these hotels (represented by different numbers of stars), the ratio of beds to tourists for these years will be as shown in Table 2.

Table 2: The ratio of beds to tourists for 2013, 2014, 2015, and 2016

YEAR	2013	2014	2015	2016
Number of Bed/ Number of Tourist	1/41	1/42	1/42	1/45

From Table 2, it is clear that the trend in building new hotels in Shiraz is not satisfactory and, using the above ratios, the managers should find a suitable target for improving the tourism industry in the city of Shiraz. Having said this, the significance of ratio data in the tourism industry becomes obvious, and it sounds necessary to study these ratios and determine suitable targets. In the late 19th century, the low productivity of the tourism industry and hospitality led to concerns among scholars such as Martin and Witt [33], and Forrester and Anderson [34]. To overcome this problem, authorities should conduct regular revisions, evaluation, and targeting in the industry, although the issue has been dealt with previously on and off. In 2004, Sigala [35],developed stepwise DEA and presented a new method for evaluating and targeting of hotels. By studying 83 hotels in Portugal and using DEA models, Neves and Lourenco [36], provided some solutions to the managers for improving performance. Sanjeev [37], evaluated a number of hotels and restaurants in India and discriminated between efficient and inefficient units. This paper is organized as follows. In section 2 a brief review of the basic concepts of DEA, value efficiency, and DEA-R will be provided. In section 3 we will propose DEA-R models with ratio data for determining the target based on value efficiency. In section 4, we studied 20 tourism companies in Shiraz using models with 3 inputs and 2 outputs. Concluding remarks are investigated in section 5.

2. Basic Concepts

In this section, a brief review of the basic concept of DEA, Value efficiency and DEA-R will be provided.

2.1 The envelopment CCR model in DEA

Suppose there are n decision-making units with m inputs and s outputs. The values related to inputs and outputs of the j^{th} unit where $j = 1, \cdots, n$ can be presented as follows:

$$X_j = (x_{1j}, x_{2j}, \cdots, x_{mj}), \quad Y_j = (y_{1j}, y_{2j}, \cdots, y_{sj}).$$

DEA is a technique that calculates the ratio efficiency for each DMU through solving a linear programming problem. Different models can be used for calculating efficiency in DEA, among which the CCR model is the most fundamental. Model (1) represents the input-oriented CCR model in which the two-phase form of the model can be utilized for calculating efficiency. In this model, t_i^- and t_r^+ are the surplus and slack variables related to input and output variables. If in the optimal solution $\varphi^* = 1$ and $t_i^- = t_r^+ = 0 \ \ (i = 1, \cdots, m, \ r = 1, \cdots, s)$, the DMU under analysis is efficient.

$$
\begin{aligned}
min \quad & \varphi - \varepsilon\Big(\sum_{i=1}^{m} t_i^- + \sum_{r=1}^{s} t_r^+\Big) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + t_i^- = \varphi x_{io} \quad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - t_r^+ = y_{ro} \quad r = 1, \cdots, s, \\
& \lambda_j \geqslant 0 \qquad\qquad j = 1, \cdots, n, \\
& t_i^-, t_r^+ \geqslant 0 \qquad i = 1, \cdots, m, \quad r = 1, \cdots, s.
\end{aligned} \tag{1}
$$

If $\lambda_j^*(j = 1, \cdots, n)$ are the optimal values obtained in the evaluation of DMU_j, then the suitable target for DMU_j can be defined as $(X_j, Y_j) = (\sum_{j=1}^{n} \lambda_j^* X_j, \sum_{j=1}^{n} \lambda_j^* Y_j)$.

2.2 The envelopment CCR model in value efficiency

The model presented for input-oriented value efficiency by Joro and Korhonen [1] is a model (2). The difference between this model and the basic models of DEA is that by applying the management's viewpoint, MPSs are selected and accordingly some variables are allowed to assume negative values.

$$
\begin{aligned}
max \quad & \sigma + \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + \sigma x_{io} + s_i^- = x_{io} \quad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \qquad r = 1, \cdots, s, \\
& s_i^-, s_r^+ \geqslant 0 \qquad i = 1, \cdots, m, \quad r = 1, \cdots, s, \\
& \lambda_j : \begin{cases} \geqslant 0 & if \quad \lambda_j^* = 0 \quad j = 1, \cdots, n. \\ = free & if \quad \lambda_j^* > 0 \quad j = 1, \cdots, n. \end{cases}
\end{aligned} \tag{2}
$$

In model (2),$\lambda_j^*(j = 1, \cdots, n)$ is calculated from the equation $(X^*, Y^*) = (\sum_{j\in MPS} \lambda_j^* x_{ij}, \sum_{j\in MPS} \lambda_j^* y_{rj})$. In addition$(X^*, Y^*)$ is located on the CRS frontier.

Definition 2.2.1. *DMU_o is called value efficient if in model (2) $\sigma^* = 0$ and the slack variable are zero in all optimal solutions, i.e., $({s_i^-}^*, {s_r^+}^*) = (0, 0)$.*

2.3 The envelopment and multiplier models in DEA-R

In some cases in the financial, educational, and health care institutes, we cannot utilize volume data. It may happen due to the following reasons:

- The volume data are confidential, e.g., the amount of company's debt or total assets.
- The concept represented by the ratio between two variables is more important than the one represented by a single variable, e.g., the ratio of the unemployed population in a region to the total population at the employment age.
- Calculating volume data is expensive and time consuming.
- Volume data are not available.

In such cases, models in which utilizing ratio data is allowed should inevitably be used. It is important to note that the term "ratio data" represents the ratio between some input data to some output data, which might not theoretically represent any formal term. For instance, the ratio of the number of faculty members to university students does not bear any specific term, whereas the ratio of total assets to total debts in an organization is termed quick ratio. Assume that the ratio of $\frac{X_j}{Y_j}$ is defined and $X_j \geqslant 0, Y_j \geqslant 0 (j = 1, \cdots, n)$. We define the production set (PPS) in the presence of ratio data as $T_R = \left\{ \left(\frac{x}{y} \right) | \sum_{j=1}^n \lambda_j \left(\frac{x_i}{y_r} \right) \leqslant \frac{x}{y}, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geqslant 0,\ j = 1, \cdots, n \right\}$. The following postulates hold in the T_R set:

Feasibility of observed data: The inclusion principle of observations related to the ratios of $\frac{x_{ij}}{y_{rj}}$ holds; $\left(\forall j = 1, \cdots, n, \quad \frac{x_{ij}}{y_{rj}} \in T_R\right)$.

Free disposability: This principle holds in the DEA-R production possibility set (See Liu et.al [6]);

$$\left(if\frac{\alpha}{\beta} \in T_R, \quad \forall\frac{x}{y}, \quad \frac{x}{y} \geqslant \frac{\alpha}{\beta} \rightarrow \frac{x}{y} \in T_R, \quad j = 1, \cdots, n\right).$$

Convexity: The convexity principle holds in the DEA-R PPS (See Liu et.al [6]);

$$(\sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) \in T_R, r = 1, \cdots, s, i = 1, \cdots, m, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geqslant 0, j = 1, \cdots, n).$$

Theorem 2.3.1. *T_R is a closed and bounded set. (See Liu et.al [6]).*

According to the above mentioned principles, in the presence of ratio data, we define the input-oriented DEA-R model under constant returns to scale (CRS) assumption as (3):

$$\begin{aligned} & min \quad \alpha \\ & s.t. \quad \alpha\left(\frac{x_o}{y_o}\right) \in T_R. \end{aligned} \tag{3}$$

According to the definition of T_R we have:

$$\begin{aligned} min \quad & \alpha \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) \leqslant \alpha \frac{x_{io}}{y_{ro}} \quad i = 1, \cdots, m, \; r = 1, \cdots, s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geqslant 0 \qquad\qquad j = 1, \cdots, n. \end{aligned} \tag{4}$$

Model (4) is a linear programming problem introduced in DEA-R to assess technologies with constant returns to scale (CRS) (Mozaffari et. al., [15]).

Dual model of (4) is the same as model (5), where $w_{ir}, i = 1, \cdots, m,\ \ r = 1, \cdots, s$ and β are the dual variables of input and output constraints and the convex combination constraint, respectively.

$$\begin{aligned} max\ \ & \beta \\ s.t.\ \ & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{io}}{y_{ro}}}\right) \geqslant \beta \qquad j = 1, \cdots, n, \\ & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1, \\ & w_{ir} \geqslant 0 \qquad i = 1, \cdots, m, \quad r = 1, \cdots, s. \end{aligned} \tag{5}$$

Wei et al. [2] suggested model (5) in which, by utilizing super efficiency, they solved the two problems of pseudo-inefficiency and requirement of the non-Archimedean number ε.

3. Value Efficiency and Target For Ratio Data

In this section, DEA-R models with ratio data are suggested for determining the target based on value efficiency. Thus, first the target with ratio data is suggested in DEA-R, and then some models are proposed for finding targets based on value efficiency in DEA-R.

3.1 Finding targets with ratio data

In this section, first according to Olesen et al. [17], the multi-objective linear programming problem (6) is proposed. In the production technology, assume that $I = \{1, \cdots, m\}$ and $O = \{1, \cdots, s\}$ are the input and output sets, respectively. O^V and O^F are the subsets of O that respectively represent the volume outputs and ratio outputs of fixed form and $O^F \cup O^V = O$. The set I^V represents volume inputs. In our proposed model there is no ratio input.

To shed light on the concept of O^F, one can restate it in the following way:

O^F are variables which do not change in case of any change in the internal ratios of DMUs. As an example, the number of newborns with congenital heart problems in a hospital does not change when the other ratios of the hospital change in size. Other variables do not exhibit such a characteristic and can be written with constraints separate from O^F.

$$
\begin{aligned}
min \quad & \left\{ \frac{\sum_{j=1}^{n} w_j x_{ij}^V}{x_{io}^V},\ i \in I^V \right\} \\
s.t. \quad & \sum_{j=1}^{n} w_j y_{rj}^V \geqslant y_{ro}^V \quad \forall r \in O^V, \\
& w_j (y_{rj}^F - y_{ro}^F) \geqslant 0 \quad \forall r \in O^F, \qquad j = 1, \cdots, n, \\
& w_j \geqslant 0 \quad j = 1, \cdots, n.
\end{aligned} \tag{6}
$$

As model (6) is a multi-objective linear programming problem, by using the infinity norm on the objective function of the model (6), the Min-Max problem for evaluating DMU_O in the input-oriented CRS technology will be as model (7).

$$
\begin{aligned}
min\ max \quad & \left\{ \frac{\sum_{j=1}^{n} w_j x_{ij}^V}{x_{io}^V},\ i \in I^V \right\} \\
s.t. \quad & \sum_{j=1}^{n} w_j y_{rj}^V \geqslant y_{ro}^V \quad \forall r \in O^V, \\
& w_j (y_{rj}^F - y_{ro}^F) \geqslant 0 \quad \forall r \in O^F, \qquad j = 1, \cdots, n, \\
& w_j \geqslant 0 \quad j = 1, \cdots, n.
\end{aligned} \tag{7}
$$

With the change of variable $\gamma = max \left\{ \frac{\sum_{j=1}^{n} w_j x_{ij}^V}{x_{io}^V},\ i \in I^V \right\}$ in model (7), we will have the linear model (8).

$$
\begin{aligned}
min \quad & \gamma \\
s.t. \quad & \sum_{j=1}^{n} w_j x_{ij}^V \leqslant \gamma x_{io}^V \quad \forall i \in I^V, \\
& \sum_{j=1}^{n} w_j y_{rj}^V \geqslant y_{ro}^V \quad \forall r \in O^V, \\
& w_j (y_{rj}^F - y_{ro}^F) \geqslant 0 \quad \forall r \in O^F, \qquad j = 1, \cdots, n, \\
& w_j \geqslant 0 \quad j = 1, \cdots, n.
\end{aligned} \tag{8}
$$

In conventional DEA models with CRS, the efficiency of input and output orientations is the same, and such a relationship also holds in the model (8).
Suppose that the optimal solution to model (8) is $w_j^*(j=1,\cdots,n)$, then the target obtained for DMU_O will be as given in (9):

$$(\hat{X}_o^V,\hat{Y}_o^V,\hat{Y}_o^F)=(\sum_{j=1}^{n}w_j^*X_j^V,\sum_{j=1}^{n}w_j^*Y_j^V,\sum_{j=1}^{n}w_j^*Y_j^F). \tag{9}$$

Theorem 3.1.1. *Improved activity (9) is Pareto efficient.*

Proof. The proof is given in the appendix. □

To obtain the Min-Max model, a multi-objective linear programming problem (MOLP) (model (10)) is proposed in which constant returns to scale technology is assumed for evaluating DMU_O and the input orientation is considered, i.e., $(\frac{x_{ij}}{y_{rj}})$.

$$\begin{aligned} min\ & \left\{\frac{\sum_{j=1}^{n}\lambda_j\frac{x_{ij}^V}{y_{rj}^V}}{\frac{x_{io}^V}{y_{ro}^V}},\ \ i\in I^V,\ r\in O^V\right\} \\ s.t.\ & \sum_{j=1}^{n}\lambda_j y_{rj}^F \geqslant y_{ro}^F \quad \forall r\in O^F, \\ & \sum_{j=1}^{n}\lambda_j=1, \\ & \lambda_j\geqslant 0 \qquad j=1,\cdots,n. \end{aligned} \tag{10}$$

Using the infinity norm on the objective function of the model (10), the Min-Max for evaluating DMU_O in the constant returns to scale technology in the input-oriented DEA-R is suggested as (11) below:

$$\begin{aligned} min\ max\ & \left\{\frac{\sum_{j=1}^{n}\lambda_j\frac{x_{ij}^V}{y_{rj}^V}}{\frac{x_{io}^V}{y_{ro}^V}},\ \ i\in I^V,\ r\in O^V\right\} \\ s.t.\ & \sum_{j=1}^{n}\lambda_j y_{rj}^F \geqslant y_{ro}^F \quad \forall r\in O^F, \\ & \sum_{j=1}^{n}\lambda_j=1, \\ & \lambda_j\geqslant 0 \qquad j=1,\cdots,n. \end{aligned} \tag{11}$$

Considering $\theta = max\left\{\frac{\sum_{j=1}^{n}\lambda_j \frac{x_{ij}^V}{y_{rj}^V}}{\frac{x_{io}^V}{y_{ro}^V}}, \quad i \in I^V,\ r \in O^V\right\}$ in model (11), the following model is obtained:

$$\begin{aligned} min\ \ & \theta \\ s.t.\ \ & \sum_{j=1}^{n}\lambda_j \frac{x_{ij}^V}{y_{rj}^V} \leqslant \theta \frac{x_{io}^V}{y_{ro}^V} \quad \forall i \in I^V,\ r \in O^V, \\ & \sum_{j=1}^{n}\lambda_j y_{rj}^F \geqslant y_{ro}^F \quad \forall r \in O^F, \\ & \sum_{j=1}^{n}\lambda_j = 1, \\ & \lambda_j \geqslant 0 \qquad\qquad j = 1, \cdots, n. \end{aligned} \tag{12}$$

Model (12) is a linear programming model which includes $n+1$ variables and $card(I^V) \times card(O^V) + card(O^F) + 1$ constraints.

By obtaining the optimal solutions to model (12), if the volume input and output data are available, the corresponding target to DMU_O is defined as (13):

$$(\hat{X}_o^V, \hat{Y}_o^V, \hat{Y}_o^F) = (\sum_{j=1}^{n}\lambda_j^* X_j^V, \sum_{j=1}^{n}\lambda_j^* Y_j^V, \sum_{j=1}^{n}\lambda_j^* Y_j^F). \tag{13}$$

If the volume input and output data are not available and there exists only the ratio between them, the corresponding target to DMU_O is defined as (14):

$$\left(\frac{\hat{X}_o^V}{\hat{Y}_o^V}, \hat{Y}_o^F\right) = \left(\sum_{j=1}^{n}\lambda_j^* \frac{X_j^V}{Y_j^V}, \sum_{j=1}^{n}\lambda_j^* Y_j^F\right). \tag{14}$$

Theorem 3.1.2. *Improved activity (14) is Pareto efficient.*

Proof. The proof is given in the appendix. □

The difference between models (1), (8), and (12) can be summarized in Table 3 below.

Table 3: Difference between models (1), (8), and (12)

Model	Data required for solving the model
Model (1)	(x_{ij}^{V}, y_{rj}^{V})
Model (8)	$(x_{ij}^{V}, y_{rj}^{V}, y_{rj}^{F})$
Model (12)	$(x_{ij}^{V}, y_{rj}^{V}, y_{rj}^{F})$ or $(\frac{x_{ij}^{V}}{y_{rj}^{V}}, y_{rj}^{F})^{*}$

*Note: In most cases, obtaining access to ratio data saves time and cost.

Figure 1 indicates the use of models (8) and (12) in different cases. As can be seen in Figure 1, model (12) is used more often in various cases as compared with the model (8).

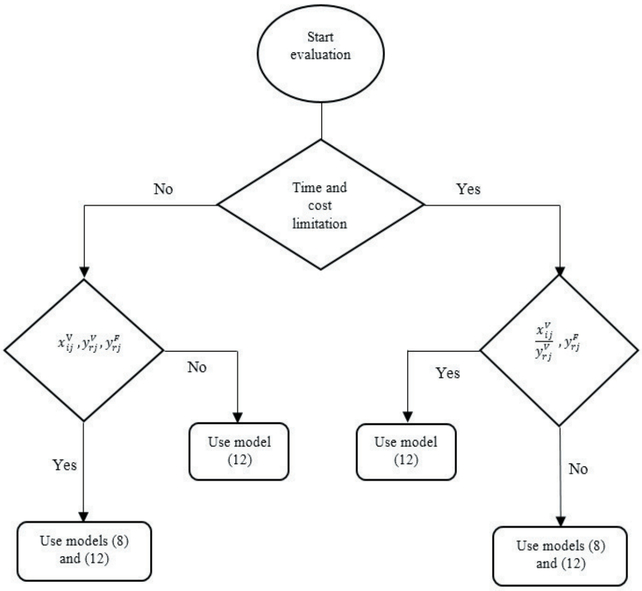
Generally speaking, models (8) and (12) are different in terms of access or lack of access to the volume of input and output parameters. If the volume were available, we could obviously use model (8) to find proper targets for our decision-making units. However, if we only had access to input/output ratios, we can use model (12) for targeting. Note that model (12) can be used in both these situations to determine the suitable targets within the framework of linear programming, a capability that model (8) lacks. It's worth mentioning that model (12) would eliminate the research gap 1 previously discussed in the introduction section.

3.2 Finding targets based on value efficiency

In this section, models (15) and (17) are proposed based on the idea of Olesen et al. [17] and combining the value efficiency and radial models (8) and (12) for applying management's viewpoints to finding targets with ratio data.

$$
\begin{aligned}
min \quad & \gamma \\
s.t. \quad & \sum_{j \notin MPS} \mu_j x_{ij}^{V} + \sum_{j \in MPS} \omega_j x_{ij}^{V} \leqslant \gamma x_{io}^{V} \quad \forall i \in I^{V}, \\
& \sum_{j \notin MPS} \mu_j y_{rj}^{V} + \sum_{j \in MPS} \omega_j y_{rj}^{V} \geqslant y_{ro}^{V} \quad \forall r \in O^{V}, \\
& \mu_j (y_{rj}^{F} - y_{ro}^{F}) \geqslant 0 \qquad \forall r \in O^{F}, \quad \forall j \notin MPS, \\
& \omega_j (y_{rj}^{F} - y_{ro}^{F}) \geqslant 0 \qquad \forall r \in O^{F}, \quad \forall j \in MPS, \\
& \mu_j \geqslant 0, \quad \omega_j : free \qquad j = 1, \cdots, n.
\end{aligned}
\tag{15}
$$

In GAMS program, model (15) will be as follows.




```
Equations
Objective , Con1(i) , Con2(r) , Con3(j,b), Con4(d,b) ;
Objective.. z=e=Teta;
Con1(i).. Sum(j,x(j,i)*mu(j))+Sum(d,xx(d,i)*omega(d))+teta*xo(i)=l=xo(i);
Con2(r).. Sum(j,y(j,r)*mu(j))+Sum(d,yy(d,r)*omega(d))-0*yo(r)=g=yo(r);
Con3(j,b).. (mu(j)*(y2(j,b)-y2o(b)))=g=0;
Con4(d,b).. (omega(d)*(yy2(d,b)-y2o(b)))=g=0;
File Results /Results.txt/;
```

Figure 1. Use of models (8) and (12)

Suppose that the optimum solution to model (15) is (μ^*, ω^*), the targets related to DMU_O can be defined as (16):

$$(\hat{X}^V_o, \hat{Y}^V_o, \hat{Y}^F_o) = (\sum_{j \notin MPS} \mu^*_j x^V_{ij} + \sum_{j \in MPS} \omega^*_j x^V_{ij},$$

$$\sum_{j \notin MPS} \mu^*_j y^V_{rj} + \sum_{j \in MPS} \omega^*_j y^V_{rj}, \sum_{j \notin MPS} \mu^*_j y^F_{rj} + \sum_{j \in MPS} \omega^*_j y^F_{rj}). \qquad (16)$$

Theorem 3.2.1. *Improved activity (16) is Pareto efficient.*

Proof. The proof is given in the appendix. □

By applying the management's opinion to model (12), model (17) below will be obtained.

$$\begin{aligned}
& min \ \ \theta \\
& s.t. \quad \sum_{j \notin MPS} \eta_j \frac{x_{ij}^V}{y_{rj}^V} + \sum_{j \in MPS} \lambda_j \frac{x_{ij}^V}{y_{rj}^V} \leqslant \theta \frac{x_{io}^V}{y_{ro}^V} \quad \forall i \in I^V, \\
& \qquad \sum_{j \notin MPS} \eta_j y_{rj}^F + \sum_{j \in MPS} \lambda_j y_{rj}^F \geqslant y_{ro}^F \quad \forall r \in O^F, \\
& \qquad \sum_{j \notin MPS} \eta_j + \sum_{j \in MPS} \lambda_j = 1 \\
& \qquad \eta_j \geqslant 0, \quad \lambda_j : free \qquad j = 1, \cdots, n.
\end{aligned} \tag{17}$$

In GAMS program, the constraints of the model (17) are as follows.

```
Equations
Objective , Con1(i,r), Con2(b) , Con3 ;
Objective.. z=e=Teta
Con1(i,r).. (teta)* (xo(i)/yo(r)) + sum(j,eta(j) * ((X(j,i))*(1/(Y(j,r)))
))+sum(d,lamda(d) * ((XX(d,i))*(1/(YY(d,r))) )) =e= (1)* (xo(i)/yo(r));
Con2(b).. sum(j,eta(j)*y2(j,b))+sum(d,lamda(d)*yy2(d,b))=g= y2o(b);
Con3.. sum(j,eta(j) ) +sum(d,lamda(d))=e= 1;
File Results /Results.txt/;
```

If (η^*, λ^*) is the optimal solution to model (17), the target presented in model (17) for DMU_O, when the volume input and output data are available and when just the ratio between the input and output data is available can be presented as models (18) and (19), respectively:

$$\begin{aligned}
& (\hat{X}_o^V, \hat{Y}_o^V, \hat{Y}_o^F) = (\sum_{j \notin MPS} \eta_j^* x_{ij}^V + \sum_{j \in MPS} \lambda_j^* x_{ij}^V, \\
& \sum_{j \notin MPS} \eta_j^* y_{rj}^V + \sum_{j \in MPS} \lambda_j^* y_{rj}^V, \sum_{j \notin MPS} \eta_j^* y_{rj}^F + \sum_{j \in MPS} \lambda_j^* y_{rj}^F),
\end{aligned} \tag{18}$$

$$\left(\frac{\hat{X}_o^V}{\hat{Y}_o^V}, \hat{Y}_o^F\right) =$$
$$\left(\sum_{j\notin MPS} \eta_j^* \frac{x_{ij}^V}{y_{rj}^V} + \sum_{j\in MPS} \lambda_j^* \frac{x_{ij}^V}{y_{rj}^V}, \sum_{j\notin MPS} \eta_j^* y_{rj}^F + \sum_{j\in MPS} \lambda_j^* y_{rj}^F\right). \quad (19)$$

Theorem 3.2.2. *Improved activity (19) is Pareto efficient.*

Proof. The proof is given in the appendix. □

Models (15) and (17) are proposed to close the research gap 2 (mentioned in the introduction section). In this regard, the most preferred solutions (MPSs) according to the manager have a great influence in efficiency measurement and targeting in model (15). Although, solving this model would require access to the volume of input and output parameters. Meanwhile, model (17) can perform the evaluations by having access to ratio data and considering the MPSs.

4. Example and Case Study

In this section, first six DMUs with one input and two outputs are evaluated by models (8), (12), (15), and (17) and the targets are determined for them. Then, in a case study for 20 tourism companies in Shiraz, Iran, the suitable target is determined. In the tables and figures presented, one can easily see the results obtained for different models and consider the difference among them.

4.1 Numerical examples

In this section, six DMUs, each with one input and two outputs, are evaluated by models (8), (12), (15), and (17). The input and output values related to these DMUs are presented in Table 4. Assume Y2 is the ratio output in the fixed form ($O2 \in O^F$).

As the inputs of all DMUs are the same, the BCC production possibility set can be shown in Figure 2. It is clear that $DMU6$ is inefficient and the other DMUs are located on the efficiency frontier.

Table 4: Inputs and outputs of six DMUs

DMU	X	Y1	Y2
1	1	4	8
2	1	6	6.8
3	1	9	5
4	1	2	8
5	1	9	3
6	1	4	5

After using models (8), (12), (15), and (17) for solving this example, one can see that the results of the two models (8) and (12) are very close, as are those of the two models (15) and (17). The efficiency measures obtained by models (8) and (12) and the CCR model (i.e., model (1)) are presented in Table 5, and the value efficiency measures obtained by models (15) and (17) are presented in Table 8.

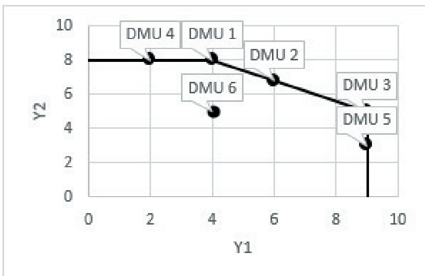


Figure 2. The production possibility set of Table 4 in DEA

The results presented in Table 5, which were obtained by using GAMS software, show that the efficiency values obtained by the model (8) and (12) are exactly the same. The columns Ref (8) and Ref (12) represent the reference DMUs introduced for the DMU under analysis by models (8) and (12). Considering these two columns, it can be understood that the reference DMUs presented by these models are almost the same. Nevertheless, in the case of DMU5, model (8) introduces DMU5 as a reference unit for it, which is a weakly efficient DMU, as can be seen in Figure 2. To be efficient, DMU5 can use a target like DMU3. Model (12) presents DMU3 as a reference for DMU5, which is both theoretically and practically more acceptable.

Table 5: The efficiency values and reference units obtained from models (1), (8), and (12)

DMU	Efficiency Value of Model(1)	Ref(1)	Efficiency Value of Model(8)	Ref(8)	Efficiency Value of Model(12)	Ref(12)
1	1	DMU1	1	DMU1	1	DMU1
2	1	DMU1,DMU3	1	DMU2	1	DMU2
3	1	DMU3	1	DMU3	1	DMU3
4	1	DMU1	0.5	DMU1	0.5	DMU1
5	1	DMU3	1	DMU5	1	DMU3
6	0.71	DMU1,DMU3	0.4444	DMU3	0.4444	DMU3

Table 6: Target obtained from models (8) and (12) using models (9) and (14)

DMU	Model(9) (X,Y1,Y2)	Model(13) (X,Y1,Y2)
1	(1,4,8)	(1,4,8)
2	(1,6,6.8)	(1,6,6.8)
3	(1,9,5)	(1,9,5)
4	(5,2,4)	(1,4,8)
5	(1,9,3)	(1,9,5)
6	(0.44,4,2.22)	(1,9,5)

In Table 6, the target presented for each DMU using models (8) and (12) is given. In this table, it is assumed that the volume inputs and outputs are available and thus related targets are calculated using models (9) and (13).

The results presented in Table 6 for DMU1, DMU2, and DMU3 are the same. According to Table 5, both models (8) and (12) introduced DMU1 as a reference for DMU4, but the target introduced by model (8) (i.e., (9)) yields values other than those of DMU1, whereas the target by model (12) (i.e., (13)) yields the values (1, 4, 8), which are exactly the values of DMU1. The target obtained for DMU5 by the model (9) is (1, 9, 3) which precisely represents DMU5. Meanwhile, DMU5 lacks 2 units in the second output. However, for DMU5, Model (13) presents the values (1,9,5), which represents DMU3; this is more accurate according to Figure 2. The targets presented by models (9) and (13) for DMU6 are actually a multiplier of each other, and it can be said that both models introduce the same target, although the values obtained from model (13) are more accurate and theoretically more justifiable, as seen in Figure 2.

Table 7: Target values obtained from models (8) and (12) using (9) and (14)

DMU	Model(9) (X/Y1,Y2)	Model(14)
1	(0.25,8)	(0.25,8)
2	(0.16,6.8)	(0.16,6.8)
3	(0.11,5)	(0.11,5)
4	(2.25,4)	(0.25,8)
5	(0.11,3)	(0.11,5)
6	(0.11,2.22)	(0.11,5)

In Table 7, column "Model (9)" (X/Y1 , Y2) represents the ratio of the target obtained from the input to the target obtained from the first output using model (9) and the target obtained from the second output using model (9). The column "Model (14)" contains the results of the model (14) if the volume input and output data are not available and there exists only the ratio between them. The results presented in these two columns for DMU1, DMU2, and DMU3 are exactly the same. The input to output ratio for DMU4 is $\frac{1}{2} = 0.5$, and the target input to output ratio by the model (9) is 2.25, which requires an increase. However, when this increase happens, Y2 decreases. The target input to output ratio by the model (9) for DMU4 is 0.25, which requires a decrease. With this decrease, Y2 remains unchanged.

Regarding DMU5, it can be stated that the second output increases when the input to the first output ratio is kept unchanged in the model (14). For DMU6, with the decrease in the input to the first output ratio in both models (14) and (9), if the target is determined by model (14), Y2 remains unchanged, and if it is given by model (9), Y2 decreases.

Table 8 shows the value efficiency obtained from models (15) and (17) for each DMU. In this table, it is assumed that DMU3 is selected by the management as MPS and all DMUs are evaluated accordingly. The results obtained from both models (15) and (17) are equal in three cases, and there is a small difference between them in the other two cases.

Table 9 presents the targets obtained from model (17) (i.e., (18)) and also from model (15) (i.e., (16)). Here, the volume input and output data are assumed to be available. In this table, DMU3 is considered as

MPS. The results for DMU1 and DMU2 are exactly equal. In DMU4, the results obtained by the two models are a multiple of each other, and in fact, both models present the same DMU. For DMU5, the same DMU (i.e., DMU5) is given by model (15) as the target, which is a weakly efficient DMU according to Figure 2. However, model (17) introduces DMU3 as a target for DMU5, which shows the superiority of model (17) over the model (15) in evaluating this DMU. We can extend the discussion for DMU4 to the results obtained for DMU6.

Table 8: The value efficiency measures obtained by models (15) and (17)

DMU	Value Efficiency of Model(15)	Value Efficiency of Model(17)
1	0	0.1852
2	0	0
3	MPS	MPS
4	0.5	0.5926
5	0	0
6	0.5556	0.5556

Table 9: Target obtained by models (15) and (17) using models (16) and (18)

DMU	Model(16) (X,Y1,Y2)	Model(18) (X,Y1,Y2)
1	(1,4,8)	(1,4,8)
2	(1,6,6.8)	(1,6,6.8)
3	MPS	MPS
4	(0.5,2,4)	(1,4,8)
5	(1,9,3)	(1,9,5)
6	(0.44,4,2.22)	(1,9,5)

In Table 10, the first column is related to the ratio of the target obtained from the input to the target obtained from the first output using model (16) and the second column presents the target obtained by the model (19). Here, it is assumed that the input and output volume data are not available and only the ratio between them is available. Comparing the results reveals that the first components in both columns are almost equal and the second components in the second column (Model (19)) are greater than or equal to those of the first column (Model (16)), which can be a more acceptable result for each DMU.

Table 10: Target obtained by models (15) and (17) using models (16) and (19)

DMU	Model(16) (X/Y1,Y2)	Model(19)
1	(0.25,8)	(0.2,8)
2	(0.16,6.8)	(0.16,6.8)
3	MPS	MPS
4	(0.25,4)	(0.20,8)
5	(0.11,3)	(0.11,5)
6	(0.11,2.22)	(0.11,5)

4.2 Case study

As was mentioned above in the Introduction, Shiraz is one of the famous tourist destinations owing to its many tourist attractions and its unique medical standing in the Middle East. An essential factor in tourist satisfaction is the accommodation and specialized services provided by travel agencies. It seems vital to evaluate the travel agencies in Shiraz in the shortest time possible and find suitable targets for improving the services provided by them. To this end, we studied 20 tourism companies in the city of Shiraz in 2015.

The available data for these companies are summarized in Table 11.

Table 11: Available data for 20 tourism companies

X1	The cost of consumed energies,costs of personnel,hotel maintenance costs
X2	The facilities provided for the travelers
X3	Advertisement,side programs for travelers,costs of special services
Y1	Current assets
Y2	Total assets

According to an online survey, the tourist satisfaction level with the services of travel agencies in Shiraz was 42 percentage on average at the beginning of the research. The agencies were considered in the following two cases.

4.2.1. Travel agency evaluation with time and cost limitation Owing to the need for the evaluation and supervision of travel agencies and their enhancement with the aim of attracting more capital, ratio data $\frac{x_{ij}^V}{y_{rj}^V}, y_{rj}^F$ were collected easily and with low cost in a limited time interval in early 2015 with expert opinion.

4.2.2. Travel agency evaluation without time and cost limitation As it was time consuming and costly to find volume input and output data for travel agencies in Shiraz, all input and output data $x_{ij}^V, y_{rj}^V, y_{rj}^F$ were collected in a four-month interval, as presented in Table 12. The following problems arose during the volume data collection.

- problems with data storage owing to the large volume of data;
- considerable number of tourists and insufficient time for data analysis;
- human error in recording input and output data in tourist software systems.

In Table 12, X1, X2, and X3 are the primary costs, the facilities provided for the travellers, and the secondary costs, respectively. Y1 and Y2 represent the current assets and total assets of each company. The primary costs include the cost of consumed energies, costs related to the personnel, and hotel maintenance costs; secondary costs include advertisement, side programs for travellers, and costs of specialized services.

Table 13 provides the efficiency values calculated by the CCR model (model (1)) and models (8) and (12). Columns Ref (1), Ref (8), and Ref (12) represent the reference DMUs related to the DMU under evaluation by model (1), model (8), and model (12), respectively.

The results obtained from models (8) and (12) are precisely equal in most cases, and there are little differences between them in other cases. The same results have been achieved in reference DMUs introduced for each DMU under analysis. In most cases, the number of reference DMUs introduced for each DMU is the same in both models, and in other cases, the number of reference DMUs in the model (12) is more significant than that in the model (8). It can be stated that the inefficient DMU can select the suitable target more freely to become effective if it uses the results of the model (12). Considering Figure 4 and the results obtained by models (8) and (12), it is clear that the efficiency values from both models are very close to each other.

Table 12: Input and output data for 20 tourism companies (Thousand Dollars)

1	43276	1298765	2809.69	34.9	98234
2	98743	459865	871.05	20.8	67849
3	12398	983424	7654.9	16.9	23457
4	76534	130985	9080.8	34.1	123698
5	239876	3578643	6723.8	2739	87643
6	120654	679844	8765.98	30.5	239870
7	514881	508735	9139.03	18.86	87981
8	15676	172947	308.67	25.66	97763
9	155799	269617	3185.7	26.27	7629
10	13707	163397	832.03	21.91	430513
11	666679	242971	11589.09	16.75	7859
12	106682	408506	2886.74	81.2	151192
13	709874	2610439	12963.87	37.75	134529
14	73560	317617	1483.69	22.74	10244
15	435248	489666	2547.02	25.5	42668
16	29165	971640	866.86	20.94	44128
17	216148	158130	4545.92	14.43	13043
18	190455	174657	1880.46	21.67	53209
19	67543	348763	4578.77	35.65	45673
20	1098	126578	3095.1	21.6	3487

In Figure 3, the total assets (Y2) of each DMU is presented in the pie chart.

Table 14 represents the targets introduced for each DMU with regard to models (8) and (12). In this table, it is assumed that the volume input and output data are available and, therefore, the related targets are calculated using (9) and (13). In this table, the target values obtained for the efficient DMUs by models (8) and (12) are the same, whereas they are different for other DMUs. For instance, in the case of DMU4, model (12) reduces the input by a little amount and consequently the outputs increase; however, model (8) substantially reduces the input so that the output increases.

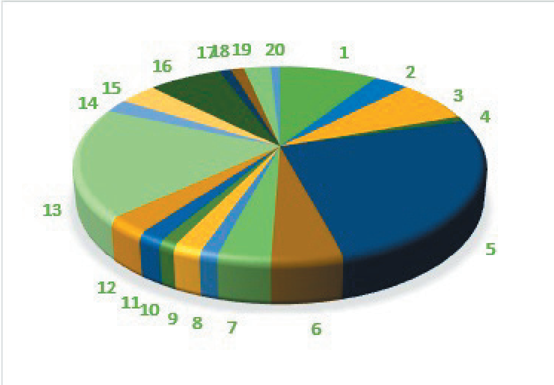


Figure 3. Total assets of each company in comparison with other companies

Table 13: The efficiency values and reference units obtained from models (1), (8), and (12)

DMU	Efficiency Value of Model(1)	Ref(1)	Efficiency Value of Model(8)	Ref(8)	Efficiency Value of Model(12)	Ref(12)
1	0.59	DMU5,DMU16	0.2813	DMU13	0.1631	DMU13,DMU15
2	0.96	DMU15,DMU16	0.6634	DMU15	0.6634	DMU15
3	1	DMU3	0.1002	DMU13	0.0356	DMU11,DMU13
4	0.11	DMU5,DMU11,DMU15	0.0957	DMU11,DMU15	0.0957	DMU11,DMU15
5	1	DMU5	1	DMU5	1	DMU5
6	0.25	DMU5,DMU11,DMU15	0.2514	DMU13	0.1722	DMU11,DMU13,DMU15
7	0.91	DMU5,DMU11,DMU15	1	DMU7	0.8912	DMU11,DMU13,DMU15
8	0.67	DMU15,DMU16	0.2972	DMU15	0.2972	DMU15
9	0.92	DMU3,DMU5,DMU11	1	DMU9	0.7023	DMU11,DMU15
10	0.23	DMU15,DMU16	0.0964	DMU15	0.0964	DMU15
11	1	DMU11	1	DMU11	1	DMU11
12	0.29	DMU15,DMU16	0.2163	DMU15	0.2163	DMU15
13	0.96	DMU5,DMU11,DMU15	1	DMU13	1	DMU13
14	0.88	DMU5,DMU11,DMU15	0.7039	DMU15	0.4888	DMU11,DMU15
15	1	DMU15	1	DMU15	1	DMU15
16	1	DMU16	0.6144	DMU13	0.2918	DMU13,DMU15
17	0.67	DMU5,DMU11,DMU15	0.6515	DMU11,DMU15	0.6515	DMU11,DMU15
18	0.59	DMU15	0.5927	DMU15	0.5927	DMU15
19	0.23	DMU5,DMU11,DMU15	0.1450	DMU15	0.1198	DMU11,DMU15
20	0.87	DMU3,DMU11	0.0585	DMU11,DMU15	0.0585	DMU11,DMU15

In DMU8, by increasing the first input and decreasing the other two inputs, model (12) yields an enormous increase in the outputs. Model (8) decreases the inputs so that the second input of this DMU becomes so small compared to that of other DMUs and thus the first output remains constant and the second output decreases. In other words, it reduces the current assets and increases the total assets.

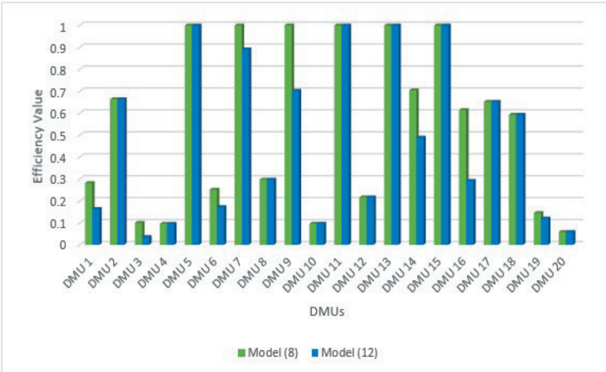


Figure 4. Comparing the efficiency values obtained from models (8) and (12)

Utilizing model (12), DMU20 can increase the output with a small increase in inputs, while using model (8), it keeps the first output constant and decreases the second output by a great decrease in inputs. Figure 5 and Figure 6 present the target values obtained from model (8) and model (12), respectively. Comparing these two figures and Figure 3, it can be concluded that model (8) leads to an increase in the difference between DMU5 and DMU13 on the one hand and other DMUs on the other hand. Meanwhile, model (12) maintains the above-mentioned difference almost constant.

In Table 15, the column "Model (9)" (X1/Y1,X2/Y1,X3/Y1,Y2) contains four components, the first three represent the ratio of the targets obtained for the three inputs to the target obtained for the first output and the fourth component is the target obtained by model (8). The column "Model (14)" contains the results obtained by the model (12) with the assumption that the volume inputs and outputs are not available and there exist only the ratios between them. The values obtained in this table, especially in the first three components, have little difference with each other. In this table, DMUs that are efficient by models (12) and (8) have exactly the same target values and DMUs that are not efficient in Table 13 but have the same efficiency values also have similar target values for the first three components. The fourth components of these DMUs in the model (14) are higher than those in the model (9).

Table 14: Targets obtained by models (8) and (12) using (9) and (13)

DMU	Model(9) (X1,X2,X3) (Y1,Y2)	Model(13) (X1,X2,X3) (Y1,Y2)
1	(790.32,2.3,8201.28) (43276,159140.01)	(6521.17,30.17,77714.02) (540020.94,1298765)
2	(577.83,5.79,9679.92) (98743,111088.6)	(2547.02,25.5,42668) (435248,489666)
3	(226.41,0.66,2349.56) (12398,45591.5)	(12019.07,23.32,47476.51) (680188.74,983424)
4	(868.79,3.26,4354.68) (76534,58340.15)	(6859.54,21.33,26066.2) (545626.7,372007.29)
5	(6723.8,27.39,87643) (239876,3578643)	(6723.8,27.39,87643) (239876,3578643)
6	(2203.41,6.42,22865.27) (120654,443684.24)	(7864.07,23.47,40594.1) (572453.61,679844)
7	(9139.03,18.86,87981) (514881,508735)	(10428.87,20.05,25171.2) (637762.81,508735)
8	(91.73,0.92,1536.74) (15676,17635.93)	(2547.02,25.5,42668) (435248,489666)
9	(3185.7,26.27,7629) (155799,269617)	(9219.29,19.04,16981.95) (606024.17,307626.31)
10	(80.21,0.8,1343.72) (13707,15420.75)	(2547.02,25.5,42668) (435248,489666)
11	(11589.09,16.75,7859) (666679,242971)	(11589.09,16.75,7859) (666679,242971)
12	(624.29,6.25,10458.19) (106682,120020.19)	(2547.02,25.5,42668) (435248,489666)
13	(12963.87,37.75,134529) (709874,2610439)	(12963.87,37.75,134529) (709874,2610439)
14	(430.46,4.31,7211.19) (73560,82757.03)	(5688.65,22.46,30573.75) (515657.76,403952.82)
15	(2547.02,25.5,42668) (435248,489666)	(2547.02,25.5,42668) (435248,489666)
16	(532.62,1.55,5527.09) (29165,107249.25)	(4914.39,28.28,63544.64) (497660.43,971640)
17	(2961.81,7.74,8497.94) (216148,131247.4)	(8703.04,19.54,18969.33) (592810.90,321711.06)
18	(1114.52,11.16,18670.58) (190455,214267.13)	(2547.02,25.5,42668) (435248,489666)
19	(395.25,3.96,6621.34) (67543,75987.74)	(4328.73,23.78,35809.02) (480850.62,441055.66)
20	(181.04,0.3,203.96) (10987,4660.71)	(10878.73,17.44,10593.67) (648497.27,262351.9)

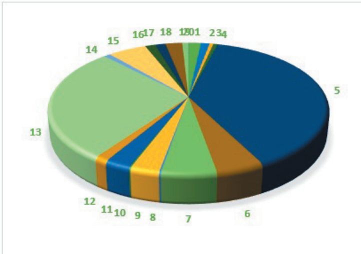


Figure 5. The total assets of each company compared with other companies using the results obtained by model (9)

Table 15: Target values obtained from models (8) and (12) using (9) and (14)

DMU	Model(9) (X1/Y1,X2/Y1,X3/Y1,Y2)	Model(14)
1	(0.1826,0.00005,0.18951,159140.01)	(0.01059,0.00006,0.13293,1298765)
2	(0.00585,0.00005,0.09803,111088.6)	(0.00585,0.00006,0.09803,489666)
3	(0.1826,0.00005,0.18951,45591.5)	(0.01766,0.00003,0.06737,983424)
4	(0.01135,0.00004,0.05689,58340.15)	(0.01135,0.00004,0.05690,372007.29)
5	(0.02803,0.00011,0.36536,3578643)	(0.02803,0.00011,0.36537,3578643)
6	(0.01826,0.00005,0.18951,443684.24)	(0.01251,0.00004,0.07392,679844)
7	(0.01774,0.00003,0.17087,508735)	(0.01582,0.00003,0.04143,508735)
8	(0.00585,0.00005,0.09803,17635.93)	(0.00585,0.00006,0.09803,489666)
9	(0.02044,0.00016,0.04896,269617)	(0.01436,0.00003,0.03439,307626.31)
10	(0.00585,0.00005,0.09803,15420.75)	(0.00585,0.00006,0.09803,489666)
11	(0.01738,0.00002,0.01178,242971)	(0.01738,0.00003,0.01179,242971)
12	(0.00585,0.00005,0.09803,120020.19)	(0.00585,0.00006,0.09803,489666)
13	(0.01826,0.00005,0.18951,2610439)	(0.01826,0.00005,0.18951,2610439)
14	(0.00585,0.00005,0.09803,82757.03)	(0.00986,0.00005,0.06807,403952.82)
15	(0.00585,0.00005,0.09803,489666)	(0.00585,0.00006,0.09803,489666)
16	(0.01826,0.00005,0.18951,107249.25)	(0.00867,0.00006,0.11882,971640)
17	(0.01370,0.00003,0.03931,131247.4)	(0.01370,0.00004,0.03932,321711.06)
18	(0.00585,0.00005,0.09803,214267.13)	(0.00585,0.00006,0.09803,489666)
19	(0.00585,0.00005,0.09803,75987.74)	(0.00812,0.00005,0.08104,441055.66)
20	(0.01647,0.00002,.01856,4660.71)	(0.01648,0.00003,0.01856,262351.9)

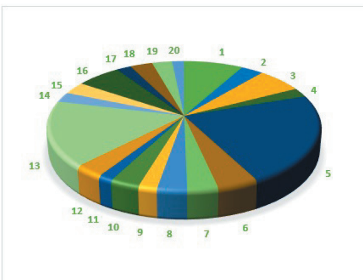


Figure 6. The total assets of each company compared with other companies using the results obtained by model (13)

Table 16 shows the value efficiency achieved by models (15) and (17) for each DMU. In this table, it is assumed that DMU11 and DMU15 are introduced as MPS by the management and all DMUs are evaluated in comparison with them. The results presented in this table are equal in some cases and have little difference in other cases as shown in Figure 7. It can be seen in the table (16) that the model (15) has a more optimistic view than the model (17) and accordingly the number of value efficient DMUs is larger.

Table 16: The value efficiency measures obtained from models (15) and (17)

DMU	Value Efficiency of Model(15)	Value Efficiency of Model(17)
1	0.7187	0.9137
2	0.6801	0.6801
3	0.8998	0.9690
4	0.9043	0.9043
5	0	0.2988
6	0.7486	0.8278
7	0	0.1088
8	0.7028	0.9269
9	0	0.2977
10	0.9036	0.9553
11	MPS	MPS
12	0.9100	0.8995
13	0	0
14	0.2961	0.5112
15	MPS	MPS
16	0.3856	0.8996
17	0.3485	0.3485
18	0.4073	0.4694
19	0.8550	0.8802
20	0.9415	0.9415

In Table 17, targets that are obtained by the model (17) using (18) and by the model (15) using (16) are listed. Here, it is assumed that the volume input and output data are available.

In Table 14, DMU13 in both models is value efficient and the target values introduced by both models are the values of DMU13 itself. For DMUs that are value efficient by the model (15), the target values introduced are precisely equal to those of the DMU themselves, whereas in other DMUs there is a considerable decrease in the input values, especially in the second input, which is not an easy task in practice.

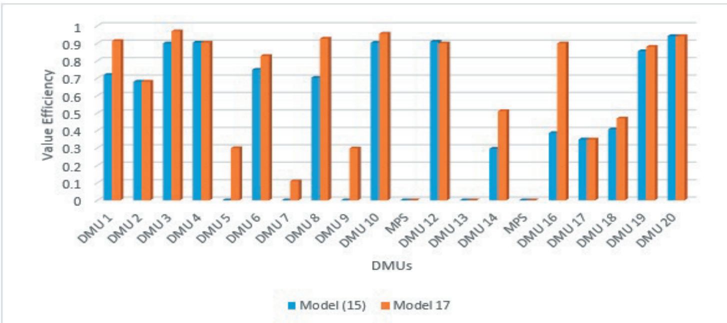


Figure 7. Comparing the results of models (15) and (17)

In Table 16, the value efficiencies of DMU2, DMU4, DMU7, and DMU20 by both models are the same, whereas in Table 18 the fourth component in model (17) is much bigger than the fourth component in model (15) although the first three components of these DMUs are almost equal, which, in fact, demonstrates the superiority of model (17).
As another example, in Table 16, DMU12 has almost the same value efficiency by both models (17) and (15), while in Table 18 there is a vast difference between the two targets in the fourth component, although the first three components are almost equal. The same is true for DMU19 and DMU18.
In Table 16, model (15) considers DMU9 as value efficient, whereas model (17) considers 0.2977 as the value efficiency measure of this DMU. In the targets obtained for the first three components in Table 18, very little difference is seen; however, there is a great difference in the fourth component and model (17) expects a higher value of DMU9.
After the targets were obtained by models (12) and (17), an on line survey was carried out regarding tourist satisfaction, yielding a result of 63 percentag

Table 17: Target values obtained by models (15) and (17) using (16) and (18)

DMU	Model(16) (X1,X2,X3) (Y1,Y2)	Model(18) (X1,X2,X3) (Y1,Y2)
1	(790.32,2.30,8201.28) (43276,159140.01)	(2583.93,32.99,86976.89) (438890.14,1298765)
2	(278.62,6.65,11917.68) (98743,130823.41)	(171,27.8,51814.91) (374433.87,554491.12)
3	(226.41,0.66,2349.56) (12398,45591.50)	(13197.93,22.48,44703.08) (710468.66,983424)
4	(868.79,3.26,4354.68) (76534,58340.15)	(6859.54,21.33,26066.2) (545626.7,372007.29)
5	(6723.8,27.39,87643) (239876,3578643)	(14341.64,45.76,184413.53) (748486.83,3578643)
6	(2203.41,6.42,22865.27) (120654,443684.24)	(7864.07,23.47,40594.1) (572453.61,679844)
7	(9139.03,18.86,87981) (514881,508735)	(10428.87,20.05,25171.2) (637762.81,508735)
8	(91.734,0.92,1536.74) (15676,17635.93)	(170,63.06,260160.59) (387050.79,4351485.92)
9	(3185.7,26.27,7629) (155799,269617)	(9219.29,19.04,16981.95) (606024.17,307626.31)
10	(80.21,0.80,1343.72) (13707,15420.75)	(170,30.54,68000.20) (375390.29,849420.43)
11	MPS	MPS
12	(259.80,7.31,13184.18) (106682,144060.65)	(170,30.35,66885.13) (375322.62,829098.71)
13	(12963.87,37.75,134529) (709874,2610439)	(12963.87,37.75,134529) (709874,2610439)
14	(430.46,4.31,7211.19) (73560,82757.03)	(5688.65,22.46,30573.75) (515657.76,403952.82)
15	MPS	MPS
16	(532.62,1.55,5527.09) (29165,107249.25)	(419.98,31.49,74118.35) (382218.13,971640)
17	(2961.81,7.74,8497.94) (216148,131247.40)	(8703.04,19.54,18969.33) (592810.90,321711.06)
18	(1114.52,11.16,18670.58) (190455,214267.13)	(2066.05,25.97,44519.58) (422937.6,502788.33)
19	(395.25,3.96,6621.34) (67543,75987.74)	(4328.73,23.78,35809.02) (480850.62,441055.66)
20	(181.04,0.30,203.96) (10987,4660.71)	(10878.73,17.44,10593.67) (648497.27,262351.90)

In Table 18, the first three components in the first column are related to the ratios of the targets obtained for the three inputs to the target obtained for the first output and the fourth component is the target obtained from the second output by the model (15) (using (16)). Assuming that the volume input and output data are not available, the second column is related to the targets introduced by the model (17) using (19).

Table 18: Target values obtained by models (15) and (17) using (16) and (19)

DMU	Model(16) (X1/Y1,X2/Y1,X3/Y1,Y2)	Model(19)
1	(0.01826,0.00005,0.18951,159140.01)	(0.00560,0.00007,0.16196,1298765)
2	(0.00282,0.00006,0.12069,130823.41)	(0.00282,0.00007,0.12069,554491.12)
3	(0.01826,0.00005,0.18951,45591.50)	(0.01915,0.00003,0.05868,983424)
4	(0.01135,0.00004,0.05689,58340.15)	(0.01135,0.00004,0.05690,372007.29)
5	(0.02803,0.00011,0.36536,3578643)	(0.01965,0.00006,0.25618,3578643)
6	(0.01826,0.00005,0.18951,443684.24)	(0.01251,0.00004,0.07392,679844)
7	(0.01774,0.00003,0.17087,508735)	(0.01582,0.00003,0.04143,508735)
8	(0.00585,0.00005,0.09803,17635.93)	(0.00144,0.00012,0.42199,4351485.92)
9	(0.02044,0.00016,0.04896,269617)	(0.01436,0.00003,0.03439,307626.31)
10	(0.00585,0.00005,0.09803,15420.75)	(0.00271,0.00007,0.14410,849420.43)
11	MPS	MPS
12	(0.00243,0.00006,0.12358,144060.65)	(0.00272,0.00007,0.14249,829098.71)
13	(0.01826,0.00005,0.18951,2610439)	(0.01826,0.00005,0.18951,2610439)
14	(0.00585,0.00005,0.09803,82757.03)	(0.00986,0.00005,0.06807,403952.82)
15	MPS	MPS
16	(0.01826,0.00005,0.18951,107249.25)	(0.00299,0.00007,0.15196,971640)
17	(0.01370,0.00003,0.03139,131247.40)	(0.01370,0.00004,0.03932,321711.06)
18	(0.00585,0.00005,0.09803,214267.13)	(0.00524,0.00006,0.10262,502788.33)
19	(0.00585,0.00005,0.09803,75987.74)	(0.00812,0.00005,0.08104,441055.66)
20	(0.01647,0.00002,0.01856,4660.71)	(0.01648,0.00003,0.01856,262351.90)

5. Results and Conclusions

In DEA, finding a suitable target for inefficient units is of great significance. When ratio data are available, DEA-R can provide the suitable target for inefficient units. By solving n linear programming problems in DEA, the suitable target for inefficient units can be calculated if these targets are "realistic" and "available". Availability of ratio data is beneficial regarding time and cost. Therefore, evaluating DMUs with ratio data in organizations and financial institutes could be of great importance. In the present article, two ratio-based models were suggested for finding targets. These models examined by using a numerical example with six DMUs and a case study in the tourism industry with 20 DMUs. The results of the study revealed that the two models present close outcomes for the efficiency value. The targets given by these models share great similarities; meanwhile, the targets obtained by the model (12) are a little more realistic. By combining value efficiency with models (8) and (12), the two models (15) and (17) were proposed, in which there is a small difference between the obtained results regarding value efficiency. Overall, in models (12) and (17) there is no need to have the volume input and output data and DMUs can be evaluated when only the ratios between inputs and outputs are available. For further research, finding targets in multi-stage networks and evaluating units in network form that contain ratio data are suggested.

Appendix A. Proofs

Proof of Theorem 3.1:
Assume that in evaluation of DMU_O through model (8), the optimal solution is obtained as expression (20):

$$(w_j^*, \gamma^*, S^{-^*}, S^{+^*}, T_j^*). \tag{20}$$

If we apply expression (20) in model (8), we will arrive at relations (21-23).

$$\sum_{j=1}^{n} w_j^* X_j^V + S^{-^*} = \gamma^* X_o^V, \tag{21}$$

$$\sum_{j=1}^{n} w_j^* Y_j^V - S^{+^*} = Y_o^V, \tag{22}$$

$$w_j^*(Y_j^F - Y_o^F) - T_j^* = 0, \quad \forall j. \tag{23}$$

If we place the third constraint set on j, we will have the relations (24-26):

$$\sum_{j=1}^{n} w_j^* X_j^V = \gamma^* X_o^V - S^{-^*} = \hat{X}_o^V, \tag{24}$$

$$\sum_{j=1}^{n} w_j^* Y_j^V = Y_o^V + S^{+^*} = \hat{Y}_o^V, \tag{25}$$

$$\sum_{j=1}^{n} w_j^* Y_j^F = \sum_{j=1}^{n} w_j^* Y_o^V + \sum_{j=1}^{n} T_j^* = \hat{Y}_o^F, \tag{26}$$

Now, consider the improved activity (9) in model (8) as problem (27):

$$\begin{aligned} min \ \ & \gamma \\ s.t. \ \ & \sum_{j=1}^{n} w_j X_j^V + S^- = \gamma \hat{X}_o^V, \\ & \sum_{j=1}^{n} w_j Y_j^V - S^+ = \hat{Y}_o^V, \\ & w_j(Y_j^F - \hat{Y}_o^F) - T_j = 0 \quad \forall j \\ & w_j \geqslant 0 \quad j = 1, \cdots, n. \end{aligned}$$

Assume that $(\hat{w}_j, \hat{\gamma}, \hat{S}^-, \hat{S}^+, \hat{T}_j)$ is the optimal solution of problem (27). If we add $(\hat{X}_o^V, \hat{Y}_o^V, \hat{Y}_o^F)$ taken from relations (24-26), we will arrive at relations (28-30):

$$\sum_{j=1}^{n} \hat{w}_j X_j^V + \hat{S}^- + \hat{\gamma} S^{-^*} = \gamma^* \hat{\gamma} X_o^V, \tag{27}$$

$$\sum_{j=1}^{n} \hat{w}_j Y_j^V - \hat{S}^+ - S^{+^*} = Y_o^V, \tag{28}$$

$$\hat{w}_j(Y_j^F - Y_o^F) - \hat{w}_j \sum_{j=1}^{n} w_j^* Y_o^F - \hat{w}_j \sum_{j=1}^{n} T_j^* + \hat{w}_j Y_o^F - \hat{T}_j = 0. \tag{29}$$

By adding $\gamma^* \hat{\gamma} = \tilde{\gamma}$, $\hat{S}^- + \hat{\gamma} S^{-^*} = \tilde{S}^-$, $\hat{S}^+ + S^{+^*} = \tilde{S}^+$ and $\hat{w}_j \sum_{j=1}^{n} w_j^* Y_o^F$ $+\hat{w}_j \sum_{j=1}^{n} T_j^* - \hat{w}_j Y_o^F + \hat{T}_j = \tilde{T}_j$, we will have:

$$\sum_{j=1}^{n} \hat{w}_j X_j^V + \tilde{S}^- = \tilde{\gamma} X_o^V,$$
$$\sum_{j=1}^{n} \hat{w}_j Y_j^V - \tilde{S}^+ = Y_o^V,$$
$$\hat{w}_j(Y_j^F - Y_o^F) - \tilde{T}_j = 0 \quad \forall j.$$

Now, $\hat{\gamma}$ must equal 1 ($\hat{\gamma} = 1$); otherwise, we will have:

$$\hat{\gamma} < 1 \rightarrow \gamma^* \hat{\gamma} < \gamma^* \rightarrow \tilde{\gamma} < \gamma^*.$$

On the other hand, $(\tilde{\gamma}, \hat{w}_j, \tilde{S}^-, \tilde{S}^+, \tilde{T}_j)$ is a feasible solution, for which we will have $\tilde{\gamma} < 1$, which contradicts the optimality of γ^* in evaluation of DMU_O. Therefore, $\hat{\gamma} = 1$. On the other hand,
$1\tilde{S}^- + 1\tilde{S}^+ + \sum_{j=1}^{n} \tilde{T}_j = (1\hat{S}^- + 1S^{-^*}) + (1\hat{S}^+ + 1S^{+^*}) + \sum_{j=1}^{n} \tilde{T}_j \leqslant 1S^{-^*} + 1S^{+^*} + \sum_{j=1}^{n} T_j^*$.
Since $1S^{-^*} + 1S^{+^*} + \sum_{j=1}^{n} T_j^*$ is at its maximum, then $1\hat{S}^- + 1\hat{S}^+ + \sum_{j=1}^{n} \hat{T}_j = 0$; therefore, $\hat{S}^- = 0$, $\hat{S}^+ = 0$, $\sum_{j=1}^{n} \hat{T}_j = 0$ and $\hat{\gamma} = 1$.
This means that the improved DMU is Pareto efficient.

If we assume $\sum_{j=1}^{n} \lambda_j^* = M$, then $\frac{\sum_{j=1}^{n} \lambda_j^*}{M} = 1$. Thus, we can define $\frac{\lambda_j^*}{M} = \tilde{\lambda}_j$.

Proof of Theorem 3.2:

Assume that in evaluation of DMU_O through model (12), the optimal solution is obtained as expression (31):

$$(\lambda_j^*, \theta^*, S^{-^*}, S^{+^*}). \tag{30}$$

If we apply expression (31) in model (12), we will arrive at relations (32-34).

$$\sum_{j=1}^{n} \lambda_j^* \frac{X_j^V}{Y_j^V} + S^{-^*} = \theta^* \frac{X_o^V}{Y_o^V}, \tag{31}$$

$$\sum_{j=1}^{n} \lambda_j^* Y_j^F - S^{+^*} = Y_o^F, \tag{32}$$

$$\sum_{j=1}^{n} \lambda_j^* = 1, \tag{33}$$

we will have the relations (35-37):

$$\sum_{j=1}^{n} \lambda_j^* \frac{X_j^V}{Y_j^V} = \theta^* \frac{X_o^V}{Y_o^V} - S^{-^*} = \frac{\hat{X}_o^V}{\hat{Y}_o^V}, \tag{34}$$

$$\sum_{j=1}^{n} \lambda_j^* Y_j^F = Y_o^F + S^{+^*} = \hat{Y}_o^F, \tag{35}$$

$$\sum_{j=1}^{n} \lambda_j^* = 1. \tag{36}$$

Now, consider the improved activity (14) in model (12) as problem (38):

$$\begin{aligned} min \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \frac{X_j^V}{Y_j^V} + S^- = \theta \frac{\hat{X}_o^V}{\hat{Y}_o^V}, \\ & \sum_{j=1}^{n} \lambda_j^* Y_j^F - S^+ = \hat{Y}_o^F, \\ & \sum_{j=1}^{n} \lambda_j^* = 1, \\ & \lambda_j \geqslant 0, \quad j = 1, \cdots, n. \end{aligned}$$

Assume that $(\hat{\lambda}, \hat{\theta}, \hat{S}^-, \hat{S}^+)$ is the optimal solution of problem (38). If we add $(\frac{\hat{X}_o^V}{\hat{Y}_o^V}, \hat{Y}_o^F)$ taken from relations (35-37), we will arrive at relations (39-41):

$$\sum_{j=1}^{n} \hat{\lambda}_j \frac{X_j^V}{Y_j^V} + \hat{S}^- + \hat{\theta} S^{-^*} = \theta^* \hat{\theta} \frac{X_o^V}{Y_o^V}, \tag{37}$$

$$\sum_{j=1}^{n} \hat{\lambda}_j Y_j^F - \hat{S}^+ - S^{+^*} = Y_o^F, \tag{38}$$

$$\sum_{j=1}^{n} \hat{\lambda}_j = 1. \tag{39}$$

By adding $\hat{S}^+ + S^{+^*} = \tilde{S}^+, \hat{S}^- + \hat{\theta} S^{-^*} = \tilde{S}^-$, and $\theta^* \hat{\theta} = \tilde{\gamma}$, now, $\hat{\theta}$ must equal $1 (\hat{\theta} = 1)$ otherwise, we will have:

$$\hat{\theta} < 1 \rightarrow \theta^* \hat{\theta} < \theta^* \rightarrow \tilde{\theta} < \theta^*.$$

On the other hand, $(\tilde{\theta}, \hat{\lambda}, \tilde{S}^-, \tilde{S}^+)$ is a feasible solution, for which we will have $\tilde{\theta} < 1$, which contradicts the optimality of θ^* in evaluation of DMU_O. Therefore, $\hat{\theta} = 1$. On the other hand,

$$1\tilde{S}^- + 1\tilde{S}^+ = (1\hat{S}^- + 1S^{-^*}) + (1\hat{S}^+ + 1S^{+^*}) \leqslant 1S^{-^*} + 1S^{+^*}$$

Since $1S^{-^*} + 1S^{+^*}$ is at its maximum, then $1\hat{S}^- + 1\hat{S}^+ = 0$; therefore, $\hat{S}^- = 0$, $\hat{S}^+ = 0$ and $\hat{\theta} = 1$. This means that the improved DMU is Pareto efficient.

Proof of Theorem 3.3: Similar to the proof of theorem 3.1.

Proof of Theorem 3.4: Similar to the proof of theorem 3.2.

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