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Stochastic Capacitated P-Median Problem with Normal Distribution

Case Study: Developing the Tanks of an Oil Refinery

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Abstract. In this paper, by developing the Capacitated P-Median Problem (CPMP) mathematical planning model, we try to model the problem in such a way that the service to customers according to the service level is effective in the optimum solution, and to some extent, brings the CPMP closer to actual circumstances. Adding the service level will convert the CPMP from a deterministic state to a stochastic model in which demands can have any distribution function. If the customer demand follows the normal distribution function, this stochastic model can be converted to a Mixed Integer Nonlinear Programming. This obtained model is used in a real case study to develop tanks of an oil refinery so that the costs are minimized and the customer demand is met at an acceptable level. In addition, comparing the new model with the traditional CPMP Model, by solving a real case study, shows the efficiency of the proposed model.

AMS Subject Classification: 65K05; 90B90; 90C06; 90C08; 90C11; 90C15; 90C30; 90C90
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1. Introduction

Facility planning, which is an important issue in industrial engineering, consists of four major components: facility allocation, facility location, facility routing, and facility design. From the combination of the first and second components, 'facility location-allocation' is generated, whose modeling is implemented by emphasizing 'mathematical programming' tools and the closer the generated model is to reality, it is more likely to achieve the goals.

One of the problems of facility location-allocation is Capacitated P-Median Problem (CPMP), which emphasizes optimum determining of p location of facilities (or median) to serve a set of n demand points, when there is 'capacity constraint' for facilities, and its objective is to minimize the total cost of servicing [28].

The most important hypotheses of the CPMP are the linearity of the cost-distance equation, the limited capacity of the facilities, the discreteness of the problem, and the determinacy of the model parameters [29]. In the real world, few variables can be defined as completely deterministic, and usually, the stochasticity of variables are in such a way that they cannot be neglected. Customer service rate is one of these parameters that is usually not deterministic and is determined by the service level. In this paper, CPMP mathematical programming model will be developed in such a way that the customer service level is effective in the optimum solution according to the service level, and taking into account the failure risk to respond to the demand and, to some extent, brings the CPMP closer to reality.

The proposed model is applicable to problems with stochastic demands. Also, if the demand follows the Normal Distribution Function, the final model can be converted to Mixed Integer Nonlinear Programming problem, which in this paper we have used to develop tanks of an oil refinery.

This article has five sections. In the first section, the introduction and general concepts are discussed. The CPMP mathematical Programming model, how it is formed, and its solving methods are presented in the second section. In the third section, the development of the CPMP Mathematical Programming model is discussed. In the fourth section, we have used the proposed model to develop the tanks of an oil refinery. Finally, in the fifth section, conclusion, and suggestions for future research are presented.

2. CPMP Mathematical Programming Model

CPMP is modeled as equations (1) to (6) [19]:

$$V(p) = Min \sum_{i \in N} \sum_{j \in M} d_{ij} x_{ij} \tag{1}$$

Subject to:

$$\sum_{j \in M} x_{ij} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{j \in M} y_j = p \tag{3}$$

$$\sum_{i \in N} q_i x_{ij} \leqslant Q_j y_j \quad \forall j \in M \tag{4}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, j \in M \tag{5}$$

$$y_j \in \{0, 1\} \quad \forall j \in M \tag{6}$$

CPMP is a Binary Integer Programming (BIP) that consists of an 'objective function' and three different types of 'constraints'.

The objective function and all of the problem constraints are linear, and all the variables, whose optimum value should be obtained, are binary.

In Table 1, the mathematical symbols of the CPMP, which exist in (1) to (6) equations, are defined:

Table 1: Definition of the mathematical symbols in the CPMP model

symbol	**Parameter type**	**Explanation**	**How to apply the model**
N	Set	The set of demand points number	The number of set members is determined by specifying the number of supply and demand points, and before modeling the problem.
M	Set	The set of supply or facilities (sites) points number	
i	Index	Use as an index for other parameters and means it is the i-th member of the set N $(i \in N)$.	
j	Index	Use as an index for other parameters and means it is the j-th member of the set M $(j \in M)$	
$V(p)$	Variable	Is the value of the objective function whose value changes with respect to the obtained solution.	Is obtained after the problem is solved.
d_{ij}	Constant Number	The cost of assigning demand i for a facility located on site j. In some cases, the cost can be defined as the distance between supply and demand points.	Is regarded as the input data of the problem.
x_{ij}	Decision Variable	Assigns itself only zero or one values. If demand i is assigned to a facility located in j site then $x_{ij} = 1$	Is obtained after the problem is solved.
y_j	Decision Variable	Assigns itself only zero or one values. Means the decision to select a facility in the site j, so if a facility is selected in the site j then $y_j = 1$	Is obtained after the problem is solved.
p	Positive constant and integer number	The set of supply or facilities (sites) points number	Is regarded as the input data of the problem.
q_i	Positive and constant number	The demand value for each customer at the i-th demand center	Is regarded as the input data of the problem.
Q_j	Positive and constant number	The capacity of each facility made j-th site	Is regarded as the input data of the problem.

In Table 2, the equations (1) to (6) are explained:

Table 2: Describing the equations (1) to (6) in the CPMP mathematical programming model

Equation Number	Equation	Explanation
(1)	$V(p) = Min \sum_{i \in N} \sum_{j \in M} d_{ij} x_{ij}$	This equation is the objective function of the problem, which aims to minimize the cost between demand points and facilities points. If the cost is a function of distance, the goal would be to minimize the distance between supply and demand points.
(2)	$\sum_{j \in M} x_{ij} = 1 \quad \forall i \in N$	This constraint, along with the equation (5), assigns each customer exactly to one facility.
(3)	$\sum_{j \in M} y_j = p$	This constraint, along with equation (6), states that the number of medians must be p, that is, precisely p of the facilities are activated, or, in other words, from the M number of the service supplier points, only p supply points is selected.
(4)	$\sum_{i \in N} q_i x_{ij} \leq Q_j y_j \quad \forall j \in M$	This constraint, along with equations (5) and (6), first ensures that the sum of demands allocated to each facility does not exceed its capacity, and second, guarantees to prevent the allocation of demand to disabled facilities.
(5)	$x_{ij} \in \{0, 1\} \quad \forall i \in N, j \in M$	The decision variable x_{ij} can only have a value of zero or one. If the i-th customer is assigned to j-th facility $x_{ij} = 1$ and otherwise $x_{ij} = 0$.
(6)	$y_j \in \{0, 1\} \quad \forall j \in M$	The decision variable y_j can only have a value of zero or one. If the new facility j is selected as the median $y_j = 1$ and otherwise $y_j = 0$.

The current mathematical Programming model of CPMP has evolved over time over the last five centuries. In the 17th century, Fermat proposed a problem whose goal was to minimize the sum of distances in a triangle. This same issue was developed by Weber and was revived in the 20th century. In this problem, Weber, by applying weight on the triangles vertices, referred to them as customer demands and to the median point as the service provider, and went on to explain that determining the location of the service provider between the three points of customer demand should be done in a way that the sum distances are minimized according to the weight given to each vertex. This is considered the first and original Facility Location-Allocation problem [20].

In the early 1960s, Hakimi [11, 12] used Weber's problem to find medians on networks and graphs. Hakimi examined the concept of 1-median and generalized it to the multi-median to determine the distribution of switching centers in the communications network. After this, p-median problems were regarded as an indispensable and integral part of the location theory and one of the most applied locating problems.

In 1970, the first linear programming model for the p-median problem (the classic model of the p-median problem) was presented by Revelle and Swain [21].

In 1977, the addition of capacity constraints to the p-median problem was introduced by Ross and Soland [22].

Later, different prblems, such as Capacitated Warehouse Location Problem, and Capacitated Clustering Problem were introduced which had a lot of similarities to the CPMP [9], and in 1998, Maniezzo et. al. considered and solved the current model of CPMP [19].

Given the fact that CPMP is an NP-Hard problem [10, 15], so far, a variety of methods such as the 'exact', the 'heuristic', the 'metaheuristic', and 'hybrid' methods, etc. have been presented to solve the problem, some of which are shown in Table 3 as an example:

Table 3: Summary of a number of published studies on solving the Capacitated P-Median Problem (CPMP)

Author(s)	Year of Publication	Number of problems solved	Solving method	Source
Maniezzo et. al.	1998	50	Metaheuristic	[19]
Baldacci et. al.	2002	60	Exact	[2]
Ceselli	2003	80	Exact	[4]
Lorena and Senne	2003	26	Hybrid	[17]
Correa et. al.	2004	1	Hybrid	[7]
Lorena and Senne	2004	11	Heuristic	[18]
Ceselli and Righini	2005	160	Exact	[5]
Ahmadi and Osman	2005	40	Metaheuristic	[1]
Diaz and Fernandez	2006	30	Hybrid	[8]
Chaves et. al.	2007	26	Hybrid	[6]
Boccia et. al.	2008	166	Exact	[3]
Fleszar and Hindi	2008	46	Heuristic	[9]
Yaghini et. al.	2010	26	Hybrid	[26]
Shamsipoor et. al.	2012	32	Artificial Neural Networks	[23]
Torres et. al.	2012	15	Hybrid	[25]
Yaghini et. al.	2013	166	Hybrid	[27]
Yaghini et. al.	2013	160	Hybrid	[28]
Stefanello et. al.	2015	65	Hybrid	[24]
Janosikova et. al.	2017	9	Hybrid	[14]
Herda	2017	3	Metaheuristic	[13]

3. Development of the Capacitated P-Median Problem Mathematical Programming Model

In this section, the CPMP mathematical Programming model will be developed in such a way that the service level to customers according to the service level would be effective in the optimum solution.

One of the most important hypotheses of CPMP is the determinacy of the model parameters, and considering the fact that most real-world problems are stochastic, this determinacy is a weakness of the current CPMP model.

One of the stochastic parameters is the customer service level, which is usually not deterministic and is determined based on the service level. In real life, customer demand is stochastic, and the facilities should have the ability and potential to match the allocated demand.

Considering that the service level is at the opposite end of the 'risk level' (α) and the service level is represented by $1-\alpha$, the equation (7) shows the relationship between the service level and the demand:

$$\begin{aligned} &Probability\ (not\ encountering\ any\ shortage) \\ &= Probability\ (offering\ the\ right\ and\ acceptable\ services\ to\ customers) \\ &\geqslant 1-\alpha \end{aligned} \tag{7}$$

As explained in the first section, the deterministic equation (4) in the CPMP model indicates the constraint on the allocation of demands to the capacitated facilities; therefore, by combining equations (4) and (7), and taking into consideration the allocation of demands to the capacitated facilities, we can make the customer service level effective in the optimum solution according to the service level, and to some extent, bring the CPMP closer to reality. Based on the above explanation, equation (4) converts to equation (8):

$$Probability\ (\sum_{i\in N} q_i x_{ij} \leqslant Q_j y_j) \geqslant 1-\alpha_j \quad \forall j \in M \tag{8}$$

Equation (8) shows that in order to achieve the desired service level, the probability of not encountering any shortage or offering the right and acceptable services to customers should be greater than, or equal to, the service level. As it is difficult to estimate the shortage cost or the cost of failing to offer the right and acceptable services to customers, managing the system using the equation (8) can provide a reasonable service level to offer services to customers. The service level has a value of between 0 and 1 and shows the likelihood of encountering shortage. For example, if the service level is equal to 80%, the likelihood of not encountering shortage will be 80%.

The developed model of CPMP is a stochastic model for solving, which it should become a deterministic model. To this end, the demand distribution function, which is stochastic, plays an important role in the problem. In this section, assuming that the demand has a normal distribution function [16], we examine the problem.

If in the problem, the demand has a normal distribution with mean μ_i and the variance σ_i^2 (for $i \in N$), then the sum of normal random distribution variables is also a normal distribution with mean $\sum_{i \in N} \mu_i x_{ij}$ and the variance $\sum_{i \in N} \sigma_i^2 x_{ij}$; therefore, with respect to equation (9), which converts the normal distribution to the standard normal distribution, the equation (8), which is a stochastic equation, converts to equation (10), which is a deterministic equation.

$$Z = \frac{X - \mu}{\sigma} \tag{9}$$

$$\frac{Q_j y_j - \sum_{i \in N} \mu_i x_{ij}}{\sqrt{\sum_{i \in N} \sigma_i^2 x_{ij}}} \geqslant Z_{(1-\alpha_j)} \quad \forall j \in M \tag{10}$$

If in the developed model of CPMP, we replace equation (8) with equation (10), the result will be a Binary Nonlinear Programming model.

Then, we define the new variable k_j^2 as the equation (11):

$$k_j^2 = \frac{\sum_{i \in N} \sigma_i^2 x_{ij}}{\sum_{i \in N} \sigma_i^2} \quad 0 \leqslant k_j \leqslant 1 \quad \forall j \in M \tag{11}$$

After that, we multiply the two sides of equation (10) in k_j to obtain the equation (12):

$$\frac{\sum_{i \in N} \mu_i x_{ij}}{\sqrt{\sum_{i \in N} \sigma_i^2}} + Z_{(1-\alpha_j)} k_j \leqslant \frac{Q_j y_j}{\sqrt{\sum_{i \in N} \sigma_i^2}} \quad \forall j \in M \tag{12}$$

Finally, by replacing equations (11) and (12) with equation (8), a Mixed Integer Nonlinear Programming, which we call Stochastic CPMP with

Normal Distribution (SCPMP-ND), is obtained:

$$V^{/}(p) = Min \sum_{i \in N} \sum_{j \in M} d_{ij} x_{ij} \tag{13}$$

Subject to:

$$\sum_{j \in M} x_{ij} = 1 \quad \forall i \in N \tag{14}$$

$$\sum_{j \in M} y_j = p \tag{15}$$

$$\frac{\sum_{i \in N} \mu_i x_{ij}}{\sqrt{\sum_{i \in N} \sigma_i^2}} + Z_{(1-\alpha_j)} k_j \leqslant \frac{Q_j y_j}{\sqrt{\sum_{i \in N} \sigma_i^2}} \quad \forall j \in M \tag{16}$$

$$k_j^2 = \frac{\sum_{i \in N} \sigma_i^2 x_{ij}}{\sum_{i \in N} \sigma_i^2} \quad \forall j \in M \tag{17}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, j \in M \tag{18}$$

$$y_j \in \{0, 1\} \quad \forall j \in M \tag{19}$$

$$0 \leqslant k_j \leqslant 1 \quad \forall j \in M \tag{20}$$

In Table 4, the CPMP and SCPMP-ND models are compared:

Table 4: Comparison of CPMP and SCPMP-ND models

Explanation	**CPMP**	**SCPMP-ND**
Model type	Linear	Nonlinear
Type of model variables	Binary	Mixed integer
Number of variables	$M(N+1)$	$M(N+2)$
Number of constraints	$N+M+1$	$N+2M+1$
Number of coefficients and constant values	$N \times M + M + N + 1$	$N \times M + 2(M+N) + 1$
Number of feasible and infeasible solutions	$2^{M(N+1)}$	$2^{M(N+2)}$

4. Results (of the case study in an oil refinery)

In this section, the SCPMP-ND Mathematical Programming Model is used to develop the tanks of an oil refinery and after coding it by the Lingo 11 Software, the solution to the problem is calculated in an exact way.

Currently, an oil refining company has a long-term contract with the government to sell petroleum products, and the demand for each product is completely clear and determined.

On the other hand, the licenses needed for selling some of the products in the stock market have also been recently obtained. Given that demand depends on the sale of the products, if a company wants to use this opportunity while complying with its obligations to the government at the same time, it must develop its tanks in a way that they are capable of responding to the indeterministic demands of the customers.

Due to the difference in the number of the demands and the demanded products, by reviewing the demands over a 24-month period, it becomes clear that demand follows the normal distribution function. Therefore, SCPMP-ND can be used to model the problem.

In this study, based on the stochastic demand of the market, we intend to develop a certain number of tanks (p) from the total number of tanks in the refinery (N) in a way that the total cost is minimized.

In order to calculate the cost of transferring petroleum products among the tanks (d_{ij}), we use the 'distance function', for calculating which, and based on the conditions governing the problem, 'the rectangular method' is used.

In this problem, there are 72 tanks with identical dimensions whose position is shown in Figure 1.

In order to be able to consider the cost of developing the tanks while taking into account the cost of transferring the products between them, defined by $\sum_{i\in N}\sum_{j\in M} d_{ij}x_{ij}$ in the objective function, we add the equation $\sum_{j\in M} f_j y_j$ to the objective function, so that the objective function is converted to equation (21):

$$Min \sum_{i \in N} \sum_{j \in M} d_{ij} x_{ij} + \sum_{j \in M} f_j y_j \tag{21}$$

In equation (21), f_j is the cost of developing the j-th tank. One of the main factors for its calculation is the distance between the tank and the loading location of the products, which is considered as rectangular.

Given that the goal is to minimize the costs and the management of the system does not know how many tanks it should develop, therefore, three questions are posed:

First question: How many tanks should be developed to minimize the costs?

Second question: After determining the number of tanks to be developed, which of the 72 tanks should be developed so that the costs are minimized?

Third Question: Which tanks should be connected with each other in order to minimize the costs?

To answer these questions, it is enough to define p, which is a constant number in the CPMP, as an integer variable according to equation (22), and solve the problem.

$$1 \leqslant p \leqslant M \quad p \text{ is integer} \tag{22}$$

All problems discussed in this section are solved by Lingo 11 software and run on a laptop with the following specifications: Intel Core 2 Duo Processor with 2.1 GHz, 4 Gigabytes of RAM and Windows 10 operating system. To solve the INLP (Integer Nonlinear Program) problem, Lingo 11 uses Branch and Bound Solver coded in the software.

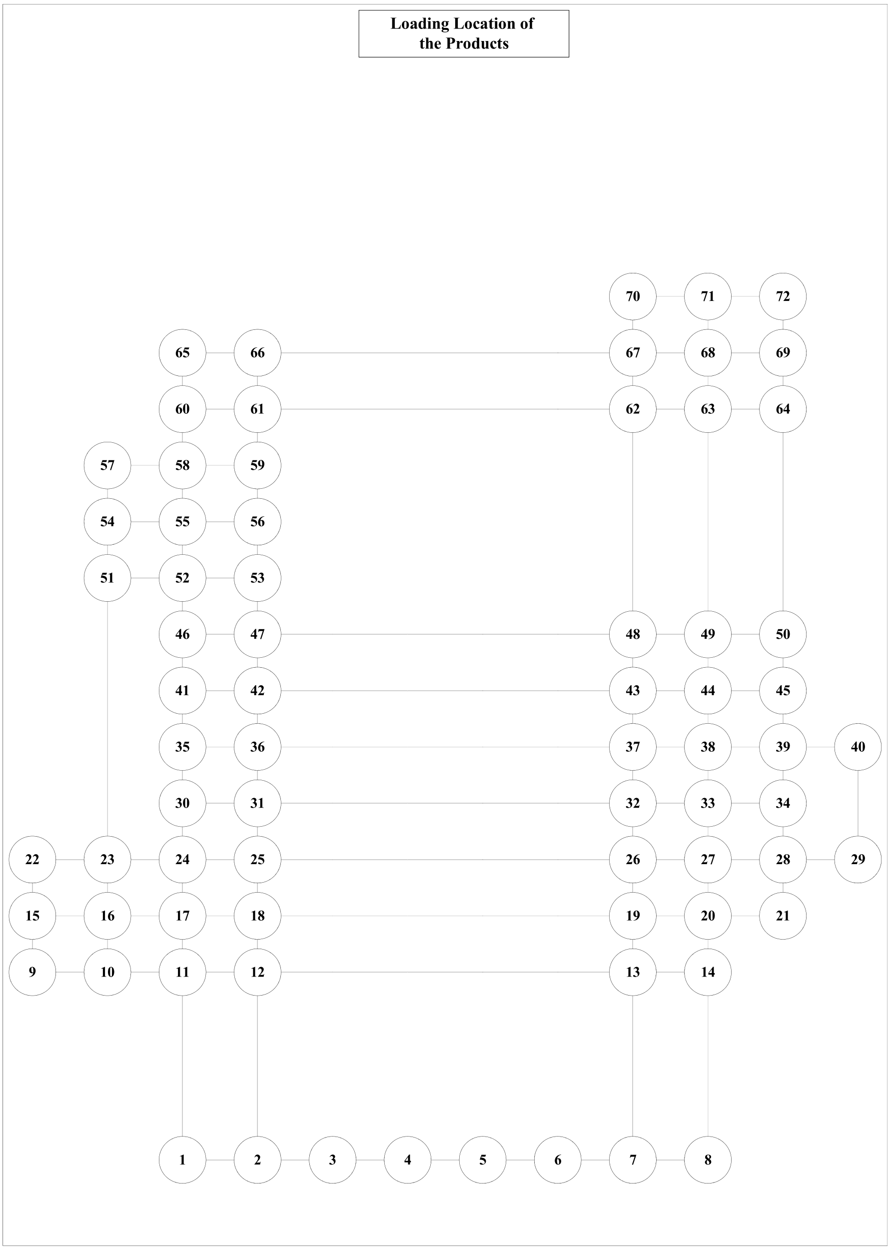

Figure 1. The location of the tanks

4.1 Problem solving

After solving the mathematical programming model for all of the service levels, it becomes clear that the problem has no feasible solution, which means that the tanks in the refinery are incapable of meeting all the customer demands and there will always be some demands that the company could not meet and a number of stock market customers would be lost.

4.2 Increasing the dimensions of the problem to meet the customer demand

Taking into consideration the fact that the 72 current tanks are not capable of meeting all of the demands at all service levels, therefore, if the company wants to have a continuous and long-term presence in the stock market, while complying with its obligations to the government at the same time, and keep the customers content, it should increase the number of available tanks. At this stage, assuming that the construction of new tanks is financially and economically justified and can convince the investors, we examine the problem again.

Considering different conditions, including the layout of the refinery equipment, land constraints, access to utilities and the ease and possibility of construction, building new tanks in order of priority in A, B, C and D locations, which is shown in Figure 2, is possible.

By adding a new tank in location A, we solve the problem again. Given the fact that the problem has justified solutions to different service levels, therefore, by building a new tank in A location, B, C and D locations are no longer required.

Then, we solve the problem by considering 73 tanks and service levels of more than 90%. As the SCPMP-ND mathematical programming model is nonlinear, the solutions obtained by Lingo in this method may be local optimum, and for a given problem, several solutions, which cannot necessarily be global optimum, may have been obtained. For this reason, the problem is solved in the interval $[90\%, 100\%]$ for 100 times, and the results, based on the lowest cost, are shown in Table 5.

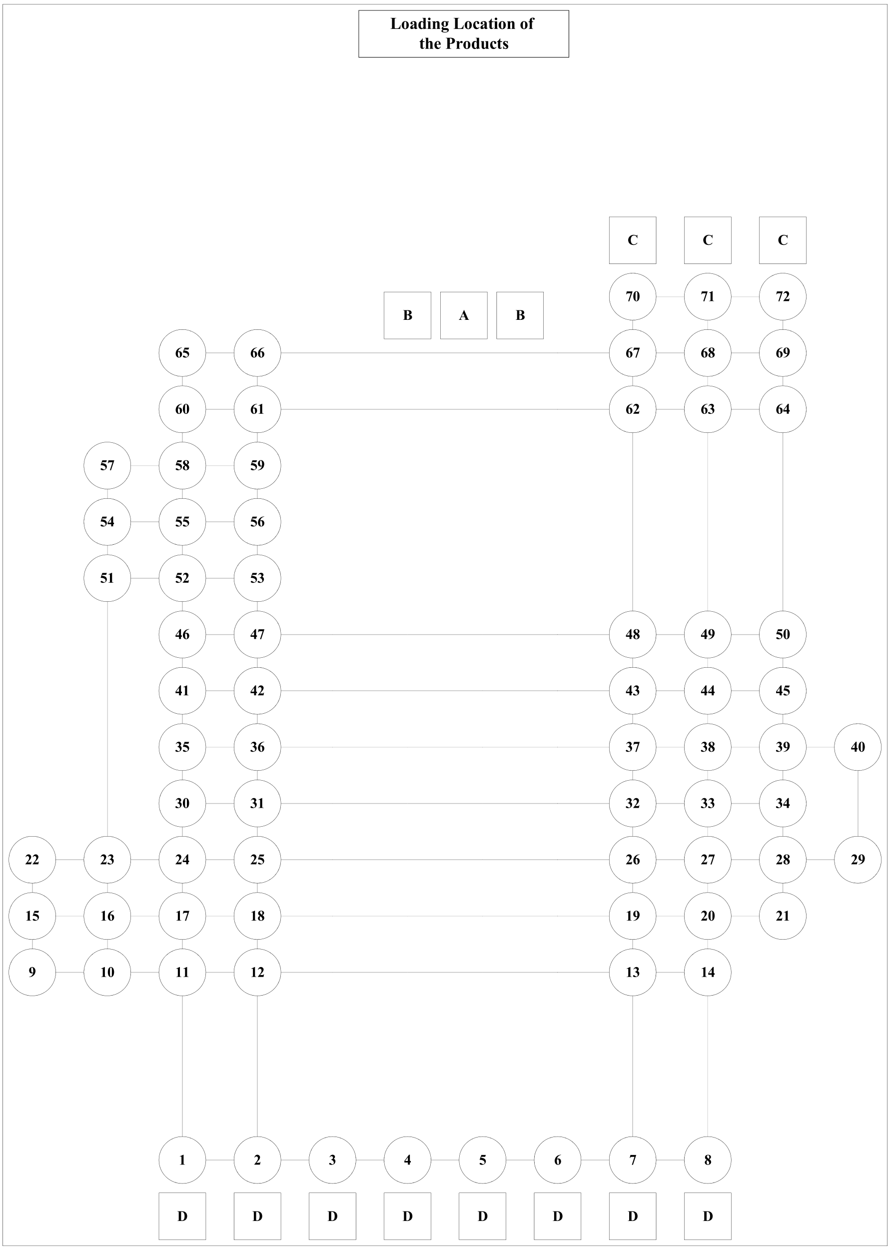

Figure 2. Prioritization to examine the construction of new tanks

Table 5: Results from problem-solving with 73 tanks with above 90% service levels

Risk Level (α)	**Service Level** ($1-\alpha$)	**Cost**	$\boldsymbol{p}$
$10.00\% \leq \alpha < 9.85\%$	$90.00\% \leq 1-\alpha < 90.15\%$	17,380	11
$9.85\% \leq \alpha < 6.81\%$	$90.15\% \leq 1-\alpha < 93.19\%$	17,860	12
$6.81\% \leq \alpha < 0.75\%$	$93.19\% \leq 1-\alpha < 99.25\%$	18,020	13
$0.75\% \leq \alpha < 0.71\%$	$99.25\% \leq 1-\alpha < 99.29\%$	18,260	12
$0.71\% \leq \alpha < 0.03\%$	$99.29\% \leq 1-\alpha < 99.97\%$	18,920	13
$0.03\% \leq \alpha < 0.00\%$	$99.97\% \leq 1-\alpha < 100.00\%$	22,520	18

If we want to meet customer demands with 90% service level (10% risk level), we should develop tanks 5, 16, 17, 19, 34, 36, 44, 52, 60, 63 and 71 at a cost of 17,380 units.

If we want to reduce the risk level in a way that the service level goes above 99.97%, we should increase the cost of developing the tanks by approximately 30% and develop 18 tanks, instead of 11, at a cost of 22,520 units.

4.3 Comparing the solutions of CPMP and SCPMP-ND

In this section, we solve the problem in section 4.2 by the CPMP model and compare its results with the solutions obtained with the SCPMP-ND method.

To do this, we first add the equation $\sum_{j\in M} f_j y_j$ to the objective function of the CPMP mathematical Programming model. Then, we consider the d_{ij} and Q_j values of the SCPMP-ND problem as the constant values of the CPMP model. Finally, assuming $\mu_i = q_i$, we solve the obtained model for CPMP. In Table 6, the results of the CPMP and SCPMP-ND mathematical programming models and their solutions are compared with each other:

Table 6: Comparing CPMP and SCPMP-ND solutions

Title	**CPMP**	**SCPMP-ND** ($99.97\% \leq 1 - \alpha < 100.00\%$)
Dimensions of problem ($M \times N$)	73×73	73×73
Model type	Linear	Nonlinear
Type of model variables	Binary	Mixed integer
Type of solutions	Global Optimum	Local Optimum
Value of the objective function	15,020	22,520
p value obtained from problem solving	7	18
Average problem solving time by software	37 second	1,276 second
Average number of iterations to reach the solution	55,144	22,744

One of the most important differences between CPMP and SCPMP-ND, also to be seen in Table 6, is the problem solving time. The problem solving time to reach the local optimum solution in SCPMP-ND is more than 33 times more than the problem solving time to reach the global optimum solution in the CPMP, which is very important and significant in large-scale problems.

5. Conclusion

One of the most important and most widely applied location-allocation problems is the CPMP, whose aim is to minimize the total cost of service. Considering that the CPMP parameters are deterministic and do not include stochastic values, this paper attempts to present a model that considers the service level to customers, which is usually stochastic, based on the service level.

If the demand follows a normal distribution, the proposed model would be a Mixed Integer Nonlinear Programming problem, which in this paper, it is attempted to use it to develop the tanks of a refinery.

For future research, two main approaches can be considered:

a) In this paper, the normal distribution is used to consider stochastic values, while in different problems these stochastic parameters may follow other statistical distributions. Therefore, we can examine the problem by taking into account other statistical distributions, such as the uniform distribution and the exponential distribution, achieve the mathematical Programming model and present a suitable solution to the problem.

b) Solving the proposed model in order to reduce the problem solving time and find the global optimum solution is one of the most important issues that researchers can consider for large-scale problems using different methods, such as heuristic and metaheuristic approaches.

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