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A Bi-Objective Formulation For Refueling Stations Selection Problem

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Abstract. Using fossil fuels in transportation sector has caused many environmental and economic problems. Therefore, the use of alternative fuel vehicles is necessary. Since such vehicles have limited fuel tank capacities, hence, frequent refueling is required. Regarding the possibility of the existence of different fuel costs in various refueling stations, the selection of suitable stations with the purpose of minimizing total cost of refueling is important. Moreover, minimizing the number of refuelings could be considered as another important criterion in refueling operations of a given trip. In this paper an integer bi-objective model is proposed to select suitable refueling stations considering two criteria of minimizing the total cost of refueling and minimizing the number of refuelings. To solve the proposed model, a new algorithm is suggested and its performance is compared with the weighted sum method of multi-objective optimization literature. The results show the superiority of the proposed solution approach.

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1. Introduction

Air pollution and global warming due to excessive consumption of fossil fuels are one of the today's human dilemmas. It is shown that 73% of all the oil consumed in Europe is used for transportation, and road transportation accounts for 22% of CO_2 emissions of the overall transportation activities [18]. The use of alternative fuel vehicles (AFVs) is a promising approach to reduce the pollution caused by the use of vehicles. Some of the alternative fuels that used in AFVs are ethanol, natural gas, liquid petroleum gas, etc. [16]. However, the capacity of fuel tank of AFVs is a major shortcoming of these type of vehicles which limits their range. Hence, AFVs may need some refuelings when their fuel capacity is not enough for traversing long distances. Therefore, it is essential to carry out precise schedules for refueling of these vehicles. In this regard, a lot of research has been done on the issue of refueling vehicles. Hodgson [8] and Berman et al. [2] presented the flow capturing location model (FCLM) to locate the refueling stations with the goal of maximizing the total flow that passes through stations. The proposed model was a good start on refueling issues, however, the limitation of driving range of vehicles was ignored. To cope with the limitation of driving range, Kuby and Lim [12] developed the FCLM as a mixed integer programming formulation named the flow refueling location model (FRLM). The purpose of the FRLM is to determine optimal locations of refueling stations for range-limited vehicles such that a maximum flow is covered. However, due to the required long time for solving the FRLM in real-world cases, Lim and Kuby [13] developed some heuristic algorithms for solving FRLM problems. Unfortunately, Lim and Kuby [13] were unable to solve the FRLM for large-scale networks. Therefore, Capar and Kuby [3] introduced a new formulation of the FRLM. This new formulation can solve large problems that have not been possibly solved by the previous formulation of the FRLM. Wang and Lin [21] proposed a refueling station location model which does not use the parameter of flow as an input data. Instead of the flow, they used a distance matrix which is easier to obtain. Using the logic of expanded network, MirHassani and Ebrazi [14] could reduce the solution time of the model that was introduced by Wang and Lin [21]. Some of other researchers on refueling vehicles are as follows. The capacity of refueling stations is one the considered issue in refueling vehicles [20]. Some studies have discussed the drivers' deviations for refueling and solution approaches for them [9, 10]. Moreover, some other topics such as comparison of p-median and FRLM [19] and dispersion of candidate sites on arcs [11] were also considered in the field of refueling.

All mentioned works have studied the establishment of infrastructures for AFVs. To the authors' knowledge, planning for drivers of AFVs has not been considered up to now in the literature. When drivers want to make a trip, due to the limited range of their vehicles they need frequent refuelings. Different cost of fuel in different refueling stations may confuse drivers in choosing refueling stations. Obviously, they are interested in selecting refueling stations with cheaper fuel. But, how they can have a trip with a minimum cost of refueling while their vehicles do not run out of fuel and at the same time have minimum stops for refueling. To cope with these challenges, a mathematical model is proposed in this paper. In this regard, suppose that there is an arbitrary route between a fixed origin and a specified destination in a network which should be traversed by a vehicle without running out of fuel. In order to encounter the proposed problem based on the mentioned criteria, a bi-objective integer programming formulation with the objectives of the minimum cost of refueling and the minimum number of refuelings is presented in this paper. To handle the model, a new solution approach is developed based on the inherent multi-objective nature of the problem. Moreover, the performance of the algorithm is compared with the weighted sum method and some valuable results have been obtained.

The remainder of the paper is organized as follows: some preliminaries are stated in Section 2. Section 3 describes the problem and proposes its mathematical programming model. In Section 4, an algorithm is presented to solve the proposed model. Computational experiments are performed in Section 5. Finally, Section 6 provides the conclusions.

2. Preliminaries

In this section, some required concepts and definitions are described.

The concept of multi-objective decision making model has been used in various fields [5, 6, 7, 15]. A basic multi-objective integer programming problem is defined as follows:

$$\begin{aligned} \min : Z(\mathbf{x}) &= \mathbf{Cx} \\ s.t.\ \ \mathbf{x} &\in X. \end{aligned} \tag{1}$$

Where, $X = \{\mathbf{x} \in \mathbb{Z}^n | A\mathbf{x} = \mathbf{b}, \mathbf{x} \geqslant 0\} \subseteq \mathbb{Z}^n$, $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$. Further, $C_{p\times n}$ is a coefficient matrix of the objective functions.

For multi-objective optimization problem, usually no single solution exists that simultaneously optimizes all objective functions. Hence, the concept of the optimal solution is rarely applied in the multi-objective problem. Therefore, the concept of efficient solution is utilized instead of optimal solution.

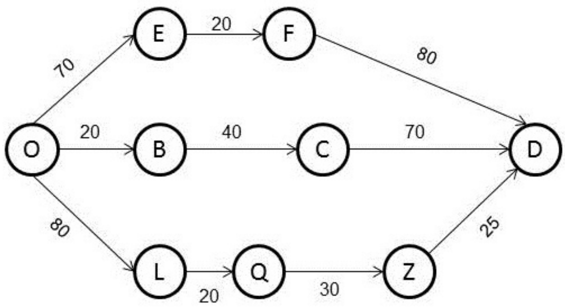

Figure 1. The network contains path p.

In efficient solutions set, no improvement in any objective function is possible without sacrificing at least one of the other objective functions. In the definition of concept of efficient solutions the order " $\preccurlyeq$ " in $\mathbb{R}^p$ is used. Let $\mathbf{a} = (a_1, \dots, a_p)^t$ and $\mathbf{b} = (b_1, \dots, b_p)^t$ be two vectors in $\mathbb{R}^p$. Then $\mathbf{a} \preccurlyeq \mathbf{b}$ if $a_i \leqslant b_i$ for $i = 1, \dots, p$ and there is also at least one $1 \leqslant j \leqslant p$ such that $a_j < b_j$.

Definition 2.1. *A feasible solution $\boldsymbol{x}^0$ of Problem (1) is efficient if there exist no $\boldsymbol{x} \in X$ such that $C\boldsymbol{x} \preccurlyeq C\boldsymbol{x}^0$. If $\boldsymbol{x}^1, \boldsymbol{x}^2 \in X$ and $C\boldsymbol{x}^1 \preccurlyeq C\boldsymbol{x}^2$ we* *say* $C\boldsymbol{x}^1$ *dominates* $C\boldsymbol{x}^2$. *If* $\boldsymbol{x}^0$ *is efficient,* $C\boldsymbol{x}^0$ *is called nondominated vector [4, 12].*

3. Problem Statement

Since the amount of fuel needed to travel on a given path is usually greater than the fuel tank capacity of a vehicle, so some refuelings may be needed to complete a trip. To find valid combinations of these refuelings, the refueling logic is described by an example. For instance, consider the shortest path p from the origin node O to the destination node D in Fig. 1 i.e. $O-B-C-D$. Each node displays a refueling station and the length of each arc is written on it. Now, suppose that a vehicle with full tank of fuel has the driving range of $R = 100$ km. This vehicle is going to have a trip from the origin O to the destination D on the shortest path p, without running out of fuel. It is assumed that the vehicle has a half-full tank of fuel at the origin so its driving range is 50 km at the origin. Moreover, it is supposed that the fuel consumption of the vehicle is directly related to the traveling distance [12], i.e. for traveling per kilometer, the vehicle needs a unit of fuel. In this regard, if the refueling is performed at the node O (the origin), then the vehicle with the full tank of fuel cannot reach the destination $(20 + 40 + 70 > 100)$ and the vehicle can reach at most a point between nodes C and D. Therefore, in this situation, to reach the destination node D, refueling at node C is necessary. However, if the vehicle does not refuel at the origin, due to the existing amount of fuel in the tank at the origin, a refueling in node B should be made. Since it is not possible to reach the destination with the full tank of fuel from node B, so refueling at node C is also needed. Therefore, $\{O, C\}$ and $\{B, C\}$ are examples of valid refueling combinations for traveling path p. Notably, the vehicle may have missed any amount of fuel e.g. M at the origin.

In order to find valid refueling combinations for path p, MirHassani and Ebrazi [14] introduced the concept of expanded network. Consider the sets N^p and E^p include nodes and arcs on path p. In order to construct the expanded network corresponding to path p, first, two virtual nodes

s^p and t^p are added prior the origin O and after the destination D of path p and connected to them, respectively. Therefore, we define set $\tilde{N}^p$ contains nodes of set N^p union nodes s^p and t^p, i.e. $\tilde{N}^p = N^p \cup \{s^p, t^p\}$. Also, arcs (s^p, O) and (t^p, D) are added to the empty set $\tilde{E}^p$. For any two nodes $i, j \in \tilde{N}^p$, define the value of f^p_{ij} which represents the amount of fuel consumption of the vehicle for traveling the subpath between nodes i and j on path p. Then, node s^p is connected to any other nodes of the path; say $i \in \tilde{N}^p$, if it is possible to begin at the origin node and reach them by the existed amount of fuel in the tank or less. In other words, $\forall i \in \tilde{N}^p \ni f^p_{Oi} \leqslant R - M \Rightarrow (s^p, i) \in \tilde{E}^p$. Moreover, define $ord_p(i)$ which shows the ordering index of node i on path p. The defination of ordering index can be given as a recursive equation i.e. $ord_p(i) = e_p(i) + 1$, where $e_p(i)$ is the number of arcs on the shortest path p between the origin and node i. For example, the ordering index of node B on the shortest path p between O and D in Fig. 1 is equal to two. Finally, each node $i \in N^p$ is connected to any other node $j \in \tilde{N}^p$ if the ordering index of node i is less than the ordering index of node j and the vehicle is able to start from node i with a full tank and reach node j. It means that, $\forall i \in N^p, j \in \tilde{N}^p \ni (ord_p(i) < ord_p(j))$ & $f^p_{ij} \leqslant R \Rightarrow (i, j) \in \tilde{E}^p$. In this way, the network $\tilde{G}^p(\tilde{N}^p, \tilde{E}^p)$ which defines an expanded network corresponding to path p with the set of nodes $\tilde{N}^p$ and arcs $\tilde{E}^p$ on path p is constructed. In the network $\tilde{G}^p$, each arc corresponds to the consecutive feasible refueling and each directed path from the node s^p to node t^p corresponds to a valid combination of stations that can refuel path p [14]. Obviously, if in the process of constructing the expanded network, we encounter a node in the set N^p, which could not be connected to the next node in the set N^p with the larger ordering index, then path p becomes infeasible i.e., this path could not be traveled due to the fuel capacity restriction.

Now, there may be different criteria for deciding and choosing appropriate refueling station combinations to pass path q. Various stations offer fuel with different costs. Therefore, refueling at the lowest cost can be one of the goals of the drivers. A minimum number of refuelings may also be of particular importance to the users of vehicles. In this regard, the following mathematical formulation is proposed for selecting of appropriate refueling stations considering the mentioned criteria.

The parameter and decision variables of the proposed model are as follows.

Parameter:

c_j: The cost of a unit of fuel in refueling station located in node j.

decision variables:

y_j^p: A binary variable with value of 1 if the vehicle is refueling at node j, otherwise 0.

x_{ij}^p: A binary variable with value of 1 if arc $(i,j) \in \tilde{E}^p$ is traveled, otherwise 0.

The proposed model is as below.

$$\min: \quad Z_1 = \sum_{\substack{(s^p,j)\in\tilde{E}^p \\ j\in\tilde{N}^p \\ j\neq t^p}} c_j\left(M + f_{s^pj}^p\right)x_{s^pj}^p + \sum_{\substack{(i,j)\in\tilde{E}^p \\ i,j\in\tilde{N}^p \\ i\neq s^p,\ j\neq t^p}} c_j f_{ij}^p x_{ij}^p, \tag{2}$$

$$\min: \quad Z_2 = \sum_{j\in N^p} y_j^p, \tag{3}$$

s.t.

$$\sum_{\{j|(i,j)\in\tilde{E}^p\}} x_{ij}^p - \sum_{\{j|(j,i)\in\tilde{E}^p\}} x_{ji}^p = \begin{cases} 1 & \text{if } i = s^p, \\ 0 & \text{if } i \neq s^p, t^p, \\ -1 & \text{if } i = t^p, \end{cases} \quad \forall i \in \tilde{N}^p, \tag{4}$$

$$\sum_{\{i|(i,j)\in\tilde{E}^p\}} x_{ij}^p \leqslant y_j^p, \qquad \forall j \in N^p, \tag{5}$$

$$x_{ij}^p \in \{0,1\}, \qquad \forall (i,j) \in \tilde{E}^p, \tag{6}$$

$$y_j^p \in \{0,1\}, \qquad \forall j \in N^p. \tag{7}$$

The first objective function (2) minimizes the total cost of refueling on path p. Note that, s^p and t^p are virtual nodes and the amount of fuel consumption from node s^p to the origin node and from the destination node to node t^p are equal to zero. The second objective function (3) minimizes the number of refuelings. Constraints (4) are the mass balance constraints, which state that the vehicle should start its trip from s^p and end it at node t^p and if the vehicle enters an intermediate node it should leave that node. Constraints (1) ensure that when the vehicle enters a node, it should refuel. Constraints (6) and (7) are zero-one variables.

4. Solution Method

In order to solve the presented model, a new algorithm is proposed based on the modified labeling algorithm [1], in this section. To this aim, according to the proposed problem, additional entries are used in node labels and the values of two objectives are considered as elements of a vector. In applying this algorithm, we first assign the label $L^p(j) = ((w_{1j}, w_{2j}), pred(j))$ to each node j of feasible path p in the expanded network. As can be seen, $L^p(j)$ contains two elements. The first element is a vector consist of two components. The first component, w_{1j}, represents the total cost of refueling for traversing from node s^p to node j. The second component, w_{2j}, shows the number of refuelings from node s^p to node j. Moreover, the second element in the node label, $pred(j)$, is a node just prior to node j.

At the beginning of the algorithm, each node of path p is labeled by the followings,

$$L^p(s^p) = ((0,0), -), \tag{8}$$

$$L^p(j) = ((\infty, \infty), -) \quad \forall j \in \tilde{N}^p \ \& \ j \neq s^p. \tag{9}$$

In the procedure of the algorithm, in addition to the set *list* which is defined initially *list* $= \{s^p\}$, set $D^p(j)$ of labels is associated to each node $j \in \tilde{N}^p$. Initially, set $D^p(j)$ contains the above mentioned labels. Set $D^p(j)$ is updated during the algorithm in such a way that, no first element of each label in $D^p(j)$ is dominated by another first element of the labels in $D^p(j)$. Now, remove the node s^p from the *list* and find all arcs $(s^p, j) \in \tilde{E}^p$. Then, to create a new label for each node j on path p where $(s^p, j) \in \tilde{E}^p$, a label scanning process is started. Since, s^p is a virtual node and no refueling is occurred on it so, the new label of node j is $L^p(j) = ((0,0), s^p)$. After that, the new label of node j is added to the set $D^p(j)$. Since $(0,0) \preccurlyeq (\infty, \infty)$, therefore the previous label of node j is omitted from the set $D^p(j)$ and the new label $((0,0), s^p)$ is added to the set $D^p(j)$ and the set is updated. Next, node j which is not in the *list* is added to it.

The general procedure of the algorithm when *list* contains nodes at which refueling could be done is described as follows. Extract a member such as i from the *list* and specify all of the arcs $(i,j) \in \tilde{E}^p$. Then the labels for all nodes j that $(i,j) \in \tilde{E}^p$ are constructed. For each $L^p(i) = ((\infty,\infty), -)$ in the set $D^p(i)$ two cases happens. If $pred(i) = s^p$, in this case due to the assumptions of the problem, the vehicle starts its trip with a $(R-M)$-full tank of fuel from the origin node and it is refueled at node i. Therefore, to find new labels for node j, the vector $H_{L^p(i)} = (w_{1j}, w_{2j}) + (c_i(M + f_{s^p i}), 1)$ is calculated. But if $pred(i) \neq s^p$, in this case, the vehicle moves from the previous station of node i with a full tank of fuel and after traversing arc $(pred(i), i)$, it reaches node i. So, to get new labels of node j, the vector $H_{L^p(i)} = (w_{1j}, w_{2j}) + (c_i f_{pred(i)i}, 1)$ is computed. After that, all obtained vectors, $H_{L^p(i)}$, are compared with respect to the order " $\preccurlyeq$ " and nondominated ones are kept. Then, the labels corresponding to the obtained vectors are added to the set $D^p(j)$. In the end, the first element of all labels in the set $D^p(j)$ are compared according to the order " $\preccurlyeq$ " and dominated ones are removed from $D^p(j)$. In this way, the set $D^p(j)$ is updated. Then, if node j is not presented in the *list*, it is added there. Again, a member is removed from *list* and this process is continued until the *list* is not empty. Thus refueling stations, which may not be unique, can be determined via post-processing by backtracking through the predecessor labels.

5. Numerical Experiments

In this section, the performance of the proposed algorithm for solving the presented model is compared with the weighted sum method. The weighted sum method entails scalar weights θ_1 and θ_2, where, $\theta_1 + \theta_2 = 1$. The weighting coefficients θ_1 and θ_2 denote the relative importance of the first and the second objective functions, respectively. The algorithm was coded in MATLAB 8.1.0.604 (R2013a) and the problem was solved by the weighted sum method using CPLEX solver of GAMS. All the experiments were done on a laptop equipped with Core i5 Processor (2.40 GHz) and 4.00 GB RAM.

Consider an example network with some nodes and arcs (Fig. 2). Nodes show the existing fuel stations and the length of each arc is written on it. The cost of a unit of fuel at each station is specified as follows:

$$c_1 = 2.7 \quad c_2 = 2.7 \quad c_3 = 3.8 \quad c_4 = 2.8 \quad c_5 = 3.1 \quad c_6 = 2.8 \quad c_7 = 3.5 \quad c_8 = 3.8 \quad c_9 = 2.7$$
$$c_{10} = 3.6 \quad c_{11} = 1.5 \quad c_{12} = 1.1 \quad c_{13} = 3.6 \quad c_{14} = 3.3 \quad c_{15} = 2 \quad c_{16} = 2.8 \quad c_{17} = 2 \quad c_{18} = 3.$$

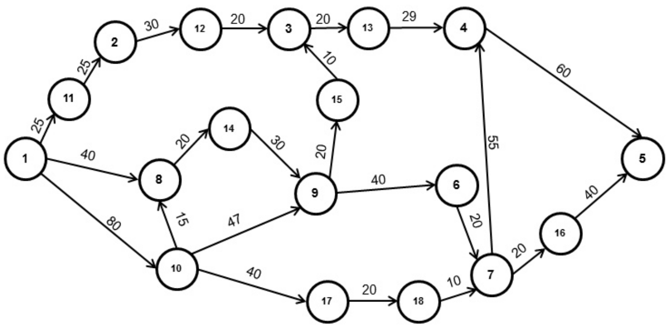

Figure 2. Network of fuel stations.

Moreover, consider a vehicle with $R = 100$ km is going to have a trip with a half-full tank of fuel at the origin. The results of the example network, when the problem is solved by the weighted sum method and the labeling algorithm are presented in Table 1. Using the labeling algorithm, efficient solutions and the value of objective functions corresponding to them in some specific $O-D$ pairs are presented in this table. Moreover, the values of objective functions that are obtained by the weighted sum method are shown in Table 1. As using different θ_1 values give similar solutions, we report the results only for two weights, i.e. $\theta_1 = 0.01$ and 0.99.

As can be seen in Table 1, the labeling algorithm obtains two efficient solutions for the $O - D$ pair $1 - 4$. The first solution $(Z_1 = 270, Z_2 = 1)$ is equal to the obtained solution by the weighted sum method for $\theta_1 = 0.01$. The second obtained solution by the labeling algorithm is

$Z_1 = 173, Z_2 = 2$ which is equal to the obtained solution by the weighted sum method for $\theta_1 = 0.99$. In fact, the labeling algorithm was able to obtain both of the solutions which are obtained by the weighted sum method for different weights.

Table 1: The results of the labeling algorithm and the weighted sum method for $R = 100$.

$O-D$	Labeling Algorithm		Weighted Sum Method		
	Z_1	Z_2	θ_1	Z_1	Z_2
$1-4$	270	1	0.01	270	1
	173	2	0.99	173	2
$8-4$	270	1	0.01	270	1
			0.99	190	2
$9-5$	252	1	0.01	252	1
			0.99	247	2
$10-4$	180	1	0.01	180	1
			0.99	180	1

For the $O-D$ pairs $8-4$ and $9-5$, the labeling algorithm obtains an efficient solution which is the same as the obtained solution by the weighted sum method in case $\theta_1 = 0.01$. Moreover, this obtained solution by the labeling algorithm in the considered $O-D$ pairs is incomparable with the obtained solution by the weighted sum method for $\theta_1 = 0.99$. For both of the considered $O-D$ pairs, by sacrificing the first objective function the labeling algorithm can obtained smaller number of refuelings than that of the weighted sum method for $\theta_1 = 0.99$.

In the $O-D$ pair $10-4$, the labeling algorithm obtains one solution.

The weighted sum method also obtains the same solutions to the labeling algorithm for $\theta_1 = 0.01$ and $\theta_1 = 0.99$.

Table 2: The results of the labeling algorithm and the weighted sum method over different ranges when $\theta_1 = 0.01$.

$O - D$	Driving Range	Labeling Algorithm		Weighted Sum Method	
		Z_1	Z_2	Z_1	Z_2
$1 - 4$	100	270	1	270	1
		173	2		
	120	297	1	188	2
		188	2		
$8 - 4$	100	270	1	270	1
	120	264	1	264	1
$9 - 5$	100	252	1	252	1
	120	162	1	162	1
$10 - 4$	100	180	1	180	1
	120	200	1	200	1

According to the presented results, the weighted sum method has some deficiencies. Setting weights to indicate the relative importance of each of the objective functions is difficult. Moreover, the weighted sum method can obtain at most one solution. In contrast, the proposed labeling algorithm is independent of weights and preserves the inherent multi-objective nature of the problem to deal with the real-world situations. Furthermore, the proposed approach can obtain multiple solutions in a one-run. In addition, according to the obtained results in Table 1, in all cases, the proposed labeling algorithm can obtain at least one of the solutions obtained by the weighted sum method for different weights.

The performances of the labeling algorithm and the weighted sum method are considered using two driving ranges $R = 100$ and 120, when $\theta_1 =$

0.01, in Table 2. The solutions and the corresponding objective functions are obtained by both solution approaches for the same $O-D$ pairs as what selected in Table 1. The results in Table 2 show that in both solution methods, in most of cases, the longer vehicle range, the equal number of refuelings are needed for traveling the path.

6. Conclusion

Considering the limited fuel tank capacity of the alternative fuel vehicles, refueling problem is an important issue in these vehicles. Regarding the possibility of the existence of different fuel costs in various refueling stations, the selection of stations which have the minimum cost of refueling is important. Moreover, minimizing the number of refuelings could be considered as another important criterion in refueling vehicles. In this paper, the problem of refueling vehicle is considered with two criteria, minimizing the total cost of refuelings as well as minimizing the number of refuelings. In this regard, an integer bi-objective programming model is presented. In order to solve the proposed model, a new algorithm is suggested and its performance is compared with the weighted sum method. The results show the superiority of the proposed solution approach. Moreover, in both solution methods, in most cases, the longer vehicle range, the equal number of refuelings are needed for traveling the path.

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