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J-Armendariz Rings

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Abstract. We introduce the notion of J-Armendariz rings, which are a generalization of weak Armendariz rings and investigate their properties. We show that local rings are J-Armendariz. Also, we prove that a ring R is J-Armendariz if and only if $R[[x]]$ is J-Armendariz. It is shown that the J-Armendariz property is not Morita invariant. As a specific case, we show that the class of J-Armendariz rings lies properly between the class of one-sided quasi-duo rings and the class of perspective rings.

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1. Introduction

Throughout this article, R denotes an associative ring with identity. For a ring R, $Nil(R)$, $M_n(R)$, $T_n(R)$ and e_{ij} denote the set of nilpotents elements in R, the $n \times n$ matrix ring over R, the $n \times n$ upper triangular matrix ring over R and the matrix with (i, j)-entry 1 and elsewhere 0, respectively. In 1997, Rege and Chhawchharia introduced the notion of an Armendariz ring. They called a ring R Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_ib_j = 0$ for all i and j. The name "Armendariz ring" is chosen because Armendariz [3, Lemma

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1] proved that reduced rings (that is a ring without nonzero nilpotents) satisfy this condition. A number of properties of Armendariz rings have been studied in [2, 3, 12, 13, 18]. So far Armendariz rings are generalized in several forms [11, 8, 16]. Liu and Zhao [16] called a ring R weak Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$, $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j \in Nil(R)$ for all i and j.

The Jacobson radical is an important tool for studying the structure of noncommutative rings, and denoted by $J(R)$. Motivated by the above definitions, we investigate a generalization of weak Armendariz rings. We call a ring R, *J-Armendariz* if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_ib_j \in J(R)$ for all i and j. Clearly, for an artinian ring, weak Armendariz rings and J-Armendariz rings are the same. Although $Nil(R)$ does not always lie in the $J(R)$, we show weak Armendariz rings are J-Armendariz and local rings are J-Armendariz too, but Example 2.4 shows that local rings are not necessarily weak Armendariz. Thus J-Armendariz rings are a proper generalization of weak Armendariz rings.

At last we study the relation of J-Armendariz rings with other classes of rings such as: right (left) quasi duo rings, perspective rings, clean rings and strongly π-regular rings. In [7], Garg et al., studied the modules whose any two isomorphic summands have a common complement. They called such modules perspective. This property in rings turns out to be left-right symmetric, that is, R_R is perspective if and only if ${}_RR$ is perspective and they called such ring a perspective ring. We show that a J-Armendariz ring R is perspective. However there exists a perspective ring which is not J-Armendariz. On the other hand a ring R is called right (left) quasi-duo if every maximal right (left) ideal of R is two-sided. We prove that a right (left) quasi-duo ring is J-Armendariz, but there exists a J-Armendariz ring R which is not right (left) quasi-duo. Therefore the class of J-Armendariz rings lies properly between the class of right (left) quasi-duo rings and the class of perspective rings.

2. J-Armendariz Property with Respect to Standard Constructions

In this section, J-Armendaiz rings are introduced as a generalization of weak Armendariz rings. We study J-Armendariz property with respect to some standard constructions like direct product, factor rings, subrings, matrix rings, corner rings, polynomial rings, etc.

Definition 2.1. *A ring R is said to be J-Armendariz if for any nonzero poly-*

nomials $f(x) = \sum_{i=0}^{n} a_i x^i$ *and* $g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$*,* $f(x)g(x) = 0$*, implies that* $a_i b_j \in J(R)$ *for each* i, j*.*

We can easily show that weak Armendariz rings are J-Armendariz. For it, let R be weak Armendariz and $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{j=0}^{m} b_j x^j \in R[x] - \{0\}$ such that $f(x)g(x) = 0$. Hence $rf(x)g(x) = 0$ for each $r \in R$ and so $ra_i b_j \in Nil(R)$ by hypothesis. This implies that $a_i b_j \in J(R)$, as desired. But Example 2.4 shows that J-Armendariz rings are not necessarily weak Armendariz.

Proposition 2.2. *Let* R *be a ring and* I *an ideal of* R *such that* R/I *is J-Armendariz. If* $I \subseteq J(R)$*, then* R *is J-Armendariz.*

Proof. It is clear after applying $J(\frac{R}{I}) = \frac{J(R)}{I}$, when $I \subseteq J(R)$. $\square$

Corollary 2.3. *Let* R *be any local ring. Then* R *is J-Armendariz.*

One may ask whether local rings are weak Armendariz, but the following gives a negative answer.

Example 2.4. Let F be a field, $R = M_2(F)$ and $R_1 = R[[t]]$. Consider the ring

$$S = \{\sum_{i=0}^{\infty} a_i t^i \in R_1 | a_0 \in kI \ for \ k \in F\},$$

where I is the identity matrix. It is obvious that S is local and so is J-Armendariz by corollary 2.3. Now for $f(x) = e_{11}t - e_{12}tx$ and $g(x) = e_{21}t + e_{11}tx \in S[x]$, we have $f(x)g(x) = 0$, but $(e_{11}t)^2$ is not nilpotent in S, and so S is not weak Armendariz.

Let R_t be a ring for each $t \in I$. Note that since $\prod_{t\in I} J(R_t) = J(\prod_{t\in I} R_t)$, then $\prod_{t\in I} R_t$ is J-Armendariz if and only if R_t is J-Armendariz, for each $t \in I$.

Theorem 2.5. *A ring* R *is J-Armendariz, if and only if* $R[[x]]$ *is J-Armendariz.*

Proof. Let R be a J-Armendariz ring. Since $R \cong \frac{R[[x]]}{\langle x \rangle}$, then by proposition 2.2, $R[[x]]$ is J-Armendariz. Conversely, assume $R[[x]]$ is J-Armendariz, and $f(y) = \sum_{i=0}^{n} a_i y^i$ and $g(y) = \sum_{j=0}^{m} b_j y^j$ are polynomials in $R[y]$, such that $f(y)g(y) = 0$. Since $a_i b_j \in R \subseteq R[[x]]$ and $R[[x]]$ is J-Armendariz, then $a_i b_j \in J(R[[x]]) \cap R$. Therefore $a_i b_j \in J(R)$, and so R is J-Armendariz. $\square$

The following example shows that the polynomial ring over a J-Armendariz ring need not be J-Armendariz in general and so the subring of a J-Armendariz ring is not necessarily J-Armendariz.

Example 2.6. Take S to be the ring as in Example 2.4. Then $S[x]$ is not J-Armendariz. For it, let $f(y) = e_{11}tx - e_{12}txy$ and $g(y) = e_{21}tx + e_{11}txy$ be polynomials in $S[x][y]$. Then $f(y)g(y) = 0$, but $(e_{11}tx)^2$ does not belong to $J(S[x])$.

Proposition 2.7. *Let R be a ring.*
(1) If $R[x]$ is J-Armendariz then R is weak Armendariz and so R is J-Armendariz.
(2) If R is a J-Armendariz ring and $J(R)[x] \subseteq J(R[x])$, then $R[x]$ is J-Armendariz.

Proof. (1) Suppose that $R[x]$ is a J-Armendariz ring. Let $f(y) = \sum_{i=0}^{n} a_i y^i$ and $g(y) = \sum_{j=0}^{m} b_j y^j$ be nonzero plynomials in $R[y]$, such that $f(y)g(y) = 0$. By the fact that $J(R[x]) = I[x]$ for some nil ideal I of R [1], $a_i b_j \in R \cap I[x] \subseteq Nil(R)$, and so R is weak Armendariz.

(2) Suppose that R is J-Armendariz and $J(R)[x] \subseteq J(R[x])$. Let $F(y) = f_0 + f_1 y + \cdots + f_n y^n$ and $G(y) = g_0 + g_1 y + \cdots + g_m y^m$ be polynomials in $R[x][y]$, with $F(y)G(y) = 0$. We also let $f_i(x) = a_{i_0} + a_{i_1}x + a_{i_2}x^2 + \cdots + a_{i_{\omega_i}} x^{\omega_i}$ and $g_j(x) = b_{j_0} + b_{j_1}x + b_{j_2}x^2 + \cdots + b_{j_{\nu_j}} x^{\nu_i} \in R[x]$ for each $0 \leqslant i \leqslant n$ and $0 \leqslant j \leqslant m$. Take a positive integer t that $t \geqslant deg(f_0(x)) + deg(f_1(x)) + \cdots + deg(f_n(x)) + deg(g_0(x)) + deg(g_1(x)) + \cdots + deg(g_m(x))$, where the degree is as polynomials in x and the degree of zero polynomial is taken to be 0. Then $F(x^t) = f_0 + f_1 x^t + \cdots + f_n x^{tn}$ and $G(x^t) = g_0 + g_1 x^t + \cdots + g_m x^{tm} \in R[x]$ and the set of coefficients of the f_i's (resp. g_j's) equals the set of coefficients of the $F(x^t)$ (resp. $G(x^t)$). Since $F(y)G(y) = 0$, then $F(x^t)G(x^t) = 0$. So $a_{is_i} b_{jr_j} \in J(R)$, where $0 \leqslant s_i \leqslant \omega_i$, $0 \leqslant r_j \leqslant \nu_j$. By hypothesis we have $J(R)[x] \subseteq J(R[x])$, and so $f_i g_j \in J(R[x])$. It implies that R is J-Armendariz. □

Note that, $M_n(R)$ is not J-Armendariz for any nonzero ring R and $n \geqslant 2$, i.e. the J-Armendariz property is not Morita invariant.

Example 2.8. Let R be a ring and $S = M_2(R)$. If $f(x) = e_{12} - e_{11}x$ and $g(x) = e_{11} + e_{12} - (e_{21} + e_{22})x$, then $f(x)g(x) = 0$. But $e_{11}(e_{11} + e_{12}) = e_{11} + e_{12}$ is not in $J(S)$. Thus S is not J-Armendariz.

Corollary 2.9. *Every J-Armendariz ring R is directly finite.*

Proof. If R is not directly finite, then R contains an infinite set of matrix units $\{e_{11}, e_{12}, e_{13}, \ldots, e_{21}, e_{22}, e_{23}, \ldots\}$ by [9, proposition 5.5]. This is a contradiction by Example 2.8. □

The next example shows that there exists a J-Armendariz ring R such that $R/J(R)$ is not J-Armendariz and so the homomorphic image of J-Armendariz rings need not to be J-Armendariz.

Example 2.10 Let R denote the localization of the ring $\mathbb{Z}$ of integers at the prime ideal $\langle 3\rangle$. Consider the quaternions $\mathbf{Q}$ over R, that is a free R-module with basis 1, i, j, k and multiplication satisfying $i^2 = j^2 = k^2 = -1$, $ij = k = -ji$. Then $\mathbf{Q}$ is a noncommutative domain with $J(\mathbf{Q}) = 3\mathbf{Q}$, and so is J-Armendariz. But $\mathbf{Q}/J(\mathbf{Q})$ is isomorphic to the 2-by-2 full matrix ring over $\mathbb{Z}_3$ and is not J-Armendariz by Example 2.8.

Let R and S be two rings and M be an (R, S)-bimodule. This means that M is a left R-module and a right S-module such that $(rm)s = r(ms)$ for all $r \in R$, $m \in M$, and $s \in S$. Given such a bimodule M we can form

$$T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix} = \left\{ \begin{pmatrix} r & m \\ 0 & s \end{pmatrix} : r \in R, m \in M, s \in S \right\}$$

and definition a multiplication on T by using formal matrix multiplication:

$$\begin{pmatrix} r & m \\ 0 & s \end{pmatrix}\begin{pmatrix} r' & m' \\ 0 & s' \end{pmatrix} = \begin{pmatrix} rr' & rm'+ms' \\ 0 & ss' \end{pmatrix}.$$

This ring construction is called triangular ring T.

Proposition 2.11. *Let R and S be two rings and M be an (R, S)-bimodule. Let T be the triangular ring $T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$. Then the rings R and S are J-Armendariz if and only if T is J-Armendariz.*

Proof. Let R and S be J-Armendariz. Take $I = \begin{pmatrix} 0 & M \\ 0 & 0 \end{pmatrix}$, therefore $T/I \cong R \times S$ is J-Armendariz and since $I \subseteq J(T) = \begin{pmatrix} J(R) & M \\ 0 & J(S) \end{pmatrix}$, then T is J-Armendariz by proposition 2.2. Conversely, let T be a J-Armendariz ring, $f_r(x) = r_0 + r_1x + \cdots + r_nx^n$, $g_r(x) = r'_0 + r'_1x + \cdots + r'_mx^m \in R[x]$, such that $f_r(x)g_r(x) = 0$, and $f_s(x) = s_0 + s_1x + \cdots + s_nx^n$, $g_s(x) = s'_0 + s'_1x + \cdots + s'_mx^m \in S[x]$, such that $f_s(x)g_s(x) = 0$. If

$$f(x) = \begin{pmatrix} r_0 & 0 \\ 0 & s_0 \end{pmatrix} + \begin{pmatrix} r_1 & 0 \\ 0 & s_1 \end{pmatrix}x + \cdots + \begin{pmatrix} r_n & 0 \\ 0 & s_n \end{pmatrix}x^n \text{ and}$$
$$g(x) = \begin{pmatrix} r'_0 & 0 \\ 0 & s'_0 \end{pmatrix} + \begin{pmatrix} r'_1 & 0 \\ 0 & s'_1 \end{pmatrix}x + \cdots + \begin{pmatrix} r'_m & 0 \\ 0 & s'_m \end{pmatrix}x^m \in T[x]$$

Then from $f_r(x)g_r(x) = 0$ and $f_s(x)g_s(x) = 0$ it follows that $f(x)g(x) = 0$. Since T is a J-Armendariz ring, $\begin{pmatrix} r_i & 0 \\ 0 & s_i \end{pmatrix}\begin{pmatrix} r'_j & 0 \\ 0 & s'_j \end{pmatrix} \in J(T) = \begin{pmatrix} J(R) & 0 \\ 0 & J(S) \end{pmatrix}$. Thus $r_ir'_j \in J(R)$ and $s_is'_j \in J(S)$ for any i, j. This shows that R and S are J-Armendariz. □

Recall that a ring R is said to be *abelian* if every idempotent of it is central. Armendariz rings are abelian [13, Lemma 7], but J-Armendariz rings need not to be abelian in general. For example, let F be a field then $R = T_2(F)$ is J-Armendariz by proposition 2.11, but it is not an abelian ring.

Proposition 2.12. *Let R be a J-Armendariz ring. Then for each idempotent e of R, eRe is J-Armendariz. The converse holds if e is a central idempotent.*

Proof. Let $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{m} b_j x^j \in (eRe)[x]$ be such that $f(x)g(x) = 0$. Since R is J-Armendariz and $a_i, b_j \in eRe \subseteq R$, then we have $a_i b_j \in J(R) \cap eRe = J(eRe)$. This means that eRe is J-Armendariz. Conversely, let eRe be a J-Armendariz ring and $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$, such that $f(x)g(x) = 0$. By the hypothesis, $0 = ef(x)eg(x)e \in (eRe)[x]$, and since eRe is J-Armendariz, we have $a_i b_j \in J(eRe) = J(R) \cap eRe$. Thus R is J-Armendariz. □

3. The Relation of J-Armendariz Rings with other Classes of Rings

Let M be a module and A, B be two summands of M. We write $A \sim B$ to denote A and B have a common complement i.e., there exists submodule C such that $M = A \oplus C = B \oplus C$. It is clear that $A \sim B$ implies that $A \cong B$. A module M is perspective when $A \cong B$ implies $A \sim B$ for any two summands A, B of M. It is clear that perspective modules satisfy the internal cancellation property in the sense that complements of isomorphic summands are isomorphic (see [6]).

In this section we give a new class of rings that are J-Armendariz.

A ring R is called right (left) quasi-duo if every maximal right (left) ideal of R is two-sided. If R is a right (left) quasi-duo ring, then $R/J(R)$ is reduced by [14, Proposition 4.3]. So $R/J(R)$ is Armendariz, and hence R is J-Armendariz by Proposition 2.2. So a right (left) quasi-duo ring is J-Armendariz but there exists a J-Armendariz ring R which is not right (left) quasi-duo by Example 3.1.

In [7, Corollary 4.8] it is proved that every right (left) quasi-duo ring is a perspective ring. Moreover, in this section we prove that every J-Armendariz ring is perspective. One may ask a perspective ring is J-Armendariz. The general answer is negative and so J-Armendariz rings lie properly between right (left) quasi duo rings and perspective rings.

The following example shows that J-Armendariz rings need not to be right quasi-duo.

Example 3.1. Take any right primitive domain R that is not a division ring (e.g. the free algebra $R = Q\langle x, y\rangle$). Then R is J-Armendariz, but R is not right quasi-duo by [14, Proposition 4.1].

Proposition 3.2. *Let R be a J-Armendariz ring, then R is perspective, but the converse is not true in general.*

Proof. Let R be a J-Armendariz ring. Then for $a, b \in R$ $ab = 0$ implies

$aNil(R)B \subseteq J(R)$. In fact, for $0 \neq c \in Nil(R)$ there exist $n \geqslant 1$ such that $c^n = 0$, and so $a(1 - cx)(1 + cx + \cdots + c^{n-1}x^{n-1})b = 0$. This implies that $acb \in J(R)$. Now taking $a = e = e^2$, $b = (1-e)$ and $c = er(1-e)$, then we have $eR(1-e) \subseteq J(R)$. Thus by [**?**, Theorem 4.7], R is a perspective ring. However there exists a perspective ring which is not J-Armendariz. Let R be a field. Then $M_n(R)$ is perspective by [7, Example 5]. But $M_n(R)$ is not J-Armendariz for $n \geqslant 2$. □

Corollary 3.3. *Let R be a J-Armendaiz ring such that idempotents lift modulo $J(R)$, then $R/J(R)$ is abelian.*

Proof. Let $\bar{e}^2 = \bar{e}$ be an idempotent in $\bar{R} = R/J(R)$. Since idempotents lift modulo $J(R)$, then for each $r \in R$, $e(r - re) \in J(R)$ and $(r - er)e \in J(R)$ by the proof of Proposition 3.2. Therefore $R/J(R)$ is abelian. □

Following [17], we define an element x of a ring R to be clean if there is an idempotent $e \in R$ such that $x - e$ is a unit of R. A clean ring is defined to be one in which every element is clean. Clean rings were initially developed in [17] as a natural class of rings which have the exchange property. A ring R is an exchange ring if for every right R-module A_R and two decompositions $A_R = M \bigoplus N = \bigoplus_{i \in I} A_i$ where $M_R \cong A_R$, and the index set I is finite, there exist submodules $A'_i \subseteq A_i$ such that $A = M \oplus (\bigoplus_{i \in I} A'_i)$. A ring R is an exchange ring if and only if for any $x \in R$ there exists an idempotent $e \in R$ such that $(1 - e) \in R(1 - x)$ (cf. [20]).
It is known [17, Proposition 1.8] that clean rings are exchange and the two concepts are equivalent for abelian rings. A ring R is said to have stable range one provided that for any $a, b \in R$, $aR + bR = R$ implies that there exists some $y \in R$ such that $a + by$ is unit in R. Now we have the following:

Proposition 3.4. *Let R be a an exchange ring. If R is a J-Armendariz ring then R is clean with stable range one.*

Proof. Let R be a J-Armendariz and exchange ring. In fact R is an exchange ring if and only if $R/J(R)$ is an exchange ring and idempotents can be lifted modulo $J(R)$ [17]. Then $R/J(R)$ is abelian by Corollary 3.3. Therefore $R/J(R)$ is clean and so R is clean by [10, Proposition 6]. Clearly $R/J(R)$ has stable rang one by [21, Theorem 6]. Hence R has stable rang one by [19, Theorem 22]ln. □

Following [4], an element $a \in R$ is called strongly π-regular if $a^n \in Ra^{n+1} \cap a^{n+1}R$ for some positive integer n. Also, an element r in a ring R is called nil clean if there is an idempotent $e \in R$ and a nilpotent $b \in R$ such that $r =$

$e+b$. The element r is further called strongly nil clean if such an idempotent and nilpotent can be chosen such that $be = eb$. A ring is called nil clean (respectively, strongly nil clean) if every one of its elements is nil clean (respectively, strongly nil clean). In [4], it is shown that every strongly nil clean ring is strongly π-regular. Now we have the following:

Proposition 3.5. *Let R a nil clean ring. If R is J-Armendariz and J-adically complete, then R is strongly π-regular.*

Proof. Let $\bar{R} = R/J(R)$. Since R is J-adically complete, then idempotents lift modulo $J(R)$ by [15, Theorem 21.31]. Therefore $\bar{R}$ is abelian by Proposition 3.3. On the other hand, since R is nil clean, then $\bar{R}$ is nil clean by [4, Corollary 3.17]. Therefore $\bar{R}$ is strongly nil clean. Suppose that $a \in R$, then for each $\bar{a} \in \bar{R}$, we may write $\bar{a} = \bar{e} + \bar{b}$ for some idempotent $\bar{e}$ and some nilpotent $\bar{b}$ which commute. By [4, Proposition 3.5], $\bar{a} = (1 - \bar{e}) + (2\bar{e} - 1 + \bar{b})$ is thus strongly π-regular decomposition of $\bar{a}$. Following [5, Corollary 6] a is strongly π-regular in R and the proof is complete. □

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