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Subspace Transitivity and Subspace Supercyclicity of Tuples of Operators in SOT and in the Norm of Hilbert-Schmidt Operators

E. Fathi
Payame Noor University

B. Yousefi*
Payame Noor University

F. Ershad
Payame Noor University

Abstract. In this paper, we investigate the subspace transitivity and subspace supercyclicity of tuples of left multiplication operators in the strong operator topology and in the norm of Hilbert-Schmidt operators.

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1. Introduction

By an n-tuple of operators we mean a finite sequence of length n of commuting continuous linear operators on a Banach space X.

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*Corresponding author

Definition 1.1. *Let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be an n-tuple of operators acting on a separable infinite dimensional Banach space X over $\mathbb{C}$ and let M be a nonzero subspace of X. We will let*

$$\mathcal{F} = \{{T_1}^{k_1}{T_2}^{k_2}...{T_n}^{k_n} : k_i \geqslant 0, i = 1, ..., n\}$$

be the semigroup generated by $\mathcal{T}$. For $x \in X$, the orbit of x under the tuple $\mathcal{T}$ is the set

$$Orb(\mathcal{T}, x) = \{Sx : S \in \mathcal{F}\}.$$

A vector x is called a subspace-hypercyclic (or M-hypercyclic) vector for $\mathcal{T}$ if $Orb(\mathcal{T}, x) \cap M$ is dense in M and in this case the tuple $\mathcal{T}$ is called subspace-hypercyclic with respect to M. The set of all M-hypercyclic vectors of $\mathcal{T}$ is denoted by $HC(\mathcal{T}, M)$. Also, a vector x is called a M-supercyclic vector for $\mathcal{T}$ if $\mathbb{C}Orb(\mathcal{T}, x) \cap M$ is dense in M and in this case the tuple $\mathcal{T}$ is called M-supercyclic. The set of all M-supercyclic vectors of $\mathcal{T}$ is denoted by $SC(\mathcal{T}, M)$.

Definition 1.2. *Suppose that $\mathcal{T} = (T_1, T_2, ..., T_n)$ is an n-tuple of operators acting on a separable infinite dimensional Banach space X over $\mathbb{C}$ and M is a nonzero subspace of X. We say that a tuple $\mathcal{T} = (T_1, T_2, ..., T_n)$ is M-transitive with respect to a tuple of nonnegative integer sequences*

$$(\{k_{j(1)}\}_j, \{k_{j(2)}\}_j, ..., \{k_{j(n)}\}_j),$$

if for every nonempty relatively open subsets U, V of X, there exists $j_0 \in \mathbb{N}$ such that ${T_1}^{-k_{j_0(1)}}{T_2}^{-k_{j_0(2)}}...{T_n}^{-k_{j_0(n)}}(U) \cap V$ contains a relatively open nonempty subset of M. Also, we say that an n-tuple $\mathcal{T}$ is M-transitive if it is M-transitive with respect an n-tuple of nonnegative integer sequences.

The first example of a hypercyclic operator on a Hilbert space was constructed by Rolewicz in 1969 ([10]). He showed that if B is the backward shift on $\ell^2(\mathbb{N})$, then λB is hypercyclic if and only if $|\lambda| > 1$. A nice criterion namely the Hypercyclicity Criterion, was developed independently by Kitai ([8]), Gethner and Shapiro ([6]). This criterion has been used to show that hypercyclic operators arise within the classes of composition operators ([2]), weighted composition operators ([14]), weighted shifts

([11]), adjoints of multiplication operators ([3]), and adjoints of subnormal and hyponormal operators ([1]). Supercyclicity was introduced by Hilden and Wallen ([7]). They showed that all unilateral backward weighted shifts are supercyclic, but there does not exist a vector that is supercyclic vector for all the unilateral backward weighted shifts.
H. Salas ([12]) give a condition for supercyclicity in Frechet spaces which is called the Supercyclicity Criterion. Supercyclicity of the operator algebra is studied in [15]. N. S. Feldman have extended these concepts for tuples of operators ([4,5]). Suprisingly, there are something that does not happen for single operators. For example, hypercyclic tuples can arise in finite dimensional, and there are operators that have somewhere dense orbits that are not everywhere dense. Also, we note that there are subspace-hypercyclic operators that are not hypercyclic ([4,5,9]). For some topics we refer to [1-19].

2. Main Results

In this section, we investigate subspace-transitivity and subspace-supercyclicity for tuples of left multiplication operators. We begin with the following theorems that are used to prove the main results.

Theorem 2.1. ([18]) *Suppose that $\mathcal{T} = (T_1, T_2, ..., T_n)$ is an n-tuple of operators acting on a separable infinite dimensional Banach space X over $\mathbb{C}$ and M is a nonzero subspace of X. Then $\mathcal{T}$ is M-transitive if and only if for any nonempty sets $U \subset M$ and $V \subset M$, both relatively open, there exists a tuple $(k_1, k_2, ..., k_n)$ of integers such that $T_1^{-k_1} T_2^{-k_2} ... T_n^{-k_n}(U) \cap V$ is nonempty and $T_1^{-k_1} T_2^{-k_2} ... T_n^{-k_n} M \subset M$.*

Theorem 2.2. ([18]) *Suppose that $\mathcal{T}$ is an n-tuple of operators acting on a separable infinite dimensional Banach space X over $\mathbb{C}$ and M is a nonzero subspace of X. If $\mathcal{T}$ is M-transitive, then $\mathcal{T}$ is M-hypercyclic.*

Theorem 2.3. ([19]) *Suppose that $\mathcal{T} = (T_1, T_2, ..., T_n)$ is an n-tuple of operators acting on a separable infinite dimensional Banach space X over $\mathbb{C}$ and M is a nonzero subspace of X. Then the following conditions are equivalent:*

i) For any nonempty sets $U \subset M$ and $V \subset M$, both relatively open, there exist a $\lambda \in \mathbb{C}\backslash\{0\}$ and a tuple $(k_1, k_2, ..., k_n)$ of integers such that

$$\lambda^{-1}T_1^{-k_1}T_2^{-k_2}...T_n^{-k_n}(U) \cap V$$

contains a relatively open nonempty subset of M.

ii) For any nonempty sets $U \subset M$ and $V \subset M$, both relatively open, there exist a $\lambda \in \mathbb{C}\backslash\{0\}$ and a tuple $(k_1, k_2, ..., k_n)$ of integers such that

$$\lambda^{-1}T_1^{-k_1}T_2^{-k_2}...T_n^{-k_n}(U) \cap V$$

is nonempty and $T_1^{k_1}T_2^{k_2}...T_n^{k_n}M \subset M$.

iii) The tuple $\mathcal{T}$ is M-supercyclic.

Recall that if $\{e_i\}$ is a basis for a separable Hilbert space H and $A \in B(H)$, then

$$||A||_2 = [\sum_{i=1}^{\infty} ||Ae_i||^2]^{\frac{1}{2}}$$

is independent of the basis chosen and hence is well defined. If $||A||_2 < \infty$, then A is called a Hilbert-Schmidt operator. The algebra of Hilbert-Schmidt operators acting on H is denoted by $B_2(H)$.

Let Q be a closed subspace of $B_2(H)$ and $L : B_2(H) \to B_2(H)$ be a continuous linear mapping. Given $T \in B_2(H)$, we note that

$$Orb(L, T) = \{L^nT : n = 0, 1, 2, \cdots\}.$$

The mapping L is called Q-hypercyclic in $||.||_2$, if there exists some $T \in B_2(H)$ such that $Orb(L, T) \cap Q$ is dense in Q with respect to the $||.||_2$-topology. In this case T is said to be a Q-hypercyclic vector for L. Similarly, L is called Q-supercyclic in $||.||_2$, if there exists some $T \in B_2(H)$ such that $\mathbb{C}Orb(L, T) \cap Q$ is dense in Q with respect to the $||.||_2$-topology.

Suppose $\{e_i\}$ is a basis for a separable Hilbert space H and suppose that S is a dense subset in H. Then $S(H)$ will denote the set of all finite rank operators E such that there exists $N \in \mathbb{N}$ satisfying $E(e_n) = 0$

for $n \geqslant N$ and $E(e_n) \in S$ for $n < N$. Clearly we can see that $S(H)$ is $|| \, ||_2$-dense in $B_2(H)$. Moreover if S is countable, then $S(H)$ is also countable. Hence $B_2(H)$ with the $|| \, ||_2$-topology is separable. Note that $B(H)$ is also separable with the strong operator topology.

Definition 2.4. *Let Q be a closed subspace of $B(H)$ and $\mathcal{L} = (L_1, L_2, ..., L_n)$ be an n-tuple of continuous linear mappings from $B(H)$ into $B(H)$. For a continuous linear mapping $S \in B(H)$, put*

$$Orb(\mathcal{L}, S) = \{L_1^{k_1} L_2^{k_2} ... L_n^{k_n} S : k_i \geqslant 0, \ i = 1, \cdots, n\}.$$

We say that $\mathcal{L}$ is Q-hypercyclic (Q-supercyclic) in strong operator topology (SOT) if there exists some $S \in B(H)$ such that the set $Orb(\mathcal{L}, S) \cap Q$ ($\mathbb{C}$ Orb(L,S)$\bigcap Q$) is dense in Q with the strong operator topology. In this case S is called a Q-hypercyclic (Q-supercyclic) vector for $\mathcal{L}$ in strong operator topology. Similarly if Q is a closed subspace of $B_2(H)$, we say that $\mathcal{L}$ is Q-hypercyclic (Q-supercyclic) in the norm of Hilbert-Schmidt operators if there exists $S \in B_2(H)$ such that $Orb(\mathcal{L}, S) \cap Q$ ($\mathbb{C}$ Orb(L,S)$\cap Q$) is dense in Q with $||.||_2$-topology and in this case S is called a Q-hypercyclic (Q-supercyclic) vector for $\mathcal{L}$ in norm of the Hilbert-Schmidt operators.

Recall that if g, $h \in H$, then operator $g \otimes h$ defines a rank one operator that is defined by $(g \otimes h)(f) =< f, h > g$ for all $f \in H$.
If $\mathcal{T} = (T_1, ..., T_n)$ is a tuple of operators on H, by $\mathcal{L}_{\mathcal{T}}$ or $\mathcal{L}_{T_1,...,T_n}$ we mean the tuple of left multiplication operators $(L_{T_1}, L_{T_2}, ..., L_{T_n})$.

Theorem 2.5. *Let Q be a subspace of $B_2(H)$ that is closed in the strong operator topology and let $\mathcal{T} = (T_1, T_2, ..., T_n)$ be an n-tuple of operators acting on H. Then the followings are equivalent:*

i)$\mathcal{L}_{T_1,...,T_n}$ *is Q-transitive on $B_2(H)$ with $|| \, ||_2$-topology,*

ii)$\mathcal{L}_{T_1,...,T_n}$ *is Q-transitive on $B(H)$ in the strong operator topology.*

Proof. (i) implies (ii): If $\mathcal{L}_{T_1,...,T_n}$ is Q-transitive on $B_2(H)$ with $|| \, ||_2$-topology, then it is Q-transitive on $B(H)$ with the strong operator topology, since $B_2(H)$ is dense in $B(H)$ in the strong operator topology.
(ii) implies (i): Suppose that U and V are relatively $|| \, ||_2$-open sets in

Q and also let $S(H)$ be defined as before. Choose $A \in U \cap S(H)$ and $B \in V \cap S(H)$ such that for certain integer $N \in \mathbb{R}$, $Ae_i = Be_i = 0$ for all $i > N$. Now let E be a finite rank operator defined by $E = \sum\limits_{i=1}^{N} e_i \otimes e_i$. Then $AE = A$ and $BE = B$. For every $k \in \mathbb{N}$, put

$$U_k = \bigcap_{i=1}^{N} \left\{ S \in B(H) : ||Se_i - Ae_i|| < \frac{1}{k} \right\} \cap Q$$

and

$$V_k = \bigcap_{i=1}^{N} \left\{ S \in B(H) : ||Se_i - Be_i|| < \frac{1}{k} \right\} \cap Q.$$

Thus U_k and V_k are relatively open subsets of Q in the strong operator topology. But $\mathcal{L}_{T_1,\dots,T_n}$ is Q-transitive in the strong operator topology. Hence $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} \dots L_{T_n}^{p_{n,k}}(U_k) \cap V_k$ is nonempty for some integers $p_{i,k} \geqslant 1,\ i = 1, \dots, n$ and so ${T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}} S_k \in V_k$ for some $S_k \in U_k$. Now we get

$$||S_k e_i - Ae_i|| < \frac{1}{k} \quad ; \quad ||{T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}} S_k e_i - Be_i|| < \frac{1}{k}$$

for $n = 1, 2, \cdots, N$. Thus we have

$$||S_k E - AE||_2^2 = \sum_{i=1}^{N} ||(S_k - A)(e_i)||^2 < \frac{N}{k^2}$$

and

$$||L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} \dots L_{T_n}^{p_{n,k}}(S_k E) - BE||_2^2 = \sum_{i=1}^{N} ||({T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}} S_k - B)(e_i)||^2 < \frac{N}{k^2}$$

for $k \in \mathbb{N}$. Hence $\{S_k\}_k$ converges to A and $\{{T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}}(S_k E)\}_k$ converges to B. Hence $S_k E \in U \cap S(H)$ and

$${T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}}(S_k E) \in V \cap S(H)$$

for some integers $p_{i,k} \geqslant 1,\ i = 1, \dots, n$. Clearly we can see that $S_k E$ and

$${T_1}^{p_{1,k}}{T_2}^{p_{2,k}} \dots {T_n}^{p_{n,k}}(S_k E)$$

are Hilbert-Schmidt operators since they are finite rank operators. Now it follows that $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}}(U) \cap V$ is nonempty. Now to complete the proof, by using Theorem 2.1, it is sufficient to show that $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} Q \subset Q$. For this let $A \in Q$ and let W be a relatively open nonempty subset of $L_{T_1}^{-p_{1,k}} L_{T_2}^{-p_{2,k}} ... L_{T_n}^{-p_{n,k}}(V) \cap U$. WE note that

$$L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} W \subset V \cap \lambda L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} U \subset V \subset Q,$$

hence $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} W \subset Q$. Since W is relatively open, so for $A_0 \in W$, we can choose $d > 0$ such that $A_0 + dA \in W$. Therefore,

$$L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}}(A_0+dA) = L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} A_0 + dL_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} A \in Q,$$

which implies that $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} A \in Q$. Thus $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} Q \subset Q$. So indeed $\mathcal{L}_{T_1,...,T_n}$ is Q-transitive with $|| \, ||_2$-topology and the proof is complete. □

The following corollaries are an immediate consequences of Theorems 2.2 and 2.5.

Corollary 2.6. *Let Q be a subspace of $B_2(H)$ that is closed in the strong operator topology. If $\mathcal{L}_{T_1,...,T_n}$ is Q-transitive on $B_2(H)$ with $||.||_2$-topology, then $\mathcal{L}_{T_1,...,T_n}$ is Q-hypercyclic on $B(H)$ in the strong operator topology.*

Corollary 2.7. *Let Q be a subspace of $B_2(H)$ that is closed in the strong operator topology. If $\mathcal{L}_{T_1,...,T_n}$ is Q-transitive on $B(H)$ in the strong operator topology, then $\mathcal{L}_{T_1,...,T_n}$ is Q-hypercyclic on $B_2(H)$ with $||.||_2$-topology.*

Proposition 2.8. *Let Q be a subspace of $B_2(H)$ that is closed in the strong operator topology and consider the n-tuple $\mathcal{L}_{T_1,...,T_n}$ of left multiplication operators acting on $B(H)$. If $\mathcal{L}_{T_1,...,T_n}$ is Q-supercyclic on $B_2(H)$ with $|| \, ||_2$-topology, then for any two nonempty relatively open subsets U, V of Q in the strong operator topology, there exist integers $p_1, ..., p_n \geqslant 1$ and $\lambda \in \mathcal{C}$ such that the set $\lambda L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} U \cap V$ is nonempty and $L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} Q \subset Q$.*

Proof. Let U and V be relatively open subsets of Q in the strong operator topology, then clearly U and V are relatively open subsets of Q in the norm of Hilbert-Schmidt operators. Now since $\mathcal{L}_{T_1,...,T_n}$ is Q-supercyclic on $B_2(H)$ with $\|\ \|_2$-topology, thus there exist integers $p_1, ..., p_n \geqslant 1$ and $\lambda \in \mathcal{C}$ such that $\lambda L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} U \cap V$ is nonempty and $L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} Q \subset Q$. So the proof is complete. □

The following theorem extends the results of Proposition 2.8.

Theorem 2.9. *Let Q be a subspace of $B_2(H)$ that is closed in the strong operator topology. For the n-tuple $\mathcal{L}_{T_1,...,T_n}$ of left multiplication operators on $B(H)$, the followings are equivalent:*

i) $\mathcal{L}_{T_1,...,T_n}$ *is Q-supercyclic on $B(H)$ with the strong operator topology.*

ii) *For any two nonempty relatively open subsets U, V of Q in the strong operator topology, there exist integers $p_1, ..., p_n \geqslant 1$ and $\lambda \in \mathcal{C}$ such that $\lambda L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} U \cap V$ is nonempty and $L_{T_1}^{p_1} L_{T_2}^{p_2} ... L_{T_n}^{p_n} Q \subset Q$.*

iii) $\mathcal{L}_{T_1,...,T_n}$ *is Q-supercyclic on $B_2(H)$ with $\|\ \|_2$-topology.*

Proof. (i) implies (ii): Let $\mathcal{L}_{T_1,...,T_n} = (L_{T_1}, L_{T_2}, ..., L_{T_n})$ be Q-supercyclic on $B(H)$ in the strong operator topology and suppose that U and V are two nonempty relatively open subsets of Q in the strong operator topology. Since $\mathcal{L}_{T_1,...,T_n}$ has a dense set in strong operator topology of Q-supercyclic vectors in Q, so U contains a Q-supercyclic vector A. On the otherhand since V is relatively open in strong operator topology and $\mathcal{C}Orb(\mathcal{L}_{T_1,...,T_n}, A) \cap Q$ is dense in Q in the strong operator topology, thus there exist integers $k_1, ..., k_n \geqslant 1$ and $\lambda \in \mathcal{C}$ such that $\lambda L_{T_1}^{k_1} L_{T_2}^{k_2} ... L_{T_n}^{k_n}(A) \in V$. This implies that $\lambda L_{T_1}^{k_1} L_{T_2}^{k_2} ... L_{T_n}^{k_n} U \cap V$ is nonempty. Also, by the same method used in the proof of Theorem 2.5, we can see that $L_{T_1}^{k_1} L_{T_2}^{k_2} ... L_{T_n}^{k_n} Q \subset Q$.
(ii) implies (iii): Suppose that U and V are relatively open subsets of Q in the norm of Hilbert-Schmidt operators and also let $S(H)$ be the set that was defined as before. Choose $A \in U \cap S(H)$ and $B \in V \cap S(H)$ such that for certain integer $N \in \mathbb{N}, Ae_i = Be_i = 0$ for $i > N$. Now let E be a finite rank operator that is defined by $E = \sum_{i=1}^{N} e_i \otimes e_i$. Then

$AE = A$ and $BE = B$. For every $k \in \mathbb{N}$, let the sets U_k and V_k be as defined in the proof of Theorem 2.5. Since $\mathcal{L}_{T_1,...,T_n}$ is Q-supercyclic in the strong operator topology, thus $\lambda_k L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}}(U_k) \cap V_k$ is nonempty for some integers $p_{i,k} \geqslant 1,\ i = 1, ..., n$ and $\lambda_k \in \mathcal{C}$. Now it follows that $\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}} S_k \in V_k$ for some $S_k \in U_k$. Hence we get

$$||S_k e_i - Ae_i|| < \frac{1}{k} \quad ; \quad ||\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}} S_k e_i - Be_i|| < \frac{1}{k}$$

for $i = 1, 2, \cdots, N$ and $k \in \mathbb{N}$. Thus

$$||S_k E - AE||_2^2 = \sum_{i=1}^{N} ||(S_k - A)(e_i)||^2 < \frac{N}{k^2}$$

and

$$||\lambda_k L_{T_1}^{p_{1,k}} ... L_{T_n}^{p_{n,k}}(S_k E) - BE||_2^2 = \sum_{i=1}^{N} ||(\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}} S_k - B)(e_i)||^2 < \frac{N}{k^2}$$

for all $k \in \mathbb{N}$. Hence $S_k E$ converges to A and $\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}}(S_k E)$ converges to B in $||.||_2$. Therefore, $S_k E \in U \cap S(H)$ and

$$\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}}(S_k E) \in V \cap S(H)$$

for some integers $p_{i,k} \geqslant 1,\ i = 1, ..., n$. Clearly, $S_k E$ and $\lambda_k {T_1}^{p_{1,k}} {T_2}^{p_{2,k}} ... {T_n}^{p_{n,k}}(S_k E)$ are Hilbert-Schmidt operators. Now it follows that $\lambda_k L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}}(U) \cap V$ is nonempty.

(iii) implies (i): It is clear since $B_2(H)$ is a dense subset of $B(H)$ in the strong operator topology. Also, we can see that $L_{T_1}^{p_{1,k}} L_{T_2}^{p_{2,k}} ... L_{T_n}^{p_{n,k}} Q \subset Q$ and so the proof is complete. $\square$

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Elham Fathi
Department of Mathematics
Ph.D Student
Payame Noor University
P.O. Box 19395-3697
Tehran, Iran
E-mail: e_fathi@phd.pnu.ac.ir

Bahmann Yousefi
Department of Mathematics
Professor of Mathematics
Payame Noor University
Box 19395-3697
Tehran, Iran
E-mail: b_yousefi@pnu.ac.ir

Fariba Ershad
Department of Mathematics
Assistant Professor of Mathematics
Payame Noor University
Box 19395-3697
Tehran, Iran
E-mail: fershad@pnu.ac.ir