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Two Classes of Multicone Graphs Determined by Their Spectra

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Abstract. A multicone graph is defined to be the join of a clique and a regular graph. In [1], new classes of multicone graphs have been characterized that are determined by their spectra. In this work, we present new classes of multicone graphs that are determined by their adjacency spectrum. Also, we show that these graphs are determined by their Laplacian spectrum. Finally, four problems for further research are proposed.

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Keywords and Phrases: Adjacency spectrum, Laplacian spectrum, DS graph, paley graph, schläfi graph

1. Introduction

In this paper, we are concerned only with finite undirected simple graph. All terminology and notations on graphs which are not defined here can be found in [3, 4, 12]. In this section we recall some definitions that will be used in the paper. Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex $v \in V(G)$, denoted by $d(v)$, is the number of neighbors of v. The adjacency matrix of G is an $n \times n$ matrix $A(G)$ whose (i, j)-entry is the number of edges between v_i and v_j. The characteristic polynomial of G is $\det(\lambda I - A(G))$, and is denoted by $P_G(\lambda)$. The signless Laplacian matrix of G is the matrix $SL(G) = A(G) + D(G)$, where $D(G)$ is the diagonal matrix with $\{d(v_1), \ldots, d(v_n)\}$ on its main diagonal. It is well-known that $SL(G)$ is positive semidefinite and so their eigenvalues are nonnegative real numbers. The eigenvalues of $A(G)$ and $L(G)$ are called the eigenvalues and

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Laplacian eigenvalues of G. The spectrum of G is the eigenvalues of G together with multiplities of its eigenvalues. We denote the spectrum of G with $Spec(G) = \{[\lambda_1]^{m_1}, [\lambda_2]^{m_2}, ..., [\lambda_s]^{m_s}\}$, where m_i denotes multiplicity of the eigenvalues of λ_i $(1 \leqslant i \leqslant s)$. The complement of a graph G, denoted by $\overline{G}$, is the graph on the vertex set of G such that two vertex of $\overline{G}$, are adjacent if and only if they are not adjacent in G. A graph is called bidegreed if the set of degrees of vertices consists of exactly two distinct elements. The largest eigenvalues of adjacency spectrum of G is called spectral radius of G and it is denoted by $\varrho(G)$. For two graphs G and H, if $Spec(G) = Spec(H)$, we say G and H are cospectral with respect to adjacency matrix. A graph H is said to be determined by its spectrum (DS for short), if for a graph H with $Spec(G) = Spec(H)$, one has G isomorphic to H. In [9], it is conjectured that almost all graphs are DS. Nevertheless, the set of graphs which are known to be DS is small and therefore it would be interesting to find more examples of DS graphs. For a survey of the subject, the reader can refer to [9, 10]. In this paper, we present new classes of multicone graphs and we show that these graphs are DS with respect to their spectra.

The plan of this paper is organized as follows. In Section 2, we review some basic information and preliminaries. In Section 3.1, we show that multicone graphs $K_w \bigtriangledown P$ are determined by their adjacency spectrum. In Section 3.2, we prove that these graphs are DS with respect to their Laplacian spectrum. Subsections 4.1 and 4.2 are the similar of the Subsections 3.1 and 3.2, respectively. In Section 5, we review what were said in the previous subsections and finally we propose four conjectures for further researches.

2. Preliminaries

Lemma 2.1. ([2, 7]). *Let G be a graph. For the adjacency matrix and Laplacian matrix, the following can be obtained from the spectrum:*

(**i**) *The number of vertices,*
(**ii**) *The number of edges.*
For the adjacency matrix, the following follows from the spectrum:
(**iii**) *The number of closed walks of any length,*
(**iv**) *Being regular or not and the degree of regularity,*
(**v**) *Being bipartite or not.*
For the Laplacian matrix, the following follows from the spectrum:
(**vi**) *The number of spanning trees,*
(**vii**) *The number of components,*
(**viii**) *The sum of squares of degrees of vertices.*

Theorem 2.2. ([4]). *If G_1 is r_1-regular with n_1 vertices, and G_2 is r_2-regular with n_2 vertices, then the characteristic polynomial of the join $G_1 \triangledown G_2$ is given by:*

$$P_{G_1 \triangledown G_2(\lambda)} = \frac{P_{G_1}(\lambda) P_{G_2}(\lambda)}{(\lambda - r_1)(\lambda - r_2)}((\lambda - r_1)(\lambda - r_2) - n_1 n_2).$$

Theorem 2.3. ([2]). *Let G be a simple graph with n vertices and m edges. Let $\delta = \delta(G)$ be the minimum degree of vertices of G and $\varrho(G)$ be the spectral radius of the adjacency matrix of G. Then*

$$\varrho(G) \leqslant \frac{\delta - 1}{2} + \sqrt{2m - n\delta + \frac{(\delta + 1)^2}{4}}.$$

Equality holds if and only if G is either a regular graph or a bidegreed graph in which each vertex is of degree either δ or $n - 1$.

Theorem 2.4. ([6]). *Let G and H be two graphs with Laplacian spectrum $\lambda_1 \geqslant \lambda_2 \geqslant ... \geqslant \lambda_n$ and $\mu_1 \geqslant \mu_2 \geqslant ... \geqslant \mu_m$, respectively. Then Laplacian spectrum of $\overline{G}$ and $G \triangledown H$ are $n - \lambda_1, n - \lambda_2, ..., n - \lambda_{n-1}, 0$ and $n + m, m + \lambda_1, ..., m + \lambda_{n-1}, n + \mu_1, ..., n + \mu_{m-1}, 0$, respectively.*

Lemma 2.5. ([6]). *Let G be a graph on n vertices. Then n is one of the Laplacian eigenvalue of G if and only if G is the join of two graphs.*

Proposition 2.6. ([1]). *For a graph G, the following statements are equivalent:*

(i) G is d-regular.

(ii) *$\varrho(G) = d_G$, the average vertex degree, that is,* $\varrho(G) = \frac{\sum_{i=1}^{n} d}{n} = \frac{nd}{n} = d$,
where n denotes the number of vertices of graph G.
(iii) *G has $\mathbf{j} = (1, 1, ..., 1)^t$ as an eigenvector for $\varrho(G)$.*

Theorem 2.7. ([4, 8]). *Let $G - j$ be the graph obtained from G by deleting the vertex j and all edges containing j. Then $P_{G-j}(\lambda) = P_G(\lambda) \sum_{i=1}^{m} \frac{\alpha_{ij}^2}{\lambda - \mu_i}$, where m, α_{ij} and $P_G(\lambda)$ are the number of distinct eigenvalues of graph G, the main angle of G (see [8]) and the characteristic polynomial of G, respectively.*

3. Main Results

In the following, we always suppose that w is a natural number. Also, P and S denote Paley graph of order 17 and Schläfi graph, respectively.

The main goal of this section is to prove that any graph cospectral with a multicone graph $K_w \bigtriangledown P$ must be bidegreed.

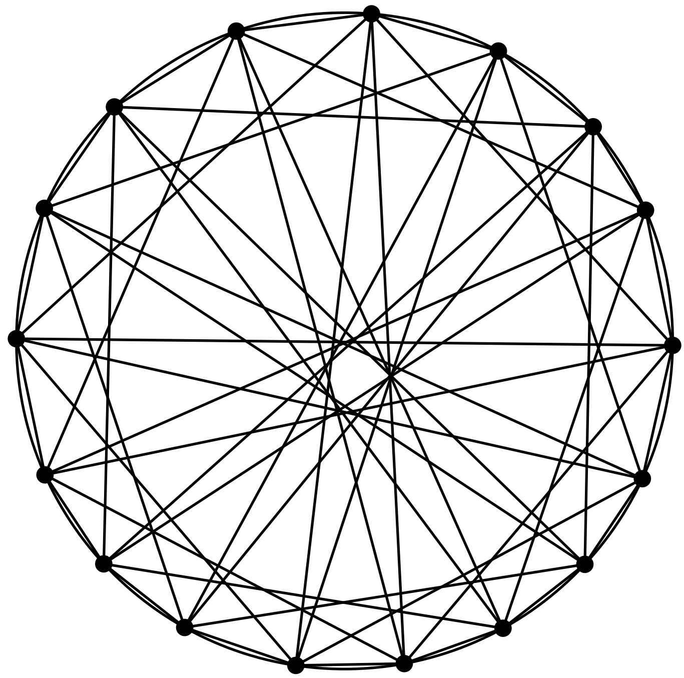

Figure 1. Paley graph of order 17

3.1 Graph cospectral with a multicone graph $K_w \bigtriangledown P$

Proposition 3.1.1. *Let G be a graph cospectral with a multicone graph $K_w \bigtriangledown P$. Then*

$$Spec(G) = \left\{ [-1]^{w-1}, \left[\frac{-1+\sqrt{17}}{2}\right]^8, \left[\frac{-1-\sqrt{17}}{2}\right]^8, \left[\frac{\theta+\sqrt{\theta^2+4\Gamma}}{2}\right]^1, \left[\frac{\theta-\sqrt{\theta^2+4\Gamma}}{2}\right]^1 \right\},$$

where $\theta = w + 7$ and $\Gamma = 9w + 8$.

Proof. By [5, page 250, Section 4] and a simple computation we can conclude that $Spec(P) = \left\{ [8]^1, \left[\frac{-1+\sqrt{17}}{2}\right]^8, \left[\frac{-1-\sqrt{17}}{2}\right]^8 \right\}$. Now, by Theorem 2.2 the proof is strightforward. $\square$

In the following lemma, we show that any graph cospectral with a multicone graph $K_w \bigtriangledown P$ must be bidegreed.

Lemma 3.1.2. *Let G be cospectral with a multicone graph $K_w \bigtriangledown P$. Then G is bidegreed in which any vertex of G is of degree $w+8$ or $w+16$.*

Proof. By Proposition 2.6, it is obvious that G cannot be regular; since regularity of a graph can be determined by its spectrum. By contrary, we suppose that the degrees sequence of graph G consists of at least three number. Hence the equality in Theorem 2.3 cannot happen for any δ; because this equality happens if and only if graph G is regular or bidegreed. But, if we put $\delta = w+8$, then the equality in Theorem 2.3 holds. Hence graphs with degrees sequence of consists of at least three number and $\delta = w+8$ must as graphs that hold the equality in Theorem 2.3 be considered. This is impossible. So, G must be bidegreed. Now, we show that $\Delta = \Delta(G) = w+16$. By contrary, we suppose that $\Delta < w+16$. Therefore, the equality in Theorem 2.3 cannot hold for any δ. But, if we put $\delta = w+8$, then this equality holds. This is a contradiction and so $\Delta = w+49$. Now, we determine another degree of vertices of G. In other words, we determine δ. For this, we note that G is bidegreed and G has $w+17$ vertices, $\Delta = w+16$ and $w(w+16)+17(w+8) = w\Delta+17(w+7) = \sum_{i=1}^{w+17} \deg v_i$. This completes the proof. $\square$

In the following lemma, we prove that the multicone graphs $K_1 \bigtriangledown P$ are DS with respect to their adjacency spectrum.

Lemma 3.1.3. *Any graph cospectral with the multicone graph $K_1 \bigtriangledown P$ is isomorphic to $K_1 \bigtriangledown P$.*

Proof. Let G be cospectral with the multicone graph $K_1 \bigtriangledown P$. Lemma 3.2 follows that G is biderreed in which any its vertex is of degree 17 or 9. We suppose that G has k vertex (vertices) of degree 17. Therefore, by Lemma 2.1 (ii) and the the spectrum of the graph G, it follows that $17k + (18-k)8 = 153$. This implies that $k = 1$. So, G has one vertex of degree 17, say j. Theorem 2.8 implies that $P_{G-j}(\lambda) = (\lambda-\mu_3)^7(\lambda-\mu_4)^7[\alpha_{1j}^2 A_1 + \alpha_{2j}^2 A_2 + \alpha_{3j}^2 A_3 + \alpha_{4j}^2 A_4]$, where

$$\mu_1 = \frac{8+\sqrt{132}}{2}, \mu_2 = \frac{8-\sqrt{132}}{2}, \mu_3 = \frac{-1+\sqrt{17}}{2} \text{ and } \mu_4 = \frac{-1-\sqrt{17}}{2}.$$

$$\begin{aligned} A_1 &= (\lambda-\mu_2)(\lambda-\mu_3)(\lambda-\mu_4), \\ A_2 &= (\lambda-\mu_1)(\lambda-\mu_3)(\lambda-\mu_4), \\ A_3 &= (\lambda-\mu_1)(\lambda-\mu_2)(\lambda-\mu_4), \\ A_4 &= (\lambda-\mu_1)(\lambda-\mu_2)(\lambda-\mu_3). \end{aligned}$$

We know that $G - j$ has 17 eigenvalues. In other words, $P_{G-j}(\lambda)$ has 17 roots. Also, if we remove the vertex j of graph G, then the number of edges that is removed of graph G is 17. Moreover, Lemma 3.2 follows that $G - j$ is regular and degree of its regularity is 8. By Lemma 2.1 (iii) for the closed walks of length 1 and 2, we have:

$$x + y + 8 = -(54\mu_3 + 20\mu_4),$$
$$y^2 + y^2 + 64 = 136 - (54\mu_3^2 + 20\mu_4^2),$$

where x and y are the eigenvalues of graph $G-j$. If we solve the above equations, then $x = \dfrac{-1+\sqrt{17}}{2}$ and $y = \dfrac{-1-\sqrt{17}}{2}$. Hence $Spec(G-j) = Spec(P)$ and so $G - j \cong P$.
This follows the result. $\square$

Up to now, we show that the multicone graph $K_1 \bigtriangledown P$ are DS with respect to their adjacency spectrum. The natural question is; what happen for multicone graph $K_w \bigtriangledown P$? we answer to this question in the next theorem.

Theorem 3.1.4. *Multicone graphs $K_w \bigtriangledown P$ are DS with respect to their adjacency spectrum.*

Proof. We solve the problem by induction on w. If $w = 1$, by Lemma 3.3 the proof is completed. Let the claim be true for w; that is, if $Spec(G_1) = Spec(K_w \bigtriangledown P)$, then $G_1 \cong K_w \bigtriangledown P$, where G_1 is an arbitrary graph cospectral with multicone graph $K_w \bigtriangledown P$. We show that the claim is true for $w+1$; that is, if $Spec(G) = Spec(K_{w+1} \bigtriangledown P)$, then $G \cong K_{w+1} \bigtriangledown P$, where G is an arbitrary graph cospectral with multicone graph$K_{w+1} \bigtriangledown P$. By Lemma 3.2, we can conclude that $G - j \cong K_w \bigtriangledown H$, where H is a 8-regular graph with 17 vertices and j is the vertex of degree $w+17$ belonging to G. On the other hand, Theorem 2.2 implies that $G - j$ has the eigenvalues $\beta_{1,2} = \dfrac{\Lambda \pm \sqrt{\Lambda^2 + 4\Gamma}}{2}$, where $\Lambda = w + 7$ and $\Gamma = 9w + 8$. Now, in a similar manner of Lemma 3.3 for $G - j$ and the closed walks of length 1, 2 and 3, we obtain $Spec(G - j) = Spec(K_w \bigtriangledown P)$. Hence induction hypothesis follows that $G - j \cong K_w \bigtriangledown P$.
Therefore, the assertion holds. $\square$

3.2 Graph cospectral with a multicone graph $K_w \bigtriangledown P$ with respect to Laplacian spectrum

In this subsection, we show that multicone graphs $K_w \bigtriangledown P$ are DS with respect to their Laplacian spectrum.

Theorem 3.2.1. Multicone graphs $K_w \bigtriangledown P$ are DS with respect to their Laplacian spectrum.

Proof. We solve the problem by induction on w. If $w = 1$, there is nothing to prove. Let the claim be true for w; that is, $Spec(L(G_1)) = Spec(L(K_w \bigtriangledown P)) = \left\{ [0]^1, [17+w]^w, \left[\frac{\sqrt{17}+17}{2}+w\right]^8, \left[\frac{-\sqrt{17}+17}{2}+w\right]^8 \right\}$ follows that $G_1 \cong K_w \bigtriangledown P$, where G_1 is a graph. We show that the claim is true for $w+1$; that is, we show that

$$Spec(L(G)) = Spec(L(K_{w+1} \bigtriangledown P)) = \left\{ [0]^1, [18+w]^{w+1}, \left[\frac{\sqrt{17}+19}{2}+w\right]^8, \left[\frac{-\sqrt{17}+19}{2}+w\right]^8 \right\}$$

follows that $G \cong K_{w+1} \bigtriangledown P$, where G is a graph. Lemma 2.6 implies that G_1 and G are the join of two graphs. On the other hand, G has one vertex and $w+17$ edges more than G_1. Therefore, we must have $G \cong K_1 \bigtriangledown G_1$. Now, induction hypothsis follows the assertion. This completes the proof. $\square$

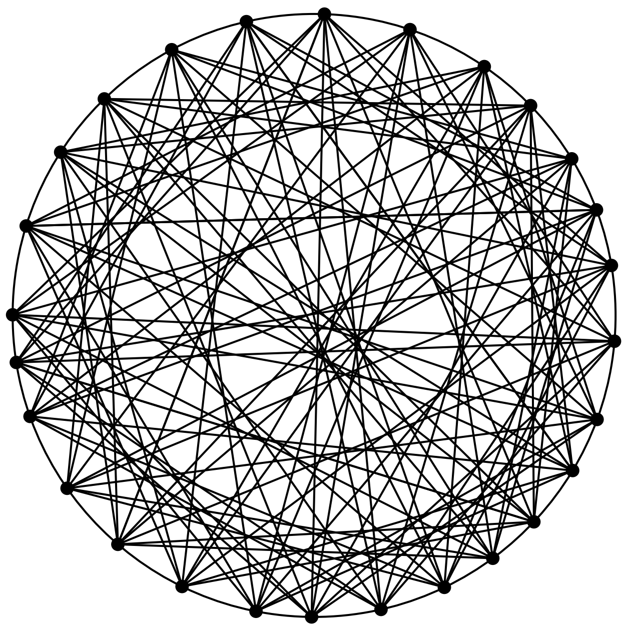

Figure 2: Schläfi graph

4. Graph Cospectral with a Multicone Graph $K_w \bigtriangledown S$

In the following section, we show that any graph cospectral with multicone graph $K_w \bigtriangledown S$ must be bidegreed.
From now on, with the similar arguments of the above results, we characterize another new classes of multicone graphs that are DS with respect to their spectra.

Proposition 4.1. *Let G be a graph cospectral with a multicone graph $K_w \bigtriangledown S$. Then*

$$Spec(G) = \left\{ [-1]^{w-1}, [1]^{20}, [-5]^{6}, \left[\frac{\Lambda + \sqrt{\Lambda^2 + 4\Gamma}}{2}\right]^{1}, \left[\frac{\Lambda + \sqrt{\Lambda^2 + 4\Gamma}}{2}\right]^{1} \right\},$$

where $\Lambda = 9 + w$ and $\Gamma = 17w + 10$.

Proof. It is well-known that $Spec(S) = \left\{ [10]^{1}, [1]^{20}, [-5]^{6} \right\}$. Now, by Theorem 2.2, the proof is obvious. $\square$

In the following lemma, we show that any graph cospectral with multicone graph $K_w \bigtriangledown S$ must be bidegreed.

Lemma 4.2. *Let G be cospectral with a multicone graph $K_w \bigtriangledown S$. Then G is bidegreed in which any vertex of G is of degree $w + 10$ or $w + 26$.*

Proof. It is obvious that G cannot be regular; since regularity of a graph can be determined by its spectrum. By contrary, we suppose that the sequence of degrees of vertices of graph G consists of at least three number. Hence the equality in Theorem 2.4 cannot happen for any δ. But, if we put $\delta = w + 10$, then the equality in Theorem 2.4 holds. So, G must be bidegreed. Now, we show that $\Delta = \Delta(G) = w+26$. By contrary, we suppose that $\Delta < w+26$. Therefore, the equality in Theorem 2.4 cannot hold for any δ. But, if we put $\delta = w + 10$, then this equality holds. This is a contradiction and so $\Delta = w + 26$. Now, $\delta = w + 10$, since G is bidegreed and G has $w + 27$ vertices, $\Delta = w + 26$ and $w(w + 26) + 27(w + 10) = w\Delta + 27(w + 10) = \sum_{i=1}^{w+27} \deg v_i$.
This completes the proof. $\square$

In the following, we show that any graph cospectral with multicone graph $K_1 \bigtriangledown S$ is isomorphic to $K_1 \bigtriangledown S$.

4.1 Graph cospectral with the multicone graph $K_1 \bigtriangledown S$

Lemma 4.1.1. *Any graph cospectral with the multicone graph $K_1 \bigtriangledown S$ is isomorphic to $K_1 \bigtriangledown S$.*

Proof. Let G be cospectral with multicone graph $K_1 \bigtriangledown S$. By Lemma 4.2 , it is easy to see that G has one vertex of degree 27, say l. Now, Theorem 2.8 implies that $P_{G-l}(\lambda) = (\lambda - \mu_3)^{19}(\lambda - \mu_4)^5[\alpha_{1j}^2 D_1 + \alpha_{2j}^2 D_2 + \alpha_{3j}^2 D_3 + \alpha_{4j}^2 D_4]$, where

$$\mu_1 = \frac{10 + \sqrt{208}}{2}, \ \mu_2 = \frac{10 - \sqrt{208}}{2}, \ \mu_3 = 1 \text{ and } \mu_4 = -5.$$
$$\begin{aligned} D_1 &= (\lambda - \mu_2)(\lambda - \mu_3)(\lambda - \mu_4), \\ D_2 &= (\lambda - \mu_1)(\lambda - \mu_3)(\lambda - \mu_4), \\ D_3 &= (\lambda - \mu_1)(\lambda - \mu_2)(\lambda - \mu_4), \\ D_4 &= (\lambda - \mu_1)(\lambda - \mu_2)(\lambda - \mu_3). \end{aligned}$$

Now, we have:

$$\begin{aligned} x + y + 10 &= -(19\mu_3 + 5\mu_4), \\ x^2 + y^2 + 100 &= 270 - (19\mu_3^2 + 5\mu_4^2), \end{aligned}$$

where x and y are the eigenvalues of graph $G - j$. If we solve above equation, then $x = 1$ and $y = -5$. Hence $Spec(G - j) = Spec(S)$ and so $G - j \cong S$.
This completes the proof. □

Theorem 4.1.2. *Multicone graphs $K_w \bigtriangledown S$ are DS with respect to their adjacency spectrum.*

Proof. We solve the problem by induction on w. If $w = 1$, there is nothing to prove. Let the claim be true for w; that is, if $Spec(G_1) = Spec(K_w \bigtriangledown S)$, then $G_1 \cong K_w \bigtriangledown S$, where G_1 is a graph. We show that the claim is true for $w + 1$; that is, if $Spec(G) = Spec(K_{w+1} \bigtriangledown S)$, then $G \cong K_{w+1} \bigtriangledown S$, where G is a graph. By Lemma 4.2 , Theorem 2.4 , Lemma 2.1 (iii) and in a similar manner of Lemma 4.3 for $G - j$, where j is a vertex of degree $w + 56$ belong to G, we obtain $Spec(G - j) = Spec(K_w \bigtriangledown S)$.
This completes the proof. □

4.2 Graph cospectral with a multicone graph $K_w \bigtriangledown S$ with respect to Laplacian spectrum

In this subsection, we show that multicone graphs $K_w \bigtriangledown S$ are DS with respect to their Laplacian spectrum.

Theorem 4.2.1. *Multicone graphs $K_w \bigtriangledown S$ are DS with respect to their Laplacian spectrum.*

Proof. We solve the problem by induction on w. If $w = 1$, there is nothing to prove. let the claim be true for w; that is, $Spec(L(G_1)) = Spec(L(K_w \bigtriangledown S)) = \left\{[w+27]^w, [w+9]^{20}, [w+15]^6, [0]^1\right\}$ follows that $G_1 \cong K_w \bigtriangledown S$, where G_1 is a graph. We show that the claim is true for $w+1$; that is, we show that $Spec(L(G)) = Spec(L(K_{w+1} \bigtriangledown S)) = \left\{[w+28]^{w+1}, [w+9]^{20}, [w+16]^6, [0]^1\right\}$ follows that $G \cong K_{w+1} \bigtriangledown S$, where G is a graph. Lemma 2.6 implies that G_1 and G are the join of two graphs. On the other hand, G has one vertex and $w+27$ edges more than G_1. Therefore, $G \cong K_1 \bigtriangledown G_1$, and the assertion holds. $\square$

5. Concluding Remarks and Open Problems

In this section, we review what were stated in the earlier sections and finally we propose four problems.

Corollary 5.1. *Let G be a graph. The following are equivalent:*
(*i*) $G \cong K_w \bigtriangledown P$.
(*ii*) $Spec(G) = Spec(K_w \bigtriangledown P)$.
(*iii*) $Spec(L(G)) = Spec((L(K_w \bigtriangledown P)))$.

Corollary 5.2. *Let G be a graph. The following are equivalent:*
(*i*) $G \cong K_w \bigtriangledown S$.
(*ii*) $Spec(G) = Spec(K_w \bigtriangledown S)$.
(*iii*) $Spec(L(G)) = Spec((L(K_w \bigtriangledown S)))$

Now, we pose the following conjectures.

Conjecture 5.3. Graphs $\overline{K_w \bigtriangledown P}$ are DS with respect to their adjacency spectrum.

Conjecture 5.4. Multicone graph $K_w \bigtriangledown P$ are DS with respect to signless Laplacian spectrum.

Conjecture 5.5. Graphs $\overline{K_w \bigtriangledown S}$ are DS with respect to their adjacency spectrum.

Conjecture 5.6. Multicone graph $K_w \bigtriangledown S$ are DS with respect to their signless Laplacian spectrum.

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