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Variational Iteration Method for Solving Hybrid Fuzzy Differential Equations

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Abstract. In this paper, we study a numerical method for hybrid fuzzy differential equations (HFDEs) by an application of the variational iteration method (VIM). We state a convergence result and give numerical examples to illustrate the theory. Numerical results are presented for some problems to demonstrate the usefulness and accuracy of this approach. The method is easy to apply and produces very accurate numerical results.

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1. Introduction

The topic of fuzzy differential equations has been rapidly growing in recent years. The fuzzy differential equations were first formulated by Kaleva [13] and Seikkala [21]. The applications of fuzzy differential equations in other fields and related mathematical tools and techniques could be found in [2, 3].

On the other hand, hybrid system is a dynamic system that exhibits both continuous and discrete dynamic behavior. The hybrid systems are devoted

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to modelling, design, and validation of interactive systems of computer programs and continuous systems. The differential equations containing fuzzy valued functions and interaction with a discrete time controller are named as hybrid fuzzy differential equations [17]. The stability properties and analytical results of HFDEs can be found in [15, 16, 20]. The numerical methods of fuzzy differential equation are studied by numerous authors such as [4, 1, 5]. Furthermore, there are some numerical techniques to solve hybrid fuzzy differential equations [17, 18, 19, 14].

Many of numerical methods have introduced discrete solutions, but we use VIM for finding analytical approximate solutions of hybrid fuzzy differential equations. In this paper, we apply VIM for solving hybrid fuzzy differential equations, based on the Seikkala's derivative.

In Section 2, we list some basic definitions of fuzzy valued functions. Section 3 reviews hybrid fuzzy differential systems. Section 4 contains the variational iteration method. The numerical examples are provided in Section 5 and conclusion is in Section 6.

2. Preliminaries

Denote by E^1 the set of all functions $u : \mathbb{R} \to [0,1]$ such that (i) u is normal, that is, there exists an $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$, (ii) u is fuzzy convex, that is, for $x, y \in \mathbb{R}$ and $0 \leqslant l \leqslant 1$, $u(lx + (1-l)y) \geqslant min\{u(x), u(y)\}$, (iii) u is upper semicontinuous, and (iv) $[u]^0 \equiv$ the closure of $\{x \in \mathbb{R} : u(x) > 0\}$ is compact. For $0 < \alpha \leqslant 1$, we define $[u]^\alpha = \{x \in \mathbb{R} : u(x) \geqslant \alpha\}$.

For later purposes, we define $\hat{0} \in E^1$ as $\hat{0}(x) = 1$ if $x = 0$ and $\hat{0}(x) = 0$ if $x \neq 0$. Next we review the Seikkala derivative [21] of $x : I \to E^1$ where $I \subset \mathbb{R}$ is an interval. if $[x(t)]^\alpha = [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]$ for all $t \in I$ and $\alpha \in [0,1]$, then $[x'(t)]^\alpha = [(\underline{x}^\alpha)'(t), (\overline{x}^\alpha)'(t)]$ if $x'(t) \in E^1$. Next consider the initial value problem (IVP)

$$\begin{cases} x'(t) = f(t, x(t)), \\ x(0) = x_0, \end{cases} \tag{1}$$

where $f : [0,\infty) \times \mathbb{R} \to \mathbb{R}$ is continuous. We would like to interpret Eq.(1) using the Seikkala derivative and $x_0 \in E^1$. Let

$$[x_0]^\alpha = [\underline{x}_0{}^\alpha, \overline{x}_0{}^\alpha] \text{and} [x(t)]^\alpha = [\underline{x}^\alpha(t), \overline{x}^\alpha(t)].$$

By Zadeh extension principle we get

$$f : [0,\infty) \times E^1 \to E^1$$

where

$$[f(t,x)]^\alpha = [\min\{f(t,u) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]\}, \max\{f(t,u) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]\}].$$

Then $x : [0,\infty) \to E^1$ is a solution of Eq.(1) using the Seikkala derivative and $x_0 \in E^1$ if

$$(\underline{x}^\alpha)'(t) = \min\{f(t,u) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]\}, \quad \underline{x}^\alpha(0) = \underline{x}_0^\alpha,$$
$$(\overline{x}^\alpha)'(t) = \max\{f(t,u) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)]\}, \quad \overline{x}^\alpha(0) = \overline{x}_0^\alpha,$$

for all $t \in [0,\infty)$ and $\alpha \in [0,1]$.

Lastly consider an $f : [0,\infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ which is continuous and the IVP

$$\begin{cases} x'(t) = f(t, x(t), k), \\ x(0) = x_0. \end{cases} \tag{2}$$

As in [3] to interpret Eq.(2) using the Seikkala derivative and $x_0, k \in E^1$, by the Zadeh extension principle we use $f : [0,\infty) \times E^1 \times E^1 \to E^1$ where

$$[f(t,x,k)]^\alpha = [\min\{f(t,u,u_k) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)], u_k \in [\underline{k}^\alpha, \overline{k}^\alpha]\},$$
$$\max\{f(t,u,u_k) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)], u_k \in [\underline{k}^\alpha, \overline{k}^\alpha]\}].$$

where $k^\alpha = [\underline{k}^\alpha, \overline{k}^\alpha]$. Then $x : [0,\infty) \to E^1$ is a solution of Eq.(2) if

$$(\underline{x}^\alpha)'(t) = \min\{f(t,u,u_k) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)], u_k \in [\underline{k}^\alpha, \overline{k}^\alpha]\}, \quad \underline{x}^\alpha(0) = \underline{x}_0^\alpha,$$
$$(\overline{x}^\alpha)'(t) = \max\{f(t,u,u_k) : u \in [\underline{x}^\alpha(t), \overline{x}^\alpha(t)], u_k \in [\underline{k}^\alpha, \overline{k}^\alpha]\}, \quad \overline{x}^\alpha(0) = \overline{x}_0^\alpha,$$

for all $t \in [0,\infty)$ and $\alpha \in [0,1]$. (see[3], p. 45).

3. The Hybrid Fuzzy Differential System

Consider the hybrid fuzzy differential system

$$\begin{cases} x'(t) = f(t, x(t), l_k(x_k)), \ t \in [t_k, t_{k+1}], \\ x(t_k) = x_k, \end{cases} \tag{3}$$

where $'$ denotes Seikkala differentiation, and
$0 \leqslant t_0 < t_1 < \cdots < t_k < \cdots, t_k \to \infty, f \in C[\mathbb{R}^+ \times E^1 \times E^1, E^1], l_k \in [E^1, E^1]$.
To be specific the system would look like

$$x'(t) = \begin{cases} x_0'(t) = f(t, x_0(t), l_0(x_0)), & x_0(t_0) = x_0, \ t_0 \leqslant t \leqslant t_1, \\ x_1'(t) = f(t, x_1(t), l_1(x_1)), & x_1(t_1) = x_1, \ t_1 \leqslant t \leqslant t_2, \\ \vdots & \\ x_k'(t) = f(t, x_k(t), l_k(x_k)), & x_k(t_k) = x_k, \ t_k \leqslant t \leqslant t_{k+1}, \\ \vdots & \end{cases}$$

Assuming that the existence and uniqueness of solution of Eq.(3) hold for each $[t_k, t_{k+1}]$, by the solution of Eq.(3) we mean the following function

$$x(t) = x(t, t_0, x_0) = \begin{cases} x_0(t), & t_0 \leqslant t \leqslant t_1, \\ x_1(t), & t_1 \leqslant t \leqslant t_2, \\ \vdots & \\ x_k(t), & t_k \leqslant t \leqslant t_{k+1}, \\ \vdots & \end{cases}$$

We note that the solution of Eq.(3) are piecewise differentiable in each interval for $t \in [t_k, t_{k+1}]$ for a fixed $x_k \in E^1$ and $k = 0, 1, 2, \cdots$.

Using a representation of fuzzy number studied by Goetschel and Voxman [6] and Wu and Ma [23], we may represent $x \in E^1$ by a pair of functions $(\underline{x}(r), \overline{x}(r)), 0 \leqslant r \leqslant 1$, such that (i) $\underline{x}(r)$ is bounded, left continuous, and nondecreasing, (ii) $\overline{x}(r)$ is bounded, left continuous, and nonincreasing, and (iii) $\underline{x}(r) \leqslant \overline{x}(r), 0 \leqslant r \leqslant 1$.

Therefore we may replace Eq.(3) by an equivalent system

$$\begin{cases} \underline{x}'(t) = \underline{f}(t, x, l_k(x_k)) = F_k(t, \underline{x}, \overline{x}), & \underline{x}(t_k) = \underline{x}_k, \\ \overline{x}'(t) = \overline{f}(t, x, l_k(x_k)) = G_k(t, \underline{x}, \overline{x}), & \overline{x}(t_k) = \overline{x}_k, \end{cases} \tag{4}$$

which possesses a unique solution $(\underline{x}, \overline{x})$ and it is a fuzzy function. That is for each t, the pair $[\underline{x}(t; r), \overline{x}(t; r)]$ is a fuzzy number, where $\underline{x}(t; r), \overline{x}(t; r)$ are respectively the solutions of the parametric form given by

$$\begin{cases} \underline{x}'(t; r) = F_k[t, \underline{x}(t; r), \overline{x}(t; r)], & \underline{x}(t_k; r) = \underline{x}_k(r), \\ \overline{x}'(t; r) = G_k[t, \underline{x}(t; r), \overline{x}(t; r)], & \overline{x}(t_k; r) = \overline{x}_k(r), \end{cases} \tag{5}$$

for $r \in [0, 1]$.

4. Variational Iteration Method

Many authors have worked on variational iteration method (VIM), cf. [7, 8, 9, 22]. VIM, was first proposed by J. H. He for solving wide rang of problems [10, 11]. VIM is based on the general Lagrange multiplier method [12].

To illustrate the basic concepts of VIM, we consider the following nonlinear differential equation:

$$Lu + Nu = g(x),$$

where L is a linear operator, N a nonlinear operator, and $g(x)$ is an inhomogeneouse term. According to VIM, we can construct a correction functional as following:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\{Lu_n(\tau) + N\tilde{u}(\tau) - g(\tau)\}\mathrm{d}\tau, \quad n \geqslant 0, \tag{6}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory, the subscript n denotes the nth-order approximation, and $\tilde{u}_n$ is considered as a restricted variation, which means $\delta\tilde{u}_n = 0$. By the correction functional Eq.(6) several approximations will be obtained and therefore, the exact solution emerges at the limit of the resulting successive approximations.

5. Numerical Examples

In this section, we solve two examples by VIM.

Example 5.1. Consider the following hybrid fuzzy differential equation

$$\begin{cases} x'(t) = x(t) + m(t)l_k(x(t_k)), \quad t \in [t_k, t_{k+1}], t_k = k, \ k = 0, 1, 2, \cdots \\ x(0, r) = [0.75 + 0.25r, 1.125 - 0.125r], \quad 0 \leqslant r \leqslant 1, \end{cases} \tag{7}$$

where

$$m(t) = \begin{cases} 2(t(mod 1)), & if\ t(mod\ 1) \leqslant 0.5, \\ 2(1 - t(mod 1)), & if\ t(mod\ 1) > 0.5, \end{cases}$$

and

$$l_k(\mu) = \begin{cases} 0, & if\ k = 0, \\ \mu, & if\ k \in \{1, 2, \cdots\}. \end{cases}$$

The parametric form of Eq.(7) is

$$\begin{cases} \underline{x}'(t) = \underline{x}(t) + \min\left\{m(t)l_k(\underline{x}(t_k)), m(t)l_k(\overline{x}(t_k))\right\} \\ \overline{x}'(t) = \overline{x}(t) + \max\left\{m(t)l_k(\underline{x}(t_k)), m(t)l_k(\overline{x}(t_k))\right\}, \end{cases} \tag{8}$$

with initial conditions

$$\begin{cases} \underline{x}(0) = 0.75 + 0.25r \\ \overline{x}(0) = 1.125 - 0.125r. \end{cases}$$

In (7), $x(t) + m(t)l_k(x(t_k))$ is a continuous function of t, x, and $l_k(x(t_k))$. Therefore by Example 6.1 of Kaleva [13], for each $k = 0, 1, 2, \cdots$. The fuzzy IVP

$$\begin{cases} x'(t) = x(t) + m(t)l_k(x(t_k)), \quad t \in [t_k, t_{k+1}], t_k = k, \\ x(t_k) = x_{tk}, \end{cases}$$

has a unique solution on $[t_k, t_{k+1}]$.

To solve this system by VIM the following formulas are obtained

$$\begin{aligned}\underline{x}_{n+1}(t) = \underline{x}_n(t) + \int_0^t \underline{l}(t,s)\Big\{\underline{x}'_n(s) - \underline{x}_n(s)\\ - \min\big\{m(s)l_k(\underline{x}(s_k)), m(s)l_k(\overline{x}(s_k))\big\}\Big\}\mathrm{d}s,\\ \overline{x}_{n+1}(t) = \overline{x}_n(t) + \int_0^t \overline{l}(t,s)\Big\{\overline{x}'_n(s) - \overline{x}_n(s)\\ - \max\big\{m(s)l_k(\underline{x}(s_k)), m(s)l_k(\overline{x}(s_k))\big\}\Big\}\mathrm{d}s,\end{aligned} \tag{9}$$

where $l(t,s)$ is a general Lagrangian multiplier which can be identified optimally via variational theory and n is the number of iteration step.
The variation is calculated with respect to $\underline{x}_n, \overline{x}_n$, respectively, then we have

$$\begin{aligned}\delta\underline{x}_{n+1}(t) &= \delta\underline{x}_n(t) + \delta\int_0^t \underline{l}(t,s)\Big\{\underline{x}'_n(s)\\ &\quad - \underline{x}_n(s) - \min\big\{m(s)l_k(\underline{x}(s_k)), m(s)l_k(\overline{x}(s_k))\big\}\Big\}\mathrm{d}s\\ &= \delta\underline{x}_n(t) + \int_0^t \underline{l}(t,s)\big\{\frac{\mathrm{d}}{\mathrm{d}s}\delta\underline{x}_n(s) - \delta\underline{x}_n(s)\big\}\mathrm{d}s\\ &= \delta\underline{x}_n(t) + \underline{l}(t,s)\delta\underline{x}_n(s)|_{s=t} - \int_0^x \big(\frac{\partial\underline{l}(t,s)}{\partial s} + \underline{l}(t,s)\big)\delta\underline{x}_n(s)\mathrm{d}s = 0.\end{aligned}$$

For arbitrary $\delta\underline{x}_n$, the following conditions are obtained

$$\underline{l}(t,s) + 1 = 0\Big|_{s=t}, \quad \frac{\partial\underline{l}(t,s)}{\partial s} + \underline{l}(t,s) = 0\Big|_{s=t}.$$

The Lagrange multiplier, can be identified as $\underline{l}(t,s) = -e^{-(s-t)}$.
Similar to above, we find $\overline{l}(t,s) = -e^{-(s-t)}$, i.e. $l(t,s) = \underline{l}(t,s) = \overline{l}(t,s) = -e^{-(s-t)}$ is a crisp function, and the following iteration formula can be obtained

as:

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \displaystyle\int_0^t e^{-(s-t)} \Big\{ \underline{x}'_n(s) \\ \qquad - \underline{x}_n(s) - \min\big\{m(s)l_k(\underline{x}(s_k)), m(s)l_k(\overline{x}(s_k))\big\} \Big\} \mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \displaystyle\int_0^t e^{-(s-t)} \Big\{ \overline{x}'_n(s) \\ \qquad - \overline{x}_n(s) - \max\big\{m(s)l_k(\underline{x}(s_k)), m(s)l_k(\overline{x}(s_k))\big\} \Big\} \mathrm{d}s. \end{cases} \tag{10}$$

Beginning with $x_0(t) = [\underline{x}_0(t), \overline{x}_0(t)]$, by the iteration formula Eq.(10), we can obtain the numerical solution of Eq.(7). For $t \in [0, 1]$, we have $k = 0$ and $l_k(x(t_k)) = 0$. Therefore, we can rewrite the iteration formula Eq.(10) in the form

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \displaystyle\int_0^t e^{-(s-t)} \{\underline{x}'_n(s) - \underline{x}_n(s)\} \mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \displaystyle\int_0^t e^{-(s-t)} \{\overline{x}'_n(s) - \overline{x}_n(s)\} \mathrm{d}s. \end{cases} \tag{11}$$

We can take an initial approximation $x_0(t) = [0.75 + 0.25r, 1.125 - 0.125r]$.
By using the above initial approximation and Eq.(11), we have

$$\begin{cases} \underline{x}_1(t) = (0.75 + 0.25r)e^t, \\ \overline{x}_1(t) = (1.125 - 0.125r)e^t. \end{cases}$$

For $t \in [0, 1]$, in the first iterative, we obtained the exact solution of Eq.(7).
In the interval [1, 1.5], we have $l_k(x(t_k)) = x(1)$. Therefore, we can rewrite the iteration formula Eq.(10) in the form

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \displaystyle\int_1^t e^{-(s-t)} \{\underline{x}'_n(s) - \underline{x}_n(s) - 2\underline{x}(1)(s-1)\} \mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \displaystyle\int_1^t e^{-(s-t)} \{\overline{x}'_n(s) - \overline{x}_n(s) - 2\overline{x}(1)(s-1)\} \mathrm{d}s. \end{cases} \tag{12}$$

We can take an initial approximation $x_0(t) = x(1)$.
By using the above initial approximation and Eq.(12), we have

$$\begin{cases} \underline{x}_1(t) = \underline{x}(1)(3e^{t-1} - 2t), \\ \overline{x}_1(t) = \overline{x}(1)(3e^{t-1} - 2t). \end{cases}$$

For $t \in [1, 1.5]$, in the first iterative, we obtained the exact solution of Eq.(7).
In the interval [1.5, 2], we have $l_k(x(t_k)) = x(1)$. Therefore, we can rewrite the

iteration formula Eq.(10) in the form

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \displaystyle\int_{1.5}^{t} e^{-(s-t)}\{\underline{x}'_n(s) - \underline{x}_n(s) - 2\underline{x}(1)(2-s)\}\mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \displaystyle\int_{1.5}^{t} e^{-(s-t)}\{\overline{x}'_n(s) - \overline{x}_n(s) - 2\overline{x}(1)(s-1)\}\mathrm{d}s. \end{cases} \tag{13}$$

We can take an initial approximation $x_0(t) = x(1)(3\sqrt{e} - 3)$.
By using the above initial approximation and Eq.(13), we have

$$\begin{cases} \underline{x}_1(t) = \underline{x}(1)(2t - 2 + e^{t-1.5}(3\sqrt{e} - 4)), \\ \overline{x}_1(t) = \overline{x}(1)(2t - 2 + e^{t-1.5}(3\sqrt{e} - 4)). \end{cases}$$

For $t \in [1.5, 2]$, in the first iterative, we obtained the exact solution of Eq.(7).

Example 5.2. Consider the following hybrid fuzzy differential equation

$$\begin{cases} x'(t) = x(t) + m(t)l_k(x(t_k)), \quad t \in [t_k, t_{k+1}], t_k = k, \ k = 0, 1, 2, \cdots \\ x(0, r) = [0.75 + 0.25r, 1.125 - 0.125r], \quad 0 \leqslant r \leqslant 1, \end{cases} \tag{14}$$

where

$$m(t) = |sin(\pi t)|, \quad k = 0, 1, 2, \cdots,$$

and

$$l_k(\mu) = \begin{cases} 0, & if\ k = 0, \\ \mu, & if\ k \in \{1, 2, \cdots\}. \end{cases}$$

The parametric form of (14) is

$$\begin{cases} \underline{x}'(t) = \underline{x}(t) + \min\{m(t)l_k(\underline{x}(t_k)), m(t)l_k(\overline{x}(t_k))\} \\ \overline{x}'(t) = \overline{x}(t) + \max\{m(t)l_k(\underline{x}(t_k)), m(t)l_k(\overline{x}(t_k))\}, \end{cases}$$

with initial conditions

$$\begin{cases} \underline{x}(0) = 0.75 + 0.25r \\ \overline{x}(0) = 1.125 - 0.125r. \end{cases}$$

Again, in (14), $x(t)+m(t)l_k(x(t_k))$ is a continuous function of t, x, and $l_k(x(t_k))$. Therefore by Example 6.1 of Kaleva [13], for each $k = 0, 1, 2, \cdots$. The fuzzy IVP

$$\begin{cases} x'(t) = x(t) + m(t)l_k(x(t_k)), \quad t \in [t_k, t_{k+1}], t_k = k, \\ x(t_k) = x_{tk}, \end{cases}$$

has a unique solution on $[t_k, t_{k+1}]$.
Similar to example 1, we find $\underline{l}(t, s) = \overline{l}(t, s) = -e^{-(s-t)}$.

For $t \in [0,1]$, we have $k = 0$ and $l_k(x(t_k)) = 0$.
Therefore, we can rewrite the iteration formula Eq.(10) in the form

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \int_0^t e^{-(s-t)}\{\underline{x}'_n(s) - \underline{x}_n(s)\}\mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \int_0^t e^{-(s-t)}\{\overline{x}'_n(s) - \overline{x}_n(s)\}\mathrm{d}s. \end{cases} \tag{15}$$

We can take an initial approximation $x_0(t) = [0.75 + 0.25r, 1.125 - 0.125r]$.
By using the above initial approximation and Eq.(15), we have

$$\begin{cases} \underline{x}_1(t) = (0.75 + 0.25r)e^t, \\ \overline{x}_1(t) = (1.125 - 0.125r)e^t. \end{cases}$$

For $t \in [0,1]$, in the first iterative, we obtained the exact solution of Eq.(14).
In the interval $[1, 2]$, we have $m(t) = -sin(\pi t)$ and $l_k(x(t_k)) = x(1)$. Therefore, we can rewrite the iteration formula Eq.(10) in the form

$$\begin{cases} \underline{x}_{n+1}(t) = \underline{x}_n(t) - \int_1^t e^{-(s-t)}\{\underline{x}'_n(s) - \underline{x}_n(s) + \underline{x}(1)sin(\pi s)\}\mathrm{d}s, \\ \overline{x}_{n+1}(t) = \overline{x}_n(t) - \int_1^t e^{-(s-t)}\{\overline{x}'_n(s) - \overline{x}_n(s) + \overline{x}(1)sin(\pi s)\}\mathrm{d}s. \end{cases} \tag{16}$$

We can take an initial approximation $x_0(t) = x(1)$. By using the above initial approximation and Eq.(16), we have

$$\begin{cases} \underline{x}_1(t) = \underline{x}(1)(\frac{sin(\pi t)+\pi cos(\pi t)}{1+\pi^2} + \frac{e^t}{e}(1 + \frac{\pi}{1+\pi^2})), \\ \overline{x}_1(t) = \overline{x}(1)(\frac{sin(\pi t)+\pi cos(\pi t)}{1+\pi^2} + \frac{e^t}{e}(1 + \frac{\pi}{1+\pi^2})). \end{cases}$$

For $t \in [1,2]$, in the first iterative, we obtained the exact solution of Eq.(14).

6. Conclusion

In this paper, variational iteration method has been successfully applied to find the solutions of hybrid fuzzy differential equations. Numerical examples show that the use of VIM may result in exact solutions by only one iteration. It can be concluded that VIM is a very powerful and easy tool for solving hybrid fuzzy differential equations.

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