Journal of Mathematical Extension
Vol. 10, No. 4, (2016), 1-18
ISSN: 1735-8299
URL: http://www.ijmex.com

Teaching-Learning-Based Modified Collaborative Optimization Algorithm

A. R. Fakharzadeh*
Shiraz University of Technology

S. Khosravi
Shiraz University of Technology

Abstract. In complex engineering systems, collaboration between different design disciplines and concurrent optimization of the whole system have given rise to an important issue in the field of optimization. In recent decades, multi-disciplinary design optimization approaches and meta-heuristic algorithms have been used in this field. In this regard, a hybrid modified collaborative optimization framework and teaching-learning-based-optimization algorithm for solving these kinds of problems is presented here. We show the efficiency of the introduced algorithm by solving test cases and making comparisons with former results.

AMS Subject Classification: 80M50
Keywords and Phrases: Teaching-learning-based optimization, collaborative optimization, multi-disciplinary design optimization

1. Introduction

One of the most important challenges of optimization that is called multidisciplinary analysis (MDA) problem is to address optimization problems of complex systems involving multiple coupled disciplines such as most real world engineering optimization problems in aerospace, automobile, architecture etc. [2, 17]. Solving these kinds of problems without

Received: September 2016; Accepted: April 2016
*Corresponding author

considering the interaction between different disciplines is impossible. In the past, a fixed point iteration process (FPI), which is just a method to make the MDA problem converge, was used for solving these kinds of problems. This method, however, does not perform overall system optimization or configure the whole system so as to reach an optimum according to one global design objective.

MDA problems are solved by introducing the multidisciplinary design optimization (MDO). In fact, MDO is capable to improve the design of the system by exploiting synergies among different disciplines [17]. Nevertheless, MDO faces two big challenges, namely, computational and organizational complexities. MDO methods are classified in two general classes: single-level and multi-level methods. In the first class, the optimization is only directed in the system level, and the disciplinary models are employed for analyses. Examples for this class include multi-discipline-feasible (MDF) [3, 4], individual-discipline-feasible (IDF) [8,4], all-at-once (AAO) [3] and simultaneous analysis and design (SAND) [7]. In the multi-level procedure, both the system and subsystem conduct optimization. Typical multi-level procedures include collaborative optimization (CO) [1, 2], bi-level integrated system synthesis (BLISS) [10], concurrent subspace optimization (CSSO) [13, 16] and analytical target cascading (ATC) [15].

Teaching-learning based optimization (TLBO) is, on the other hand, one of the new meta-heuristic algorithms based on the natural phenomenon of teaching-learning process. TLBO was proposed in 2011 by Rao et al. [11] to solve continuous non-linear programming (NLP) large scale optimization. This algorithm does not require algorithm-specific parameters or their own algorithm-specific control parameters. In addition, from the literature survey on evolutionary algorithms, it is observed that the step of duplicate solutions removal is not explicitly found in the application of different evolutionary algorithms; while, as is presented in the literature of TLBO, the step of duplicate solutions removal.

In this paper, a new algorithm to solve MDO problems is introduced which is a hybrid of TLBO algorithm and collaborative optimization framework that is called Teaching-learning-based modified collaborative optimization (TLBMCO) algorithm. The paper is organized as follows. First, the TLBMCO is described and explained how it is combined by modified collaborative optimization (MCO). Then, three famous test problems of MDO are solved by the TLBMCO algorithm and the obtained results are compared with those of other methods. Finally, conclusions are drawn.

2. Teaching-Learning-Based Modified Collaborative Optimization

The TLBMCO algorithm has two cycles: internal and external. The internal cycle, which is a bi-level MDO framework, decomposes the original problem to system and sub-system levels. These obtained problems are then optimized and explored, and the synergy of coupling between various interacting disciplines in each iteration is exploited. The internal cycle is described in Sub-section 2.1 and is explained in section 2.2.

2.1 Internal cycle of the TLBMCO

The internal cycle is based on CO, which was proposed by Kroo et al. [2]. Here modified CO (MCO), which is a bi-level framework developed by A. V. Demiguel and W. Murray [5] is used. First, the internal cycle receives stochastic values of variables (design and coupling variables). Then, this cycle transfers these values into feasible ones. Since MCO has both level system and subsystem, in the internal cycle two kinds of problems are solved, namely level system and subsystem. The level system problem is formulated as follows:

System level:

$$Min \sum_{i=1}^{N_D} (Z^{i*} - Z^0) + (Y_{ij}^{i*} - Y_{ij}^0)$$

$$S.t.\ X_{LB}^0 \leqslant X^0 \leqslant X_{UB}^0, X^0 = [Z^0, Y_{ij}^0],$$

i-th subsystem:

$$Min\ d_i(X^i) = (Z^i - \widehat{Z}^0) + (Y^i_{ij} - \widehat{Y^0_{ij}}),$$

$$S.to : g_i(X^i) \leqslant 0,$$

$$X^i_{LB} \leqslant X^i \leqslant X^i_{UB},\ X^i = [Z^i, Y^i_{ij}, X^i_{loc}],$$

where Z is the vector of shared variables, Y_{ij} is the coupling variable vector (whose value is sent from the i-th subsystem to the j-th subsystem), X^i_{loc} is the vector of the i-th subsystem local variables, and X^0_{LB}, X^0_{UB}, X^i_{LB} and X^i_{UB} are vectors of lower and upper bounds of the variables respectively for system level and the i-th subsystem.

The internal cycle starts when the system level receives new values for variables (Z^i and Y_{ij}). In the first step, these values are forwarded to the subsystem level. Then, using these values, subsystem problems are optimized and in each subsystem an optimal solution (e.g. $X^{i*} = [Z^{i*}, Y^{i*}_{ij}, X^{i*}_{loc}]$) is produced. Then, this obtained optimal solution is sent to the system level. Now, using the received values, optimization of system level is performed. The next iteration of this cycle starts by sending the achieved optimal solution system level to the subsystem level. The inter cycle is terminated when the changes in the objective function in the system level are smaller than a predefined value.

2.2 External cycle

During the internal cycle, compatible solutions, that is, solutions with equal values of variables at both subsystem and level system, were attempted to be attained, But, in some iterations of the internal cycle, the objective function of the system level cannot be minimized to zero, or does not even reach a value near zero. Therefore, one main goal of the external cycle is to counteract these incompatible solutions in the next generation. In order to achieve this aim, a penalty value for the objective function is assigned. The TLBO algorithm is used [10, 6] in the external cycle. TLBO is an algorithm with few specific parameters. The implementation of TLBO does not require the determination of any controlling parameters, and this makes the algorithm robust and powerful. The process of TLBMCO algorithm for MDO problems, which is shown in Fig. 1, is described in the following.

Step 1: In the beginning, an initial population (students) is created by selecting the values of the coupling and shared variables randomly. Then, these values are sent to the internal cycle.

Step 2: In the internal cycle, the system level receives the values from the previous step. Next, these values are sent to the sub-systems. In the next step, the sub-systems are optimized and the obtained optimal solutions are forwarded to the system level. Now, the system level is optimized and its optimal solution is delivered to the subsystems. This process is continued until the objective function value of the system level is gets less than a predefined small positive number (ε).

Step 3: In the internal cycle, the goal in both the system level and sub-systems is to minimize the difference between values of variables Z^0, Y_{ij}^0 and Z^i, Y_{ij}^i. However, the obtained solutions from some iterations are incompatible. To exclude these kinds of solutions from the next generation, a large penalty value is assigned to the corresponding design objective function.

Step 4: The objective function used in evaluation of the external cycle, is the designing objective function. After checking the incompatible solutions, the teacher phase starts with evaluating the values of the objective function of the compatible solutions. Then, the mean of each variable is calculated. In the next step, the best solution is identified as the teacher. The teacher will try to move the students toward its own level. The solutions (students) are hence modified based on the best solution (teacher). Afterwards, the modified solutions are sent to the internal cycle. In the next step, each obtained solution of the internal cycle is improved by mutual interaction with other obtained solutions (phase learner, see [11]). Again, the improved solutions are delivered to the internal cycle. Finally, the termination criterion is checked.

Each subsystem objective is added to the system level in CO as a compatibility constraint. These constraints lead to difficult convergence during optimization. Since in TLBMCO the original objective function of the problem is optimized in the external cycle, the sum of compatibility constraints corresponding to the MCO algorithm is taken as the objective function of system level in TLBMCO. For the internal cycle of the TLBMCO algorithm, in some cases, the compatible objective cannot be minimized to zero or near to zero. Hence, the external cycle enables these incompatible solutions to be excluded in the next generation. Therefore, the TLBMCO algorithm overcomes convergence problem of CO.

3. Numerical Results

To show the efficiency of the presented algorithm for MDO problems, the TLBMCO algorithm was coded in MATLAB and run on three benchmark problems available in the literature. The analyses were performed on a system with Intel(R) Core (TM) i5-2430M processor and 8GB RAM.

Example 3.1. Consider an analytical example of an MDO problem involving two disciplines. The coupling between the two disciplines is described by equations below:

$$\begin{aligned} y_{12} &= z_1^2 + x_1 + z_2 - 0.2y_{21}, \\ y_{21} &= \sqrt{y_{12}} + z_1 + z_2. \end{aligned}$$

The related optimization problem is described by the following formulation:

$$\begin{aligned} &Min \;\; x_1^2 + z_2 + y_{12} + \exp(-y_{21}), \\ &S.t. 1 - \frac{y_{12}}{8} \leqslant 1, \\ &\quad \frac{y_{21}}{10} - 1 \leqslant 1 \\ &\quad -10 \leqslant z_1 \leqslant 10, \quad 0 \leqslant x_1 \leqslant 10, \quad 0 \leqslant z_2 \leqslant 10, \end{aligned}$$

where y_{12} and y_{21} are coupling variables, z_1 and z_2 are shared variables and x_1 is local variable. For $\varepsilon = 10^{-8}$, all optimizations from different initial populations stably converge to the optimum point [3.026800] with the objective $f = 8.0027$. Table 1 presents the obtained results and the results of other methods. These results show the efficiency and benefits of the new algorithm in comparison with other methods.

Table 1: Comparison of different MDO method for example 3.1

Method	MDF	IDF	CSSO	BLISS	CO	TLBMCO
f	8.02537	8.00347	8.19881	8.1278	21.99930	8.002712
(1)	3	5	11	3	9	9
(2)	0	2	0	0	2	0
(3)	231	0	208	178	0	0
(4)	0	48	44	30	10137	215
(5)	0	48	226	27	8522	198
(6)	462	96	686	413	19259	413
(7)	2.8×10^{-3}	7.62×10^{-5}	2.4×10^{-2}	1.5×10^{-2}	1.75	1.85×10^{-5}

(1): Number of design variables

(2): Number of equality constraints

(3): Function calls of system analysis

(4): Function calls of discipline 1

(5): Function calls of discipline 2

(6): Total function calls

(7): Relative error

Speed reducer optimization problem is selected as the second example.

Example 3.2. The speed reducer (Fig. 2) optimization problem is taken from [9]. The objective of this problem is to minimize the volume of a speed reducer (Fig. 7) subjected to stress, deflection, and geometric constraints. Hence, the problem is stated as follows:

$$\begin{aligned} Min \quad f(x) &= 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \\ &\quad -1.5079x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2), \\ S.t.\ g_1 &: \frac{27.0}{x_1x_2^2x_3} - 1 \leqslant 0, \\ g_2 &: \frac{397.5}{x_1x_2^2x_3^2} - 1 \leqslant 0, \end{aligned}$$

$$
\begin{aligned}
& g_3 : \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leqslant 0, \\
& g_4 : \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leqslant 0, \\
& g_5 : \frac{A_1}{B_1} - 1100 \leqslant 0, \\
& g_6 : \frac{A_2}{B_2} - 1100 - 850 \leqslant 0, \\
& g_7 : x_2x_3 - 40.0 \leqslant 0, \\
& g_8 : 5.0 \leqslant \frac{x_1}{x_2}, \\
& g_9 : \frac{x_1}{x_2} \leqslant 12.0, \\
& g_{10} : \frac{(1.5x_6 + 1.9)}{x_4} - 1 \leqslant 0, \\
& g_{11} : \frac{(1.5x_7 + 1.9)}{x_5} - 1 \leqslant 0,
\end{aligned}
$$

$$
\begin{aligned}
& A_1 = \left[(\frac{745.0x_4}{x_2x_3}) + 16.9 \times 10^6 \right]^{0.5}, \; B_1 = 0.1x_6^3, \\
& A_2 = \left[(\frac{745.0x_5}{x_2x_3}) + 16.9 \times 10^6 \right]^{0.5}, \; B_2 = 0.1x_7^3, \\
& 2.6 \leqslant x_1 \leqslant 3.6, \; 0.7 \leqslant x_2 \leqslant 0.8, \; 17 \leqslant x_3 \leqslant 28 \\
& 7.3 \leqslant x_4 \leqslant 8.3, \; 7.3 \leqslant x_5 \leqslant 8.3, \; 2.9 \leqslant x_6 \leqslant 3.9, \; 5.0 \leqslant x_7 \leqslant 5.5.
\end{aligned}
$$

According to [14, 17], the problem above is decomposed into three subsystems (disciplines). Sub-system 1 concerns designing the gear, while sub-systems 2 and 3 are associated with the design of shafts 1 and 2:

Subsystem 1:

$$
\begin{aligned}
& Min \;\; (x_1 - \tilde{x_1})^2 + (x_2 - \tilde{x_2})^2 + (x_3 - \tilde{x_3})^2 + (y_1 - \tilde{y_1})^2, \\
& S.t. \;\; g_1 : \frac{27.0}{x_1x_2^2x_3} - 1 \leqslant 0, \\
& \qquad g_2 : \frac{397.5}{x_1x_2^2x_3^2} - 1 \leqslant 0, \\
& \qquad g_7 : x_2x_3 - 40.0 \leqslant 0, \\
& \qquad g_8 : 5.0 \leqslant \frac{x_1}{x_2},
\end{aligned}
$$

$$g_9 : \frac{x_1}{x_2} \leqslant 12.0,$$
$$2.6 \leqslant x_1 \leqslant 3.6,\ 0.7 \leqslant x_2 \leqslant 0.8,\ 17 \leqslant x_3 \leqslant 28,$$
$$y_1 = x_2 x_3.$$

Subsystem 2:

$$Min\ \ (x_1-\tilde{x_1})^2+(x_2-\tilde{x_2})^2+(x_3-\tilde{x_3})^2+(x_4-\tilde{x_4})^2+(x_6-\tilde{x_6})^2+(y_2-\tilde{y_2})^2$$
$$1.5079x_1(x_6^2 + x_7^2) + 7.477x_1(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2),$$

$$S.t.\ \ g_3 : \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leqslant 0,$$
$$g_5 : \frac{A_1}{B_1} - 1100 \leqslant 0,$$
$$g_{10} : \frac{(1.5x_6 + 1.9)}{x_4} - 1 \leqslant 0,$$
$$A_1 = \left[(\frac{745.0x_4}{x_2x_3}) + 16.9 \times 10^6\right]^{0.5},\ B_1 = 0.1x_6^3,$$
$$2.6 \leqslant x_1 \leqslant 3.6,\ 0.7 \leqslant x_2 \leqslant 0.8,\ 17 \leqslant x_3 \leqslant 28,$$
$$7.3 \leqslant x_4 \leqslant 8.3,\ 7.3 \leqslant x_5 \leqslant 8.3,\ 2.9 \leqslant x_6 \leqslant 3.9,$$
$$y_2 = -1.5079x_1x_6^2 + 7.477x_6^3 + 0.7854x_4x_6^2.$$

Subsystem 3:

$$Min\ \ (x_1-\tilde{x_1})^2+(x_2-\tilde{x_2})^2+(x_3-\tilde{x_3})^2+(x_5-\tilde{x_5})^2+(x_7-\tilde{x_7})^2+(y_3-\tilde{y_3})^2,$$

$$S.t.\ \ g_4 : \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leqslant 0,$$
$$g_6 : \frac{A_2}{B_2} - 1100 - 850 \leqslant 0,$$
$$g_{11} : \frac{(1.5x_7 + 1.9)}{x_5} - 1 \leqslant 0,$$
$$A_2 = \left[(\frac{745.0x_5}{x_2x_3}) + 16.9 \times 10^6\right]^{0.5},\ B_2 = 0.1x_7^3,$$
$$2.6 \leqslant x_1 \leqslant 3.6,\ 0.7 \leqslant x_2 \leqslant 0.8,\ 17 \leqslant x_3 \leqslant 28,$$
$$7.3 \leqslant x_4 \leqslant 8.3,\ 7.3 \leqslant x_5 \leqslant 8.3,\ 5.0 \leqslant x_7 \leqslant 5.5,$$

$$y_3 = -1.5079x_1x_7^2 + 7.477x_7^3 + 0.7854x_5x_7^2.$$

The optimization problem is solved by the TLBMCO algorithm. The optimal solution is obtained as [3.5. 0.7, 17, 7.3, 7.7153, 3.3512, 5.2866] with the objective value f = 2994.3451. For comparing this solution with other methods, from [18, 19], Table. 2 is presented.

Example 3.3. As the last example, the Electrical Package problem is solved by TLBMCO, which is a bench-mark multidisciplinary problem with strong coupling between electrical and thermal subsystems. Operating temperatures in thermal subsystem affect component resistances on electrical subsystem , while temperatures depend on resistances. The sub-systems are demonstrated in Figure 3. This problem is one of the ten test cases used by NASA. The Electrical Package problem optimization problem has 8 design variables that are the following:
x_1: heat sink width(m)

x_2: Heat sink length (m)

x_3: Fin length (m)

x_4: Fin width (m)

x_5: Resistance 1 at temperature $T^\circ(\Omega)$

x_6: Temperature coefficient of electrical resistance 1 ($^\circ K^{-1}$)

x_7: Resistance 2 at temperature 2 ($^\circ K - 1$)

x_8: Temperature coefficient of electrical resistance 2 ($^\circ K^{-1}$)

The thermal and electrical state variables (linking variables) are:

y_1: Negative of watt density (watts/m3)

y_2: Resistance 1 at temperature T_1° (Ω)
y_3: Resistance 1 at temperature T_2° (Ω)

y_4: Current in resistor 1 (amps)

y_5: Current in resistor 2 (amps)

y_6: Power dissipation in resistor 1 (watts)

y_7: Power dissipation in resistor 2 (watts)

y_8: Total circuit current (amps)
y_9: Total circuit resistance (Ω)

y_{10}: Total current power (watts)

y_{11}: Component temperature T_1 of resistor 1 ($^\circ C$)

y_{12}: Component temperature T_2 of resistor 2 ($^\circ C$)

y_{13}: Heat sink volume (m^3)

The Electrical Package problem optimization problem is modeled as follows:

$Min \;\; f = y_1,$

$$\begin{aligned} S.t.\;\; & g_1 = y_{11} - 85 \leqslant 0, \\ & g_2 = y_{12} - 85 \leqslant 0, \\ & h_1 = y_4 - y_5 = 0 \leqslant 0, \\ & 0.05 \leqslant x_1 \leqslant 0.15,\; 0.05 \leqslant x_2 \leqslant 0.15,\; 0.01 \leqslant x_3 \leqslant 0.1, 0.005 \leqslant x_4 \leqslant 0.05, \\ & 10 \leqslant x_5 \leqslant 1000,\; 0.004 \leqslant x_6 \leqslant 0.009,\; 10 \leqslant x_7 \leqslant 1000,\; 0.004 \leqslant x_8 \leqslant 0.009, \end{aligned}$$

where objective function is negative watt density. Also, coupling between two disciplines, mainly Thermal and Electrical, are formulate by equations as follow:

$$y_4 = \tfrac{y_3 y_8}{y_2 + y_3},$$
$$y_5 = \tfrac{y_2 y_8}{y_2 + y_3},$$
$$y_6 = (y_4)^2 y_2,$$
$$y_7 = (y_5)^2 y_3,$$
$$y_8 = \frac{voltage}{y_9},$$
$$y_9 = \frac{1}{\frac{1}{y_2} + \frac{1}{y_3}},$$
$$y_{10} = (y_8)^2 y_9,$$
$$y_{11} = f_{11}(y_6, y_7, x_1, x_2, x_3, x_4),$$
$$y_{12} = f_{12}(y_6, y_7, x_1, x_2, x_3, x_4),$$
$$y_{13} = x_1 x_2 x_3,$$
$$y_1 = -\tfrac{y^{10}}{y_{13}},$$
$$y_2 = x_5[1 + x_6(y_{11} - T^0)],$$
$$y_3 = x_5[1 + x_8(y_{12} - T^0),$$

Table 2: Comparison of different MDO method for Example 3.2

Method	MDF	IDF	CSSO	BLISS	CO	TLBMCO
f	2994.3632	2994.5276	2995.2934	2995.6027	2998.4935	2994.3547
1	3	5	11	3	9	9
2	0	2	0	0	2	0
3	252	0	208	247	0	0
4	0	106	218	197	5460	405
4	0	106	170	187	7528	413
5	0	106	190	217	6550	371
6	756	318	1202	1342	19538	1189
re	2.7×10^{-6}	5.8×10^{-5}	3.1×10^{-4}	4.2×10^{-4}	1.4×10^{-3}	1.1×10^{-7}

(1): Number of equality constraints
(2): Function calls of system analysis
(3): Function calls of discipline 1
(4): Function calls of discipline 2
(5): Function calls of discipline 3
(6): Total function calls
(7): Relative error

where voltage and T_0 are constant parameters, which are set as follows: voltage $= 10V$ and $T_0 = 10°C$, and f_{11} and f_{12} are implicitly functions.

Table 3: Results of solving Example 3.3 by TLBMCO

Optimal value	-655462.654
(1)	4328
(2)	8543
(3)	12871

(1): Function calls of discipline E
(2): Function calls of discipline T
(3): Total function calls

Table.3 demonstrates the obtained result of solving Example 3 by the TLBMCO algorithm.

According to Table.1 and Table. 2, IDF has the least function calls, and that of CO is the most. The last column of both tables show the error for the various methods. To compare convergence histories, the relative error of the objective function value was used. Least error is given by the TLBMCO method. Also, Table. 3 confirms the ability of the TLBMCO method in solving the Electrical Package problem.

4. Conclusion

The proposed algorithm, TLBMCO, which is a hybrid of the TLBO algorithm and the MCO framework, has two cycles. Its performance is checked by experimenting two benchmark problems with different characteristics. However, the number of the function calls of the TLBMCO is more than the number of that of IDF. But, the error of the TLBMCO is the least. Therefore, the results show the better performance of TLBMCO over other MDO frameworks, namely, MDF, IDF, CSSO, BLISS and CO. In addition, TLBMCO shows a satisfactory performance with less error.

References

[1] R. D. Braun, *Collaborative Optimization: an Architecture for Large-scale Distributed Design*, Ph.D. thesis, Stanford University, 1996.

[2] R. D. Braun, A. A. Moore, and I. M. Kroo, Collaborative approach to launch vehicle design, *J. Spacecr Rockets*, 34 (1997), 478-486.

[3] A. Chiralaksanakul and S. Mahadevan, Decoupled approach to multidisciplinary design optimization under uncertainty, *Optimization and Engineering*, 8 (2007), 21-42.

[4] E. J. Cramer, J. E. Dennis, P. D. Frank, R. M. Lewis, and G. R. Shubin, Problem formulation for multidisciplinary optimization, *SIAM J. Optim*, 4 (4) (1994), 754-776.

[5] A. V. DeMiguel and W. Murray, *An Analysis of collaborative optimization methods*, 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, CA, AIAA, (1994), 2000-4720.

[6] N. Dib, A. Sharaqa, Synthesis of thinned concentric circular antenna arrays using teaching-learning-based optimization,*International Journal of RF and Microwave Computer-Aided Engineering*, 24 (4) (2014), 443-450.

[7] R. T. Haftka, Simultaneous analysis and design, *AIAA J.*, 23 (7) (1985), 1099-1103.

[8] H. S. Lee, *Sequential Approximate Individual Discipline Feasible Method Using Enhanced Two-point Diagonal Quadratic Approximation Method*, Master Thesis, Hanyang University (in Korean), 2004.

[9] S. L. Padula, N. Alexandrov, and L. L. Green, *MDO Test Suite at NASA Langley Research Center*, In: Proceedings of the 6th AIAA/NASA/ISSMO symposium on multidisciplinary analysis and optimization, Bellevue, WA, AIAA, 1996-4028.

[10] S. J. Patel, A. K. Panchal, and V. Kheraj, Extraction of solar cell parameters from a single currentvoltage characteristic using teaching learning based optimization algorithm, *Applied Energy*, 119 (C) (2014), 384-393.

[11] R. V. Rao, V. J. Savsani, and D. P. Vakharia, Teachinglearning-based optimization: A novel method for constrained mechanical design optimization problems, *Computer-Aided Design*, 43 (2011), 303-315.

[12] J. Sobieszczanski-Sobieski, T. D. Altus, M. Phillips, and R. Sandusky, Bilevel integrated system synthesis for concurrent and distributed processing, *AIAA. J.*, 41 (10) (2003), 1996-2003.

[13] J. Sobieszczanski-Sobieski, J. Agte, and R. Sandusky, *Bi-level Integrated System Synthesis*, Proceedings of AIAA/USAF/ NASA/ISSMO symposium on multidisciplinary analysis and optimization AIAA, 1998, Paper AIAA-98-4916.

[14] S. Tosserams, L. F. P. Etman, and J. E. Rooda, An augmented Lagrangian decomposition method for quasi-separable problems in mdo., *Structural and Multidisciplinary Optimization*, 34 (2007), 211-227.

[15] F. Xiong, X. Yin, W. Chen, and S. Yang, *Enhanced Probabilistic Analytical Target Cascading with Application to Multiscale Design*, In: 8th

World congress on structural and multidisciplinary optimization, Lisbon, Portugal, 2009.

[16] W. Yao, X. Chen, Q. Ouyang, and M. van Tooren, A surrogate based multistage-multilevel optimization procedure formultidisciplinary design optimization, *Structural and Multidisciplinary Optimization*, 45 (2012), 559-574.

[17] W. Yao, X. Chen, Q. Ouyang, and M. van Tooren, A reliability-based multidisciplinary design optimization procedure based on combined probability, *Structural and Multidisciplinary Optimization*, 48 (2) (2013), 339-354.

[18] S. I. Yi, J. K. Shin, and G. J. Park, Comparison of MDO methods with mathematical examples, *Structural and Multidisciplinary Optimization*, 35 (2008), 391-402.

[19] M. Zhao and W. Cui, On the development of Bi-Level Integrated System Collaborative Optimization, *Structural and Multidisciplinary Optimization*, 73-84.

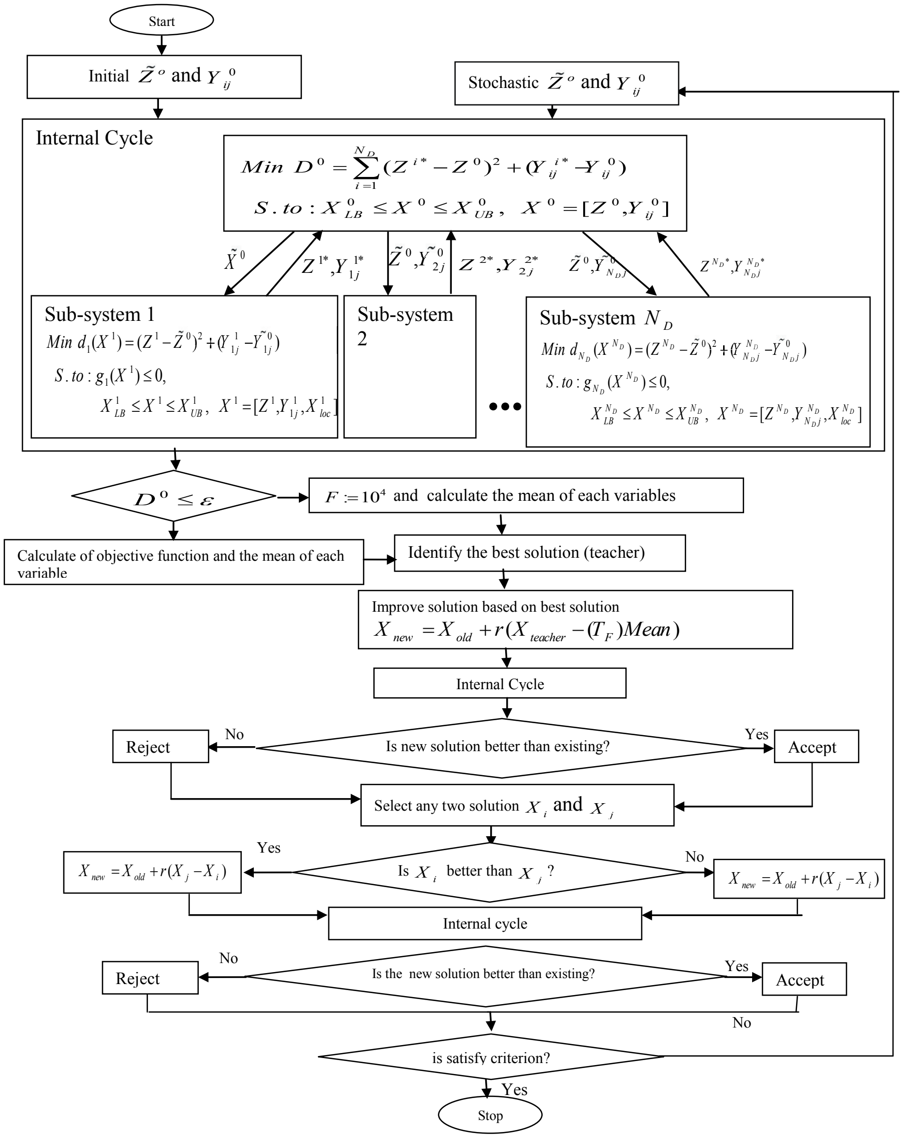

Figure 1. The TLBMCO algorithm for solving MDA proble

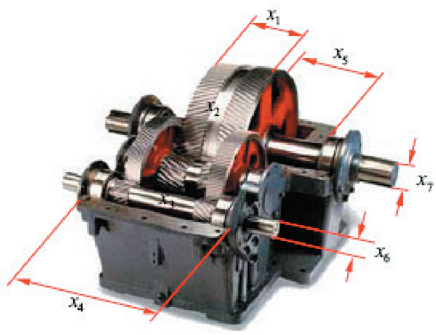

Figure 2. speed reducer

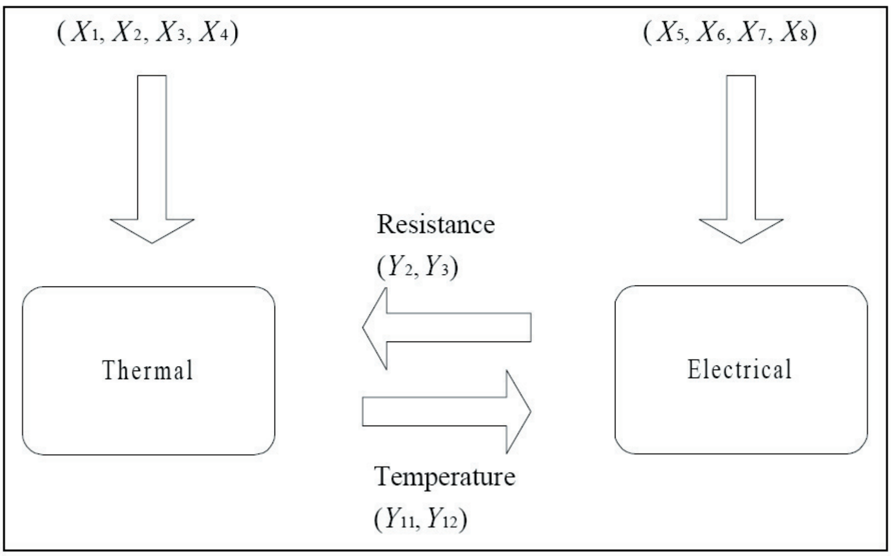

Figure 3. Interdisciplinary Interactions of Electronic Packaging

Alireza Fakharzadeh
Department of Mathematics
Professor of Mathematics
Shiraz University of Technology
Shiraz, Iran
E-mail: a_fakharzadeh@sutech.ac.ir

Somayeh Khosravi
Department of Mathematics
Ph.D of Mathematics
Shiraz University of Technology
Shiraz, Iran
E-mail: s.khosravi@sutech.ac.ir