Journal of Mathematical Extension

Teaching-learning- based modified collaborative optimization algorithm

Alireza Fakharzadeh. J*
Shiraz University of Technology Department of Mathematics

Somayeh Khosravi
Shiraz University of Technology Department of Mathematics

Abstract. In complex engineering systems, collaboration between different design disciplines and optimization of whole system concurrently are caused an important issue in optimization. In recent decades the multi-disciplinary design optimization approaches and meta-heuristic algorithms have been used in this field. In this regard, a hybrid modified collaborative optimization framework and teaching-learning -based-optimization algorithm, for solving these kind of problems is presented here. We show efficiency of the introduced algorithm by solving test cases and comparison to former results.

AMS Subject Classification: MSC code1; MSC code 2; more
Keywords and Phrases: Teaching-learning- based optimization, collaborative optimization, multi-disciplinary design optimization.

1 Introduction

One of the very important challenge of optimization is addressing the optimization problems of complex systems involving multiple coupled disciplines, that called multidisciplinary analysis (MDA) problem, such as the most real world engineering optimization problems in aerospace, automotive, architecture and etc.[2,17], solving these kind of problems

Received: XXXX
*Corresponding Author

without considering interaction between different disciplines is impossible. In the past, fixed point iteration process (FPI) were used for solving these kind of problems, which is just a method to converge the MDA problem and does not perform overall system optimization or the configuring of the whole system, such as to reach an optimum according to one global design objective.

MDA problems were solved by introducing the multidisciplinary design optimization (MDO); indeed, MDO is capable to improve system design by exploiting synergies among different disciplines[17]. Nevertheless, MDO is faced with two big challenges, computational and organizational complexities. MDO methods are classified in two general classes: single level and multi level methods. In the first class, the optimization is only directed in the system level, and the disciplinary models are employed for analysis, such as multi-discipline-feasible (MDF) [3,4], individual -discipline-feasible (IDF) [8,4], all-at-once (AAO) [3] and simultaneous analysis and design (SAND) [7]. In multi level procedure, both the system and subsystem conduct optimization; typical multi level procedure includes collaborative optimization (CO) [1,2], bi-level integrated system synthesis (BLISS) [10], concurrent subspace optimization (CSSO) [13,16] and analytical target cascading (ATC) [15].

In the other side teaching-learning based optimization (TLBO) is one of the new meta-heuristic algorithm based on the natural phenomenon of teaching-learning process; TLBO was proposed in 2011 by Rao et.al [11], to solve continuous non linear programming (NLP) large scale optimization. This algorithm does not require the algorithm-specific parameters and their own algorithm-specific control parameters. In addition, from the literature survey on evolutionary algorithm, it is observed that step of duplicate solutions removal is not explicitly found in the application of different evolutionary algorithms, whereas, in literature of TLBO it is presented that the step of duplicate solutions removal.

In this paper, we introduce a new algorithm to solve MDO problems, which is a hybrid of TLBO algorithm and collaborative optimization framework that is called Teaching-learning- based modified collaborative optimization (TLBMCO) algorithm. The paper is organized as follows. First, the TLBMCO is described and it is explained that how it is combined by modified collaborative optimization (MCO); then, two famous test problem of MDO are solved by TLBMCO algorithm and the obtained results are comprised with results of other methods. Finally, conclusion will be drawn.

2 Teaching-learning- based modified collaborative optimization

The TLBMCO algorithm has two cycles: internal cycle and external cycle. The internal cycle, which is a bi-level MDO framework, decomposes original problem to system level and sub-system level; then these obtained problems are optimize problem and explored and exploited the synergistic of coupling between various interacting disciplines in each iteration. We describe internal cycle in sub-section 2.1. But external cycle optimize original problem by TLBO.

2.1 Internal cycle of the TLBMCO

The internal cycle is based on CO, which was proposed by Kroo et al. (1994) [2]. We use a modified Co (MCO), which is a bi-level framework developed by A. V. Demiguel and W. Murray [5]. First internal cycle receives stochastic value of variables (design and coupling variables); then it transfers these values into feasible values. Since, MCO has both level system and subsystem, in the internal cycle two kind of problems are solved, namely level system problem and subsystem problems. Level system problem is formulated as follows:
System level:

$$
\begin{aligned}
&Min \sum_{i=1}^{N_D} (Z^{i*} - Z^0) + (Y_{ij}^{i*} - Y_{ij}^0) \\
&S.to : X_{LB}^0 \leq X^0 \leq X_{UB}^0, X^0 = \left[Z^0, Y_{ij}^0 \right]
\end{aligned}
$$

i-th subsystem:

$$\begin{aligned} &Min\ d_i(X^i) = (Z^i - \widehat{Z}^0) + (Y^i_{ij} - \widehat{Y^0_{ij}}) \\ &S.to : g_i(X^i) \leq 0 \\ &\qquad X^i_{LB} \leq X^i \leq X^i_{UB}, X^i = \left[Z^i, Y^i_{ij}, X^i_{loc}\right] \end{aligned}$$

where Z is vector of shared variable, Y_{ij} is coupling variable vector (i-th subsystem sends value to j-th subsystem), X^i_{loc} is vector of i-th subsystem local variables, and X^0_{LB}, X^0_{UB}, X^i_{LB} and X^i_{UB} are vectors of variable's lower and upper bounds for respectively system level and i-th subsystem.

The internal cycle starts when the system level receives new value of variable (Z^i and Y_{ij}); in the first step, these values are forwarded to subsystem level. Then, by these values subsystem problems are optimized and in each subsystem optimal solution (e.g. $X^{i*} = \left[Z^{i*}, Y^{i*}_{ij}, X^{i*}_{loc}\right]$) is produced. Then, this obtained optimal solution is sent to the system level. Now, by the received values, optimization system level is done. The next iteration of this cycle, starts by sending achieved optimal solution system level to subsystem level. The inter cycle is terminated when the changes in the objective function in the system level are smaller than a predefined value.

2.2 External cycle

During internal cycle we are attempted to attain compatible solutions, which have an equal values with regard to the same variable at both subsystem and level system. But in some iteration of internal cycle, the objective function of system level cannot be minimized to zero or near zero. Therefore, one main goal of the external cycle is to counteract these incompatible solutions in the next generation. In order to achieve this aim, a penalty value for objective function is assigned. We use the TLBO algorithm [10,6] in the external cycle. TLBO is an algorithm-specific parameter-less. The implementation of TLBO does not require the determination of any controlling parameters which makes the algorithm robust and powerful. The process TLBMCO algorithm for MDO

problems, which is showed in Fig.1, is described in continue.
Step 1: In the beginning, an initial population (students) is created by selecting the values of the coupling and shared variables at stochastic; then these values are sent to the internal cycle.
Step 2: In the internal cycle, system level receives these values. Then, these values are sent to the subsystems. In next step, sub-systems are optimized and the obtained optimal solutions are forwarded to the system level. Now, system level is optimized and its optimal solution is delivered to subsystems. This process is continued until objective function value of system level is less than a predefined small positive number (ε).
Step 3: In the internal cycle, the goal in both system level and subsystems are minimizing difference between values of variables Z^0, Y_{ij}^0 and Z^i, Y_{ij}^i . However, the obtained solutions of some iterations are incompatible; to exclude these kind of solutions from the next generation, the corresponding design objective function is assigned with a large penalty value.
Step 4: Objective function in evaluating of external cycle, is designing objective. After checking incompatible solutions, teacher phase starts by evaluating values of objective function for compatible solutions. Then the mean of each variable is calculated. In the next step, best solution is identified as teacher. Teacher will try to move students toward its own level; therefore, solutions (students) are modified based on the best solution (teacher). Then, the modified solutions are sent to internal cycle. In the next step, each obtained solution of internal cycle is improved by mutual interaction with other obtained solutions (phase learner, see [11]). Again improved solutions are delivered to internal cycle. Finally the termination criterion are checked.

3 Numerical Results

To show the efficiency of the presented algorithm for problems for MDO problems, the TLBMCO algorithm was coded in Matlab and run on two benchmark problems available in the literature on a system based on a Intel(R) Core (TM) i5-2430M processor with 8GB RAM.

Example 3.1. Consider an analytical example of an MDO problem involving two disciplines. The coupling between two disciplines is described by equations as follows:

$$y_{12} = z_1^2 + x_1 + z_2 - 0.2y_{21}$$
$$y_{21} = \sqrt{y_{12}} + z_1 + z_2$$

The related optimization problem by following formulation is described

$$
\begin{aligned}
Min\ :& x_1^2 + z_2 + y_{12} + \exp(-y_{21}) \\
S.to\ :& 1 - \frac{y_{12}}{8} \leq 1 \\
& \frac{y_{21}}{10} - 1 \leq 1 \\
& -10 \leq z_1 \leq 10, 0 \leq x_1 \leq 10, 0 \leq z_2 \leq 10
\end{aligned}
$$

where y_{12} and y_{21} are coupling variables, z_1 and z_2 are shared variables and x_1 is local variable. For $\varepsilon = 10^{-8}$,all the optimizations from different initial populations stably converge to the optimum point [3.026800] with the objective $f = 8.0027$. Table 1 presents the obtained results and the results of other methods from [18,19]. These results show the efficiency and benefits of the new algorithm in comparison with the others method.

Example 3.2. Example 2: The speed reducer (Fig. 2) optimization problem is taken in the reference [9]. The objective of this problem is to minimize the volume of a speed reducer (Fig. 2) subjected to stress, deflection, and geometric constraints; hence, the problem is stated as

Table 1: Comparison of different MDO method for example 3.1

Method	MDF	IDF	CSSO	BLISS	CO	TLBMCO
f	8.02537	8.00347	8.19881	8.1278	21.99930	8.002712
Number of design variables	3	5	11	3	9	9
Number of equality constraints	0	2	0	0	2	0
Function calls of system analysis	231	0	208	178	0	0
Function calls of discipline	0	48	44	30	10137	215
Function calls of discipline	0	48	226	27	8522	198
relative errore	0.00281274	7.62228×10^{-5}	0.02448501	0.0156182	1.748930	1.84934×10^{-5}

follow:

$$
\begin{aligned}
Min f(x) &= 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \\
&\quad - 1.5079x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \\
S.to : & g_1 : \frac{27.0}{x_1x_2^2x_3} - 1 \leq 0 \\
& g_2 : \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0 \\
& g_3 : \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0 \\
& g_4 : \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0 \\
& g_5 : \frac{A_1}{B_1} - 1100 \leq 0 \\
& g_6 : \frac{A_2}{B_2} - 1100 - 850 \leq 0 \\
& g_7 : x_2x_3 - 40.0 \leq 0 \\
& g_8 : 5.0 \leq \frac{x_1}{x_2} \\
& g_9 : \frac{x_1}{x_2} \leq 12.0 \\
& g_{10} : \frac{(1.5x_6 + 1.9)}{x_4} - 1 \leq 0 \\
& g_{11} : \frac{(1.5x_7 + 1.9)}{x_5} - 1 \leq 0 \\
& A_1 = \left[(\frac{745.0x_4}{x_2x_3}) + 16.9 \times 10^6 \right]^{0.5}, \ B_1 = 0.1x_6^3 \\
& A_2 = \left[(\frac{745.0x_5}{x_2x_3}) + 16.9 \times 10^6 \right]^{0.5}, \ B_2 = 0.1x_7^3 \\
& 2.6 \leq x_1 \leq 3.6, \ 0.7 \leq x_2 \leq 0.8, \ 17 \leq x_3 \leq 28 \\
& 7.3 \leq x_4 \leq 8.3, \ 7.3 \leq x_5 \leq 8.3, \ 2.9 \leq x_6 \leq 3.9, \ 5.0 \leq x_7 \leq 5.5
\end{aligned}
$$

According to [14,17], the above problem is decomposed into three sub-system (discipline). Sub-system 1 is concerned with designing the gear, while sub-systems 2 and 3 is associated with the design of shafts 1 and 2:

Subsystem1:

$$
\begin{aligned}
&Min(x_1-\tilde{x_1})^2+(x_2-\tilde{x_2})^2+(x_3-\tilde{x_3})^2+(y_1-\tilde{y_1})^2\\
S.to: &g_1: \frac{27.0}{x_1x_2^2x_3}-1\leq 0\\
&g_2: \frac{397.5}{x_1x_2^2x_3^2}-1\leq 0\\
&g_7: x_2x_3-40.0\leq 0\\
&g_8: 5.0\leq \frac{x_1}{x_2}\\
&g_9: \frac{x_1}{x_2}\leq 12.0\\
&2.6\leq x_1\leq 3.6,\ 0.7\leq x_2\leq 0.8,\ 17\leq x_3\leq 28\\
&y_1=x_2x_3
\end{aligned}
$$

Subsystem2:

$$
\begin{aligned}
&Min(x_1-\tilde{x_1})^2+(x_2-\tilde{x_2})^2+(x_3-\tilde{x_3})^2+(x_4-\tilde{x_4})^2+(x_6-\tilde{x_6})^2+(y_2-\tilde{y_2})^2\\
&1.5079x_1(x_6^2+x_7^2)+7.477x_1(x_6^3+x_7^3)+0.7854(x_4x_6^2+x_5x_7^2)\\
S.to: &g_3: \frac{1.93x_4^3}{x_2x_3x_6^4}-1\leq 0\\
&g_5: \frac{A_1}{B_1}-1100\leq 0\\
&g_{10}: \frac{(1.5x_6+1.9)}{x_4}-1\leq 0\\
&A_1=\left[(\frac{745.0x_4}{x_2x_3})+16.9\times 10^6\right]^{0.5},\ B_1=0.1x_6^3\\
&2.6\leq x_1\leq 3.6,\ 0.7\leq x_2\leq 0.8,\ 17\leq x_3\leq 28\\
&7.3\leq x_4\leq 8.3,\ 7.3\leq x_5\leq 8.3,\ 2.9\leq x_6\leq 3.9\\
&y_2=-1.5079x_1x_6^2+7.477x_6^3+0.7854x_4x_6^2
\end{aligned}
$$

Subsystem3:

$$Min(x_1-\tilde{x_1})^2+(x_2-\tilde{x_2})^2+(x_3-\tilde{x_3})^2+(x_5-\tilde{x_5})^2+(x_7-\tilde{x_7})^2+(y_3-\tilde{y_3})^2$$

$$S.to: g_4: \frac{1.93x_5^3}{x_2x_3x_7^4}-1\leq 0$$

$$g_6: \frac{A_2}{B_2}-1100-850\leq 0$$

$$g_{11}: \frac{(1.5x_7+1.9)}{x_5}-1\leq 0$$

$$A_2=\left[(\frac{745.0x_5}{x_2x_3})+16.9\times 10^6\right]^{0.5},\ B_2=0.1x_7^3$$

$$2.6\leq x_1\leq 3.6,\ 0.7\leq x_2\leq 0.8,\ 17\leq x_3\leq 28$$

$$7.3\leq x_4\leq 8.3,\ 7.3\leq x_5\leq 8.3,\ 5.0\leq x_7\leq 5.5$$

$$y_3=-1.5079x_1x_7^2+7.477x_7^3+0.7854x_5x_7^2$$

The optimization problem is solved by TLBMCO algorithm. The optimal solution is obtained as [3.5. 0.7, 17, 7.3, 7.7153, 3.3512, 5.2866] with the objective value f = 2994.3451. For comparing this solution with other methods, from [18,19], Table.2 is presented.
According to Table. 1. and Table. 2. IDF has the least of the number of the function calls, and that of CO is the most. The last of both tables show the error for the various methods. To compare convergence histories, we used the relative error of the objective function value, least error is given by the TLBMCO method.

4 Conclusion

A proposed algorithm, TLBMCO, which is a hybrid of TLBO algorithm and MCO framework, has two cycles. Its performance is checked by experimenting with two benchmark problems with different characteristics. However, the number of the function calls of the TLBMCO is more than IDF. But, the error of the TLBMCO is least. There for, the results show the better performance of TLBMCO over other MDO frameworks, namely MDF, IDF, CSSO, BLISS and CO. Also, TLBMCO shows a satisfied performance with less error.

Table 2: Comparison of different MDO method for example 3.2

Method	f	Number of equality constraints	Function calls of system analysis	Function calls of discipline 1	Function calls of discipline 2	Function calls of discipline 3	relative error
MDF	8.02537	0	0	0	756		2.72980×10^{-6}
IDF	8.00347	2	106	106	318		0.0000576331
CSSO	8.19881	0	170	190	1202		0.000313381
BLISS	8.12785	0	187	217	1342		0.000416675
CO	21.99930	2	7528	6550	19538		0.00138209
TLBMCO	8.002712	0	413	371	1189		1.08871×10^{-7}

References

[1] R. D. Braun, 1996. "Collaborative optimization: an architecture for large-scale distributed design", Ph.D. thesis, Stanford University.

[2] R. D. Braun, A. A. Moore, I.M. Kroo, 1997. "Collaborative approach to launch vehicle design", *J Spacecr Rockets*, Vol. 34, pp 478486.

[3] A . Chiralaksanakul, S. Mahadevan, 2007."Decoupled approach to multidisciplinary design optimization under uncertainty", *Optimization and Engineering*, Vol 8, pp 2142.

[4] E.J. Cramer, J.E. Dennis, P.D. Frank, R.M. Lewis, G.R. Shubin, 1994. "Problem formulation for multidisciplinary optimization", *SIAM J Optim*, Vol.4, No.4, pp 754776.

[5] A.V. DeMiguel, W. Murray, "An Analysis of Collaborative Optimization Methods", 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, CA, AIAA-2000-4720.

[6] N. Dib, A. Sharaqa, 2014. " Synthesis of thinned concentric circular antenna arrays using teaching-learning-based optimiza-

tion",*International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 24, Issue 4, pp 443450.

[7] R.T. Haftka, "Simultaneous analysis and design",*AIAA J*, Vol. 23, No.7, pp 1099-1103, 1985.

[8] H.S. Lee, "Sequential approximate individual discipline feasible method using enhanced two-point diagonal quadratic approximation method", Master Thesis, Hanyang University (in Korean), 2004.

[9] S.L. Padula, N. Alexandrov, L.L. Green, 1996. "MDO test suite at NASA Langley research center", In: Proceedings of the 6th AIAA/NASA/ISSMO symposium on multidisciplinary analysis and optimization, Bellevue, WA, AIAA, pp 1996-4028.

[10] S.J. Patel, A.K. Panchal, V. Kheraj, 2014. " Extraction of solar cell parameters from a single currentvoltage characteristic using teaching learning based optimization algorithm", *Applied Energy*, Vol. 119, issue C, pp. 384-393.

[11] R.V. Rao, V.J. Savsani, D.P. Vakharia, 2011. " Teachinglearning-based optimization: A novel method for constrained mechanical design optimization problems", *Computer-Aided Design*, Vol. 43, pp 303-315.

[12] J. Sobieszczanski-Sobieski, T.D. Altus, M. Phillips, R. Sandusky, 2003. "Bilevel integrated system synthesis for concurrent and distributed processing",*AIAA J*, Vol. 41, No. 10, pp 19962003.

[13] J. Sobieszczanski-Sobieski, J. Agte, R. Sandusky, 1998. "Bilevel integrated system synthesis", Proceedings of AIAA/USAF/ NASA/ISSMO symposium on multidisciplinary analysis and optimization AIAA, Paper AIAA-98-4916.

[14] S. Tosserams, L. F. P. Etman, J. E. Rooda 2007. "An augmented Lagrangian decomposition method for quasi-separable problems in mdo". *Structural and Multidisciplinary Optimization*, Vol. 34, pp 211227.

[15] F . Xiong, X. Yin, W. Chen, S. Yang, 2009. "Enhanced probabilistic analytical target cascading with application to multiscale design", In: 8th World congress on structural and multidisciplinary optimization, Lisbon, Portugal.

[16] W. Yao, X. Chen, Q. Ouyang, M. van Tooren, 2012. "A surrogate based multistage-multilevel optimization procedure formultidisciplinary design optimization", *Structural and Multidisciplinary Optimization*, Vol. 45, pp 559574.

[17] W. Yao, X. Chen, Q. Ouyang, M. van Tooren, 2013. " A reliability-based multidisciplinary design optimization procedure based on combined probability", *Structural and Multidisciplinary Optimization*, Vol. 48, Issue 2, pp 339-354.

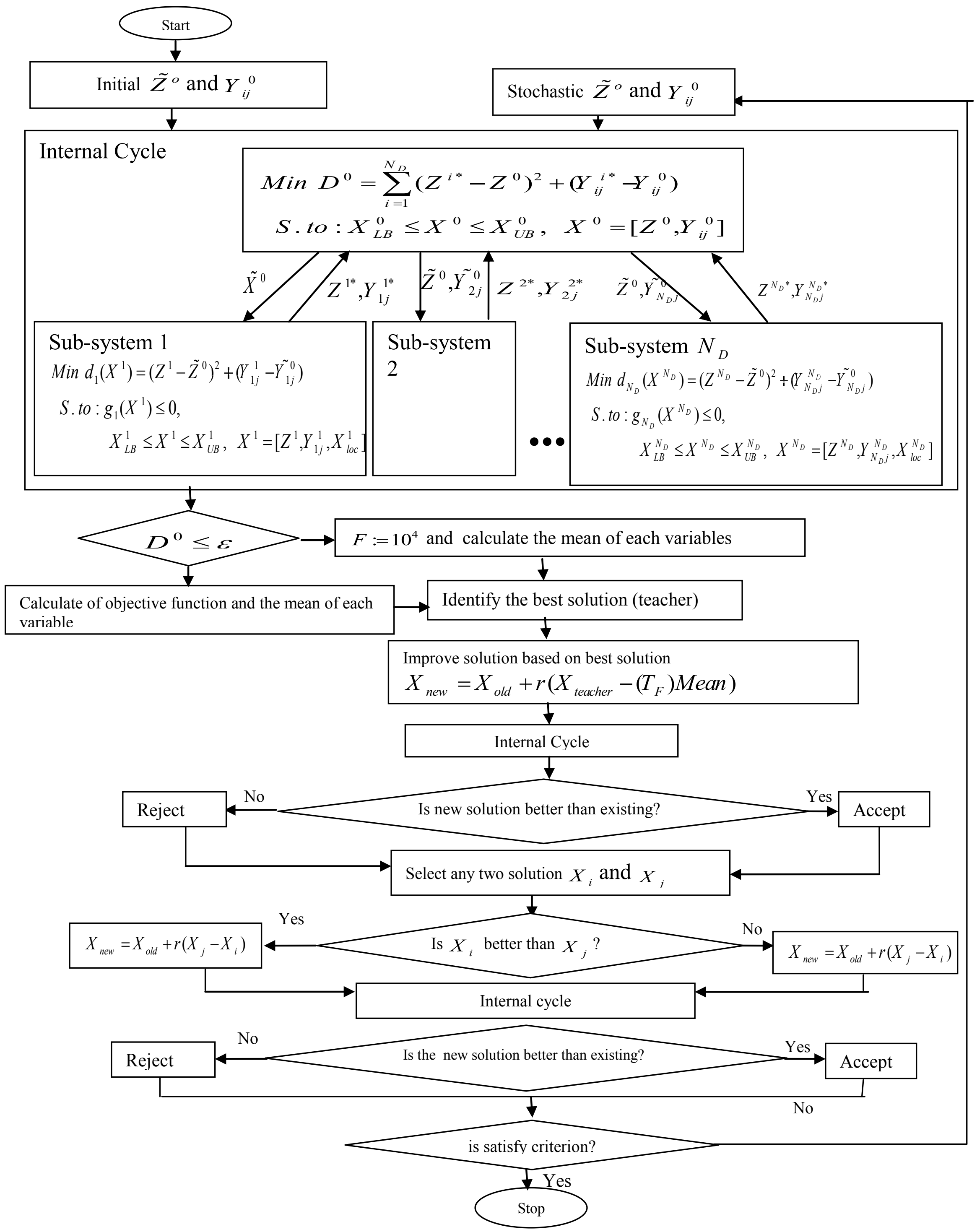
[18] S. I. Yi J. K. Shin and G. J. Park, 2008. "Comparison of MDO methods with mathematical examples",*Struct Multidisc Optim*, Vol. 35, pp 391402.

[19] M. Zhao, W. Cui, 2011. "On the development of Bi-Level Integrated System Collaborative Optimization", *Struct Multidisc Optim*, Vol. 43, pp7384.

Alireza Fakharzadeh. J
Department of Mathematics
Assistant Professor of Mathematics
Shiraz University of Technology
Shiraz, Iran
E-mail: a_fakharzadeh@sutech.ac.ir

Somayeh Khosravi
Department of Mathematics
PHD Student
Shiraz University of Technology
Shiraz, Iran
E-mail: s.khosravi@sutech.ac.ir

Figure 1: The TLBMCO algorithm for solving MDA proble



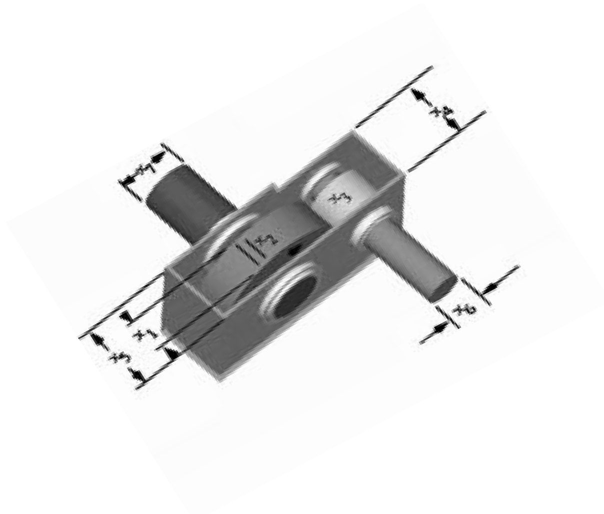


Figure 2: speed reducer