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An Efficient Method for Solving a Class of Nonlinear Fuzzy Optimization Problems

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Abstract. In this paper, a new method is presented for solving nonlinear fuzzy optimization problems (NFOP) where all coefficients of the problem are triangular fuzzy numbers. First, we convert NFOP problem to an interval nonlinear programming problem (INP) by α-cuts and in general case, we determine INP based on α. Then by solving INP model, the optimal solution of the main problem is obtained. To illustrate the proposed method numerical examples are solved and the obtained results are discussed.

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1. Introduction

In traditional optimization problems, the coefficients of the problems are evermore treated as deterministic values. However, uncertainty always exits in practical engineering problems. In order to deal with the

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uncertain programming, fuzzy and stochastic approaches are generally used to describe the imprecise characteristics. In stochastic programming (e.g. [3] (1959); [9] (1982); [14] (2003); [5] (2005)) the uncertain coefficients are regarded as random variables and their probability distributions are assumed to be known. In fuzzy programming (e.g. [22] (1986); [6] (1989); [15] (1989); [13] (2001)) the constraints and objective function are viewed as fuzzy sets and their membership functions need to be known. In these methods, the membership functions and probability distributions play important roles. However, it is sometimes difficult to specify an appropriate membership function or accurate probability distribution in an uncertain environment.
Newly, the interval analysis method was developed to model the uncertainty in uncertain optimization problems, in which the bounds of the uncertain coefficients are only required, not necessarily knowing the probability distributions or membership functions. Many researchers (Tanaka et al. (1984), Rommelfanger (1989), Chanas and Kuchta (1996a,b), Tong (1994), Liu and Da (1999), Sengupta et al. (2001), Zhang et al. (1999) and etc.) studied the linear interval number programming problems. Nevertheless, for most of the engineering problems, the objective function and constraints are nonlinear, and they are always obtained through numerical algorithms such as finite element method (FEM) instead of the explicit expressions. The reference (Ma, 2002), seems the first publication on nonlinear interval number programming (NINP). In this reference, a deterministic optimization method is used to obtain the interval of the nonlinear objective function. As a result, an effective method still have not been developed to deal with the general NINP problem in which there exit not only uncertain nonlinear objective function but also uncertain nonlinear constraints, so far.

Fuzzy set theory has been applied to many disciplines such as control theory and operation research, mathematical modeling and industrial applications. Tanaka, et al [25], first proposed the concept of fuzzy optimization on general level. Zimmerman [29] proposed the first formatting of fuzzy linear programming. An optimal solution of fuzzy nonlinear programming problems introduced by A. Kumar and J. Kaur [11] and B. Kheirfam [10]. In their works, they have taken all coefficients and decision variables to be fuzzy numbers and all the constraints to be linear. In [1, 18] authors have developed KKT conditions for solving fuzzy nonlinear programming problems with continuous and differentiable objective function and constraints.

In this paper, we focus on solving fuzzy nonlinear optimization problems. we take all coefficients of the objective function and constraints to be fuzzy numbers. We convert the NFOP into a crisp form with using the α-cuts. The crisp form will be interval nonlinear programming problem and this form of the problem will be free of the compulsion membership functions for solve. This paper is organized as follows: in Section 2., some basic definitions and arithmetic operations of triangular fuzzy numbers and intervals are reviewed. In Section 3., formulations of fuzzy nonlinear programming problems for solving INP problems are discussed and Interval nonlinear programming method is proposed. In Section 4., to demonstrate the effectiveness of the proposed method, some examples are solved. The conclusion appears in Section 5..

2. Preliminaries

Definition 2.1. *Let* $I = \{K : K = [a, b]\,,\ a, b \in R\}$ *and let* $A, B \in I$ *then the interval arithmetic operations are defined by*

$$A * B = \{\alpha * \beta :\ \alpha \in A,\ \beta \in B\},$$

where $* \in \{+, -, \times, /\}$. *(note that: / is undefined when* $0 \in B$*).*

Letting $A = [a, b]$ *and* $B = [c, d]$ it can be shown that it is equivalent to

$$\begin{aligned}
&A + B = [a, b] + [c, d] = [a + c, b + d]\,, \quad \text{(Minkowski addition)}\\
&A - B = [a, b] - [c, d] = [a - d, b - c]\,, \quad \text{(Minkowski difference)}\\
&A \cdot B = [a, b] \cdot [c, d] = [\min(ac, ad, bc, bd)\ , \max(ac, ad, bc, bd)\ ]\,,\\
&\frac{A}{B} = \frac{[a, b]}{[c, d]} = [a, b] \,.\left[\frac{1}{d}, \frac{1}{c}\right] \quad \text{if}\ \ 0 \notin [c, d]\,,\\
&kA = \{ka : a \in A\}. \quad \text{(Scalar multiplication)}
\end{aligned}$$

This means that each interval operation $* \in \{+, -, \times, /\}$ is reduced to real operations and comparisons.

Note. [24] If $k = -1$, scalar multiplication gives the opposite $-A = (-1)\,A = \{-a \,:\, a \in A\}$ but, in general, $A + (-A) \neq 0$, i.e. the opposite of A is not the inverse of A in Minkowski addition (unless $A = \{a\}$ is a singleton). Minkowski difference is $A - B = A + (-1)\,B = \{a - b :\, a \in A,\ b \in B\}$. A first implication of this fact is that, in general, even if it is true that $(A + C = B + C) \Leftrightarrow A = B$, addition/subtraction simplification is not valid, i.e. $(A + B) - B \neq A$.
To partially overcome this situation, the following H-difference was introduced:

Definition 2.2. [24] *Let $X = R^n,\ n \geqslant 1$, of real vectors equipped with standard addition and scalar multiplication operations. Following Diamond and Kloeden (see [7]), denote by $K(X)$ and $K_C\,(X)$ the spaces of nonempty compact and compact convex sets of X. Then, the H-difference of A and B is defined as:*

$$A \ominus B = C \Longleftrightarrow A = B + C \tag{1}$$

and an important property of $\ominus$ is that $A \ominus A = \{0\}\,,\ \forall\ A \in K(X)$ and $(A + B) \ominus B = A,\ \forall\ A, B \in K\,(X)$; H-difference is unique, but a necessary condition for $A \ominus B$ to exist is that A contains a translate $\{c\} + B$ of B. In general, $A - B \neq A \ominus B$.

Now, some definitions and notations of fuzzy set theory are reviewed.

Definition 2.3. [18] *Let R be the set of real numbers and $\tilde{a} : R \to [0, 1]$ be a fuzzy set. We say that $\tilde{a}$ is a fuzzy number if it satisfies the following properties:*

1. *$\tilde{a}$ is normal, that is, there exists $x_0 \in R$ such that $\tilde{a}\,(x_0) = 1$.*
2. *$\tilde{a}$ is fuzzy convex, that is,*

$$\tilde{a}\,(tx + (1 - t)\,y) \geqslant \min\{\tilde{a}\,(x)\,,\ \tilde{a}\,(y)\}\,; \quad \forall\ x,\ y\ \in R\ ,\ t \in [0, 1]$$

3. *$\tilde{a}$ is upper semi continuous on R, that is, $\{x|\, \tilde{a}(x) \geqslant \alpha\}$ is a closed subset of R for each $\alpha \in [0,1]$;*

4. *$cl\{x \in R|\tilde{a}\,(x) > 0\}$ forms a compact set.*

F(R) denotes the set of all fuzzy numbers on R. For all $\alpha \in (0,1]$, α-level set $\widetilde{a}_\alpha$ of any $\tilde{a} \in F(R)$ is defined as $\widetilde{a}_\alpha = \{x \in R \mid \widetilde{a}(x) \geqslant \alpha\}$. The 0-level set $\widetilde{a}_0$ is defined as the closure of the set $\{x \in R \mid \widetilde{a}\,(x) > 0\}$. By definition of fuzzy numbers, it was proved that, for any $\tilde{a} \in F\,(R)$ and for each $\alpha \in (0,1]$, $\widetilde{a}_\alpha$ is compact convex subset of R, and we write $\widetilde{a}_\alpha = \left[\tilde{a}^L_\alpha, \tilde{a}^U_\alpha\right]$.

Definition 2.4. [20] *According to Zadeh's extension principle, we have addition and scalar multiplications in fuzzy number space $F(R)$ by their α-cuts are as follows:*

$$\begin{aligned} (\tilde{a} \oplus \tilde{b})_\alpha &= \left[\tilde{a}^L_\alpha + \tilde{b}^L_\alpha, \tilde{a}^U_\alpha + \tilde{b}^U_\alpha\right] \\ (\mu \otimes \tilde{a})_\alpha &= \left[\mu\tilde{a}^L_\alpha,\ \mu\tilde{a}^U_\alpha\right] \end{aligned}$$

We define difference of two fuzzy numbers by their α-cuts by using H-difference as follows:

$$(\tilde{a} - \tilde{b})_\alpha = \widetilde{a}_\alpha \ominus \tilde{b}_\alpha,$$

where $\tilde{a}, \tilde{b} \in F\,(R)$, $\mu \in R$ and $\alpha \in [0,1]$.

Proposition 2.5. [18] *For $\tilde{a} \in F\,(R)$, we have*

1. *$\widetilde{a}^L_\alpha$ is bounded left continuous nondecreasing function on $(0,1]$;*

2. *$\widetilde{a}^U_\alpha$ is bounded left continuous nonincreasing function on $(0,1]$;*

3. *$\widetilde{a}^L_\alpha$ and $\widetilde{a}^U_\alpha$ are right continuous at $\alpha = 0$;*

4. *$\widetilde{a}^L_\alpha \leqslant \widetilde{a}^U_\alpha$.*

Moreover, if the pair of functions $\widetilde{a}^L_\alpha$ and $\widetilde{a}^U_\alpha$ satisfy the conditions (1)-(4), then there exists a unique $\tilde{a} \in F\,(R)$ such that $\widetilde{a}_\alpha = \left[\tilde{a}^L_\alpha, \tilde{a}^U_\alpha\right]$ for each $\alpha \in [0,1]$.

We define here a partial order relation on fuzzy number space.

Definition 2.6. [18] *For* $\tilde{a}, \tilde{b} \in F(R)$ *and* $\widetilde{a}_\alpha = \left[\tilde{a}^L_\alpha, \tilde{a}^U_\alpha\right]$, $\widetilde{b}_\alpha = \left[\tilde{b}^L_\alpha, \tilde{b}^U_\alpha\right]$, *are two closed intervals in R, for all* $\alpha \in [0,1]$, *we define*

1. $\tilde{a} \preceq \tilde{b} \;\Leftrightarrow\; \tilde{a}^L_\alpha \leqslant \tilde{b}^L_\alpha,\; \tilde{a}^U_\alpha \leqslant \tilde{b}^U_\alpha$
2. $\tilde{a} \prec \tilde{b}$ *if and only if for all* $\alpha \in [0,1]$:

$$\begin{cases} \tilde{a}^L_\alpha < \tilde{b}^L_\alpha \\ \tilde{a}^U_\alpha < \tilde{b}^U_\alpha \end{cases} \quad or \quad \begin{cases} \tilde{a}^L_\alpha \leqslant \tilde{b}^L_\alpha \\ \tilde{a}^U_\alpha < \tilde{b}^U_\alpha \end{cases} \quad or \quad \begin{cases} \tilde{a}^L_\alpha < \tilde{b}^L_\alpha \\ \tilde{a}^U_\alpha \leqslant \tilde{b}^U_\alpha \end{cases}$$

Definition 2.7. [20] *The membership function of a triangular fuzzy number* $\tilde{a}$ *is definedby*

$$\mu_{\tilde{a}}(r) = \begin{cases} \frac{r-a}{b-a}, & if\, a \leqslant r \leqslant b \\ \frac{c-r}{c-b}, & if\, b < r \leqslant c \end{cases}$$

which is denoted by $\tilde{a} = (a,b,c)$. *The* α*-level set of* $\tilde{a}$ *is then:*

$$\tilde{a}_\alpha = [(1-\alpha)\,a + \alpha b,\; (1-\alpha)\,c + \alpha b].$$

Definition 2.8. [18]*Let* V *be a real vector space and* $F(R)$ *be a fuzzy number space. Then a function* $\tilde{f} : V \to F(R)$ *is called fuzzy-valued function defined on* V. *Corresponding to such a function* $\tilde{f}$ *and* $\alpha \in [0,1]$, *we define two real-valued functions* $\tilde{f}^L_\alpha$ *and* $\tilde{f}^U_\alpha$ *on* V *as* $\tilde{f}^U_\alpha(x) = (\tilde{f}(x))^U_\alpha$ *and* $\tilde{f}^L_\alpha(x) = (\tilde{f}(x))^L_\alpha$ *for all* $x \in V$.

3. Fuzzy Nonlinear Optimization

Let $T \subseteq R^n$ be an open subset of R^n and $f_j(X)$, $g_j(X)$ be nonlinear (or linear) real-valued functions on T. Consider the following nonlinear fuzzy optimization problem:

$$\begin{array}{rl} \min \tilde{f}(X) & = \sum_{j=1}^n \tilde{c}_j f_j(X), \\ \text{s.t.} & \\ & \sum_{j=1}^n \tilde{a}_{ij} g_j(X) \preccurlyeq \tilde{b}_i, \qquad i = 1,\ldots,m; \\ & X \geqslant 0, \end{array} \tag{2}$$

where $\tilde{c}_j$ $(j = 1, \ldots, n)$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ $(i = 1, \ldots, m)$ are triangular fuzzy numbers.

Definition 3.1. *Let* $X^0 \in S = \left\{X \in T : \sum_{j=1}^{n} \tilde{a}_{ij} g_j(X) \preccurlyeq \tilde{b}_i,\ i{=}\,1,\ldots,m,\ X \geqslant 0\right\}$ *we say* X^0 *is an optimal solution of NFOP* (2) *if there exists no* $X^1 (\neq X^0) \in S$ *such that:*

$$\tilde{f}(X^1) \prec \tilde{f}\left(X^0\right).$$

Now, we can convert NFOP (2) to interval nonlinear programming (INP) by α-cuts technique. Let $\alpha \in [0,1]$ and

$$\tilde{c}_j = \left(c_j^1, c_j^2, c_j^3\right),\ \tilde{a}_{ij} = \left(a_{ij}^1,\ a_{ij}^2,\ a_{ij}^3\right),\ \tilde{b}_i = (b_i^1,\ b_i^2,\ b_i^3),$$

be triangular fuzzy numbers.
According to the Definition 2.7, we have

$$\tilde{f}_\alpha(X) = \left[\sum_{j=1}^{n} ((c_j^2 - c_j^1)\,\alpha + c_j^1) f_j(X),\ \sum_{j=1}^{n} (c_j^3 - (c_j^3 - c_j^2)\,\alpha) f_j(X)\right].$$

In addition, the constraints can be converted to

$$\left[\sum_{j=1}^{n} ((a_{ij}^2 - a_{ij}^1)\,\alpha + a_{ij}^1) g_j(X),\ \sum_{j=1}^{n} (a_{ij}^3 - (a_{ij}^3 - a_{ij}^2)\,\alpha) g_j(X)\right]$$
$$\leqslant \left[\left(b_j^2 - b_j^1\right)\alpha + b_j^1, b_j^3 - \left(b_j^3 - b_j^2\right)\alpha\right].$$

Therefore, NFOP (2) is converted to INP problem as

$$\begin{aligned} \min f(X) &= \textstyle\sum_{j=1}^{n} \left[\underline{c}_j, \overline{c}_j\right]\ f_j(X), \\ \text{s.t.} & \\ & \textstyle\sum_{j=1}^{n} \left[\underline{a}_{ij}, \overline{a}_{ij}\right] g_j(X) \leqslant \left[\underline{b}_i, \overline{b}_i\right], \qquad i = 1, 2, \ldots, m; \\ & X \geqslant 0. \end{aligned} \tag{3}$$

where for $j = 1,\ \ldots\ ,\ n$ and $i = 1,\ \ldots\ ,\ m$:

$$\begin{aligned} \underline{c}_j &= \left(c_j^2 - c_j^1\right)\alpha + c_j^1, & \overline{c}_j &= c_j^3 - \left(c_j^3 - c_j^2\right)\alpha, \\ \underline{a}_{ij} &= \left(a_{ij}^2 - a_{ij}^1\right)\alpha + a_{ij}^1, & \overline{a}_{ij} &= a_{ij}^3 - \left(a_{ij}^3 - a_{ij}^2\right)\alpha, \\ \underline{b}_i &= \left(b_j^2 - b_j^1\right)\alpha + b_j^1, & \overline{b}_i &= b_j^3 - \left(b_j^3 - b_j^2\right)\alpha. \end{aligned}$$

By setting $\alpha = 1$ in the problem (3), the following nonlinear programming will be obtained:

$$\begin{array}{ll} \min\ f_1(X') & = \sum_{j=1}^{n} c_j^2 f_j(X'), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} a_{ij}^2 g_j(X') \leqslant b_j^2, \qquad i = 1, \ldots, m; \\ & X' \geqslant 0. \end{array} \tag{4}$$

By setting $\alpha = 0$ in the problem (3),the following interval nonlinear programming will be obtained:

$$\begin{array}{ll} \min\ z & = \sum_{j=1}^{n} \left[c_j^1, c_j^3\right] f_j(X), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} \left[a_{ij}^1, a_{ij}^3\right] g_j(X) \leqslant \left[b_j^1, b_j^3\right], \quad i = 1, 2, \ldots, m; \\ & X \geqslant 0. \end{array} \tag{5}$$

Theorem 3.2. [17] *For INP Problem* (5) *the best and worst optimum values can be obtained by solving the following problems respectively:*

$$\begin{array}{ll} \min\ \underline{z} & = \sum_{j=1}^{n} c'_j\ f_j(X), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} a''_{ij} g_j(X) \leqslant \overline{b}_i, \quad i = 1, 2, \ldots, m; \\ & X \geqslant 0. \end{array} \tag{6}$$

$$\begin{array}{ll} \min\ \overline{z} & = \sum_{j=1}^{n} c''_j\ f_j(X), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} a'_{ij} g_j(X) \leqslant \underline{b}_i, \quad i = 1, 2, \ldots, m; \\ & X \geqslant 0. \end{array} \tag{7}$$

where

$$c'_j = \begin{cases} c_j^1, & f_j(X) \geqslant 0 \\ c_j^3, & f_j(X) \leqslant 0 \end{cases}, \qquad a''_{ij} = \begin{cases} a_{ij}^1, & g_j(X) \geqslant 0 \\ a_{ij}^3, & g_j(X) \leqslant 0 \end{cases},$$

$$c''_j = \begin{cases} c_j^3, & f_j(X) \geqslant 0 \\ c_j^1, & f_j(X) \leqslant 0 \end{cases}, \qquad a'_{ij} = \begin{cases} a_{ij}^3, & g_j(X) \geqslant 0 \\ a_{ij}^1, & g_j(X) \leqslant 0 \end{cases}.$$

Theorem 3.3. [17] *If the objective function for Problem* (5) *is changed to '*max*', the best and worst optimum values can be obtained by solving the following problems respectively:*

$$\begin{array}{rl} \max\ \overline{z} & = \sum_{j=1}^{n} c''_j\ f_j\left(X\right), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} a''_{ij} g_j\left(X\right) \leqslant \overline{b}_i, \quad i=1,2,\ldots,m; \\ & X \geqslant 0. \end{array} \tag{8}$$

$$\begin{array}{rl} \max\ \underline{z} & = \sum_{j=1}^{n} c'_j\ f_j\left(X\right), \\ \text{s.t.} & \\ & \sum_{j=1}^{n} a'_{ij} g_j\left(X\right) \leqslant \underline{b}_i, \quad i=1,2,\ldots,m; \\ & X \geqslant 0, \end{array} \tag{9}$$

where a'_{ij}, a''_{ij}, c'_j and c''_j are as defined in theorem 3.2.

Definition 3.4. *If $X'^{*} = \left(x'^{*}_1, x'^{*}_2, \ldots, x'^{*}_n\right)^T$, $\underline{X}^{*} = \left(\underline{x}^{*}_1, \underline{x}^{*}_2, \ldots, \underline{x}^{*}_n\right)^T$ and $\overline{X}^{*} = \left(\overline{x}^{*}_1, \overline{x}^{*}_2, \ldots, \overline{x}^{*}_n\right)^T$ are the optimal solutions of the problems* (4), (6) *and* (7) *respectively and z'^{*}, $\underline{z}^{*}$ and $\overline{z}^{*}$ are the optimum value of the problems* (4), (6) *and* (7) *respectively,then the fuzzy optimal solution and the fuzzy optimum value of the problem* (2) *are as follow respectively:*

$$X^{*} = \left[\left(\underline{x}^{*}_1, x'^{*}_1, \overline{x}^{*}_1\right), \left(\underline{x}^{*}_2, x'^{*}_2, \overline{x}^{*}_2\right), \ldots, \left(\underline{x}^{*}_n, x'^{*}_n, \overline{x}^{*}_n\right)\right]^T \quad \textit{and} \quad z^{*} = \left(\underline{z}^{*},\ z'^{*}, \overline{z}^{*}\right).$$

Definition 3.5. *If $\left(\underline{x}^{*}_i, x'^{*}_i, \overline{x}^{*}_i\right)$, $1 \leqslant i \leqslant n$, are all triangular fuzzy numbers then $X^{*}$is called a strong fuzzy solution. Otherwise, if $\exists i;;1 \leqslant i \leqslant n$, $\left(\underline{x}^{*}_i, x'^{*}_i, \overline{x}^{*}_i\right)$ is not a triangular fuzzy number, then by reordering $\left(\underline{x}^{*}_i, x'^{*}_i, \overline{x}^{*}_i\right)$ such that all elements of X^{*} remain fuzzy numbers, the solution is called a weak fuzzy solution.*

Therefore, by using Theorem 3.2 and Definitions 3.4 and 3.5, we can obtain the optimal solution of the problem (2).

4. Numerical Examples

In this section, we will explain previous method with presenting several examples. Note that for obtaining the optimal solutions of the nonlinear programming problems, the function *fmincon* of MATLAB is used.

Example 4.1. [1] Consider the following nonlinear fuzzy programming problem

$$
\begin{aligned}
\max\ \tilde{z} \ &= (1,3,4)\, x_1^2 + (1,2,3)\, x_2^2, \\
\text{s.t.}\ & \\
& (0,1,3)\, x_1 + (2,3,5)\, x_2 \preccurlyeq (3,4,6)\,, \\
& (1,2,4)\, x_1 - (0,1,2)\, x_2 \preccurlyeq (1,2,5)\,, \\
& x_1, x_2 \geqslant 0.
\end{aligned}
\tag{10}
$$

Now, we convert the problem (10) to an interval nonlinear programming by α-cuts:

$$
\begin{aligned}
\max\ z_\alpha \ &= [2\alpha+1,\ 4-\alpha]\, x_1^2 + [\alpha+1,\ 3-\alpha]\, x_2^2, \\
\text{s.t.}\ & \\
& [\alpha,\ 3-2\alpha]\, x_1 + [\alpha+2,\ 5-2\alpha]\, x_2 \leqslant [\alpha+3,\ 6-2\alpha]\,, \\
& [\alpha+1,\ 4-2\alpha]\, x_1 - [\alpha,\ 2-\alpha]\, x_2 \leqslant [\alpha+1,\ 5-3\alpha]\,, \\
& x_1,\ x_2 \geqslant 0,\ \alpha \in [0,1]\,.
\end{aligned}
\tag{11a}
$$

Setting $\alpha = 1$, the following nonlinear programming problem will be obtained:

$$
\begin{aligned}
\max\ z' \ &= 3{x'}_1^2 + 2{x'}_2^2, \\
\text{s.t.}\ & \\
& {x'}_1 + 3{x'}_2 \leqslant 4, \\
& 2{x'}_1 - {x'}_2 \leqslant 2, \\
& {x'}_1,\ {x'}_2 \geqslant 0.
\end{aligned}
\tag{11b}
$$

The optimal solution of this problem is obtained:

$$
{z'}^* = 7.597,\ {x'}_1^* = 1.429, {x'}_2^* = 0.857.
$$

with cut $\alpha = 0$, we have:

$$
\begin{aligned}
\max\ z \ &= [1,\ 4]\, x_1^2 + [1,\ 3]\, x_2^2, \\
\text{s.t.}\ & \\
& [0,\ 3]\, x_1 + [2,\ 5]\, x_2 \leqslant [3,\ 6]\,, \\
& [1,\ 4]\, x_1 - [0,\ 2]\, x_2 \leqslant [1,\ 5]\,, \\
& x_1,\ x_2 \geqslant 0.
\end{aligned}
\tag{11c}
$$

Now, by considering the Theorem 3.3, we have two problems as below:

$$\begin{aligned}
\max\ \overline{z}\ &= 4\overline{x_1^2} + 3\overline{x_2^2},\\
\text{s.t.}\ &\\
&2\overline{x_2} \leqslant 6,\\
&\overline{x_1} - 2\overline{x_2} \leqslant 5,\\
&\overline{x_1},\ \overline{x_2} \geqslant 0.
\end{aligned} \tag{11d}$$

The optimal solution of this problem is obtained:

$$\overline{x_1^*} = 11,\ \overline{x_2^*} = 3,\ \overline{z}^* = 511.$$

$$\begin{aligned}
\max\ \underline{z}\ &= \underline{x_1^2} + \underline{x_2^2},\\
\text{s.t.}\ &\\
&3\underline{x_1} + 5\underline{x_2} \leqslant 3,\\
&4\underline{x_1} \leqslant 1,\\
&\underline{x_1}, \underline{x_2} \geqslant 0.
\end{aligned} \tag{11e}$$

The optimal solution of this problem is:

$$\underline{x_1^*} = 0,\ \underline{x_2^*} = 0.6,\ \underline{z}^* = 0.36.$$

Therefore, by using definition 3.2, the strong fuzzy optimal solution of the problem (10) is:

$$x_1^* = \left(\underline{x_1^*}, {x'}^*_1, \overline{x_1^*}\right) = (0, 1.429, 11)\,, \quad x_2^* = \left(\underline{x_2^*}, {x'}^*_2, \overline{x_2^*}\right) = (0.6,\ 0.857, 3)$$

and the fuzzy optimum value of the objective function is:

$$z^* = \left(\underline{z}^*,\ {z'}^*, \overline{z}^*\right) = (0.36, 7.597, 511)\,.$$

Example 4.2. [9] Consider the following nonlinear fuzzy programming problem:

$$\begin{aligned}
\min\ \tilde{z}\ &= (1,2,3)\,x_1^2 + (0,1,2)\,x_2^2,\\
\text{s.t.}\ &\\
&(x_1-2)^2 + (x_2-2)^2 \leqslant 1.\\
&x_1,\ x_2 \geqslant 0.
\end{aligned} \tag{12}$$

Now, we convert the problem (12) into an interval nonlinear programming by α-cuts:

$$\begin{aligned} \min\ z_\alpha \ &= [\alpha+1, 3-\alpha]\, x_1^2 + [\alpha, 2-\alpha]\, x_2^2, \\ \text{s.t.} \ & \\ & (x_1-2)^2 + (x_2-2)^2 \leqslant 1, \\ & x_1,\ x_2 \geqslant 0. \end{aligned} \tag{13a}$$

Setting $\alpha = 1$, the following intervalnonlinear programming will be obtained:

$$\begin{aligned} \min\ z' \ &= 2{x'}_1^2 + {x'}_2^2, \\ \text{s.t.} \ & \\ & ({x'}_1-2)^2 + ({x'}_2-2)^2 \leqslant 1, \\ & {x'}_1,\ {x'}_2 \geqslant 0. \end{aligned} \tag{13b}$$

The optimal solution of this problem is:

$$z'^* = 4.814,\ {x'}_1^* = 1.155, {x'}_2^* = 1.465.$$

Setting $\alpha = 0$, we have:

$$\begin{aligned} \min\ z \ &= [1,\ 3]\, x_1^2 + [0,\ 2]\, x_2^2, \\ \text{s.t.} \ & \\ & (x_1-2)^2 + (x_2-2)^2 \leqslant 1, \\ & x_1,\ x_2 \geqslant 0. \end{aligned} \tag{13c}$$

Now, by considering the theorem **??**, we have two problems as below:

$$\begin{aligned} \min\ \overline{z} \ &= 3\overline{x_1^2} + 2\overline{x_2^2}, \\ \text{s.t.} \ & \\ & (\overline{x_1}-2)^2 + (\overline{x_2}-2)^2 \leqslant 1, \\ & \overline{x_1},\ \overline{x_2} \geqslant 0. \end{aligned} \tag{13d}$$

The optimal solution of this problem is:

$$\overline{x_1^*} = 1.207,\ \overline{x_2^*} = 1.391,\ \overline{z}^* = 8.239.$$

$$\begin{aligned} \min\ \underline{z} \ &= \underline{x_1^2}, \\ \text{s.t.} \ & \\ & (\underline{x_1}-2)^2 + (\underline{x_2}-2)^2 \leqslant 1, \\ & \underline{x_1},\ \underline{x_2} \geqslant 0. \end{aligned} \tag{13e}$$

The optimal solution of this problem is:

$$\underline{x_1^*} = 1,\ \underline{x_2^*} = 2,\ \underline{z}^* = 1.$$

Therefore by using of Definition 3.4, the optimal solution of the problem (12) is:

$$x_1^* = \left(\underline{x_1^*}, {x'}^*_1, \overline{x_1^*}\right) = (1, 1.155, 1.391)\,,\ \ x_2^* = \left(\underline{x_2^*}, {x'}^*_2, \overline{x_2^*}\right) = (2,\ 1.465, 1.391)$$

and the fuzzy optimum value of objective function is:

$$z^* = \left(\underline{z}^*,\ {z'}^*, \overline{z}^*\right) = (1, 4.814, 8.239)\,.$$

However, x_2^* is not a triangular fuzzy number. Therefore, by reordering elements of x_2^*, we have:

$$u_2^* = \left(\overline{x_2^*}, {x'}^*_2,\ \underline{x_2^*}\right) = (1.391,\ 1.465, 2)\,.$$

Hence, the optimal solution $X^* = (x_1^*,\ u_2^*)^T$ of this problem according to definition 3.3 is a weak fuzzy solution.

5. Conclusion

In this paper, a new method was presented for solving nonlinear fuzzy programming problems. First, this problem was converted into an interval nonlinear programming problem by α-cuts. Then the cuts $\alpha = 0$ and $\alpha = 1$ were used. In general, we have three nonlinear programming problems; to solve the problems *fmincon* function in Matlab may be used. Then according to Definition 3.4 and Definition 3.5 the fuzzy optimal solution and fuzzy optimal value of the main problem were obtained.

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