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Generalized Vector Variational-Like Inequalities

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Abstract. In this paper, we consider different types of generalized vector variational-like inequalities and study the relationships between their solutions. We study the general forms of Stampacchia and Minty type vector variational inequalities for bifunctions and establish the existence of their solutions in the setting of topological vector spaces. We extend these vector variational inequalities for the Clarke's subdifferential of non-differentiable locally Lipschitz functions and prove the existence of their solutions. As applications, we establish some existence results for a solution of the vector optimization problem by using Stampacchia and Minty type vector variational inequalities.

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1. Introduction

Let X be a real Banach space endowed with a norm $\|.\|$ and X^* its dual space with a norm $\|.\|_*$. We denote by 2^{X^*} and $\langle .,. \rangle$ the family of all nonempty subsets of X^* and the dual pair between X and X^*, respectively. Let K be a non-empty open subset of X, $\eta : K \times K \to X$ a vector-valued function. We suppose that C is a closed, convex and pointed cone of $\mathbb{R}^n$; i.e., $C \cap \{-C\} = \emptyset$ with int$C \neq \emptyset$, where int$C$

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denotes the interior of C. Let note that $0 \notin \text{int}C$.
Let X_1 and Y be real topological vector spaces, K a nonempty subset of X_1 and $C \subseteq Y$ a proper, closed, convex cone with int$C$, Note that $C \neq Y$ if and only if $0 \notin \text{int}C$. Let $g : K \times X_1 \to Y$ be a vector-valued bifunction and $\eta : K \times K \to X$ be a map. We consider the following Stampacchia Vector Variational Inequality problem (SVVI) consists of finding a vector $x \in K$ such that

$$g(x, \eta(x, y)) \notin -\text{int}C, \qquad \forall y \in K,$$

and Minty Vector Variational Inequality problem (MVVI) consists of finding a vector $x \in K$ such that

$$g(y, \eta(x, y)) \notin \text{int}C, \qquad \forall y \in K.$$

The solution sets of $(SVVI)$ and $(MVVI)$ are denoted by S_g and M_g, respectively. $(SVVI)$ and $(MVVI)$ contain the Stampacchia and Minty vector variational inequalities considered in [1,12,14,19] and references therein as special cases.

If $g : K \times K \to Y$ and $\eta(y, x) = y$ for all $x, y \in K$, then Stampacchia vector variational inequality reduces to the vector equilibrium problem considered and studied in ([1,5,6,11,13,18,19]) and references therein. In this case, Minty vector variational inequality becomes the so called dual vector equilibrium problem considered and studied by Konnov and Schaible ([13]) for scalar-valued bifunctions. It is worth mentioning that the vector equilibrium problem includes vector variational inequalities, the vector complementarity problem, the vector saddle point problem and the Nash equilibrium problem for vector-valued functions as special cases.
In the sequel we adopt the following ordering relations:

$$x \geqslant_C y \Leftrightarrow x - y \in C \text{and} x >_C y \Leftrightarrow x - y \in \text{int}C.$$

In this section, we recall some known definitions and results which will be used in the sequel. Throughout the article, for a nonempty subset A of a vector space, we denote by coA the convex hull of A. If A is a subset of a topological space, intA and A (or clA) denote the interior and closure of A, respectively. We denote by 2^A the family of all subsets of A.

Let X and Y be real topological vector spaces. A set-valued map $\Gamma : X \to 2^Y$ is said to be upper semicontinuous (u.s.c) on X if for each $x_0 \in X$ and for any open set V in Y containing $\Gamma(x_0)$, there exists an open neighborhood U in X such that $\Gamma(x) \subseteq V$ for all $x \in U$.

A set-valued map $\Gamma : K \to X$ is called a KKM map if and only if for every finite subset $x_1, ..., x_n$ of K

$$co\{x_1, x_2, ..., x_n\} \subseteq \bigcup_{i=1}^{n} \Gamma(x_i).$$

Mohan and Neogy in ([16]) introduced a Condition (C) defined as follows.

Condition (C). Let $\eta : X \times X \to X$. We say the function η satisfies the Condition (C) if, for any $x, y \in X$, $\lambda \in [0, 1]$,

$(C)(a)$ $\quad \eta(y, y + \lambda\eta(x, y)) = -\lambda\eta(x, y),$

$(C)(b)$ $\quad \eta(x, y + \lambda\eta(x, y)) = (1 - \lambda)\eta(x, y).$

Definition 1.1. *The map $\eta : K \times K \to X$ is said to be* skew *if for all $x, y \in K$,*

$$\eta(y, x) + \eta(x, y) = 0.$$

The following concepts and results are taken from Clarke et al. ([8]).

Definition 1.2. *The function $f : K \to \mathbb{R}^n$ is locally Lipschitz if for each $x \in K$ there exists a neighborhood of x and $k > 0$ such that for all y, z in this neighborhood we have $\|f(y) - f(z)\| \leqslant k\|y - z\|$.*

Let $f : K \to \mathbb{R}$ be a locally Lipschitz function. The Clarke's generalized derivative of $f : K \to \mathbb{R}$ at x in direction $v \in X$ is defined by

$$f^o(x; v) = \limsup_{\substack{y \to x \\ \lambda \to 0^+}} \frac{f(y + \lambda v) - f(y)}{\lambda},$$

and the Clarke's generalized subdifferential of f at $x \in X$ is defined by

$$\partial^c f(x) = \{\xi \in X^* : \langle \xi, v\rangle \leqslant f^o(x; v) \quad, \forall v \in X\}.$$

Definition 1.3. *A subset K of X is said to be invex with respect to $\eta : K \times K \to X$ if , for any $x, y \in K$ and $\lambda \in [0,1]$, we have $y + \lambda\eta(x,y) \in K$.*
Throughout this paper, $f_i : K \to \mathbb{R}$ the components of $f : K \to \mathbb{R}^n$ are non-differentiable locally Lipschitz functions and K stands for an invex subset of X. The definition of Clarke's generalized derivative can be extended to a locally Lipschitz vector-valued function $f : K \longrightarrow \mathbb{R}^n$. In fact the Clarke's generalized derivative of f at x in direction v is

$$f^o(x;v) = f_1^o(x;v) \times f_2^o(x;v) \times ... \times f_n^o(x;v),$$

and the Clarke's generalized subdifferential of f at $x \in X$ is the set

$$\partial^c f(x) = \partial^c f_1(x) \times \partial^c f_2(x) \times ... \times \partial^c f_n(x).$$

Remark 1.4. *Similar to the real-valued case, one can show that the set-valued mapping $\partial^c f : K \to X^{*n}$ of a function $f : K \to \mathbb{R}^n$ is $(\|.\| - w^*)$-u.s.c.(See; [8]).*

The following results will play a crucial role in establishing existence results for solutions of vector variational inequalities.

Lemma 1.5. ([10]) *Let K be a nonempty convex subset of a Hausdorff topological vector space X. Let $\Gamma : K \to 2^X$ be a KKM map such that for all $y \in K$, $\Gamma(y)$ is closed and $\Gamma(y^*)$ is compact for some $y^* \in K$. Then $\bigcap_{y \in K} \Gamma(y) \neq \emptyset$.*

The following result is a particular form of Corollary 3.2. in [15].

Theorem 1.6. ([15]) *Let K be a nonempty convex subset of a Hausdorff topological vector space X. Let $\Gamma : K \to 2^K$ be a set-valued map such that*

(i) for all $x \in K$, $\Gamma(x)$ is a nonempty convex subset of K and $x \notin \Gamma(x)$;

(ii) for all $y \in K$, $\Gamma^{-1}(y)$ is open in K;

(iii) there exist a nonempty compact convex subset B of K and a nonempty compact subset D of K such that for all $x \in K \setminus D$, there exists $\tilde{y} \in B$ such that $x \in int_K \Gamma^{-1}(\tilde{y})$.

Then, there exists a point $\bar{x} \in K$ such that $\Gamma(\bar{x}) = \emptyset$.

Lemma 1.7.([9]) *Let (Y, C) be an ordered topological vector space with a closed, convex and pointed cone C with $intC \neq \emptyset$. Then foe each $x, y \in Y$, one has*

(1) $y - x \in \text{int}C$ and $y \notin \text{int}C \Rightarrow x \notin \text{int}C$.

(2) $y - x \in C$ and $y \notin \text{int}C \Rightarrow x \notin \text{int}C$.

(3) $y - x \in -\text{int}C$ and $y \notin -\text{int}C \Rightarrow x \notin -\text{int}C$.

(4) $y - x \in -C$ and $y \notin -\text{int}C \Rightarrow x \notin -\text{int}C$.

2. Existence Results for Generalized Vector Variational-Like Inequalities (VVLI)

Let Let $f^o : K \times X \to \mathbb{R}^n$ be a vector-valued bifunction, the Clarke's generalized derivative of $f : K \to \mathbb{R}^n$, and $\eta : K \times K \to X$ be a map. We supposed that C is a closed, convex and pointed cone in $\mathbb{R}^n$ with $intC \neq \emptyset$. We consider the following *Stampacchia Vector Variational Inequality* (SVVI) problem in terms of the bifunction f^o:
SVVI(f, η)

$$\text{Find } \bar{x} \in K \text{ such that } f^o(\bar{x}; \eta(y, \bar{x})) \notin -\text{int}C, \text{ for all } y \in K,$$

and *Minty Vector Variational Inequality* (MVVI) problem in terms of the bifunction f^o:
MVVI(f, η)

$$\text{Find } \bar{x} \in K \text{ such that } f^o(y; \eta(\bar{x}, y)) \notin \text{int}C, \text{ for all } y \in K.$$

The solution sets of SVVI(f, η) and MVVI(f, η) are denoted by S_f and M_f, respectively. The following results will play a crucial role in establishing existence results for solutions of SVVI(f, η) and MVVI(f, η).

Definition 2.1. *Let $\eta : K \times K \to X$ be a map. A bifunction $f^o : K \times X \to \mathbb{R}^n$ is said to be* *(i)* C-pseudomonotone with respect to η *if for all* $x, y \in K$,

$$f^o(x;\eta(y,x)) \notin -intC \quad \Rightarrow \quad f^o(y;\eta(x,y)) \notin intC;$$

(ii) C-subodd with respect to η *if for all* $x, y \in K$, $f^o(x;\eta(x,y)) + f^o(x;\eta(y,x)) \in C$.

The following results will play a crucial role in establishing existence results for solutions of SVVI(f,η) and MVVI(f,η).

Definition 2.2. *Let $K \subseteq X$ be nonempty invex w.r.t $\eta : K \times K \to X$. A bifunction $f^o : K \times X \to \mathbb{R}^n$ is said to be*
(i)u-hemicontinuous *if for all $x, y \in K$ and $t \in [0,1]$, the mapping $t \mapsto f^o(x + t\eta(y,x);\eta(y,x))$ is continuous at 0^+.*
(ii)C-subodd with respect to η if for all $x, y \in K$, $f^o(x,\eta(x,y)) + f^o(x,\eta(y,x)) \in C$. If $\eta(y,x) = y - x$ for all $x, y \in K$, the definition of C-pseudomonotone w.r.t. η reduces to the definition of C-pseudomonotone on K.
The following result is a generalization of well-know Minty lemma.

Proposition 2.3. *Let $f^o : K \times X \to \mathbb{R}^n$ be a vector-valued bifunction and $\eta : K \times K \to X$ be a map. The following statements hold:*
(a) If f^o is C-pseudomonotone w.r.t. η, then $S_f \subseteq M_f$;
(b) Let $K \subseteq X$ be a nonempty invex set and the map $\eta : K \times K \to X$ be skew and satisfy Condition (C)(a), then $M_f \subseteq S_f$.

Proof.(a) It directly follows from the definition of C-pseudomonotonicity of f.
(b) Let $\bar{x} \in K$ be a solution of MVVI(f,η). Then for every $y \in K$, we have $f^o(y;\eta(\bar{x},y)) \notin \text{int}C$. Since K is an invex set, for any $\lambda \in (0,1)$ we have $\bar{x} + \lambda\eta(y,\bar{x}) \in K$ and so

$$f^o\left(\bar{x} + \lambda\eta(y,\bar{x}), \eta(\bar{x}, \bar{x} + \lambda\eta(y,\bar{x}))\right) \notin \text{int}C. \tag{1}$$

By Condition *(C)(a),*

$$\eta(\bar{x}, \bar{x} + \lambda\eta(y,\bar{x})) = -\lambda\eta(y,\bar{x}).$$

Since η is skew, we have $-\lambda\eta(y,\bar{x}) = \lambda\eta(\bar{x},y)$ and by positive homogeneity of f^o, we have

$$\begin{aligned} f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(\bar{x},\bar{x}+\lambda\eta(y,\bar{x}))) &= f^o(\bar{x}+\lambda\eta(y,\bar{x});\lambda\eta(\bar{x},y)) \\ &= \lambda f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(\bar{x},y)). \end{aligned}$$

It follows from (1) that

$$f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(\bar{x},y)) \notin \mathrm{int}C. \tag{2}$$

By C-suboddness of f^o, we have

$$f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(y,\bar{x})) + f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(\bar{x},y)) \in C. \tag{3}$$

From (2)-(3) and the fact that if $b \notin \mathrm{int}C$ and $a+b \in C$ then $a \notin \mathrm{int}C$, we obtain

$$f^o(\bar{x}+\lambda\eta(y,\bar{x});\eta(y,\bar{x})) \notin -\mathrm{int}C,$$

Letting $\lambda \to 0^+$ and using u-hemicontinuity of f for any $y \in K$, we get

$$f^o(\bar{x};\eta(y,\bar{x})) \notin -\mathrm{int}C.$$

Thus $M_f \subseteq S_f$. □

The following notion of proper C-suboddness generalizes the concept of C-suboddness given by Lalitha and Mehta ([14]).

Definition 2.4. *A bifunction $f^o : K \times X \to \mathbb{R}^n$ is said to be proper C-subodd with respect to $\eta : K \times K \to X$ if for every $x, y_1, \dots, y_n \in K$ with $\sum_{i=1}^n \eta(y_i, x) = 0$, we have*

$$f^o(x,\eta(y_1,x)) + \cdots + f^o(x,\eta(y_n,x)) \in C.$$

When K is a nonempty convex subset of X, we derive the following result from Proposition 2.3. (b).

Corollary 2.5. *Let K be a nonempty convex subset of X and $\eta : K\times K \to X$ be a skew map such that $\eta(y, y+\lambda(x-y)) = -\lambda\eta(x,y)$ for all $x, y \in K$ and all $\lambda \in [0,1]$. If vector-valued bifunction $f^o : K \times X \to Y$ is proper C-subodd and positive homogeneous such that for all $x, y \in K$*

and all $t \in [0,1]$, the map $t \mapsto f^o(x+t(y-x);\eta(y,x))$ is continuous at 0^+, then $M_f \subseteq S_f$.

Theorem 2.6. *Let K be a nonempty compact convex subset of X, the map $\eta : K \times K \to X$ be affine in the first argument and the bifunction $f^o : K \times X \to \mathbb{R}^n$ be C-pseudomonotone w.r.t. η and $f^o(x;\eta(x,x)) \notin -intC$ and for all $y \in K$, the map $x \mapsto f^o(y;\eta(x,y))$ is continuous. Then MVVI(f,η) has a solution.*

Proof. For all $y \in K$, define set-valued maps $M, S : K \to 2^K$ by

$$M(y) = \{x \in K : f^o(y;\eta(x,y)) \notin \text{int}C\}$$

and

$$S(y) = \{x \in K : f^o(x;\eta(y,x)) \notin -\text{int}C\}.$$

Since for all $x \in K$, $f^o(x;\eta(x,x)) \notin -\text{int}C$, we have $S(y)$ is nonempty for each $y \in K$. We claim that S is a KKM map on K. If S is not a KKM map, then there exists $x_0 \in \text{co}\{y_1,...,y_n\}$ such that for all $\lambda_1,\dots,\lambda_n \in [0,1]$ with $\sum_{i=1}^n \lambda_i = 1$, we have $x_0 \notin \bigcup_{i=1}^n S(y_i)$. Thus,

$$f^o(x_0;\eta(y_i,x_0)) \in -\text{int}C, \quad \text{for each } i = 1,...,n.$$

Since $-\text{int}C$ is convex, we have

$$\sum_{i=1}^n \lambda_i f^o(x_0;\eta(y_i,x_0)) \in -\text{int}C. \tag{4}$$

On the other hand, by affineness of η in the first argument and positive homogeneity of f^o in the second argument, we have

$$\sum_{i=1}^n \lambda_i f^o(x_0;\eta(y_i,x_0)) = f^o(x_0;\eta(x_0,x_0)) \notin -\text{int}C,$$

which contradicts to (4). Hence, S is a KKM map.
By virtue of C-pseudomonotone of f, $S(y) \subseteq M(y)$ for all $y \in K$. Thus, M is also a KKM map and, of course, $M(y)$ is nonempty for all $y \in K$.

Now we show that for all $y \in K$, $M(y)$ is closed. Let $\{x_n\}$ be a sequence in $M(y)$ such that $\{x_n\}$ converges to $x_0 \in K$. Then,

$$f^o(y, \eta(x_n, y)) \notin \text{int}C.$$

Since the map $x \mapsto f^o(y; \eta(x, y))$ is continuous, we have

$$f^o(y; \eta(x_n, y)) \to f^o(y; \eta(x_0, y)) \notin \text{int}C.$$

We conclude that $x_0 \in M(y)$, that is, $M(y)$ is a closed subset of a compact set K and hence compact. Thus all the conditions of Lemma 1.5 are fulfilled and hence

$$M_f = \bigcap_{y \in K} M(y) \neq \emptyset.$$

Therefore, there exists $\bar{x} \in K$ such that

$$f^o(y; \eta(\bar{x}, y)) \notin \text{int}C, \quad \forall y \in K,$$

and hence MVVI(f, η) has a solution. □

Remark 2.7. *The condition '$f^o(x; \eta(x, x)) \notin -intC$ for all $x \in K$' in Theorem 2.6 holds if $f^o : K \times X \to Y$ is proper C-subodd and $\eta(x, x) = 0$ for all $x \in K$.*

When K is not necessarily compact, we have following result.

Theorem 2.8. *Let K be a nonempty convex subset of X, the map $\eta : K \times K \to X$ be affine in the first argument and the bifunction $f^o : K \times X \to \mathbb{R}^n$ be proper C-subodd, C-pseudomonotone w.r.t. η . Assume that the following conditions hold:*

(i) *For all $x \in K$, $\eta(x, x) = 0$;*

(ii) *For all $y \in K$, the map $x \mapsto f^o(y; \eta(x, y))$ is continuous;*

(iii) *There exists a nonempty compact convex subset D of K such that for all $x \in K \setminus D$, there exists $\tilde{y} \in D$ satisfying $f^o(\tilde{y}; \eta(x, \tilde{y})) \in intC$.*

Then MVVI(f,η) has a solution.

Proof. Let $\{y_1, \dots y_k\}$ be a finite subset of K and let $Q = \mathrm{co}(D \cup \{y_1, \dots y_k\})$. Then Q is compact and convex. By Theorem 2.6, there exists $\bar{x} \in Q$ such that

$$f^o(y; \eta(\bar{x}, y)) \notin \mathrm{int}C, \quad \forall y \in Q.$$

From condition (iii), $\bar{x} \in D$. In particular, we have $\bar{x} \in D$ such that

$$f^o(y_i; \eta(\bar{x}, y_i)) \notin \mathrm{int}C, \quad \forall i = 1, 2, \dots, k.$$

Since D is compact and convex, by continuity of the map $x \mapsto f^o(y; \eta(x, y))$, we have

$$G(y) = \{x \in D : f^o(y; \eta(x, y)) \notin \mathrm{int}C\}$$

is closed in D and hence compact. Therefore, $\{G(y)\}_{y \in K}$ has a nonempty intersection property and hence

$$\bigcap_{y \in K} G(y) \neq \emptyset.$$

Thus, there exists $\bar{x} \in D$ such that $f^o(y; \eta(\bar{x}, y)) \notin \mathrm{int}C$ for all $y \in K$. $\square$

By combining Corollary 2.5. and Theorem 2.8., we obtain the existence result for a solution of SVVI(f, η).

Theorem 2.9. *Let K be a nonempty convex subset of X and the map $\eta : K \times K \to X$ be skew, affine in the first argument such that $\eta(x, x) = 0$ and $\eta(y, y + \lambda(x - y)) = -\lambda\eta(x, y)$ for all $x, y \in K$ and all $\lambda \in [0, 1]$. Let the vector-valued bifunction $f^o : K \times X \to \mathbb{R}^n$ be proper C-subodd, C-pseudomonotone and the map $t \mapsto f^o(x + t(y - x); \eta(y, x))$ is continuous at 0^+. Assume that there exists a nonempty compact convex subset D of K such that for all $x \in K \setminus D$, there exists $\tilde{y} \in D$ satisfying $f^o(\tilde{y}; \eta(x, \tilde{y})) \in intC$. Then SVVI($f, \eta$) has a solution.*

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