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The Observational Modelling of Information Function with Finite Partitions

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Abstract. In this paper we study the notions of relative information function and relative conditional information function on a relative probability measure space. We present some examples and prove some theorems about them. Also the concept of relative information function for a relative measure preserving transformation is introduced and some of its properties, are proved. Finally, it is proved that the relative information function of relative measure preserving transformations is invariant under isomorphism.

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1. Introduction

Molaie in [11] has studied the notion of one dimensional observer. This notion has been applied in dynamical systems [9,13], topology [5,8,9], geometry [12], and mathematical physics [12]. Let X be a non-empty set, then any function $\eta : X \to [0,1]$ is called a one-dimensional observer of X. In this paper we assume that $g : X \to X$ is a mapping, $\eta : X \to [0,1]$ is an observer of X and E is an arbitrary subset of X. The relative probability measure of E with respect to an observer η is the function

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$m_\eta^g(E) : X \to [0,1]$ which is defined in [13] by

$$m_\eta^g(E)(x) = \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \chi_E(g^i(x))\eta(g^i(x)).$$

The notation $m_\eta^g(E)(x)$ is the measure of E according to an observer viewpoint when it looks at x [13]. In this paper we will use the notion of observer to define the relative information function for relative measure preserving transformations.

2. Relative Information Function for Partitions

We assume that (X, m_η^g) is a relative probability space.
A partition of X is a disjoint collection of elements of $P(X)$ whose union is X, where $P(X)$ is the power set of X.
Let $A = \{A_1, ..., A_n\}$ and $C = \{C_1, ..., C_m\}$ be two finite partitions of X. Their join is defined in [16] as the partition:

$$A \vee C = \{A_i \cap C_j : A_i \in A, C_j \in C\}.$$

If $T : X \to X$ is a mapping, then T is called relative probability measure preserving if $T^{-1}E \subseteq E$ and $m_\eta^g(T^{-1}E)(x) = m_\eta^g(E)(x)$, for all $x \in X$. Also we say D is a refinement of C, and write $C \preccurlyeq_\eta^g D$, when we can write each element of C, as union of some elements of D.

Definition 2.1. *Let $A = \{A_1, ..., A_n\}$ be a finite partition of X. Then the relative information function of A is defined by*

$$I_{(\eta,g)}(A, x) = -\sum_{i=1}^{n} \chi_{A_i}(x) \log m_\eta^g(A_i)(x).$$

Example 2.2. Let $X = [0,1]$. If $A = \{A_1, A_2, A_3, A_4\}$ such that

$$A_1 = [0, \frac{1}{4}], A_2 = (\frac{1}{4}, \frac{1}{2}], A_3 = (\frac{1}{2}, \frac{3}{4}], A_4 = (\frac{3}{4}, 1].$$

Then A is a partition of X. Now let $g : X \to X$ be defined by $x \mapsto \frac{1}{2}$. Let $\eta : X \to [0, 1]$ be defined by $x \mapsto x$. Then

$$\begin{aligned} m_\eta^g(A_i)(x) &= \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \chi_{A_i}(g^i(x))\eta_(g^i(x)) \\ &= \limsup_{n\to\infty} \frac{1}{n} [\chi_{A_i}(x).x + \sum_{i=1}^{n-1} \chi_{A_i}(\frac{1}{2}).(\frac{1}{2})]. \end{aligned}$$

So $m_\eta^g(A_i)(x) = 0$ for $i \neq 2$ and $m_\eta^g(A_2)(x) = \frac{1}{2}$. Thus

$$I_{(\eta,g)}(A, \frac{1}{3}) = -\sum_{i=1}^{4} \chi_{A_i}(\frac{1}{3}) \log m_\eta^g(A_i)(\frac{1}{3}) = -\log(\frac{1}{2}) = \log 2.$$

Theorem 2.3. *If $T : X \to X$ is a relative probability measure preserving map then $I_{\eta,g}(T^{-1}A, x) = I_{\eta,g}(A, x)$, for all $x \in X$.*

Proof. Let $A = \{A_1, ..., A_n\}$. Since $T^{-1}A$ is a partition of X, there exists $T^{-1}A_{i_0} \in T^{-1}A$, such that $x \in T^{-1}A_{i_0}$ and $T^{-1}A_{i_0}$ is uniqe. So $x \in A_{i_0}$ and we have

$$\begin{aligned} I_{\eta,g}(T^{-1}A, x) &= -\sum_{i=1}^{n} \chi_{T^{-1}A_i}(x) \log m_\eta^g(T^{-1}A_i)(x) \\ &= -\log m_\eta^g(T^{-1}A_{i_0})(x) \\ &= -\log m_\eta^g(A_{i_0})(x) \\ &= -\sum_{i=1}^{n} \chi_{A_i}(x) \log m_\eta^g(A_i)(x) \\ &= I_{\eta,g}(A, x). \quad \square \end{aligned}$$

Now we want to study the concept relative conditional information function for finite partitions. Suppose that A and C are two finite partitions of X.

Definition 2.4. [6] *The relative information function of A given C is the number:*

$$I_{\eta,g}(A/C, x) = -\sum_{i=1}^{n}\sum_{j=1}^{m} \chi_{A_i \cap B_j}(x) \log \frac{m_\eta^g(A_i \cap B_j)(x)}{m_\eta^g(B_j)(x)}.$$

Theorem 2.5. *Let $D = \{\emptyset, X\}$. Then*

$$I_{\eta,g}(A/D, x) = I_{\eta,g}(A, x) + \log m_\eta^g(X)(x).$$

Proof.

$$\begin{aligned} I_{\eta,g}(A/D, x) &= -\sum_{i=1}^{n} \chi_{A_i}(x) \log \frac{m_\eta^g(A_i)(x)}{m_\eta^g(X)(x)}. \\ &= I_{\eta,g}(A, x) + \sum_{i=1}^{n} \chi_{A_i}(x) \log m_\eta^g(X)(x) \\ &= I_{\eta,g}(A, x) + \chi_{\cup_{i=1}^n A_i}(x) \log m_\eta^g(X)(x) \\ &= I_{\eta,g}(A, x) + \log m_\eta^g(X)(x). \quad \square \end{aligned}$$

Corollary 2.6. *If $D = \{\emptyset, X\}$ and $m_\eta^g(X)(x) = 1$ then*

$$I_{\eta,g}(A/D, x) = I_{\eta,g}(A, x).$$

Theorem 2.7. *If (X, m_η^g) is a relative probability space, and A, C and D are finite partitions of X then*
i) $I_{\eta,g}(A \vee C/D, x) = I_{\eta,g}(A/D, x) + I_{\eta,g}(C/A \vee D, x)$;
ii) $I_{\eta,g}(A \vee C, x) = I_{\eta,g}(A, x) + I_{\eta,g}(C/A, x)$;
iii) If $A \preccurlyeq_\eta^g C$ then $I_{\eta,g}(A, x) \leqslant I_{\eta,g}(C, x)$;
iv) If $A \preccurlyeq_\eta^g C$, then $I_{\eta,g}(A/D, x) \leqslant I_{\eta,g}(C/D, x)$.

Proof. See [6]. $\square$

Corollary 2.8. *If (X, m_η^g) is a relative probability space, T is a relative probability measure-preserving map, and A, C are countable partitions of X then $I_{\eta,g}(T^{-1}(A)/T^{-1}(C, x) = I_{\eta,g}(A/C, x)$.*

Proof. For each $A_i \in A, C_j \in C$, we have $T^{-1}(A_i \cap C_j) = T^{-1}A_i \cap T^{-1}C_j$, so $T^{-1}(A \vee C) = T^{-1}A \vee T^{-1}C$. By Theorems 2.5 and 2.7 (part ii), we can write

$$\begin{aligned} I_{\eta,g}(T^{-1}A/T^{-1}C,x) &= I_{\eta,g}(T^{-1}A\vee T^{-1}C,x) - I_{\eta,g}(T^{-1}C,x) \\ &= I_{\eta,g}(T^{-1}(A\vee C),x) - I_{\eta,g}(T^{-1}C,x) \\ &= I_{\eta,g}(A\vee C,x) - I_{\eta,g}(C,x) \\ &= I_{\eta,g}(A/C,x). \quad \square \end{aligned}$$

Two finite partitions A and C are called independent if $m^g_\eta(A\cap C)(x) = m^g_\eta(A)(x)m^g_\eta(C)(x)$ for all $A\in A, C\in C$, and $x\in X$.

Theorem 2.9. *Let A and C be two finite partitions of (X, m^g_η), and let A, C be independent. Then $I_{\eta,g}(A\vee C,x) = I_{\eta,g}(A,x) + I_{\eta,g}(C,x)$.*

Proof. Let $A = \{A_1, ..., A_n\}$ and $C = \{C_1, ..., C_m\}$. Since A, C and $A\vee C$ are some partitions of X, there exist $A_{i_0}\in A$, and $C_{j_0}\in C$ such that $x\in A_{i_0}\cap C_{j_0}$ and A_{i_0}, C_{j_0} are uniqe. So by definition of the notion Independence for A, C, we can write

$$\begin{aligned} I_{\eta,g}(A\vee C,x) &= -\sum_{i=1}^{n}\sum_{j=1}^{m}\chi_{A_i\cap C_j}(x)\log m^g_\eta(A_i\cap C_j)(x) \\ &= -\log m^g_\eta(A_{i_0}\cap C_{j_0})(x) \\ &= -\log(m^g_\eta(A_{i_0})(x)\times m^g_\eta(C_{j_0})(x)) \\ &= -\log m^g_\eta(A_{i_0})(x) - \log m^g_\eta(C_{j_0})(x) \\ &= -\sum_{i=1}^{n}\chi_{A_i}(x)\log m^g_\eta(A_i)(x) - \sum_{j=1}^{m}\chi_{C_j}(x)\log m^g_\eta(C_j)(x) \\ &= I_{\eta,g}(A,x) + I_{\eta,g}(C,x). \quad \square \end{aligned}$$

Corollary 2.10. *If A and C are two independent finite partitions of (X, m^g_η), then $I_{\eta,g}(A/C,x) = I_{\eta,g}(A,x)$.*

Proof. From Theorems 2.7 and 2.9, we have

$$I_{\eta,g}(A/C,x) = I_{\eta,g}(A\vee C,x) - I_{\eta,g}(C,x) = I_{\eta,g}(A,x). \quad \square$$

Theorem 2.11. *Let A, C and D be finite partitions of (X, m^g_η). If A and $C\vee D$ are independent then*

$$I_{\eta,g}(A \vee C/D, x) = I_{\eta,g}(A, x) + I_{\eta,g}(C/D, x).$$

Proof. Let $A = \{A_1, ..., A_n\}, C = \{C_1, ..., C_m\}$, and $D = \{D_1, ..., D_k\}$. By definition the notion of partition, there exist $A_{i_0} \in A, C_{j_0} \in C$ and $D_{k_0} \in D$ such that $x \in A_{i_0} \cap C_{j_0} \cap D_{k_0}$ and A_{i_0}, C_{j_0}, and D_{k_0} are uniqe. Now since A and $C \vee D$ are independent, we have

$$\begin{aligned}
I_{\eta,g}(A \vee C/D, x) &= -\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{r} \chi_{A_i \cap C_j \cap D_k}(x) \log \frac{m_\eta^g(A_i \cap C_j \cap D_k)(x)}{m_\eta^g(D_k)(x)} \\
&= -\log \frac{m_\eta^g(A_{i_0} \cap C_{j_0} \cap D_{k_0})(x)}{m(D_{k_0})(x)} \\
&= -\log \frac{m_\eta^g(A_{i_0})(x) \times m_\eta^g(C_{j_0} \cap D_{k_0})(x)}{m(D_{k_0})(x)} \\
&= -\log m_\eta^g(A_{i_0})(x) - \log \frac{m_\eta^g(C_{j_0} \cap D_{k_0})(x)}{m(D_{k_0})(x)} \\
&= -\sum_{i=1}^{n} \chi_{A_i}(x) \log m_\eta^g(A_i)(x) \\
&\quad -\sum_{j=1}^{m}\sum_{k=1}^{r} \chi_{C_j \cap D_k}(x) \log \frac{m_\eta^g(C_j \cap D_k)(x)}{m_\eta^g(D_k)(x)} \\
&= I_{\eta,g}(A, x) + I_{\eta,g}(C/D, x). \quad \square
\end{aligned}$$

3. Relative Information Function of a Relative Measure Preserving Transformation

In this section we assume that (X, m_η^g) is a relative probability space and $T : X \longrightarrow X$ is a relative measure preserving transformation.

Definition 3.1. *Let A be a finite partition of (X, m_η^g). The relative information function of T with respect to A is defined as*

$$I_{\eta,g}(T, A, x) = \limsup_{n\to\infty} \frac{1}{n} I_{\eta,g}(\vee_{i=0}^{n-1} T^{-i} A, x).$$

Theorem 3.2. *Let A, B, C and D are finite partitions of X. Then the following holds.*
i) $I_{\eta,g}(T, A, x) \geqslant 0$;
ii) if $A \preccurlyeq_\eta^g C$ *and* $B \preccurlyeq_\eta^g D$, *then* $A \vee B \preccurlyeq_\eta^g C \vee D$;
iii) $A \preccurlyeq_\eta^g C$ *implies that* $I_{\eta,g}(T, A, x) \leqslant I_{\eta,g}(T, C, x)$;
iv) $I_{\eta,g}(T, T^{-1}A, x) = I_{\eta,g}(T, A, x)$.

Proof.

i) It is trivial.

ii) Let $A = \{A_1, ..., A_n\}, B = \{B_1, ..., B_k\}, C = \{C_1, ..., C_m\}$, and $D = \{D_1, ..., D_r\}$. Since $A \preccurlyeq_\eta^g C$, there exists a partition $I(1), ...I(n)$ of the set $\{1, ..., m\}$ such that $A_i = \bigcup_{j\in I(i)} C_j$ for every $i = 1, ..., n$. Also since $B \preccurlyeq_\eta^g D$, there exists a partition $Q(1), ..., Q(k)$ of the set $\{1, ..., r\}$ such that $B_q = \bigcup_{s\in Q(q)} D_s$ for every $q = 1, ..., k$. For each i and q, we have

$$A_i \bigcap B_q = (\bigcup_{j\in I(i)} C_j) \bigcap (\bigcup_{s\in Q(q)} D_s) = \bigcup_{j\in I(i), s\in Q(q)} (C_j \bigcap D_s),$$

and this means $A \vee B \preccurlyeq_\eta^g C \vee D$.

iii) Let $A = \{A_1, ..., A_n\}$ and $C = \{C_1, ..., C_m\}$. Since $A \preccurlyeq_\eta^g C$, there exists a partition $I(1), ..., I(n)$ of the set $\{1, ..., m\}$ such that $A_i = \bigcup_{j\in I(i)} C_j$ for every $i = 1, ..., n$. So for each i, $T^{-1}A_i = T^{-1}(\bigcup_{j\in I(i)} C_j) = \bigcup_{j\in I(i)} T^{-1}C_j$, therefore $T^{-1}A \preccurlyeq_\eta^g T^{-1}C$. Thus we obtain $T^{-i}A \preccurlyeq_\eta^g T^{-i}C$, for every $i \in \mathbb{N}$. Since $A \preccurlyeq_\eta^g C$ by part ii), we get $A \vee T^{-1}A \preccurlyeq_\eta^g C \vee T^{-1}C$. Now by induction we conclude $\vee_{i=0}^{n-1} T^{-i}A \preccurlyeq_\eta^g \vee_{i=0}^{n-1} T^{-i}C$, for $n \geqslant 1$. Since for each i, $T^{-i}A$ and $T^{-i}C$ are partitions of X, $\vee_{i=0}^{n-1} T^{-i}A$ and $\vee_{i=0}^{n-1} T^{-i}C$, are partitions of X, too. Therefore by Theorem 2.5, (iii), $I_{\eta,g}(\vee_{i=0}^{n-1} T^{-i}A, x) \leqslant I_{\eta,g}(\vee_{i=0}^{n-1} T^{-i}C, x)$. Thus $\frac{1}{n} I_{\eta,g}(\vee_{i=0}^{n-1} T^{-i}A, x) \leqslant \frac{1}{n} I_{\eta,g}(\vee_{i=0}^{n-1} T^{-i}C, x)$. Hence $I_{\eta,g}(T, A, x) \leqslant I_{\eta,g}(T, C, x)$.
iv) By Theorem 2.3, we have

$$I_{\eta,g}(\bigvee_{i=1}^{n} T^{-i}A, x) = I_{\eta,g}(T^{-1}(\bigvee_{i=0}^{n-1} T^{-i}A), x) = I_{\eta,g}(\bigvee_{i=0}^{n-1} T^{-i}A, x).$$

So we can write

$$\begin{aligned} I_{\eta,g}(T,T^{-1}A,x) &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{i=0}^{n-1}T^{-i}(T^{-1}A),x) \\ &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{i=1}^{n}T^{-i}A,x) \\ &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{i=0}^{n-1}T^{-i}A,x) \\ &= I_{\eta,g}(T,A,x). \quad \square \end{aligned}$$

$I_{\eta,g}(T,x)=\sup_A I_{\eta,g}(T,A,x)$, is called the relative information function of T at x, where the supremum is taken over all finite partitions of (X,m_η^g).

Theorem 3.3.

i) $I_{\eta,g}(id,x)=0$;

ii) For $k\geqslant 1$, $I_{\eta,g}(T^k,x)=kI_{\eta,g}(T,x)$.

Proof.

i) Since $T=id$, we have $\vee_{i=0}^{n-1}T^{-i}A=A$, for any $n\in\mathbf{N}$. Therefore,

$$I_{\eta,g}(id,A,x)=\limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(A,x)=0.$$

ii) Let A be an arbitrary finite partition. We can write

$$\begin{aligned} I_{\eta,g}(T^k,\vee_{i=0}^{k-1}T^{-i}A,x) &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{j=0}^{n-1}(T^k)^{-j}(\vee_{i=0}^{k-1}T^{-i}A),x) \\ &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{j=0}^{n-1}\vee_{i=0}^{k-1}T^{-(kj+i)}A,x) \\ &= \limsup_{n\to\infty}\frac{1}{n}I_{\eta,g}(\vee_{i=0}^{nk-1}T^{-i}A,x) \\ &= \limsup_{n\to\infty}\frac{nk}{n}\frac{1}{nk}I_{\eta,g}(\vee_{i=0}^{nk-1}T^{-i}A,x) \\ &= kI_{\eta,g}(T,A,x). \end{aligned}$$

So

$$kI_{\eta,g}(T,x) = k\sup_A I_{\eta,g}(T,A,x) = \sup_A I_{\eta,g}(T^k, \vee_{i=0}^{k-1}T^{-i}A,x)$$
$$\leqslant \sup_A I_{\eta,g}(T^k,A,x) = I_{\eta,g}(T^k,x).$$

On the other hand, since $A \prec \vee_{i=0}^{k-1}T^{-i}A$, we have

$$I_{\eta,g}(T^k,A,x) \leqslant I_{\eta,g}(T^k, \vee_{i=0}^{k-1}T^{-i}A,x) = kI_{\eta,g}(T,A,x). \quad \square$$

Corollary 3.4. *Let $T^k = id$, for some $k \in \mathbb{N}$, then $I_{\eta,g}(T,x) = 0$.*

Proof. $T^k = id$, implies that $I_{\eta,g}(T^k,x) = 0$. Therefore $I_{\eta,g}(T,x) = \frac{1}{k}I_{\eta,g}(T^k,x) = 0$. $\square$

Definition 3.5. *Let $T_1 : X \longrightarrow X$ and $T_2 : X \longrightarrow X$ be two relative measure preserving transformations. We say that T_1 and T_2 are isomorphic if there exists a bijective relative measure preserving transformation $\varphi : X \longrightarrow X$ such that $\varphi o T_1 = T_2 o \varphi$.*

Theorem 3.6. *If $T_1 : X \longrightarrow X$ and $T_2 : X \longrightarrow X$ are isomorphic, then $I_{\eta,g}(T_1,x) = I_{\eta,g}(T_2,x)$.*

Proof. By definition, there exists a bijective relative measure preserving transformation $\varphi : X \longrightarrow X$ such that $\varphi o T_1 = T_2 o \varphi$. We can write

$$\begin{aligned}
I_{\eta,g}(T_2,A,x) &= \limsup_{n\to\infty} \frac{1}{n} I_{\eta,g}(\bigvee_{i=0}^{n-1} T_2^{-i}A,x) \\
&= \limsup_{n\to\infty} \frac{1}{n} I_{\eta,g}(\varphi^{-1}(\bigvee_{i=0}^{n-1} T_2^{-i}A),x) \\
&= \limsup_{n\to\infty} \frac{1}{n} I_{\eta,g}(\bigvee_{i=0}^{n-1} \varphi^{-1}(T_2^{-i}A),x) \\
&= \limsup_{n\to\infty} \frac{1}{n} I_{\eta,g}(\bigvee_{i=0}^{n-1} T_1^{-1}(\varphi^{-i}A),x) \\
&= I_{\eta,g}(T_1,\varphi^{-1}A,x).
\end{aligned}$$

So

$$\begin{aligned} I_{\eta,g}(T_2,x) &= \sup_A I_{\eta,g}(T_2,A,x) = \sup_A I_{\eta,g}(T_1,\varphi^{-1}A,x) \\ &\leqslant \sup_A I_{\eta,g}(T_1,A,x) = I_{\eta,g}(T_1,x). \end{aligned}$$

Therefore $I_{\eta,g}(T_2,x) \leqslant I_{\eta,g}(T_1,x)$. Similarly we obtain $I_{\eta,g}(T_1,x) \leqslant I_{\eta,g}(T_2,x)$. $\square$

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