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A Step by Step Method to Improve the Performance of Decision Making Units

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Abstract. In this paper, we present the concept of context-dependent DEA based on the FDH model by introducing the FDH-attractiveness and FDH-progress for each DMU. By the presented method, not only we can improve the performance of inefficient DMUs, but we can find a target for improvement among the existing efficient DMUs. These targets are observed DMUs and are not some virtual points on the efficiency frontier. Also, the paper presents a step by step method to improve the performance of DMUs by measuring FDH-attractiveness and FDH-progress. One numerical example and a case study consists of 20 Iranian bank branches are given for illustration.

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Keywords and Phrases: DEA, context-dependent, FDH, FDH-attractiveness, FDH-progress

1. Introduction

Data envelopment analysis (DEA) was originally developed to measure the relative efficiency of peer decision making units (DMUs) in multiple input and multiple output settings (Charnes et al. [4] and Banker et al. [3]). E1-Mahgary and Lahdelma [6] proposed a methodology for examining various two-dimensional charts for illustrating the DEA efficiency results. DEA is widely used in the evaluation of organizations. Hsiao et al. [8], proposed an entropy-based weighted Russell

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measure in data envelopment analysis, and illustrated their method using data gathered from 24 of Taiwan's commercial banks in order to rank and compare it with the conventional DEA models. Paradi et al. [11] proposed a two-stage Data Envelopment Analysis approach for simultaneously benchmarking the performance of operating units along different dimensions (for line managers) and a modified Slacks-Based Measure model is applied to aggregate the obtained efficiency scores from stage one and generate a composite performance index for each unit. Their approach was illustrated by using the data from a major Canadian bank with 816 branches operating across the nation.

The main goal of the DEA is to classify the DMUs into two classes: efficient and inefficient DMUs. Efficient DMUs are only characterized by an efficiency score of one and performance of inefficient DMUs depends on the efficient DMUs, that is, the inefficiency scores change only if the efficiency frontier is altered.

Although the performance of efficient DMUs is not influenced by the presence of inefficient DMUs, it is often influenced by the context. The context-dependent DEA [10, 12] is introduced to measure the relative attractiveness of a particular DMU when compared to others. We know that the DMUs in the reference set can be used as benchmark targets for inefficient DMUs. The context-dependent DEA provides several (benchmarks) targets by setting evaluation context [10, 12].

Consider a system that consists of some DMUs, such that the combination of DMUs in the system is not acceptable notion, that is we can't choose more than one target unit for each DMU. For example, consider the system of some machines, in such systems, efficient machines are not chosen as a point on a continuous efficiency frontier and the target machine must be one of already existing machines. In such cases, target unit must be one of the observed units and an inefficient unit must try to achieve a performance like the performance of this target unit. Because of some restrictions such as costs, personnel and etc. an inefficient unit may not be able to achieve this performance and become one of the efficient units in one step. However, we can design a method to improve the efficiency level of inefficient DMUs step by step to achieve efficiency frontier. For this reason, we define some efficiency levels. This discussion

is also true for a system including a number of teachers.
In such cases, that discussed above, the reference set of each inefficient DMU can not be chosen as a point on a continuous efficiency frontier, but it should be choosing among the existing DMUs, therefore, we use the FDH model to measure the relative attractiveness and progress of a particular DMU. Free Disposal Hull (FDH) is a well-known empirical approximation of the production possibility set (PPS) which relies on the sole assumption that PPS satisfies free disposability. These models were first formulated by Deprins et al. [5], and they ensure that efficiency evaluations are effected from only actually observed performances. Tulkens [15] introduced a relative efficiency for non-convex set free disposal hull (FDH) of the observed data, defined by Deprins et al. [5], and formulated a mixed integer programming which is different from the CCR and BCC models.
Simar [13] showed how to adapt the Hall-Simar [7] methodology to a multivariate frontier setup, providing stochastic versions of DEA/FDH estimators that improve the performance of the standard DEA/FDH estimators in the presence of noise. Simar and Zelenyuk [14] proposed an approach which allows to estimate nonparametric stochastic frontier in a more general setup, with no restriction on the size of the noise and presented the stochastic version of the DEA/FDH estimators.

As a result, in this paper, we present an algorithm to measure the relative FDH-attractiveness and relative FDH-progress. For this purpose, we use the linear reformulation of the output-oriented FDH model given by Agrell et al. [1]. By calculating FDH-attractiveness and FDH-progress for each DMU, we can design a computational method to achieve a better level (and improve performance of DMU) for each inefficient DMU and do this approach step by step to achieve efficiency frontier(or best level).
The rest of the paper is organized as follows: next section introduces the original FDH model and its linearization. Section (3) presents the concept of context-dependent DEA based on FDH model and also, DMUs improvement. In section (4), we illustrate our proposed DEA method with some examples. Finally, some conclusions are pointed out at the end of this paper.

Table 1: Nomenclatures.

Symbol	Definition	Symbol	Definition
DMU_o	DMU under evaluation	DMU_j	jth DMU
m	number of inputs	s	number of outputs
x_{ij}	ith input of DMU_j	y_{rj}	rth output of DMU_j
x_j	vector of inputs of DMU_j	y_j	vector of outputs of DMU_j
s_i^-	ith input slack	s_r^+	rth output slack
λ_j	intensity		
φ	efficiency score	φ_j	efficiency score for DMU_j
E^k	set of DMUs	J^k	efficient DMUs in E^k
ϵ	non-archimedean infinitesimal positive number		

2. FDH Model and its Linearization

2.1 FDH model

FDH (Free Disposal Hall) model was first formulated by Deprins et al. [5] and developed and extended by Tulkens et al. [15]. The basic motivation of FDH is to ensure that efficiency evaluations are effected from only actually observed performances. Assume that there are n DMUs, each DMU_j (j=1,. . . ,n) uses the input vector $X_j \in R^m$ to produce the output vector $Y_j \in R^s$. Matrices of all inputs and all outputs are denoted by X and Y, respectively. We assume that the data are all positive. Table 1 reports the used nomenclatures.
For our purpose and for evaluating $DMU_o, o \in \{1, \ldots, n\}$, we work with output oriented FDH model under variable returns to scale which is stated as follows:

$$\begin{array}{lll} \max & \phi + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\ s.t. & \phi y_{ro} - \sum_{r=1}^{s} \lambda_j y_{rj} + s_r^+ = \mathbf{0}, & r = 1, \ldots, s, \\ & x_{io} - \sum_{i=1}^{m} \lambda_j x_{ij} - s_i^- = \mathbf{0}, & i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j = 1, & \\ & \lambda_j \in \{0, 1\}, & \\ & s_i^-, s_r^+, \lambda_j \geqslant 0, & \end{array} \tag{1}$$

Where, λ is the intensity boolean vector, s_i^- and s_r^+ are vectors of input and output slack, respectively, and ϵ is a non-archimedean infinitesimal positive number. The efficiency surface is a staircase based on those given DMUs that are not dominated by other given DMUs. Thus, the efficiency analysis is done relative to the other given DMUs instead of a hypothetical efficiency frontier. This has the advantage, that the achievement goal for an inefficient DMU given by its efficient reference point will be more credible, than in the cases of CCR and the BCC models. The reference point will simply be one of the already existing operating DMUs.

2.2 Linearization

Note that model (1) is a mixed integer programming, and it is difficult to use. In order to avoid this problem, we use the linear reformulation of the output oriented FDH model (1), as given in Agrell et al. [1], which is expressed as follows:

$$\begin{array}{lll} \max & \sum_{j=1}^{n} \phi_j + \epsilon \sum_{j=1}^{n} (\sum_{r=1}^{s} s_{rj}^+ + \sum_{i=1}^{m} s_{ij}^-) & \\ s.t. & \phi_j y_{ro} - y_{rj}\lambda_j + s_{rj}^+ = 0, & r = 1,\ldots,s,\ j = 1,\ldots,n \\ & (x_{ij} - x_{io})\lambda_j + s_{ij}^- = 0, i = 1,\ldots,m, & j = 1,\ldots,n \\ & \sum_{j=1}^{n} \lambda_j = 1, & \\ & \lambda_j, s_{rj}^+, s_{ij}^- \geqslant 0. & \end{array} \tag{2}$$

Finding a reference set for each DMU and recognition strongly efficient DMUs are not our main purpose, but, we try to find the efficiency score for each DMU with respect to other evaluation contexts. Therefore, we work with the following relaxation model:

$$\begin{array}{lll} \max & \sum_{j=1}^{n} \phi_j & \\ s.t. & \phi_j y_{ro} - y_{rj}\lambda_j + s_{rj}^+ = 0, & r = 1,\ldots,s,\ j = 1,\ldots,n \\ & (x_{ij} - x_{io})\lambda_j + s_{ij}^- = 0, i = 1,\ldots,m, & j = 1,\ldots,n \\ & \sum_{j=1}^{n} \lambda_j = 1, & \\ & \lambda_j, s_{rj}^+, s_{ij}^- \geqslant 0. & \end{array} \tag{3}$$

Definition 2.1. (FDH efficiency)*A* DMU_o *is FDH efficient if* ϕ_o^**=1. We know that, FDH is a special case of DEA. With the FDH formulation, each DMU is evaluated by comparing it to the other DMUs on a one-to-one basis.*

3. Context-Dependent DEA Based on FDH Model

The context-dependent DEA [10, 12] is introduced to measure the relative attractiveness of a particular DMU when compared to others. Relative attractiveness depends on the evaluation context constructed from alternative DMUs.

3.1 FDH context-dependent DEA

Assume that there are n DMUs which use m inputs to produce s outputs. In order to improve the performance of inefficient DMU, when the target of improvement should be given among the efficient observed DMUs, we define $\mathbf{J}^1$ as the set of all DMUs and $\mathbf{E}^1$ as the set of efficient DMUs in $\mathbf{J}^1$ by model (3). Now, consider the following model:

$$\begin{array}{lll} \max & \displaystyle\sum_{j \in F(\mathbf{J}^k)} \phi_j & \\ s.t. & \phi_j y_{ro} - y_{rj}\lambda_j + s_{rj}^+ = 0, & r = 1, \dots, s,\ j \in F(\mathbf{J}^k) \\ & (x_{ij} - x_{io})\lambda_j + s_{ij}^- = 0, & i = 1, \dots, m,\ j \in F(\mathbf{J}^k) \\ & \sum_{j \in F(\mathbf{J}^k)} \lambda_j = 1, & \\ & \lambda_j, s_{rj}^+, s_{ij}^- \geqslant 0, & \end{array} \tag{4}$$

where $j \in F(\mathbf{J}^k)$ means $DMU_j \in \mathbf{J}^k$. Then, we define $\mathbf{J}^{k+1} = \mathbf{J}^k - \mathbf{E}^k$. When k=1, then first-level efficient frontier is defined by DMUs in $\mathbf{E}^1$, when k=2, model (4) give the second-level efficient frontier after the exclusion the first-level efficient DMUs. In this manner we can identify several levels of the efficient frontiers, where $\mathbf{E}^k$ consist the k*th*-level of efficient frontier. The efficient frontiers can be obtained by similar manner to Seiford et al. [12]. Assume that L levels of efficient frontiers are identified by the above algorithm. Now, based upon these evaluation contexts $\mathbf{E}^k, (k = 1, \dots, L)$, the context dependent DEA measures the relative FDH-attractiveness of each DMUs as follows:

$$
\begin{array}{lll}
H_o^*(k) = \max & \sum\limits_{j \in F(\mathbf{E}^{k+k_o})} H_j & k = 1, \dots, L - k_o \\
s.t. & H_j y_{ro} - y_{rj}\lambda_j + s_{rj}^+ = 0, & r = 1, \dots, s,\ j \in F(\mathbf{E}^{k+k_o}) \\
 & (x_{ij} - x_{io})\lambda_j + s_{ij}^- = 0, & i = 1, \dots, m,\ j \in F(\mathbf{E}^{k+k_o}) \\
 & \sum_{j \in F(\mathbf{E}^{k+k_o})} \lambda_j = 1, & \\
 & \lambda_j, s_{rj}^+, s_{ij}^- \geqslant 0, &
\end{array}
\tag{5}
$$

where $DMU_o \in \mathbf{E}^{k_o}$. Clearly, $H_o^*(k+1) < H_o^*(k)$.

Theorem 3.1. *In the assessment of $DMU_o \in \boldsymbol{E}^{k_o}$ by model (5), for each $k \in 1, \dots, L - k_o$, we have $H_o^*(k) < 1$.*

Proof. Let $(H_j^*, \lambda_j^*, s_{rj}^{+*}, s_{ij}^{-*}, j \in F(\mathbf{E}^{k+k_o}))$ be optimal solution of model (5). By summation on $j \in F(\mathbf{E}^{k+k_o})$ for first constraint we have

$$y_o \sum_{j \in F(\mathbf{E}^{k+k_o})} H_j^* \leqslant \sum_{j \in F(\mathbf{E}^{k+k_o})} \lambda_j y_j$$

and therefore

$$y_o H_o^*(k) \leqslant \sum_{j \in F(\mathbf{E}^{k+k_o})} \lambda_j y_j.$$

If we define $(\hat{x}_o^k, \hat{y}_o^k)$ as projection of (x_o, y_o) on efficiency frontier $\mathbf{E}^{k+k_o}$, where $\hat{x}_o^k = x_o, \hat{y}_o^k = H_o^*(k) y_o$, then we know $(\hat{x}_o^k, \hat{y}_o^k) \in \mathbf{E}^{k+k_0}$ and is inefficient DMU with respect to efficiency frontier $\mathbf{E}^{k_o}$. If we solve model (4) for this DMU and assume that the optimal objective $\bar{H}_o(k_o)$, then $\bar{H}_o(k_o) > 1$. It must noted that, projection of $(\hat{x}_o^k, \hat{y}_o^k)$ on efficiency frontier $\mathbf{E}^{k_o}$, i,e. $(\hat{x}_o^k, \hat{y}_o^k \bar{H}_o(k_o))$ coincides previous (original) DMU, namely (x_o, y_o), and therefore $\bar{H}_o(k) H_o^*(k) = 1$. Since $\bar{H}_o(k_o) > 1$ we conclude that $H_o^*(k) < 1$. $\square$

Definition 3.2. *We call $FA_o^*(k) \equiv \frac{1}{H_o^*(k)}$ as k-degree FDH-attractiveness of DMU_o from a specific level $\boldsymbol{E}^{k_o}$.*

Note that, the larger value of , shows that, DMUo is more attractive.

In fact DMUs in $\mathbf{E}^{k+k_o}$ must try more to obtain the levels of input and output of DMU_o ($\in \mathbf{E}^{k_o}$) in comparing the other DMUs in $\mathbf{E}^{k_o}$.
Now, for obtaining the FDH-progress measure for each DMU, we introduce the following context-dependent DEA model:

$$\begin{array}{lll} P_o^*(k) = \max & \displaystyle\sum_{j \in F(\mathbf{E}^{k_o-k})} P_j & k = 1, \dots, k_o - 1 \\ s.t. & P_j y_{ro} - y_{rj}\lambda_j + s_{rj}^+ = 0, & r = 1, \dots, s, \ j \in F(\mathbf{E}^{k_o-k}) \\ & (x_{ij} - x_{io})\lambda_j + s_{ij}^- = 0, & i = 1, \dots, m, \ j \in F(\mathbf{E}^{k_o-k}) \\ & \sum_{j \in F(\mathbf{E}^{k_o-k})} \lambda_j = 1, & \\ & \lambda_j, s_{rj}^+, s_{ij}^- \geqslant 0, & \end{array} \tag{6}$$

where $DMU_o \in \mathbf{E}^{k_0}$. Clearly, $P_o^*(k+1) > P_o^*(k)$.

Theorem 3.3. *In the assessment of $DMU_o \in \mathbf{E}^{k_0}$ by model (6), for each $k \in 1, \dots, k_o - 1$, we have $P_o^*(k) > 1$.*

Proof. The proof is similar to the proof of Theorem 3.1 and is omitted. □

Definition 3.4. *We call $FP_o^*(k) \equiv P_o^*(k)$ as k-degree FDH-progress of DMU_o from a specific level $\mathbf{E}^{k_o}$.*

To improve the performance of an inefficient DMU_o in $\mathbf{E}^{k_o}$, we can consider an individual DMU in each efficient frontier, $\mathbf{E}^{k_o-k}, k = 1, \dots, k_o - 1$, as a possible target. Note that, a smaller value of $FP_o^*(k)$ is preferred, that is, $DMU_o(\in \mathbf{E}^{k_o})$ must try less than other DMUs in $\mathbf{E}^{k_o}$ to improve its input and output levels to obtain the performance of some DMUs in the efficient frontier, $\mathbf{E}^{k_o-k}, k = 1, \dots, k_o - 1$.

Remark 3.5. *In fact, the attractiveness score for $DMU_o \in E^i$ shows the distance between DMU_o and DMUs in lower levels, that is, the worse DMUs and the progress score for DMU_o shows the distance between DMU_o and DMUs in upper levels, that is, the better DMUs. See figure 1 for more details.*

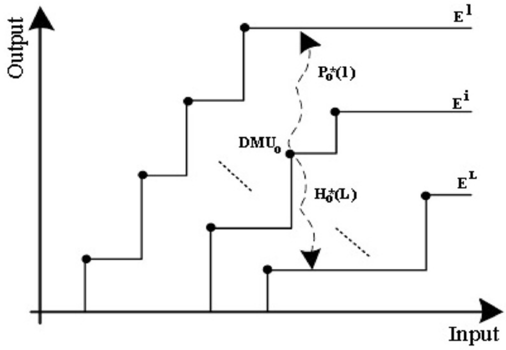

Figure 1: Attractiveness and Progress score of DMU_o

3.2 Layer improvement of DMUs

To improve the performance of DMU_o, instead of comparing it with DMUs on level $\mathbf{E}^1$ and efforts for DMU_o to reach this efficiency level, it is better to do the given improvement step by step. That is, the method suggests DMU_o, to choose the nearest accessible layer, namely $\mathbf{E}^{k_o-1}$ as the initial step for improving its performance. Next it should adjust its inputs and outputs in order to achieve that efficiency level and then having the required capacity, to make itself more efficient step by step in the same way, to finally reach the efficiency level $\mathbf{E}^1$.
Regarding the model (6), in order to improve the efficiency level of DMU_o and shifting it from $\mathbf{E}^{k_0}$ to $\mathbf{E}^{k_0-k}$, we will have the following changes in its inputs and outputs:

$$\hat{x}_o \rightarrow x_o - \sum_{j\in F(\mathbf{E}^{k_o-k})} s_j^- = \sum_{j\in F(\mathbf{E}^{k_o-k})} \lambda_j x_j,$$

$$\hat{y}_o \rightarrow y_o \sum_{j\in F(\mathbf{E}^{k_o-k})} P_j + \sum_{j\in F(\mathbf{E}^{k_o-k})} s_j^+ = y_o P_o^*(k)$$

$$+ \sum_{j\in F(\mathbf{E}^{k_o-k})} s_j^+ = \sum_{j\in F(\mathbf{E}^{k_o-k})} \lambda_j y_j,$$

3.3 Proposed algorithm

In short, we can present the proposed algorithm for step by step improving inefficient DMUs as the following steps:
Step 1: By using the model (3.1), we obtain the levels of efficiency frontiers.
Step 2: By using the model (3.2), we obtain FDH-attractiveness of DMUs when different efficient frontiers are chosen as evaluation contexts.
Step 3: By using the model (3.3), we obtain FDH-progress of DMUs when different efficient frontiers are chosen as evaluation contexts.
Step 4: By using these results, target DMU on the first upper layer for each DMU can be characterized.
Therefore, by this algorithm, a step by step method to improve the performance of DMUs can be calculated.

4. Numerical Example

In this section, we consider a group of some DMUs and apply the proposed algorithm and then discuss about step by step improvement of DMUs.

4.1 Data and results

Consider a group of 10 DMUs with two outputs and one single input of one. The data set of DMUs are given in Table 2.
By using the DEA model (4), we obtain four levels of efficient frontiers. They are: The following Statements
$\mathbf{E}^1 = \{DMU_j | j = 1, 5, 8\}$,
$\mathbf{E}^2 = \{DMU_j | j = 2, 6, 7, 9\}$,
$\mathbf{E}^3 = \{DMU_j | j = 4, 10\}$,
$\mathbf{E}^4 = \{DMU_j | j = 3\}$.
By models (5) and (6), we consider the FDH-attractiveness and FDH-progress of DMUs when different efficient frontiers are chosen as evaluation contexts.

Table 2: Sample DMUs.

DMU	1	2	3	4	5	6	7	8	9	10
Output 1	10	8	2	4	8	6	3	3	2	1
Output 2	2	1	1	2	6	4	4	8	6	4
Input	1	1	1	1	1	1	1	1	1	1

Table 3: Attractiveness Scores.

Level	DMU_j	Evaluation context			
		$\mathbf{E}^1$	$\mathbf{E}^2$	$\mathbf{E}^3$	$\mathbf{E}^4$
1	1	1	1.67	2.5	5
	5	1	1.50	3.0	6
	8	1	1.50	3.0	8
2	2	—	1.00	2.0	4
	6	—	1.00	2.0	4
	7	—	1.00	2.0	4
	9	—	1.00	2.0	6
3	4	—	—	1.0	2
	10	—	—	1.0	4
4	3	—	—	—	1

In Tables 3 and 4, the results are given. Table 3 shows the FDH-attractiveness scores for the sample DMUs based upon evaluation context $\mathbf{E}^1$, $\mathbf{E}^2$, $\mathbf{E}^3$ and $\mathbf{E}^4$. Table 4 shows the FDH-progress scores for the sample DMUs based upon evaluation context $\mathbf{E}^1$, $\mathbf{E}^2$, $\mathbf{E}^3$ and $\mathbf{E}^4$. Also, in Table 4 target DMU on the first upper layer for each DMU is characterized.

Table 4: FDH-Progress Scores.

Level	DMU_j	Evaluation context				Optimal lambda	Nearest benchmark
		$\mathbf{E}^1$	$\mathbf{E}^2$	$\mathbf{E}^3$	$\mathbf{E}^4$		
1	1	1.00	—-	—	—	—	—
	5	1.00	—-	—	—	—	—
	8	1.00	—-	—	—	—	—
2	2	1.25	1.00	—	—	$\lambda_1^* = 1$	DMU_1
	6	1.33	1.00	—	—	$\lambda_5^* = 1$	DMU_5
	7	1.50	1.00	—	—	$\lambda_5^* = 1$	DMU_5
	9	1.33	1.00	—	—	$\lambda_8^* = 1$	DMU_8
3	4	2.00	1.50	1	—	$\lambda_6^* = 1$	DMU_6
	10	2.00	1.50	1	—	$\lambda_9^* = 1$	DMU_9
4	3	5.00	3.0	2.0	1	$\lambda_4^* = 1$	DMU_4

It should be noted that, by using the results of these two Tables manager can decide that, what combination of these two measures are appropriate for ranking DMUs. If $DMU_o \in \mathbf{E}^{k_o}$, then its projection on the efficient frontiers $\mathbf{E}^k, (k = k_o+1, \ldots, L)$ is an actually observed DMU such as, for example, DMU_t, that is, $A_o^*(k)$ shows that, how much $DMU_o(\in \mathbf{E}^{k_0})$ makes itself distinctive from $DMU_t(\in \mathbf{E}^k)$. Similar discussion is true for progress measure.

4.2 Discussion about layer improvement

Considering the fact that the FDH-attractiveness and FDH-progress for each DMU with respect to all layers have been calculated, we can present some DMUs improvement approaches. For example, the method suggests $DMU_3 \in \mathbf{E}^4$ to choose layer $\mathbf{E}^3$ as the initial step for improving its performance and then adjusts and improves its output levels from (2,1) to (4,2). That is DMU_3 achieves a performance like a performance of DMU_4 (See Table 4). Then, having the required capacity, again this DMU can improve its output levels from (4,2) to (6,4), which is the same as output levels of $DMU_6 \in \mathbf{E}^2$. In this manner this DMU (and also, other DMUs) can improve its efficiency level step by step.

5. Empirical Example

To illustrate the performance of the proposed approach, we employ the above DEA methodology and our proposed algorithm on the empirical example used in [9] and [2], with the assumption of variable returns to scale. As can be seen in Table 5, the data set consists of 20 DMUs with 3 inputs and 3 outputs. The data are originally reported by Amirteimoori et al. [2] which consist of 20 Iranian bank branches (DMUs) in 2005. Three outputs include Deposits, Loans and Charges. Three inputs include Staff, Computer terminals and Space. In Table 5, we can see the results of output oriented BCC model and also the results of output oriented FDH model. It is obvious that more DMUs identified as efficient by output oriented FDH model than output oriented BCC model.

By using the DEA model (4), we obtain two levels of efficient frontiers, that are:

$\mathbf{E}^1 = \{DMU_j | j = 01, 02, 03, 04, 05, 06, 07, 08, 09, 11, 12, 13, 15, 16, 17, 19, 20\}$
$\mathbf{E}^2 = \{DMU_j | j = 10, 14, 18\}$.

By model (5), we consider the FDH-attractiveness of DMUs that belong to E^1 with respect to the DMUs in E^2. The results appear in Table 6. Also, by using model (6) we consider the FDH-progress of DMUs that belong to E^2 with respect to the DMUs in E^1. Again, we can see the results in Table 6. The results in Table 6 show that the definitions 3.1 and 3.2 are well-defined. From the results of Table 6, it is clear that the nearest target for each inefficient DMUs, i.e. DMUs in E^2, is DMU_9.

Therefore, we can present some DMUs improvement approaches. For example, the method suggests DMU_{10}, DMU_{14} and $DMU_{18} \in \mathbf{E}^2$ to choose layer $\mathbf{E}^1$ as the initial step for improving. However, this is the only step for improving their performances. Therefore, the presented methodology suggests, they should adjust and improve their current levels of input and output into the new levels, i.e. (0.476,0.600,0.135) and (0.080,0.364,0.244), respectively. That is, these DMUs achieve a performance like a performance of DMU_9. Therefore, these inefficient DMUs can improve their efficiency level in one step.

Table 5: The data for 20 Iranian bank branches.

DMU	I 1	I 2	I 3	O 1	O 2	O 3	BCC-O Results	FDH Results
01	0.950	0.700	0.155	0.190	0.521	0.293	1.000	1.000
02	0.796	0.600	1.000	0.227	0.627	0.462	1.114	1.000
03	0.798	0.750	0.513	0.228	0.970	0.261	1.000	1.000
04	0.865	0.550	0.210	0.193	0.632	1.000	1.000	1.000
05	0.815	0.850	0.268	0.233	0.722	0.246	1.105	1.000
06	0.842	0.650	0.500	0.207	0.603	0.569	1.278	1.000
07	0.719	0.600	0.350	0.182	0.900	0.716	1.000	1.000
08	0.785	0.750	0.120	0.125	0.234	0.298	1.000	1.000
09	0.476	0.600	0.135	0.080	0.364	0.244	1.000	1.000
10	0.678	0.550	0.510	0.082	0.184	0.049	1.682	1.341
11	0.711	1.000	0.305	0.212	0.318	0.403	1.585	1.000
12	0.811	0.650	0.255	0.123	0.923	0.628	1.000	1.000
13	0.659	0.850	0.340	0.176	0.645	0.261	1.156	1.000
14	0.976	0.800	0.540	0.144	0.514	0.243	1.758	1.264
15	0.685	0.950	0.450	1.000	0.262	0.098	1.000	1.000
16	0.613	0.900	0.525	0.115	0.402	0.464	1.555	1.000
17	1.000	0.600	0.205	0.090	1.000	0.161	1.000	1.000
18	0.634	0.650	0.235	0.059	0.349	0.068	1.925	1.043
19	0.372	0.700	0.238	0.039	0.190	0.111	1.000	1.000
20	0.583	0.550	0.500	0.110	0.615	0.764	1.000	1.000

Table 6: Results for 20 Iranian bank branches.

DMU	FDH-Attractiveness	FDH-Progress	Optimal lambda	Nearest benchmark
01	83.33	—	—	—
02	9.43	—	—	—
03	3.95	—	—	—
04	3.95	—	—	—
05	3.95	—	—	—
06	8.37	—	—	—
07	8.37	—	—	—
08	8.37	—	—	—
09	8.37	—	—	—
10	—	1.34146341463	$\lambda_9^* = 1$	DMU_9
11	5.93	—	—	—
12	9.24	—	—	—
13	3.84	—	—	—
14	—	1.26388888889	$\lambda_9^* = 1$	DMU_9
15	16.95	—	—	—
16	16.95	—	—	—
17	16.95	—	—	—
18	—	1.04297994269	$\lambda_9^* = 1$	DMU_9
19	16.95	—	—	—
20	16.95	—	—	—

6. Conclusion

Considering the fact that, to improve the performance of an inefficient DMU, sometimes, the target of improvement should be given among the efficient observed DMUs, in this paper we have extended the concept of context-dependent DEA based on the FDH model by using the output oriented linear form of FDH model. We have introduced the FDH-attractiveness and FDH-progress for each DMU. For this purpose, we used the linear form of FDH model and combined it with the context-dependent DEA models. By this manner, we proposed some new DEA models and presented some theorems. Finally, an algorithm for step by step improving the efficiency levels of inefficient DMUs is given. Implementation of a case study consists of 20 Iranian bank branches and also a numerical example, verified the potential of the proposed methodology.
As a future research, the scope of this work can be expanded in the presence of imprecise and incomplete data.

Therefore, if DMU_p and DMU_q belong to same efficiency level, and attractiveness of DMU_p is larger than the attractiveness of DMU_q and progress of DMU_p is smaller than progress of DMU_q, then better ranking is expected for DMU_p. Thus, we can use this concept for rank DMUs.

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