Journal of Mathematical Extension
Vol. 11, No. 2, (2017), 1-17
ISSN: 1735-8299
URL: http://www.ijmex.com

A Numerical Method to Compute the Complex Solution of Nonlinear Equations

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Abstract. In this paper, it is attempted to approximate the real and complex roots of nonlinear equations. For this reason, by considering the convergence conditions of Adomian decomposition method (ADM) for solving functional equations, a new appropriate method is presented. It will be shown that the proposed method can be computed suitable approximate real and complex roots of a given function more efficient than Maple software. Furthermore, with providing some examples the aforementioned cases are dealt with numerically.

AMS Subject Classification: 65E05; 65Y20
Keywords and Phrases: Complex solution of nonlinear equations, Adomian decomposition method (ADM), convergence conditions of ADM, Banach's fixed point theorem

1. Introduction

Finding the location of roots of equations and finding the solutions of a system of equations frequently appear in scientifics works. As all of these questions are not solvable by analytic methods, the study of numerical methods has provided an attractive field for researchers of mathematical sciences and caused the appearance of difierent numerical methods

Received: April 2015; Accepted: August 2015
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like ADM [5, 6, 10, 12, 13, 15], iterative method and other techniques [1, 4, 8, 9, 11, 17]. Theoretical treatment of the convergence of the decomposition series to the ADM has been considered in [2,7, 14, 16, 18, 20]. The numerical solution of nonlinear equations with real coefficients based on ADM has been considered in [5, 6, 10]. Also, the authors in [12, 13, 15] have applied convergence conditions of ADM to solve nonlinear equations and system of nonlinear equations.

Purpose of this paper is to introduce a modification of ADM based on convergence conditions of the ADM [14] and the Banach's fixed point theorem to compute the complex solution of nonlinear equations $f(x) = 0$. The proposed method doesnot need to get initial solution and the method produce different initial solutions for converging to different roots. Here, it is focused to compute complex solutions whereas the standard ADM and its modifications [5, 6, 10, 12, 15] can only obtain real solutions of the nonlinear equations. The proposed method is numerically performed through Maple programming, it will be shown that the proposed method can be computed suitable approximate real and complex roots of a given function more efficient than Maple software.

This paper is organized into following sections of which this introduction is the first. ADM and its convergence conditions are described in Section (2). Section (3) derives the method and peresent an efficent algorithm. Also, some numerical examples to illustrate the efficiency and reliability of the perposed algorithm is presented in Section (4). Finally, the paper is concluded with conclusion.

2. ADM and its Convergence Conditions

Consider the following functional equation,

$$y - Ny = g, \tag{1}$$

where N is a nonlinear operator from a Hilbert space H in to H, g is a given function in H and we are looking for $y \in H$ satisfying (1).

Adomian process assumes $y = \sum_{i=0}^{\infty} y_i$ and substitutes it in (1), to obtain $y_i(t)$ recursively by,

$$\begin{cases} y_0 = g, \\ y_{i+1} = Ny_i. \end{cases} \tag{2}$$

Theorem 2.1. (See [14]) *Let N be an operator from a Hilbert space H in to H and y be the exact solution of* (1). $\sum_{i=0}^{\infty} y_i$*, which is obtained by* (2)*, converges to y when:*

$$\exists \ 0 \leqslant \alpha < 1 , \qquad \| \, y_{k+1} \| \leqslant \alpha \, \| \, y_k \| \ , \quad \forall k \in \mathrm{N} \bigcup \{0\}.$$

Definition 2.2. *For every $i \in \mathrm{N} \bigcup \{0\}$ we define:*

$$\alpha_i = \begin{cases} \frac{\|y_{i+1}\|}{\|y_i\|} , & \| \, y_i \, \| \neq 0 \\ 0 \, . & , \ \| \, y_i \, \| = 0 \end{cases}$$

Corollary 2.3. *In Theorem (2.1), $\sum_{i=0}^{\infty} y_i$ converges to exact solution y, when* $0 \leqslant \alpha_i < 1 \quad , \quad \mathrm{i} = 1 \, , 2 \, , 3 \, , ...$

3. Numerical Method to Solve Nonlinear Equations

In this section, the computation of the roots of a given nonlinear equation is considered by modification of ADM. The proposed method can appropriately obtain the real and complex solutions whereas the existing ADMs can only obtain real solutions. Now consider the nonlinear equation of the form,

$$0 = f(x) = F(x) + c \, , \tag{3}$$

where F is a nonlinear function and c is a constant. The ADM decomposes the solution x by an infinite series of components,

$$x = \sum_{n=0}^{+\infty} x_n \, , \tag{4}$$

and the nonlinear term F(x) by an infinite series,

$$F(x) = \sum_{n=0}^{+\infty} A_n \, , \tag{5}$$

where the components of A_n are the so-called Adomian polynomials [20]. Note that, the authors in [15] have peresented a Maple procedure to construct Adomian polynomials of nonlinear term $F(x)$. As it was seen in [14], $\sum_{i=0}^{\infty} x_i$ converges to the exact solution x, when,

$$\exists\ 0 \leqslant \alpha < 1\,, \qquad |\,x_{k+1}| \leqslant \alpha\ |\,x_k|\ , \quad \forall k \in \mathrm{N}\bigcup\{0\}\,. \tag{6}$$

By adding $\beta\, x$ on both sides of (3), we have,

$$\beta x = F(x) + \beta x + c\,, \tag{7}$$

where, β is an unknown real or complex constant and it will be determined such that the convergence conditions (6) will be approximately held. Now, (7) implies that,

$$x = \frac{F(x) + \beta x}{\beta} + \frac{c}{\beta}\,, \tag{8}$$

and by standard ADM [14], we have,

$$\begin{cases} x_0 = \frac{c}{\beta}\,, \\ x_1 = \frac{F(x_0)+\beta x_0}{\beta}\,. \end{cases} \tag{9}$$

Replacing $F(x)$ with three terms of Taylor series of $F(x)$, at $x = 0$, becomes,

$$\begin{cases} x_0 = \frac{c}{\beta}\,, \\ x_1 \approx \frac{F(0)+x_0 F'(0)+\frac{x_0^2}{2!}F''(0)+\beta x_0}{\beta} = \frac{\beta^2 F(0)+\beta\, cF'(0)+\frac{c^2}{2!}F''(0)}{\beta^3} + \frac{c}{\beta}\,. \end{cases} \tag{10}$$

For an arbitrary number α, $0 < \alpha < 1$, and by attention to (6), we set:

$$x_1 = \alpha x_0\,. \tag{11}$$

Substituting (10) into (11) yields,

$$\beta_1 = \frac{-cF'(0) + \sqrt{c^2\, F'^2(0) - 2\, c^2 F''(0)\,[F(0) - c\,(\alpha - 1)]}}{2\,[\,F(0) - c(\alpha - 1)\,]}\,, \qquad (a)\ \ (12)$$

or

$$\beta_2 = \frac{-cF'(0) - \sqrt{c^2\, F'^2(0) - 2\, c^2 F''(0)\,[F(0) - c\,(\alpha - 1)]}}{2\,[\,F(0) - c(\alpha - 1)\,]}\,, \qquad (b)\ \ (12)$$

where β_1 and β_2 can be complex number. Now, we are able to compute an initial approximation for the nonlinear equation (3) as below.
For given number $m \in N \bigcup \{0\}$, set:

$$\begin{cases} x_0 = \frac{c}{\beta}\,, \\ x_1 = A_0, \\ x_2 = A_1, \\ . \\ . \\ . \\ x_m = A_{m-1}\,, \end{cases}$$

when A_i be Adomian polynomials, $\sum_{i=0}^{+\infty} A_i = \frac{F(x)+\beta x}{\beta}$, and β is chosen by (12.a) or (12.b). So, $\bar{x} = x_0 + x_1 + ... + x_m$ is an initial solution of (3). In similar way and by attention to Restarted ADM [6], the initial approximation $\bar{x}$ will be improved. For this reason, we rewrite (3) as below,

$$x = \frac{f(x)}{\beta} + x - \bar{x} + \bar{x}. \tag{13}$$

Here, β is an unknown real or complex constant and it will be determined such that the convergence conditions (6) will be approximately held.
As it is seen in (9) and (10), we have,

$$\begin{cases} x_0 = \bar{x}\,, \\ x_1 = A_0 = \frac{f(x_0)+\beta x_0}{\beta} - \bar{x} = \frac{f(\bar{x})}{\beta}\,, \\ x_2 = \frac{x_1 f'(x_0)+\beta x_1}{\beta} = \frac{f(\bar{x})f'(\bar{x})}{\beta^2} + \frac{f(\bar{x})}{\beta}\,, \\ x_3 = \frac{x_2 f'(x_0)+\frac{x_1^2 f''(x_0)}{2}+\beta x_2}{\beta}\,. \end{cases} \tag{14}$$

For an arbitrary number α , $0 < \alpha < 1$, and by attention to (6), we set,

$$x_3 = \alpha x_2. \tag{15}$$

Substituting (14) into (15) yields an quadratic equation as,

$$2\,(\alpha-1)\,f(\bar{x})\beta^2 + 2\,(\alpha-2)\,f(\bar{x})f'(\bar{x})\,\beta - f(\bar{x})\left(2\,f'(\bar{x})^2 + f(\bar{x})f''(\bar{x})\right) = 0,$$

this equation has two roots as follows,

$$\beta_1 = \frac{f'(\bar{x})\,(2-\alpha) + \sqrt{\alpha^2\, f'^2(\bar{x}) + 2\ f(\bar{x})\, f''(\bar{x})\,(\alpha-1)}}{2(\alpha-1)}\,, \qquad (a) \quad (16)$$

or

$$\beta_2 = \frac{f'(\bar{x})\,(2-\alpha) - \sqrt{\alpha^2\, f'^2(\bar{x}) + 2\ f(\bar{x})\, f''(\bar{x})\,(\alpha-1)}}{2(\alpha-1)}\,, \qquad (b) \quad (16)$$

Thus, we are able to improve the initial approximation $\bar{x}$, of the nonlinear equation (3) by using an efficent algorithm which is peresented in the next section.

3.1. The Proposed Design Algorithm

The above result is summarized in the following algorithm. The main idea of this algorithm is compute the complex solution of nonlinear equations.

Proposed Algorithm:

Input: Nonlinear equation $0 = f(x) = F(x) + c$, $\alpha(0 < \alpha < 1)$, $\varepsilon > 0$, $m \in N \bigcup \{0\}$and $n \in N$.

Output : appropriate approximation $\tilde{x}$.

Step 1:

Choose β by (12.a) or (12.b).

Step 2:

Consider Adomian polynomials A_i, $\sum_{i=0}^{+\infty} A_i = \frac{F(x)+\beta x}{\beta}$, and set $x_0 = \frac{c}{\beta}$, $x_1 = A_0, \ldots, x_m = A_{m-1}$ and $\bar{x} = x_0+x_1+...+x_m$(as an initial solution).

Step 3:

If $|\,f(\bar{x})\,| \leqslant \varepsilon$ then go to step 7.

Step 4:

Choose β by (16.a) or (16.b).

Step 5:

Consider Adomian's polynomials B_i, $\sum_{i=0}^{+\infty} B_i = \frac{f(x)+\beta x}{\beta}$, and set $x_0 = \bar{x}$, $x_1 = B_0 - \bar{x}$, $x_2 = B_1, \ldots, x_n = B_{n-1}$ and $\bar{x} = x_0 + x_1 + ... + x_n$ (as an improved solution)

Step 6:

If $|\,f(\bar{x})\,| \leqslant \varepsilon$ then go to step 7 else go to step 4.

Step 7: Set $\tilde{x} = \bar{x}$(as an appropriate solution) and stop.

Remark 3.1. By using different obtained real or complex constants β which are appeared in steps 1 and 4, it is possible that the obtained approximation solutions converge to different real or complex solutions of equation (3).

4. Test Problems

In this section, five examples are solved by the proposed method. The obtained results show that the method can appropriately obtain the real and complex solutions of the nonlinear equation (3) more efficient than Maple software. Here, we let $n = 2$, $\varepsilon = 10^{-8}$ and $\alpha = 0.1$.

Example 4.1. Consider the nonlinear equation,

$$1.2\,x^4 + 4.3\,x^3 + 6.945751311\,x^2 + 5.745751311\,x + 2.645751311 = 0, \quad (17)$$

which has four solutions,

$$z_1 = -1.291666667 + 0.7323864973\ I, \quad (a)\ (18)$$

$$z_2 = -1.291666667 - 0.7323864973\ I, \quad (b)\ (18)$$

$$z_3 = -0.5 + 0.8660254038\ I, \quad (c)\ (18)$$

and

$$z_4 = -0.5 - 0.8660254038\ I. \quad (d)\ (18)$$

According to (3), we have,

$$f(x) = 1.2\,x^4 + 4.3\,x^3 + 6.945751311\,x^2 + 5.745751311\,x + 2.645751311,$$

$$F(x) = 1.2\,x^4 + 4.3\,x^3 + 6.945751311\,x^2 + 5.745751311\,x,$$

and

$$c = 2.645751311.$$

To use algorithm (3.1), with (12.a), yields,

$$\beta = -3.192084061 + 3.198310327I,$$
$$x_0 = -0.4136162563 - 0.4144230285I,$$
$$x_1 = -0.2104116605 - 0.0200096997I,$$
$$x_2 = -0.1176618792 - 0.05559463413I.$$

So, the initial approximation is,

$$\bar{x} = \sum_{i=0}^{2} x_i = -0.7416897960 - 0.4900273623I.$$

Here, algorithm (3.1), step 4, with (16.a) is considered to improve the above initial approximation $\bar{x}$ and the results are shown in Table 1.

Table 1: To improve the initial approximation solution (algorithm (3.1), step 4, with (16.a))

No. It.	Improved approximation	$\mid f(\bar{x}) \mid$
1	x_0 =-0.7416897960 - 0.4900273623 I x_1 =-0.6323000426 - 0.07584196153 I x_2 =-0.3275481791 - 0.4849568431 I and $\bar{x} = \sum_{i=0}^{2} x_i$ =-1.701538018 - 1.050826167 I	3.13
2	$\bar{x} = \sum_{i=0}^{2} x_i$ =-1.421545904 - 0.9056422876 I	7.86 e^{-1}
3	$\bar{x} = \sum_{i=0}^{2} x_i$ =-1.287675516 - 0.7908821270 I	1.56 e^{-1}
4	$\bar{x} = \sum_{i=0}^{2} x_i$ =-1.291239765 - 0.7358933724 I	8.91 e^{-3}
5	$\bar{x} = \sum_{i=0}^{2} x_i$ =-1.291674796 - 0.7324270519 I	1.04 e^{-4}
6	$\bar{x} = \sum_{i=0}^{2} x_i$ =-1.291666747 - 0.7323869020 I	1.04 e^{-6}
7	$\tilde{x} = \sum_{i=0}^{2} x_i = -1.291666667 - 0.7323865025I$	1.30 e^{-8}

In fact, Table 1, yields,

$$\tilde{x} = \underbrace{{}^{1}x_0 + {}^{1}x_1 + {}^{1}x_2 + {}^{2}x_1 + {}^{2}x_2 + \cdots + {}^{7}x_1 + {}^{7}x_2}_{15\ terms}$$
$$= -1.291666667 - 0.7323865025I,$$

which is quite near to the exact solution (18.b). Again, consider the initial approximation,

$$\bar{x} = -0.7416897960 - 0.4900273623I,$$

which was obtained by algorithm (3.1), step 1, (12.a). To use algorithm (3.1), step 4, (16.b), yields,

$$\tilde{x} = \underbrace{{}^{1}x_0 + {}^{1}x_1 + {}^{1}x_2 + {}^{2}x_1 + {}^{2}x_2 + \cdots + {}^{6}x_1 + {}^{6}x_2}_{13\ terms}$$
$$= -0.5000000137 - 0.8660253935I,$$

which is quite near to the exact solution (18.d). Furthermore, to use algorithm (3.1), step 1, with (12.b), yields,

$$\beta = -3.192084061 - 3.198310327I,$$

and the initial approximation is,

$$\bar{x} = \sum_{i=0}^{2} x_i = -0.7416897960 + 0.4900273623I.$$

Now, by using algorithm (3.1), step 4, with (16.a), we have,

$$\tilde{x} = \underbrace{{}^{1}x_0 + {}^{1}x_1 + {}^{1}x_2 + {}^{2}x_1 + {}^{2}x_2 + \cdots + {}^{7}x_1 + {}^{7}x_2}_{15\ terms}$$
$$= -1.291666667 + 0.7323865025I,$$

and by using algorithm (3.1), step 4, with (16.b), we have,

$$\tilde{x} = \underbrace{{}^{1}x_0 + {}^{1}x_1 + {}^{1}x_2 + {}^{2}x_1 + {}^{2}x_2 + \cdots + {}^{6}x_1 + {}^{6}x_2}_{13\ terms}$$
$$= -0.5000000137 + 0.8660253935I,$$

which are quite near to the exact solutions (18.a) and (18.c), respectively. So, all solutions of the equation (17) were appropriately obtained by using the proposed algorithm (3.1). Note that the ADM and its modifications [5, 6, 10, 12, 15] cannot solve this example.

Example 4.2. Consider the nonlinear equation,

$$3\,x^6 + 29.5\,x^5 + 107\,x^4 + 221\,x^3 + 177.5\,x^2 + 10\,x - 42 = 0, \qquad (19)$$

which has two real solutions,

$$z_1 = 0.3722813233, \qquad (a)\ (20)$$

$$z_2 = -5.372281323, \qquad (b)\ (20)$$

and four complex solutions,

$$z_3 = -1.5 + 2.179449472\ I, \qquad (c)\ (20)$$

$$z_4 = -1.5 - 2.179449472\ I, \qquad (d)\ (20)$$

$$z_5 = -0.9166666667 + 0.3996526269\,I, \qquad (e)\ (20)$$

$$z_6 = -0.9166666667 - 0.3996526269\,I. \qquad (f)\ (20)$$

By attention to (3), we have,

$$f(x) = 3\,x^6 + 29.5\,x^5 + 107\,x^4 + 221\,x^3 + 177.5\,x^2 + 10\,x - 42,$$

$$F(x) = 3\,x^6 + 29.5\,x^5 + 107\,x^4 + 221\,x^3 + 177.5\,x^2 + 10\,x,$$

and

$$c = -42.$$

To use algorithm (3.1), step 1, with (12.a), yields,

$$\beta = -96.73777713,$$

$$x_0 = 0.4341633770,$$

$$x_1 = -0.1877593704,$$

$$x_2 = 0.4520572430,$$

and since $|\,x_0| \leqslant |\,x_1|$ and $|\,x_1\,| > |\,x_2\,|$, so the initial approximation is,

$$\bar{x} = \sum_{i=0}^{1} x_i = 0.2464040066.$$

Now, algorithm (3.1), step 4, with (16.a) and (16.b) is considered to improve the above initial approximation $\bar{x}$, and the obtained results are as below,

$$\tilde{x} = \underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + {}^2x_2 + \cdots + {}^5x_1 + {}^5x_2}_{11\ terms}$$
$$= 0.3722813234 + 0.29e^{-9}I,$$

and

$$\tilde{x} = \underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + {}^2x_2 + \cdots + {}^8x_1 + {}^8x_2}_{17\ terms}$$
$$= -0.9166666655 + 0.3996526208I,$$

which are quite near to the real exact solution (20.a) and the complex exact solution (20.c). In continue, to apply algorithm (3.1), step 1, with (12.b) yields,

$$\beta = 85.62666602,$$

furthermore the initial approximation is,

$$\bar{x} = \sum_{i=0}^{2} x_i = -0.9118518596.$$

Again, algorithm (3.1), step 4, with (16.a) and (16.b) is used to improve the above initial approximation $\bar{x}$, and the obtained results are as below,

$$\tilde{x} = \underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + {}^2x_2 + \cdots + {}^5x_1 + {}^5x_2}_{11\ terms}$$
$$= -0.9166666672 - 0.3996526228I,$$

and

$$\tilde{x} = \underbrace{{}^1x_0 + {}^1x_1 + {}^1x_2 + {}^2x_1 + {}^2x_2 + \cdots + {}^7x_1 + {}^7x_2}_{15\ terms}$$
$$= -0.9166666674 + 0.3996526270I,$$

which are quite near to the complex exact solutions (20.d) and (20.c), respectively. Hence, one real and two complex solutions of the equation

(19) were appropriately obtained by using the proposed method(Note that , $|\,f(\tilde{x})\,| < 10^{-6}$ is held for all above approximation solutions, $\tilde{x}$). It must be noted that the ADM and its modifications [5, 6, 10, 12, 15] are not able to find complex solutions of this example.

Example 4.3. Consider the nonlinear equation,

$$(2 - I)\,x + 2.5\,I - 4.5 + e^{Ix^2} = 0. \tag{21}$$

Here, according to algorithm (3.1) we have,

$$f(x) = (2 - I)\,x + 2.5\,I - 4.5 + e^{Ix^2},$$

$$F(x) = (2 - I)\,x + e^{Ix^2},$$

and

$$c = 2.5\,I - 4.5. \tag{22}$$

The algorithm (3.1) is applied to solve (21) and the results are shown in Table 2.

Table 2: Different roots of example 4.3

β in algorithm(3.1)	m	Approximate solution,$\tilde{x}$	$\|\,f(\tilde{x})\,\|$
(12.a)&(16.a)	0	2.179819048+0.1271110185 I	6.0 e^{-9}
(12.a)&(16.b)	0	-0.3916190983+2.778327692 I	9.1 e^{-9}
(12.b)&(16.a)	0	0.8039553334-0.8149692140 I	5.0e^{-10}
	2	-0.8171635125+1.239908970 I	3.6e^{-9}
(12.b)&(16.b)	1	3.000432324-0.07487328602 I	5.7 e^{-9}

Table 2 shows that we can obtain different appropriate approximate roots by choosing different parameter, m. In fact, algorithm (3.1), first, provide appropriate initial solutions based on constant c (22), then approve these initial solutions. Whereas using Maple software, directly, yields:

```
> solve(2*x-I*x+(-4.5+2.5*I)+exp(I*x^2),x);

> fsolve(2*x-I*x+(-4.5+2.5*I)+exp(I*x^2),x);
                              0.8039553333-0.8149692140I
```

i.e., "*solve*" function does not present solution and "*fsolve*" function presents only one solution.

Example 4.4. Consider the nonlinear equation,

$$(1-2I)x^2+(4+3I)x-((2+I)x-1)\sinh(x)-(x+2I-I\sinh(x))\,e^{Ix}-2=0. \tag{23}$$

Here, according to algorithm (3.1) we have,

$$f(x)=(1-2I)x^2+(4+3I)x-((2+I)x-1)\sinh(x)-(x+2I-I\sinh(x))\,e^{Ix}-2,$$

$$F(x)=(1-2I)x^2+(4+3I)x-((2+I)x-1)\sinh(x)-(x+2I-I\sinh(x))\,e^{Ix},$$

and

$$c=2.$$

The algorithm (3.1)is applied to solve (23) and the results are shown in Table 3.

Table 3: Different roots of example 4.4

β in algorithm(3.1)	m	Approximate solution,$\tilde{x}$	$\mid f(\tilde{x})\mid$
(12.a)&(16.a)	0	1.302129885-0.7201017727I	6.7 e^{-9}
	1	0.6427282968-2.583539902I	2.0e^{-9}
	2	0.4173678159+0.1750106379I	1.8e^{-9}
(12.a)&(16.b)	2	-3.772756439-2.351131754I	0.1 e^{-7}
(12.b)&(16.a)	0	0.4173678156+0.1750106381I	3.5 e^{-9}

Note that for above problem, choosing (12.b)&(16.b) yield $\tilde{x}=0.4173678156+0.1750106381I$, too (The same as (12.b)&(16.a)). Also, using Maple software, directly, yields,

```
> solve(1*x^2+3*I*x-1*I*x*sinh(x)-2*I*x^2+4*x-2*x*sinh(x)-2+sinh(x)
                    -exp(I*x)*x-2*I*exp(I*x)+I*exp(I*x)*sinh(x),x);

> fsolve(1*x^2+3*I*x-1*I*x*sinh(x)-2*I*x^2+4*x-2*x*sinh(x)-2+sinh(x)
                    -exp(I*x)*x-2*I*exp(I*x)+I*exp(I*x)*sinh(x),x);
                    0.4173678160+0.1750106377I
```

i.e., "*solve*" function does not present solution and "*fsolve*" function presents only one solution.

Example 4.5. Consider the nonlinear equation,

$$\begin{aligned} &3x^3 - 15.14602679x^2 + 17.23368794x + \\ &I\,(1 - \sinh(x))\,(x^2 - 5.048675598\;x + 5.744562647) = 0. \end{aligned} \tag{24}$$

Here, according to algorithm (3.1) we have,

$$\begin{aligned} f(x) &= 3x^3 - 15.14602679x^2 + 17.23368794x + \\ &\qquad I\,(1 - \sinh(x))\,(x^2 - 5.048675598\;x + 5.744562647), \\ F(x) &= 3x^3 - 15.14602679x^2 + 17.23368794x + \\ &\qquad I\,(1 - \sinh(x))\,(x^2 - 5.048675598)\;x, \end{aligned}$$

and

$$c = 5.744562647\,I.$$

The algorithm (3.1) is applied to solve (24) and the results are shown in Table 4.

Table 4: Different roots of example 4.5

β in algorithm(3.1)	m	Approximate solution,$\tilde{x}$	$\lvert f(\tilde{x}) \rvert$
(12.a)&(16.a)	0	0.09950948073-0.3016084986I	1.2 e^{-9}
(12.b)&(16.a)	2	3.316624789	1.3 e^{-7}
(12.b)&(16.b)	0	1.732050809	1.0 e^{-8}

Here, two real and one complex solution are obtained by proposed algorithm (3.1). Whereas using Maple software, directly, yields,

```
> solve(3*x^3-15.14602679*x^2+17.23368794*x+I*x^2-(5.048675598*I)*x+
5.744562647*I-I*sinh(x)*x^2+(5.048675598*I)*sinh(x)*x-(5.744562647*I)
*sinh(x), x)
                    0.9950948071e-1-.3016084986I
> fsolve(3*x^3-15.14602679*x^2+17.23368794*x+I*x^2-(5.048675598*I)*x+
5.744562647*I-I*sinh(x)*x^2+(5.048675598*I)*sinh(x)*x-(5.744562647*I)
*sinh(x), x)
                     0.9950948071e-1-.3016084986I
```

i.e., "solve" and "fsolve" functions present only one complex solution.

5. Conclusion

In this paper, to solve nonlinear equation in the form $F(x) + c = 0$, the appropriate algorithm (3.1) is presented. The algorithm, based on constant c, can be produce different parameters β which by using these parameters we are able to obtain different suitable approximate solutions. Although two different β may be conclude the same approximate solution but algorithm (3.1), usually, can obtain more than one (if exist) approximate root. Whereas, Maple software without getting appropriate initial solution, usually, can not present more than one (see examples (4.1)-(4.5)).

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