Journal of Mathematical Extension
Vol. 10, No. 1, (2016), 1-22
ISSN: 1735-8299
URL: http://www.ijmex.com

A Common Fixed Point Theorem for Multi-valued Mappings in Ultrametric Spaces

Z. Alinejad
Ferdowsi University of Mashhad

A. K. Mirmostafaee *
Ferdowsi University of Mashhad

Abstract. We will introduce a new class of implicit functions to prove a common fixed point theorem for multi-valued mappings in ultrametric spaces. Our result enables us to give a genuine generalization of some known fixed point theorems, provided that the underling space is ultrametric.

AMS Subject Classification: 47H10; 54E50; 54E35
Keywords and Phrases: Fixed point, multi-valued mappings, ultrametric space

1. Introduction

Since Banach's paper [4] on the existence of a unique fixed point for a strict contraction, the study of contractive mappings has been an important topic in metric spaces. Banach's celebrated theorem also yields convergence of iterates to the unique fixed point. Interesting results have also been obtained regarding set-valued mappings. In 1969, Nadler [14] employed Banach's iterative method to establish the existence of a fixed point for a strictly contractive set-valued mapping. Nadler's fixed point theorem recently extended by some mathematicians [3, 10, 13, 19].

Received: January 2015; Accepted: June 2015
*Corresponding Author

Kikkawa and Suzuki extended Nadler's fixed point theorem for multi-valued mappings as follows.

Theorem 1.1. [10, Theorem 2] *Let (X, d) be a complete metric space and let T be a mapping from X into $CB(X)$. Assume that there exists $r \in [0, 1)$ and $\eta(r) = \frac{1}{1+r}$ such that*
$\eta(r)d(x, Tx) \leq d(x, y)$ implies $\mathcal{H}(Tx, Ty) \leq rd(x, y)$ for all $x, y \in X$. Then there exists $z \in X$ such that $z \in Tz$.

In 2013, Popescu gave a new condition for mappings in a metric space, which guarantees the existence of its fixed point.

Theorem 1.2. [16, Theorem 7] *Let (X, d) be a complete metric space, and let T be a mapping on X. Assume that there exist $r \in [0, 1)$, $a \in [0, 1]$, $b \in [0, 1)$, $(a + b)r^2 + r \leq 1$ if $r \in [\frac{1}{2}, \frac{1}{\sqrt{2}}]$, $a + (a + b)r \leq 1$ if $r \in [\frac{1}{\sqrt{2}}, 1)$ such that*
$ad(x, Tx) + bd(y, Tx) \leq d(x, y)$ implies $d(Tx, Ty) \leq rd(x, y)$, for all $x, y \in X$. Then there exists a unique fixed point z of T. Moreover, $\lim_n T^n x = z$ for all $x \in X$.

The generalized contractions and implicit relations in metric spaces have been considered by several authors in connection with the existence of fixed points (see, for instance, [1, 2, 5] and the references therein). It is interesting to study the existence of fixed points for multi-valued mappings on ultrametric spaces; see for example [7, 11, 12, 15].

The aim of this paper is to prove a general common fixed point theorem for multi-valued mappings in ultrametric spaces. We will introduce a new class of implicit functions to prove a common fixed point theorem for multi-valued mappings in ultrametric spaces.

This result enables us to give a simultaneous generalization of Theorems 1.1 and 1.2 and some well-known fixed point theorems in the literature [6, 8, 13] provided that the underlying space metric is non-Archimedean. By presenting some examples, we will show that our results are genuine generalization of some old results and may fail in usual metric spaces.

2. Preliminaries

In this section, we introduce some preliminary results that will be used in the sequel. We begin by recalling some definitions.

Definition 2.1. *Let X be any nonempty set and let $T : X \to 2^X$ be a multi-valued mapping. A point $z \in X$ is called a fixed point of T if $z \in Tz$, where 2^X denotes the collection of all nonempty subsets of X.*

Let (X, d) be a metric space. Throughout this paper, we assume that $CB(X)$ is the family of all nonempty closed bounded subsets of X.

Definition 2.2. *Let (X, d) be a metric space. For every $A, B \in CB(X)$, the Hausdorff metric $\mathcal{H}$ induced by the metric d of X is defined by*

$$\mathcal{H}(A, B) = \max\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\},$$

where $d(x, B) = \inf_{y\in B} d(x, y)$.

Definition 2.3. *An ultrametric space(a non-Archimedean metric space) [9] is a metric space (X, d) in which the triangle inequality is replaced with*

$$d(x, y) \leq \max\{d(x, z), d(z, y)\}, \quad (x, y, z \in X).$$

In an ultrametric space X, for any sequence $\{x_n\}$, we have

$$d(x_n, x_m) \leq \max\{d(x_{j+1}, x_j) : m \leq j \leq n-1\} \quad (n > m).$$

This implies that $\{x_n\}$ is Cauchy if and only if $\{d(x_{n+1}, x_n)\}$ converges to zero. The following result will be frequently used in the sequel. Since we couldn't find any reference for it, we give it here for the sake of completion.

Proposition 2.4. *If (X, d) be an ultrametric space, then so is $(CB(X), \mathcal{H})$.*

Proof. It follows from the definition that $\mathcal{H}$ is symmetric and $\mathcal{H}(A, B) = 0$ if and only if $A = B$. In order to prove the ultrametric inequality, we will show first that

$$d(x, A) \leq \max\{d(x, y), d(y, A)\}, \quad (x, y \in X; A \in CB(X)).$$

Let $x, y \in X$ and $A \in CB(X)$. If $d(x,y) > d(y,A)$. By the definition, we can find $a_0 \in A$ such that $d(x,y) \geq d(y,a_0)$. Then we have

$$\begin{aligned} d(x,A) &\leq d(x,a_0) \leq \max\{d(x,y), d(y,a_0)\} \\ &= d(x,y) = \max\{d(x,y), d(y,A)\}. \end{aligned}$$

Let $d(x,y) \leq d(y,A)$, then $d(x,y) \leq d(a,y)$ for all $a \in A$. Hence for each $a \in A$,

$$d(x,A) \leq d(x,a) \leq \max\{d(x,y), d(y,a)\} = d(a,y).$$

Therefore $d(x,A) \leq \inf_{a\in A} d(a,y) = d(y,A) = \max\{d(x,y), d(y,A)\}$. Let $A, B, C \in CB(X)$ and take some $a \in A$. If there exists $b \in B$ such that $d(a,b) \leq d(b,C)$, then

$$\begin{aligned} d(a,C) &\leq \max\{d(a,b), d(b,C)\} = d(b,C) \\ &\leq \mathcal{H}(B,C) \leq \max\{\mathcal{H}(A,B), \mathcal{H}(B,C)\}. \end{aligned}$$

Otherwise, $d(a,b) > d(b,C)$ for each $b \in B$. Hence

$$d(a,C) \leq \max\{d(a,b), d(b,C)\} = d(a,b).$$

Therefore

$$d(a,C) \leq \inf_{b\in B} d(a,b) = d(a,B) \leq \mathcal{H}(A,B) \leq \max\{\mathcal{H}(A,B), \mathcal{H}(B,C)\}.$$

Similarly, $d(c,A) \leq \max\{\mathcal{H}(A,B), \mathcal{H}(B,C)\}$ for each $c \in C$ and therefore

$$\mathcal{H}(A,C) \leq \max\{\mathcal{H}(A,B), \mathcal{H}(B,C)\}.$$

□

3. Results

In recent years, many authors utilized implicit functions instead of contraction conditions to prove common fixed point theorems. Implicit functions are proving fruitful due to their unifying power besides admitting new contraction conditions. Altun [2] also proved some results on common fixed points of multi-valued mappings using implicit function. In this section, we generalize the definition of implicit function in [2] for ultrametric space to prove a general common fixed point theorem, which generalizes some known results in the literature.

Definition 3.1. *Let $\mathbb{R}_+$ denote the set of all nonnegative real numbers and $\mathcal{G}$ denote the set of all functions $g : \mathbb{R}_+^6 \to \mathbb{R}_+$ with the following properties:*

$\mathfrak{g}_0$*: $g\left(\liminf_{n\to\infty}(p_n)\right) \le \liminf_{n\to\infty} g(p_n)$ for every sequence $\{p_n\}$ in $\mathbb{R}_+^6$, where $\liminf p_n$ means componentwise $\liminf$.*

$\mathfrak{g}_1$ *: $0 \le t_i \le t_i'$ for $i = 1, 2, 3, 4$ implies that for each $w \in \mathbb{R}_+$,*

$$g(w, t_1, t_2, t_3, t_4, 0) \ge g(w, t_1', t_2', t_3', t_4', 0),$$

and

$$g(w, t_1, t_2, t_3, 0, t_4) \ge g(w, t_1', t_2', t_3', 0, t_4'),$$

$\mathfrak{g}_2$ *: there is a continuous strictly increasing function $\varphi : \mathbb{R}+ \to \mathbb{R}+$ with $\varphi(t) < t$ for $t > 0$ such that the inequalities $0 \le u \le w$ and*

$$g(w, v, v, u, \max\{u, v\}, 0) \le 0 \quad or \quad g(w, v, u, v, 0, \max\{u, v\}) \le 0$$

imply $w \le \varphi(v)$.

Inspired the results of Popescu and Altun, we establish a new type of fixed point theorem in the framework of ultrametric spaces.

Theorem 3.2. *Let (X, d) be a complete ultrametric space and $T, S : X \to CB(X)$ be two mappings such that for some $g \in \mathcal{G}$ and $r > 0$, the sequence $\{\varphi^n(r)\}$ converges to zero, where φ is the function in $\mathfrak{g}_2$. Suppose that for some $0 \le a, b \le 1$ with $a + b \le 1$,*

$$ad(x, Tx) + bd(y, Tx) \le d(x, y) \ \text{or} \ ad(y, Sy) + bd(x, Sy) \le d(x, y)$$

implies that

$$g\left(\mathcal{H}(Tx, Sy), d(x, y), d(x, Tx), d(y, Sy), d(x, Sy), d(y, Tx)\right) \le 0. \qquad (1)$$

Then S and T have a common fixed point z and $Tz = Sz$.

Proof. We divide the proof into four steps.

Step 1. Either $\inf_{x\in X} d(x,Tx)=0$ or $\inf_{x\in X} d(x,Sx)=0$.

Let $\alpha = \inf_{x\in X} d(x,Tx)$ and $\beta = \inf_{x\in X} d(x,Sx)$. Without loss of generality, we may assume that $\alpha \le \beta$. Suppose that $\alpha > 0$ and let φ be the function in $\mathfrak{g}_2$. Then the inequality $\varphi(\alpha) < \alpha$ together with continuity of φ imply that there is some $\varepsilon > 0$ such that $\varphi(t) < \alpha$ for all $t \in [\alpha, \alpha+\varepsilon)$. Choose some $x_0 \in X$ with $\alpha \le d(x_0,Tx_0) < \alpha+\varepsilon$ and find some $x_1 \in Tx_0$ such that $\alpha \le d(x_0,x_1) < \alpha+\varepsilon$. Then

$$d(x_1,Tx_0)=0,\ d(x_0,Tx_0)\le d(x_0,x_1) \text{ and } a\le 1.$$

Hence

$$ad(x_0,Tx_0)+bd(x_1,Tx_0)=ad(x_0,Tx_0)\le d(x_0,Tx_0)\le d(x_0,x_1).$$

By our assumption, we have

$$g\left(\mathcal{H}(Tx_0,Sx_1),d(x_0,x_1),d(x_0,Tx_0),d(x_1,Sx_1),d(x_0,Sx_1),d(x_1,Tx_0)\right)\\ \le 0.$$

Since

$$d(x_0,Tx_0)\le d(x_0,x_1) \text{ and } d(x_0,Sx_1)\le \max\{d(x_0,x_1),d(x_1,Sx_1)\},$$

by $\mathfrak{g}_1$,

$$g(\mathcal{H}(Tx_0,Sx_1),d(x_0,x_1),d(x_0,x_1),d(x_1,Sx_1),\\ \max\{d(x_0,x_1),d(x_1,Sx_1)\},0)\le 0.$$

The above inequality and the fact that $d(x_1,Sx_1)\le \mathcal{H}(Tx_0,Sx_1)$ by $\mathfrak{g}_2$ imply that

$$\mathcal{H}(Tx_0,Sx_1)\le \varphi(d(x_0,x_1))<\alpha.$$

Therefore

$$d(x_1,Sx_1)\le \mathcal{H}(Tx_0,Sx_1)<\alpha.$$

Hence $\beta \le d(x_1,Sx_1) < \alpha$. This contradiction proves our claim.

Step 2. There is a Cauchy sequence $\{x_n\}$ in X such that

$$x_{2n-1}\in Tx_{2n-2} \quad \textit{and} \quad x_{2n}\in Sx_{2n-1} \quad (n>1).$$

By step 1, we may assume that $\inf_{x\in X} d(x,Tx)=0$. Choose some $x_0\in X$ with $d(x_0,Tx_0)<r$ and select some $x_1\in Tx_0$ such that $d(x_0,x_1)<r$. Since $ad(x_0,Tx_0)+bd(x_1,Tx_0)\le ad(x_0,Tx_0)\le d(x_0,Tx_0)\le d(x_0,x_1)$, by our assumption

$$g\left(\mathcal{H}(Tx_0,Sx_1),d(x_0,x_1),d(x_0,Tx_0),d(x_1,Sx_1),d(x_0,Sx_1),d(x_1,Tx_0)\right)\\ \le 0.$$

Since $d(x_1,Tx_0)=0$, the inequalities

$$d(x_0,Tx_0)\le d(x_0,x_1) \text{ and } d(x_0,Sx_1)\le \max\{d(x_0,x_1),d(x_1,Sx_1)\}$$

together with $\mathfrak{g}_1$ imply that

$$g\left(\mathcal{H}(Tx_0,Sx_1),d(x_0,x_1),d(x_0,x_1),d(x_1,Sx_1),\max\{d(x_0,x_1),d(x_1,\\ Sx_1)\},0\right)\le 0.$$

Put

$$w_1=\mathcal{H}(Tx_0,Sx_1),u_1=d(x_1,Sx_1),v_1=d(x_0,x_1).$$

Then

$$g(w_1,v_1,v_1,u_1,\max\{u_1,v_1\},0)\le 0 \text{ and } u_1\le w_1.$$

By $\mathfrak{g}_2$, we obtain

$$d(x_1,Sx_1)\le \mathcal{H}(Tx_0,Sx_1)\le \varphi(d(x_0,x_1))<\varphi(r).$$

Choose $\epsilon_1>0$ such that $d(x_1,Sx_1)+\epsilon_1<\varphi(r)$. Hence there exists $x_2\in Sx_1$ such that $d(x_1,x_2)\le d(x_1,Sx_1)+\epsilon_1<\varphi(r)$. Then we have

$$d(x_2,Sx_1)=0 \text{ and } d(x_1,Sx_1)\le d(x_1,x_2).$$

Therefore

$$ad(x_1,Sx_1)+bd(x_2,Sx_1)\le ad(x_1,Sx_1)\le d(x_1,Sx_1)\le d(x_1,x_2).$$

By our assumption

$$g\left(\mathcal{H}(Tx_2,Sx_1),d(x_1,x_2),d(x_2,Tx_2),d(x_1,Sx_1),d(x_2,Sx_1),d(x_1,Tx_2)\right)\\ \le 0.$$

Since

$$d(x_2,Sx_1)=0,d(x_1,Sx_1)\le d(x_1,x_2)$$

and

$$d(x_1, Tx_2) \leq \max\{d(x_1, x_2), d(x_2, Tx_2)\},$$

by $\mathfrak{g}_1$, we obtain

$$g(w_2, v_2, u_2, v_2, 0, \max\{u_2, v_2\}) \leq 0 \text{ and } u_2 \leq w_2,$$

where

$$w_2 = \mathcal{H}(Tx_2, Sx_1), v_2 = d(x_1, x_2), u_2 = d(x_2, Tx_2).$$

By $\mathfrak{g}_2$, $w_2 \leq \varphi(v_2)$. Therefore

$$d(x_2, Tx_2) \leq \mathcal{H}(Tx_2, Sx_1) \leq \varphi(d(x_1, x_2)) < \varphi^2(r).$$

Similarly, we can choose $\epsilon_2 > 0$ such that $d(x_2, Tx_2) + \epsilon_2 < \varphi^2(r)$. Then select some $x_3 \in Tx_2$ with $d(x_2, x_3) \leq d(x_2, Tx_2) + \epsilon_2$. Hence

$$d(x_2, x_3) \leq d(x_2, Tx_2) + \epsilon_2 < \varphi^2(r).$$

By continuing this procedure, we can inductively find a sequence $\{x_n\}$ in X such that for each $n \in \mathbb{N}$

$$d(x_{2n}, Tx_{2n}) \leq \varphi(d(x_{2n}, x_{2n-1})), d(x_{2n-1}, Sx_{2n-1}) \leq \varphi(d(x_{2n-1}, x_{2n-2})),$$

$$x_{2n-1} \in Tx_{2n-2}, \quad x_{2n} \in Sx_{2n-1}, \quad \text{and } d(x_n, x_{n+1}) < \varphi^n(r).$$

Since $\lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} \varphi^n(r) = 0$, the sequence $\{x_n\}$ is Cauchy.

Step 3. *There is $z \in X$ such that $z \in Tz$ or $z \in Sz$.*

By the completeness of (X, d), $z = \lim_{n\to\infty} x_n$ exists. We claim that for each $n \in \mathbb{N}$ either

$$ad(x_{2n}, Tx_{2n}) + bd(z, Tx_{2n}) \leq d(z, x_{2n})$$

or

$$ad(x_{2n+1}, Sx_{2n+1}) + bd(z, Sx_{2n+1}) \leq d(z, x_{2n+1}).$$

Suppose that for some $n \in \mathbb{N}$,

$$ad(x_{2n+1}, Sx_{2n+1}) + bd(z, Sx_{2n+1}) > d(x_{2n+1}, z)$$

and

$$ad(x_{2n}, Tx_{2n}) + bd(z, Tx_{2n}) > d(x_{2n}, z).$$

By the ultrametric inequality,

$$d(x_{2n}, x_{2n+1}) \leq \max\{d(x_{2n}, z), d(x_{2n+1}, z)\}.$$

If $\max\{d(x_{2n}, z), d(x_{2n+1}, z)\} = d(x_{2n}, z)$, since $a + b \leq 1$ we obtain

$$\begin{aligned} d(x_{2n}, z) &< ad(x_{2n}, Tx_{2n}) + bd(z, Tx_{2n}) \\ &\leq ad(x_{2n}, x_{2n+1}) + bd(z, x_{2n+1}) \\ &\leq ad(x_{2n}, z) + bd(x_{2n}, z) \leq d(x_{2n}, z). \end{aligned}$$

If $\max\{d(x_{2n}, z), d(x_{2n+1}, z)\} = d(x_{2n+1}, z)$, we have

$$\begin{aligned} d(x_{2n+1}, Sx_{2n+1}) &\leq d(x_{2n+1}, x_{2n+2}) \leq \varphi(d(x_{2n}, x_{2n+1})) \\ &< d(x_{2n}, x_{2n+1}) \leq d(x_{2n+1}, z) \end{aligned}$$

and

$$d(z, Sx_{2n+1}) \leq \max\{d(x_{2n+1}, z), d(x_{2n+1}, Sx_{2n+1})\} \leq d(x_{2n+1}, z).$$

Hence

$$\begin{aligned} d(x_{2n+1}, z) &< ad(x_{2n+1}, Sx_{2n+1}) + bd(z, Sx_{2n+1}) \\ &\leq (a + b)d(x_{2n+1}, z) \leq d(x_{2n+1}, z). \end{aligned}$$

Since in each case, we get into a contradiction, our claim is proved. Thus by assumption we have for each $n \in \mathbb{N}$, either

$$g(\mathcal{H}(Tx_{2n}, Sz), d(x_{2n}, z), d(x_{2n}, Tx_{2n}), d(z, Sz), d(x_{2n}, Sz), d(z, Tx_{2n})) \\ \leq 0$$

or

$$g(\mathcal{H}(Tz, Sx_{2n+1}), d(z, x_{2n+1}), d(z, Tz), d(x_{2n+1}, Sx_{2n+1}), d(z, Sx_{2n+1}), \\ d(x_{2n+1}, Tz)) \leq 0.$$

Therefore, one of the following cases happens:

(a) There exists a subsequence $\{x_{n_j}\} \subseteq \{x_n\}$ such that for all $j \in \mathbb{N}$,

$$ad(x_{2n_j}, Tx_{2n_j}) + bd(z, Tx_{2n_j}) \leq d(x_{2n_j}, z).$$

Therefore

$$g(\mathcal{H}(Tx_{2n_j}, Sz), d(x_{2n_j}, z), d(x_{2n_j}, Tx_{2n_j}), d(z, Sz), d(x_{2n_j}, Sz), d(z, Tx_{2n_j})) \leq 0,$$

for all $j > 1$. We have

$$\lim_{j\to\infty} d(x_{2n_j}, Tx_{2n_j}) \leq \lim_{j\to\infty} d(x_{2n_j}, x_{2n_j+1}) = 0$$

and

$$\lim_{j\to\infty} d(x_{2n_j}, Sz) \leq \lim_{j\to\infty} d(x_{2n_j}, z) + d(z, Sz) \leq d(z, Sz).$$

Since $\lim_{j\to\infty} d(z, Tx_{2n_j}) \leq \lim_{j\to\infty} d(z, x_{2n_j+1}) = 0$, by $\mathfrak{g}_0$,

$$g\left(\liminf_{n\to\infty} \mathcal{H}(Tx_{2n_j}, Sz), 0, 0, d(z, Sz), d(z, Sz), 0\right) \leq 0.$$

Put $w = \liminf_{n\to\infty} \mathcal{H}(Tx_{2n_j}, Sz)$, $u = d(z, Sz)$ and $v = 0$. Then

$$d(z, Sz) \leq d(x_{2n_{j+1}}, Sz) + d(x_{2n_{j+1}}, z) \leq \mathcal{H}(Tx_{2n_j}, Sz) + d(x_{2n_j+1}, z).$$

Hence

$$u = d(z, Sz) \leq \liminf_{n\to\infty} \mathcal{H}(Tx_{2n_j}, Sz) + \lim_{n\to\infty} d(x_{2n_j+1}, z) \leq w + 0 = w.$$

By $\mathfrak{g}_2$, $w \leq \varphi(0) = 0$. By the above inequality, $d(z, Sz) \leq w$. Hence $d(z, Sz) = 0$. This means that $z \in Sz$.

(b) There exists a subsequence $\{x_{n_k}\} \subseteq \{x_n\}$ such that for all $k \in \mathbb{N}$

$$ad(x_{2n_k+1}, Sx_{2n_k+1}) + bd(z, Sx_{2n_k+1}) \leq d(x_{2n_k+1}, z).$$

In this case, by our assumption

$$g(\mathcal{H}(Tz, Sx_{2n_k+1}), d(x_{2n_k+1}, z), d(z, Tz), d(x_{2n_k+1}, Sx_{2n_k+1}), \\ d(z, Sx_{2n_k+1}), d(x_{2n_k+1}, Tz)) \le 0.$$

By a similar argument as was used in (a), we can show that

$$g\left(\liminf_{n\to\infty} \mathcal{H}(Tz, Sx_{2n_k+1}), 0, d(z, Tz), 0, 0, d(z, Tz)\right) \le 0$$

and

$$d(z, Tz) \le \liminf_{n\to\infty} \mathcal{H}(Tz, Sx_{2n_k+1}).$$

By $\mathfrak{g}_2$, $\liminf_{n\to\infty} \mathcal{H}(Tz, Sx_{2n_k+1}) \le \varphi(0) = 0$. Therefore $d(z, Tz) = 0$. This means that $z \in Tz$.

Step 4. *$z \in Tz$ or $z \in Sz$ if and only if $Tz = Sz$.*

Let $z \in Tz$, then $ad(z, Tz) + bd(z, Tz) = 0 \le d(z, z)$. By our assumption,

$$g\left(\mathcal{H}(Tz, Sz), d(z, z), d(z, Tz), d(z, Sz), d(z, Sz), d(z, Tz)\right) \le 0.$$

That is

$$g\left(\mathcal{H}(Tz, Sz), 0, 0, d(z, Sz), d(z, Sz), 0\right) \le 0.$$

Let $w = \mathcal{H}(Tz, Sz), v = 0$ and $u = d(z, Sz)$. Then $u \le w$ and $g(w, v, v, u, \max\{u, v\}, 0) \le 0$. By $\mathfrak{g}_2$, $\mathcal{H}(Tz, Sz) \le \varphi(0) = 0$. This means that $Tz = Sz$. A similar argument shows that $Tz = Sz$ if $z \in Sz$. □

Corollary 3.3. *Let (X, d) be a complete ultrametric space and let T be mapping from X into $CB(X)$. Suppose that there exists $g \in \mathcal{G}$ such that $ad(x, Tx) + bd(y, Tx) \le d(x, y)$ implies*

$$g\left(\mathcal{H}(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\right) \le 0$$

for all $x, y \in X$, where $a, b \in [0, 1]$ and $a + b \le 1$. Moreover, assume that $\lim_{n\to\infty} \varphi^n(r) = 0$ for some $r > 0$, where φ is the function in $\mathfrak{g}_2$. Then there exists $z \in X$ such that $z \in Tz$.

Proof. It is enough to take $S = T$ in Theorem 3.2. □

Definition 3.4. Let (X,d) be a metric space and $T: X \to 2^X$ a multi-valued mapping. Then T is called a multi-valued weakly Picard operator if for all $x \in X$ and all $y \in Tx$, there exists a sequence $\{x_n\}$ such that $x_0 = x$, $x_1 = y$, $x_{n+1} \in Tx_n$ for all $n \in \mathbb{N}$ and $\{x_n\}$ converges to a fixed point of T.

We refer the reader to [17] and [18] for further information about Picard and weakly Picard operators.

Corollary 3.5. *Let (X,d) be a complete ultrametric space and let T be mapping from X into $CB(X)$. Suppose that there exists $g \in \mathcal{G}$ such that $ad(x,Tx) + bd(y,Tx) \le d(x,y)$ implies*

$$g\left(\mathcal{H}(Tx,Ty), d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\right) \le 0$$

for all $x, y \in X$, where $a, b \in [0,1]$ and $a+b \le 1$. Moreover, assume that $\lim_{n\to\infty} \varphi^n(r) = 0$ for all $r > 0$, where φ is the function in $\mathfrak{g}_2$. Then T is a multi-valued weakly Picard operator.

Proof. Since the function φ satisfies

$$\lim_{n\to\infty} \varphi^n(t) = 0 \quad \text{for all } t > 0,$$

by omitting step 1 of the proof of the Theorem 3.2 and mimicking the rest of the proof, we can show that T is a multi-valued weakly Picard operator. $\square$

Corollary 3.6. *Let (X,d) be a complete ultrametric space and $T, S: X \to CB(X)$ be two mappings such that for some $g \in \mathcal{G}$ and $r > 0$, the sequence $\{\varphi^n(r)\}$ converges to zero, where φ is the function in $\mathfrak{g}_2$. Suppose that $d(x,Tx) \le d(x,y)$ or $d(y,Sy) \le d(x,y)$ implies that $g\left(\mathcal{H}(Tx,Sy), d(x,y), d(x,Tx), d(y,Sy), d(x,Sy), d(y,Tx)\right) \le 0$. Then there is $z \in X$ such that $z \in Tz = Sz$.*

Proof. Let $a = 1$ and $b = 0$ in Theorem 3.2. $\square$

It is known that in an ultrametric space, every point inside a ball is its center, i.e. if $d(x,y) < r$ then $B(x;r) = B(y;r)$.

In the next result, we use this property to prove a common fixed point theorem for multi-valued mappings with restricted domain.

Theorem 3.7. *Let (X,d) be a complete ultrametric space, $r>0$, $x_0\in X$ and $T,S:B(x_0,r)\to CB(X)$. Let for some $g\in\mathcal{G}$,*

$$x,y\in B(x_0,r) \text{ and } ad(x,Tx)+bd(y,Tx)\le d(x,y) \text{ or } \\ ad(y,Sy)+bd(x,Sy)\le d(x,y)$$

imply that

$$g\left(\mathcal{H}(Tx,Sy),d(x,y),d(x,Tx),d(y,Sy),d(x,Sy),d(y,Tx)\right)\le 0, \tag{2}$$

where $a,b\in[0,1]$ and $a+b\le 1$. If for some $x\in B(x_0;r)$

$$d(x,Tx)<r \quad \text{or} \quad d(x,Sx)<r$$

and $\lim_{n\to\infty}\varphi^n(r)=0$, where φ is the function in $\mathfrak{g}_2$. Then there is $z\in B(x_0;r)$ such that $z\in Tz=Sz$.

Proof. Without loss of generality, we can assume that $d(x,Tx)<r$ for some $x\in B(x_0;r)$. Choose some $x_1\in Tx$ such that $d(x,x_1)<r$. Then $B(x;r)=B(x_0;r)=B(x_1;r)$ and

$$ad(x,Tx)+bd(x_1,Tx)\le ad(x,Tx)\le d(x,Tx)\le d(x,x_1).$$

By our assumption

$$g\left(\mathcal{H}(Tx,Sx_1),d(x,x_1),d(x,Tx),d(x_1,Sx_1),d(x,Sx_1),d(x_1,Tx)\right)\le 0.$$

Put $w_1=\mathcal{H}(Tx,Sx_1)$, $u_1=d(x_1,Sx_1)$ and $v_1=d(x,x_1)$. Since

$$d(x_1,Tx)=0,\ d(x,Tx)\le d(x,x_1) \text{ and} \\ d(x,Sx_1)\le\max\{d(x,x_1),d(x_1,Sx_1)\},$$

from $\mathfrak{g}_1$ we obtain

$$g(w_1,v_1,v_1,u_1,\max\{u_1,v_1\},0)\le 0 \text{ and } u_1\le w_1.$$

By $\mathfrak{g}_2$, we have

$$d(x_1,Sx_1)\le\mathcal{H}(Tx,Sx_1)\le\varphi(d(x,x_1))<\varphi(r).$$

Choose some $\epsilon_1>0$ with $d(x_1,Sx_1)+\epsilon_1<\varphi(r)$ and find some $x_2\in Sx_1$ such that $d(x_1,x_2)\le d(x_1,Sx_1)+\epsilon_1<\varphi(r)$. Then $B(x_2;r)=B(x_1;r)$.

Thus $x_2 \in B(x_0; r)$. Since $ad(x_1, Sx_1) + bd(x_2, Sx_1) \leq d(x_1, x_2)$, by our assumption

$$g\left(\mathcal{H}(Tx_2, Sx_1), d(x_1, x_2), d(x_2, Tx_2), d(x_1, Sx_1), d(x_2, Sx_1),\right.$$

$$\left. d(x_1, Tx_2)\right) \leq 0.$$

From $\mathfrak{g}_1$,

$$g(w_2, v_2, u_2, v_2, 0, \max\{u_2, v_2\}) \leq 0 \text{ and } u_2 \leq w_2,$$

where

$$w_2 = \mathcal{H}(Tx_2, Sx_1), v_2 = d(x_1, x_2), u_2 = d(x_2, Tx_2).$$

So that $w_2 \leq \varphi(v_2)$, by $\mathfrak{g}_2$. That is, $d(x_2, Tx_2) \leq \mathcal{H}(Tx_2, Sx_1) \leq \varphi(d(x_1, x_2)) < \varphi^2(r)$. Similarly, we can find some $x_3 \in Tx_2$ such that $d(x_2, x_3) < \varphi^2(r)$. Hence $B(x_3; r) = B(x_2; r)$. Therefore $x_3 \in B(x_0, r)$.

Continuing this way, we get a sequence $\{x_n\}$ in $B(x_0, r)$ such that

$$x_{2n-1} \in Tx_{2n-2}, \quad x_{2n} \in Sx_{2n-1} \quad \text{and } d(x_n, x_{n+1}) < \varphi^n(r) \quad (n > 1).$$

Since $\lim_{n\to\infty} \varphi^n(r) = 0$, the sequence $\{x_n\}$ is Cauchy. Hence there is $z \in B(x_0, r)$ with $x_n \to z$. Since open balls are closed in an ultrametric space, $z \in B(x_0; r)$.

Using the proofs of Step 3 and Step 4 in the proof of Theorem 3.2, we can show that $z \in Tz = Sz$. □

Corollary 3.8. *Let (X, d) be a complete ultrametric space and let T be mapping from $B(x_0, r)$ into $CB(X)$ for some $x_0 \in X$ and $r > 0$. Suppose that there is $g \in \mathcal{G}$ such that $ad(x, Tx) + bd(y, Tx) \leq d(x, y)$ implies that*

$$g\left(\mathcal{H}(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\right) \leq 0,$$

for all $x, y \in B(x_0, r)$, where $a, b \in [0, 1]$ and $a + b \leq 1$. If $d(x, Tx) < r$ for some $x \in B(x_0; r)$ and $\lim_{n\to\infty} \varphi^n(r) = 0$, then T has a fixed point.

Proof. Take $S = T$ in Theorem 3.7. □

4. Applications

In this section, we give some applications of our main theorem. We show that our results in section 3, enable us to generalize and improve some well-known fixed point theorems. We also present an example to support our results.

In 2009, Moţ and Petruşel proved the following generalization of Nadler's fixed point theorem.

Theorem 4.1. [13, Theorem 6.6] *Let (X, d) be a complete metric space and let T be a mapping from X into $CB(X)$. Assume that there exists $\lambda_1, \lambda_2, \lambda_3 \in [0, 1)$ such that $\lambda_1 + \lambda_2 + \lambda_3 < 1$ and $\frac{1-\lambda_2-\lambda_3}{1+\lambda_1} d(x, Tx) \leq d(x, y)$ implies that*

$$\mathcal{H}(Tx, Ty) \leq \lambda_1 d(x, y) + \lambda_2 d(x, Tx) + \lambda_3 d(y, Ty),$$

for all $x, y \in X$. Then T has a fixed point.

Recently, Gordji et al. proved the following theorem.

Theorem 4.2. [8, Theorem 2.1] *Let (X, d) be a complete metric space, and T be a map from X into $CB(X)$ such that*

$$\mathcal{H}(Tx, Ty) \leq \lambda_1 d(x, y) + \lambda_2 (d(x, Tx) + d(y, Ty)) + \lambda_3 (d(x, Ty) + d(y, Tx))$$

for all $x, y \in X$, where $\lambda_1, \lambda_2, \lambda_3 \geq 0$ and $\lambda_1 + 2\lambda_2 + 2\lambda_3 < 1$. Then, T has a fixed point.

Theorem 3.2, enables us to prove the following results which are simultaneous generalizations of the above results in ultrametric spaces.

Theorem 4.3. *Let (X, d) be a complete ultrametric space and $T, S : X \to CB(X)$ be such that $ad(x, Tx) + bd(y, Tx) \leq d(x, y)$ or $ad(y, Sy) + bd(x, Sy) \leq d(x, y)$ implies that*

$$\mathcal{H}(Tx, Sy) \leq \lambda_1 d(x, y) + \lambda_2 d(x, Tx) + \lambda_3 d(y, Sy) + \lambda_4 d(x, Sy) + \lambda_5 d(y, Tx),$$

where $\lambda_i \geq 0$ for each $1 \leq i \leq 5$ and $\sum_{i=1}^{5} \lambda_i < 1$ and $a, b \in [0, 1]$, with $a + b \leq 1$. Then there is $z \in X$ such that $z \in Tz = Sz$.

Proof. By Theorem 3.2, it is enough to prove that the function

$$g(t_1,\dots,t_6)=t_1-\sum_{i=2}^{6}\lambda_{i-1}t_i,\quad (t_i\geq 0, 1\leq i\leq 6),$$

together with $\varphi(t)=\left(\sum_{i=1}^{5}\lambda_i\right)t$ satisfies $\mathfrak{g}_0$, $\mathfrak{g}_1$ and $\mathfrak{g}_2$. Clearly $\mathfrak{g}_0$ and $\mathfrak{g}_1$ hold. Let $0\leq u\leq w$ and $g(w,v,v,u,\max\{u,v\},0)\leq 0$. If $u\leq v$, then

$$\begin{aligned} w &\leq (\lambda_1+\lambda_2)v+\lambda_3 u+\lambda_4\max\{u,v\}\\ &\leq \sum_{i=1}^{4}\lambda_i v\leq\varphi(v). \end{aligned}$$

If $v<u$, then $w<\sum_{i=1}^{4}\lambda_i u\leq u$, which is a contradiction. Hence in this case $\mathfrak{g}_2$ holds. A similar proof shows that when $u\leq w$ and $g(w,v,u,v,0,\max\{u,v\})\leq 0$, the relation $w\leq\varphi(v)$ holds. Hence g satisfies $\mathfrak{g}_0,\mathfrak{g}_1$ and $\mathfrak{g}_2$. $\square$

Corollary 4.4. *Let (X,d) be a complete ultrametric space and $T:X\to CB(X)$ be such that $ad(x,Tx)+bd(y,Tx)\leq d(x,y)$ implies that*

$$\mathcal{H}(Tx,Ty)\leq\lambda_1 d(x,y)+\lambda_2 d(x,Tx)+\lambda_3 d(y,Ty)+\lambda_4 d(x,Ty)+\lambda_5 d(y,Tx)$$

where $\lambda_i\geq 0$ for each $1\leq i\leq 5$ and $\sum_{i=1}^{5}\lambda_i<1$ and $a,b\in[0,1]$, with $a+b\leq 1$. Then T is a multi-valued weakly Picard operator.

Proof. Let $\varphi(t)=\left(\sum_{i=1}^{5}\lambda_i\right)t$. Since $\lim_{n\to\infty}\varphi^n(t)=0$ for all $t>0$, the result follows from Corollary 3.5 and the proof of Theorem 4.3. $\square$

Corollary 4.5. *Let (X,d) be a complete ultrametric space and T,S be self-mapping on X such that $ad(x,Tx)+bd(y,Tx)\leq d(x,y)$ or $ad(y,Sy)+bd(x,Sy)\leq d(x,y)$ implies that*

$$d(Tx,Sy)\leq\lambda_1 d(x,y)+\lambda_2 d(x,Tx)+\lambda_3 d(y,Sy)+\lambda_4 d(x,Sy)+\lambda_5 d(y,Tx)$$

where $\lambda_i\geq 0$ for each $1\leq i\leq 5$ and $\sum_{i=1}^{5}\lambda_i<1$ and $a,b\in[0,1]$, with $a+b\leq 1$. Then T and S have a unique common fixed point.

Proof. By Theorem 4.3, T and S have a common fixed point. To prove the uniqueness, suppose that z, z' are common fixed points of T and S. Then

$$ad(Tz, z) + bd(z', Tz) = bd(z', z) \leq d(z, z').$$

If $z \neq z'$, by our assumption,

$$\begin{aligned} d(z, z') &= d(Tz, Sz') \leq \lambda_1 d(z, z') + \lambda_2 d(z, Tz) + \lambda_3 d(z', Sz') \\ &+ \lambda_4 d(z, Sz') + \lambda_5 d(z', Tz) = (\lambda_1 + \lambda_4 + \lambda_5)\, d(z, z') < d(z, z') \end{aligned}$$

which is contradiction. Hence $z = z'$. □

The following example is due to Suzuki [19, Theorem 3]. This example shows that Corollary 4.5 and hence Theorem 4.3 is not true in general for complete metric spaces.

Example 4.6. Define a complete subset X of the Euclidean space $\mathbb{R}$ as follows: $X = \{0, 1\} \cup \{x_n : n \in \mathbb{N} \cup \{0\}\}$, where $x_n = (\frac{1}{4})(-\frac{3}{4})^n$ for $n \in \mathbb{N} \cup \{0\}$. Define a mapping T on X by $T0 = 1$, $T1 = x_0$ and $Tx_n = x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$. Clearly, T does not have a fixed point. However,

$$d(x, Tx) \leq d(x, y) \quad \text{implies} \quad d(Tx, Ty) \leq \frac{3}{4} d(x, y).$$

for all $x, y \in X$ [19, Theorem 3].

The following example shows that Theorem 3.2 is a genuine generalization of Theorem 1.1 in ultrametric spaces.

Example 4.7. Let $X = \{a, b, c, e\}$ and $d(a, c) = d(a, e) = d(b, c) = d(b, e) = 1$ and $d(a, b) = d(c, e) = \frac{3}{4}$. It is easy to verify that X is a complete ultrametric space. Define $T : X \to CB(X)$ by $T(a) = T(b) = T(c) = \{a\}$ and $T(e) = \{b\}$. For $r = \frac{3}{4}$, we have $\eta(r) = \frac{4}{7}$. Since $\eta(r)d(c, Tc) = \frac{4}{7} \leq \frac{3}{4} = d(c, e)$ and $\mathcal{H}(Tc, Te) = \frac{3}{4} > \frac{9}{16} = rd(c, e)$, T does not satisfy in assumption in Theorem 1.1. We prove that

$$d(x, Tx) \leq d(x, y) \quad \text{implies} \quad \mathcal{H}(Tx, Ty) \leq \frac{3}{4} d(x, y).$$

for all $x, y \in X$. Since $\mathcal{H}(Ta, Tb) = \mathcal{H}(Ta, Tc) = \mathcal{H}(Tb, Tc) = 0$, we have $\mathcal{H}(Tx, Ty) \leq \frac{3}{4}d(x, y)$ for $x, y \in \{a, b, c\}$. Also,

$$\mathcal{H}(Ta, Te) = \frac{3}{4} \leq \frac{3}{4} = \frac{3}{4}d(a, e) \quad , \quad \mathcal{H}(Tb, Te) = \frac{3}{4} \leq \frac{3}{4} = \frac{3}{4}d(b, e).$$

Note that $d(e, Te) = 1 > \frac{3}{4} = d(c, e)$. We can apply Theorem 4.3 to deduce existence of a fixed point for T. However, Theorem 1.1 can not be used in this example.

$\square$ In 2009, Dhompongsa and Yingtaweesittikul proved the following theorem.

Theorem 4.8. [6, Theorem 4.1] *Let (X, d) be a complete metric space. Suppose that $r \in [0, 1)$ and $T : X \to CB(X)$ are such that*
$\theta(r)d(x, Tx) \leq d(x, y)$ implies

$$\mathcal{H}(Tx, Ty) \leq r \max\{d(x, y), d(x, Tx), d(y, Ty)\}$$

for all $x, y \in X$ and the function $x \to d(x, Tx)$ is lower semicontinuous, where $\theta : [0, 1) \to (\frac{1}{2}, 1]$ is the function

$$\theta(r) = \begin{cases} 1 & 0 < r < \frac{\sqrt{5}-1}{2} \\ (1-r)r^{-2} & \frac{\sqrt{5}-1}{2} \leq r < 2^{-1/2} \\ (1+r)^{-1} & 2^{-1/2} \leq r < 1. \end{cases}$$

Then T has a fixed point.

We give the following extension of Theorem 4.8 for ultrametric spaces.

Theorem 4.9. *Let (X, d) be a complete ultrametric space and $T, S : X \longrightarrow CB(X)$. Assume that there is an increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(t) < t$ for each $t > 0$ and $\{\varphi^n(r)\}$ converges to zero for some $r > 0$. Suppose that*

$$\mathcal{H}(Tx, Sy) \leq \varphi(\max\{d(x, y), d(x, Tx), d(y, Sy), d(x, Sy), d(y, Tx)\})$$

provided that $ad(x, Tx) + bd(y, Tx) \leq d(x, y)$ or $ad(y, Sy) + bd(x, Sy) \leq d(x, y)$, where $a, b \in [0, 1]$ with $a + b \leq 1$. Then T, S have a common fixed point.

Proof. Define $g : \mathbb{R}_+^6 \to \mathbb{R}_+$ by

$$g(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \varphi\left(\max\{t_2, t_3, t_4, t_5, t_6\}\right).$$

We will show that $g \in \mathcal{G}$. Clearly $\mathfrak{g}_0$ and $\mathfrak{g}_1$ hold. Let $0 \le u \le w$ and $g(w, v, v, u, \max\{u, v\}, 0) \le 0$. If $u \le v$, then

$$\begin{aligned} w &\le \varphi\left(\max\{v, v, u, \max\{u, v\}, 0\}\right) \\ &\le \varphi(v). \end{aligned}$$

If $v < u$, then

$$\begin{aligned} w &\le \varphi\left(\max\{v, v, u, \max\{u, v\}, 0\}\right) \\ &\le \varphi(u) < u, \end{aligned}$$

which is a contradiction. Hence in this case $\mathfrak{g}_2$ holds. Similarly, if $u \le w$ and

$$g(w, v, u, v, 0, \max\{u, v\}) \le 0,$$

one can show that $w \le \varphi(v)$. Hence $g \in \mathcal{G}$. From Theorem 3.2, the result follows. $\square$

Corollary 4.10. *Let (X, d) be a complete ultrametric space and $T, S : X \longrightarrow CB(X)$. Assume that there is $0 \le r < 1$ such that*

$$\mathcal{H}(Tx, Sy) \le r \max\{d(x, y), d(x, Tx), d(y, Sy), d(x, Sy), d(y, Tx)\}$$

provided that $ad(x, Tx) + bd(y, Tx) \le d(x, y)$ or $ad(y, Sy) + bd(x, Sy) \le d(x, y)$, where $a, b \in [0, 1]$ such that $a + b \le 1$. Then T and S have a common fixed point.

Proof. Take $\varphi(t) = rt$ in Theorem 4.9. $\square$

Corollary 4.11. *Let (X, d) be a complete ultrametric space and $T : X \longrightarrow CB(X)$. Assume that there is $0 \le r < 1$ such that*

$$\mathcal{H}(Tx, Ty) \le r \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$$

provided that $ad(x, Tx) + bd(y, Tx) \le d(x, y)$, where $a, b \in [0, 1]$ such that $a + b \le 1$. Then T is a multi-valued weakly Picard operator.

Proof. Since $\lim_{n\to\infty} \varphi^n(t) = \lim_{n\to\infty} r^n t = 0$ for all $t > 0$, the result follows from Corollary 3.5 and Corollary 4.10. □

Note that the above result is not true in general for complete metric spaces. In fact, Suzuki proved the following theorem.

Theorem 4.12. [19, Theorem 3] *Define a function θ as in Theorem 4.8. Then for each $r \in [0, 1)$, there exist a complete metric space (X, d) and a mapping T on X such that T does not have a fixed point and*

$$\theta(r)d(x, Tx) < d(x, y) \quad \textit{implies} \quad d(Tx, Ty) \leq rd(x, y),$$

for all $x, y \in X$.

Acknowledgements
The authors would like to thank the referees for their valuable comments and suggestions.

References

[1] S. M. A. Aleomraninejad, Sh. Rezapour, N. Shahzad, On fixed point generalizations of Suzuki's method, *Appl. Math. Lett.*, 24 (2011), 1037-1040.

[2] I. Altun, Fixed point and homotopy results for multivalued maps satisfying an implicit relation, *J. Fixed Point Theory Appl.*, 9 (2011), 125-134.

[3] J. P. Aubin, J. Siegel, Fixed points and stationary points of dissipative multivalued maps, *Proc. Amer. Math. Soc.*, 78 (1980), 391-398.

[4] S. Banach, Sur les opérations dans les ensemble abstraits et leur application aux équations intégrales, *Fund. Math.*, 3 (1922), 133-181.

[5] A. Constantin, A random fixed point to multifunctions, *Stoch. Anal. Appl.*, 12 (1) (1994), 65-73.

[6] S. Dhompongsa, H. Yingtaweesittikul, Fixed point for multivalued mappings and the metric completeness, *Fixed Point Theory Appl.*, 2009, 15 pages, Article ID 972395.

[7] L. Gajic, On ultra metric spaces, *Novi Sad J. Math.*, 31 (2) (2001), 69-71.

[8] M. E. Gordji, H. Baghani, H. Khodaei, M. Ramezani, A generalization of Nadlers fixed point theorem, *J Nonlinear Sci. Appl.*, 3 (2) (2010), 148-151.

[9] J. de Groot, Non-Archimedean metrics in topology, *Proc. Amer. Math. Sot.*, 9 (1956), 948-953.

[10] M. Kikkawa, T. Suzuki, Three fixed point theorems for generalized contraction with constants in complete metric spaces, *Nonlinear Anal.*, 69 (2007), 2942-2949.

[11] W. A. Kirk and N. Shahzad, Some fixed point results in ultrametric spaces, *Topology Appl.*, 159 (2012), 3327-3334.

[12] J. Kubiaczyk and N. Mostafa Ali, A Multivalued Fixed Point Theorem in Non-Archimedean Vector Spaces, *Novi Sad J. Math.*, 26 (2) (1996), 111-116.

[13] G. Moţ, A. Petruşel, Fixed point theory for a new type of contractive multivalued operators, *Nonlinear Anal.* 70 (2009), 3371-3377.

[14] S. B. Nadler, Multi-valued contraction mappings, *Pacific J. Math.*, 30 (1969), 475-488.

[15] C. Petalas and F. Vidalis, A fixed point theorem in non-Archimedaen vector spaces, *Proc. Amer. Math. Soc.*, 118 (1993), 819-821.

[16] O. Popescu, Some generalizations of Suzuki and Edelstein type theorems, *Fixed Point Theory Applications*, 319 (2013), 11 pages.

[17] I. A. Rus, Weakly Picard mappings, *Comment. Math. Univ. Caroline*, 34 (4) (1993), 769-773.

[18] I. A. Rus, A. Petruşel, A. Sîntămărian, Data dependence of the fixed point set of some multivalued weakly Picard operators, *Nonlinear Anal.*, 52 (2003), 1947-1959.

[19] T. Suzuki, A generalized Banach contraction principle that characterizes metric completeness, *Proc. Amer. Math. Soc.*, 136 (5) (2008), 1861-1869.

Zahra Alinejad
Department of Pure Mathematics
PhD student of Mathematics
Ferdowsi University of Mashhad
Mashhad, Iran
E-mail: alinejad_z@yahoo.com

Alireza Kamel Mirmostafaee
Center of Excellence in Analysis on Algebraic Structures
Department of Pure Mathematics
Professor of Mathematics
Ferdowsi University of Mashhad
P. O. Box 1159, Mashhad 91775
Mashhad, Iran
E-mail: mirmostafaei@ferdowsi.um.ac.ir