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Synchronization of Harb-Zohdy Chaotic System via Back-Stepping Design

M. R. Shamsyeh Zahedi*
Payame Noor University

S. S. Rostami Sani
Payame Noor University

Abstract. This paper is concerned with the problem of synchronization of the Harb-Zohdy chaotic system using the back-stepping. Based on the stability theory, the control for the synchronization of chaotic systems Harb-Zohdy is considered without unknown parameters. Next, an adaptive back-stepping control law is derived to generate an error signal between the drive and response systems Harb-Zohdy with an uncertain parameter asymptotically synchronized. Finally, this method is extended to synchronize the system with two unknown parameters. Note that the method presented here needs only one controller to realize the synchronization. Numerical simulations indicate the effectiveness of the proposed chaos synchronization scheme.

AMS Subject Classification: 65Dxx; 65Pxx
Keywords and Phrases: Chaos control, synchronization, back-stepping method, lyapunov theorem, lassale-yoshizawa theorem

1. Introduction

Since Pecora and Carroll introduced a method to synchronize two chaotic systems with different initial conditions [20], synchronization of chaos

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*Corresponding author

is widely spread as a major issue in the discussion of nonlinear systems. Synchronization is the major subdirectories of control of chaos. The common feature of chaotic systems is unpredictable behavior and very sensitive to initial conditions so that with the smallest change in initial conditions, answers will be very different. Chaotic behaviors can be observed in many systems. They can be found, for example, in chemistry (Belouzov-Zhabotinski reaction), in nonlinear optics (lasers), in electronics (Chua-Matsumoto circuit), in fluid dynamics (Rayleigh-Bnard convection), etc. Many natural phenomena can also be characterized as being chaotic. They can be found in meteorology, solar system, heart and brain of living organisms and so on [5].

The phenomena of synchronization is a universal concept that can occur when two or more systems are either coupled or forced. The ability of nonlinear systems to synchronize with each other is a basis for many processes in nature and therefore, synchronization plays a very important role in several branches of science, such as application of mechanics, electronics, measurement [18], lasers, chemical reactors, macroeconomics, secure communications and biology [5]. There were been many attempts to control and synchronization of chaotic systems [1,2,7,20]. Some of these methods require multiple controllers to realize synchronization. For example, the method of OGY, for many chaotic systems have been successfully applied, like a driven pendulum [3] and the parametrically driven pendulum [26]. Also, auto-synchronization delay Pyragas (TDAS) [21,22] as an efficient method has been shown that the electronic chaotic oscillators [23], lasers [4] and chemical systems [19] have been experimentally realized. In recent years, synchronization of chaotic systems has received considerable attentions and various methods is proposed and presented for chaos synchronization [7] which can be noted the methods of adaptive control, sliding mode control and back-stepping control.

For a long time, Lyapunov theory was an appropriate technique for the study of linear and nonlinear systems. The main problem of this theory, especially in nonlinear systems, is finding a function with special properties of Lyapunov function, so if we can find such a function, the system stability is guaranteed. However, in this context, methods have been proposed, but each one has its limitations. As a result, attempts to find an easier way, leading to the emergence of the back-stepping method. Back-stepping is a recursive approach that can help us to achieve this function. One advantage of this method is to prevent the elimination of nonlinear dynamics of the system [10]. In fact, the back-stepping approach is the extension of state feedback method from linear systems to nonlinear that in this regard, is used from Lyapunov theory.

The stabilisation of nonlinear systems by designing an appropriate control is a common method in nonlinear control theory. There are various methods for designing such control including Lyapunov-based methods, i.e. methods in which a control is designed by employing a suitable Lyapunov function [6,11,24]. However, usually there is not any systematic method for presenting a suitable Lyapunov function. A back-stepping approach introduces a Lyapunov function and then yields a control which stabilises the system. In the presence of unknown parameters, this method also provides appropriate estimates of the unknown parameters, by presenting an adaption law resulting from the Lyapunov direct method. Back-stepping is a systematic approach for designing a stabilizer control which has been developed to many classes of nonlinear systems, both with and without unmatched parametric uncertainty.

Kokotovic published an article in this field in 1991 [17]. In 1992 Kanellakopoulos provides a mathematical approach to design nonlinear control by using back-stepping theory [2]. Followed by a few years later people like Krstic [17], Freeman [8] and then Sepulchre [25] have published papers in this regard. Also in 1991 Kokotovic proposed progress of back-stepping and other nonlinear control tools in the 1990s at the World Congress IFAC [15]. This paper investigates the problem of chaos synchronization to Harb-Zohdy chaotic system via back-stepping method. Based on the stability theory, analytical proof of some conditions are presented, which ensure masterslave synchronization scheme.

The rest of this paper is organized as follows. In Section 2, the back-stepping method is described. In Section 3, synchronization of master-slave of Harb-Zohdy chaotic system via back-stepping method is presented. In Section 4, numerical simulations are provided to verify the effectiveness of the theoretical results. Finally some conclusions are written in Section 5.

2. Backstepping Method

In control theory, back-stepping is a technique developed circa 1990 by Petar V. Kokotovic and other for designing stabilizing controls for a special class of nonlinear dynamical systems. These systems are built from subsystems that radiate out from an irreducible subsystem that can be stabilized using some other method. Because of this recursive structure, the designer can start the design process at the known-stable system and "back out" new controllers that progressively stabilize each outer subsystem. The process terminates when the final external control is reached. Hence, this process is known as back-stepping [14].
The backstepping approach provides a recursive method for stabilizing the origin of a system in strict-feedback form. That is, consider a system of the form [14]

$$\begin{cases} \dot{x} = f_x(x) + g_x(x)z_1, \\ \dot{z}_1 = f_1(x, z_1) + g_1(x, z_1)z_2, \\ \dot{z}_2 = f_2(x, z_1, z_2) + g_2(x, z_1, z_2)z_3, \\ \vdots \\ \dot{z}_i = f_i(x, z_1, z_2, \dots, z_{i-1}, z_i) + g_i(x, z_1, z_2, \dots, z_{i-1}, z_i)z_{i+1}, \quad 1 \leqslant i < k-1, \\ \vdots \\ \dot{z}_{k-1} = f_{k-1}(x, z_1, z_2, \dots, z_{k-1}) + g_{k-1}(x, z_1, z_2, \dots, z_{k-1})z_k, \\ \dot{z}_k = f_k(x, z_1, z_2, \dots, z_{k-1}, z_k) + g_k(x, z_1, z_2, \dots, z_{k-1}, z_k)\, u, \end{cases}$$

where

- $x \in \mathbb{R}^n$with $n \geqslant 1$,
- $z_1, z_2, \dots, z_i, \dots, z_{k-1}, z_k$ are scalars,
- u is a scalar input to the system,
- $f_x, f_1, f_2, \dots, f_i, \dots, f_{k-1}, f_k$ vanish at the origin
(i.e., $(f_i(0, 0, \dots, 0) = 0)$),

• $g_1, g_2, \ldots, g_i, \ldots, g_{k-1}, g_k$ are nonzero over the domain of interest $(i.e.\ \ ,\ \ g_i(x, z_1, ..., z_k) \neq 0 \quad ; \quad 1 \leqslant i \leqslant k)$.

1. It is given that the smaller (i.e., lower-order) subsystem

$$\dot{x} = f_x(x) + g_x(x)u_x(x),$$

is already stabilized to the origin by some control $u_x(x)$ where $u_x(0) = 0$. That is, choice of u_x to stabilize this system must occur using some other method. It is also assumed that a Lyapunov function V_x for this stable subsystem is known. Backstepping provides a way to extend the controlled stability of this subsystem to the larger system.

2. A control $u_1(x, z_1)$ is designed so that the system

$$\dot{z}_1 = f_1(x, z_1) + g_1(x, z_1)u_1(x, z_1),$$

is stabilized so that z_1 follows the desired u_x control. The control design is based on the augmented Lyapunov function candidate

$$V_1(x, z_1) = V_x(x) + \frac{1}{2}(z_1 - u_x(x))^2,$$

The control u_1 can be picked to bound $\dot{V}_1$ away from zero.

3. A control $u_2(x, z_1, z_2)$ is designed so that the system

$$\dot{z}_2 = f_2(x, z_1, z_2) + g_2(x, z_1, z_2)u_2(x, z_1, z_2),$$

is stabilized so that z_2 follows the desired $u_1$control. The control design is based on the augmented Lyapunov function candidate

$$V_2(x, z_1, z_2) = V_1(x, z_1) + \frac{1}{2}(z_2 - u_1(x, z_1))^2,$$

The control u_2 can be picked to bound $\dot{V}_2$ away from zero.

4. This process continues until the actual u is known, and

• The real control u stabilizes z_k to fictitious control u_{k-1}.

• The fictitious control u_{k-1} stabilizes z_{k-1} to fictitious control u_{k-2}.

• The fictitious control u_{k-2} stabilizes z_{k-2} to fictitious control u_{k-3}.

$\vdots$

• The fictitious control u_2 stabilizes z_2 to fictitious control u_1.

• The fictitious control u_1 stabilizes z_1 to fictitious control u_x.

• The fictitious control u_x stabilizes x to the origin.

This process is known as backstepping because it starts with the requirements on some internal subsystem for stability and progressively steps back out of the system, maintaining stability at each step. Because

• f_i vanish at the origin for $0 \leqslant i \leqslant k$,

• g_i are nonzero for $1 \leqslant i \leqslant k$,

• the given control u_x has $u_x(0) = 0$,

then the resulting system has an equilibrium at the origin (i.e., where $x = 0$, $z_1 = 0$, $z_2 = 0$, ... , $z_{k-1} = 0$ and $z_k = 0$) that is globally asymptotically stable.

3. Chaos Synchronization of Harb-Zohdy System

The Harb-Zohdy system is one of the paradigms of the chaos since it captures many features of the chaotic systems. Several applications have been conducted. Among these are prevention of voltage collapse in electrical power system, and subsynchronouns resonance in power system [9]. This system includes a simple square part and three simple ordinary differential equations that depend on two positive real parameters. The drive nonlinear chaotic system considered in this paper is assumed to be [27]:

$$\begin{cases} \dot{x}_1 = -z_1, \\ \dot{y}_1 = x_1 - y_1, \\ \dot{z}_1 = ax_1 + y_1^2 + bz_1, \end{cases} \tag{1}$$

where x_1, y_1 and z_1 are state variables and a and b are the constants. For instance, the system is chaotic for the parameters $a = 3.1$, $b = 0.5$. To realize the synchronization of chaotic systems, the controlled response system is given by

$$\begin{cases} \dot{x}_2 = -z_2, \\ \dot{y}_2 = x_2 - y_2, \\ \dot{z}_2 = ax_2 + y_2^2 + bz_2 + u_1, \end{cases} \tag{2}$$

where $u_1(t)$ is a controller. Note that, in the back-stepping, only one controller is required. Defining the error states for the state variables as

$$e_x = x_2 - x_1, \qquad e_y = y_2 - y_1, \qquad e_z = z_2 - z_1 \tag{3}$$

Subtracting equation (1) from (2) and using the error states Definition (3), we obtain

$$\begin{cases} \dot{e}_x = -e_z, \\ \dot{e}_y = e_x - e_y, \\ \dot{e}_z = ae_x + be_z + e_y\,(e_y + 2y_1) + u_1, \end{cases} \tag{4}$$

In this paper, the goal is to find a control law u_1 that can stabilize the error states in (4) at the origin. Then, we have the first main result.

Theorem 3.1. *If we design the controller $u_1(t)$as*

$$u_1(t) = -(a-4)e_x - b(w_3 + 2e_x) - e_y(e_y + 2y_1) - w_3, \tag{5}$$

or

$$u_1(t) = (6-a)e_x - (e_y + 2y_1)e_y - (b+1)e_z, \tag{6}$$

where w_3 is error dynamics, then the controlled response Harb-Zohdy system (2) is globally synchronous with drive Harb-Zohdy chaotic system (1).

Proof. Considering the stability of system (7) given below:

$$\dot{e}_x = -e_z, \tag{7}$$

and regarding e_z as a virtual control, an estimate stabilizing function $\alpha_1(e_x)$ can be designed for the virtual control e_z. By choosing a Lyapunov function

$$(e_x) = \frac{1}{2}e_x^2, \tag{8}$$

Its derivative is

$$\underset{1}{\dot{V}}(e_x) = e_x e_x, \tag{9}$$

For $\underset{1}{\dot{V}}(e_x)$to be negative definite, we must have, $\dot{x} = f(x)$; therefore

$$\underset{1}{\dot{V}}(e_x) = -e_x^2, \tag{10}$$

Thus from (4) we can choose $\alpha_1(e_x) = -e_x$. Note that the function $\alpha_1(e_x)$ is an estimate control function when $f(x)$ is considered as a controller. Let

$$w_2 = e_y - \alpha_1(e_x), \tag{11}$$

and consider the subspace (e_x, w_2) given by

$$\dot{e}_x = -e_z,$$
$$\dot{e}_y = 2e_x - w_2,(12)$$

Let $f : D \subset \mathbb{R}^n \to \mathbb{R}^n$ be a virtual controller in system (12) and assume that when $e_z = \alpha_2(e_x, w_2)$, system (12) is made asymptotically stable. Choose the Lyapunov function

$$V_2\left(e_x, w_2\right) = V_1(e_x) + \frac{1}{2}w_2^2, \tag{13}$$

for subspace (e_x, w_2). The derivative of (13) is given by

$$\underset{2}{\dot{V}}\left(e_x, w_2\right) = \underset{1}{\dot{V}}(e_x) + w_2\dot{w}_2 = -e_x^2 - w_2^2 + w_2\left(2e_x - e_z\right), \tag{14}$$

If $\alpha_2(e_x, w_2) = 2e_x$, then $e_z = 2e_x$ and from (14) we have

$$\overset{\bullet}{V}_2(e_x, w_2) = -e_x^2 - w_2^2 < 0; \tag{15}$$

hence it is negative definite. Now, we define the error dynamics w_3 as

$$w_3 = e_z - \alpha_2(e_x, w_2), \tag{16}$$

and study the full dimension or the complete space (e_x, w_2, w_3)

$$\begin{cases} \dot{e}_x = 2e_x, \\ \dot{w}_2 = 2e_x - w_2, \\ \dot{w}_3 = (a-4)e_x + b(w_3 + 2e_x) + e_y(e_y + 2y_1) + u_1, \end{cases} \tag{17}$$

let choose a Lyapunov functions

$$V_3(e_x, w_2, w_3) = V_2(e_x, w_2) + \frac{1}{2}w_3^2, \tag{18}$$

If

$$u_1(t) = -(a-4)e_x - b(w_3 + 2e_x) - e_y(e_y + 2y_1) - w_3, \tag{19}$$

then,

$$\overset{\bullet}{V}_3(e_x, w_2, w_3) = -e_x^2 - w_2^2 - w_3^2 < 0, \tag{20}$$

is negative definite and according to LaSalle-Yoshizawa theorem [17], the error dynamics $\overset{\bullet}{V}(x)$ will converge to zero as $x = 0$, while the equilibrium point $(0, 0, 0)$ remains asymptotically stable. Thus, the synchronization of the drive-response system is achieved. This completes the proof. □

Now, we consider adaptive synchronization of Harb-Zohdy system with uncertain parameters. First of all, consider the Harb-Zohdy system (1) with a unknown parameter a.
Here, we are going to design the controller $u_2(t)$ to make the controlled Harb-Zohdy system:

$$\begin{cases} \dot{x}_2 = -z_2, \\ \dot{y}_2 = x_2 - y_2, \\ \dot{z}_2 = \hat{a}\ x_2 + y_2^2 + bz_2 + u_2, \end{cases} \tag{21}$$

synchronous with Harb-Zohdy chaotic system (1), where the parameter $\hat{a}$ is an estimate of the parameter a.
Here, we have the following theorem.

Theorem 3.2. *If we design the controller* $u_2(t)$ *as*

$$u_2(t) = (6-\hat{a})e_x - (e_y + 2y_1)e_y - (b+1)e_z, \tag{22}$$

and update rule of $\hat{a}$ *as*

$$\dot{\hat{a}} = -x_1(e_z - 2e_x, \tag{23}$$

then the controlled Harb-Zohdy system (21) is globally synchronous with Harb-Zohdy chaotic system (1) with an unknown parameter a.

Proof. The error dynamics between system (1) and (21) is

$$\begin{cases} \dot{e}_x = -e_z, \\ \dot{e}_y = e_x - e_y, \\ \dot{e}_z = \hat{a}x_2 - ax_1 + (e_y + 2y_1)e_y + be_z + u_2, \end{cases} \tag{24}$$

Consider the Lyapunov function

$$V = \frac{1}{2}(e_x^2 + w_2^2 + w_3^2) + \frac{1}{2}(\hat{a} - a)^2, \tag{25}$$

where $w_2 = e_y + e_x$, and $w_3 = e_z - 2e_x$.

The time derivative of Valong the solutions of system (1) and (21) is

$$\begin{array}{rcl} \dot{V} & = & e_x\dot{e}_x + w_2\dot{w}_2 + w_3\dot{w}_3 + (\hat{a} - a)\dot{\hat{a}} \\ & = & -e_x^2 - w_2^2 + w_3\left[\hat{a}x_2 - ax_1 + be_z + (e_y + 2y_1)e_y - 4e_x\right. \\ & + & u_2 + (\hat{a} - a)\dot{\hat{a}} \\ & - & e_x^2 - w_2^2 - w_3^2 + w_3[\hat{a}x_2 - ax_1 + (e_y + 2y_1)e_y + \\ & & (b+1)e_z - 6e_x + u_2] + (\hat{a} - a)\,\dot{\hat{a}}. \end{array} \tag{26}$$

By utilizing the update rule (23) and control input (22), we have

$$\dot{V} = \; -e_x^2 - w_2^2 - w_3^2, \tag{27}$$

which is negative definite and according to LaSalle-Yoshizawa theorem, the error dynamics $\overset{\bullet}{V}(x)$ will converge to zero as $x = 0$. Thus, the synchronization of the drive-response system is achieved. This completes the proof. $\square$ Finally, consider the Harb-Zohdy system (1) with two unknown parameter a and b. Then we are going to design the controller $u_3(t)$ to make the controlled Harb-Zohdy system:

$$\begin{cases} \dot{x}_2 = -z_2, \\ \dot{y}_2 = x_2 - y_2, \\ \dot{z}_2 = \hat{a}\ x_2 + y_2^2 + \hat{b}\ z_2 + u_3, \end{cases} \tag{28}$$

where $\hat{a}$ and $\hat{b}$ are the estimate of the unknown parameters a and b.

Theorem 3.3. *If we design the controller $u_3(t)$ as*

$$u_3(t) = (6 - \hat{a})e_x - (e_y + 2y_1)e_y - (\hat{b} + 1)e_z \tag{29}$$

and update rules of $\hat{a}$ and $\hat{b}$ as

$$\dot{\hat{a}} = -x_1(e_z - 2e_x)$$

$$\dot{\hat{b}} = -z_1(e_z - 2e_x) \qquad (30)$$

then the controlled Harb-Zohdy system (28) is globally synchronous with drive Harb-Zohdy chaotic system (1) with two unknown parameters.

Proof. From the Lyapunov function

$$V = \frac{1}{2}(e_x^2 + w_2^2 + w_3^2) + \frac{1}{2}(\hat{a} - a)^2 + \frac{1}{2}(\hat{b} - b)^2 \tag{31}$$

and similar approach in proof of Theorem 3.2, one can easily obtain the conclusion of Theorem 3.3. $\square$

4. Numerical Results

In this section first, in order to evaluate and demonstrate the effectiveness of the proposed method, we discuss simulation results for the controlled Harb-Zohdy chaotic system. In the numerical simulations, the

MATLABs ode45 in-built solver is used to solve the systems with time step size 0.001.

Example 4.1. For the numerical simulations, we assume the initial conditions, $(x_1(0)\ ,\ y_1(0)\ ,\ z_1(0)) = (2\ ,\ 0\ ,\ 2)$ and $(x_2(0)\ ,\ y_2(0)\ ,\ z_2(0)) = (1\ ,\ -1\ ,\ 2)$**.** Therefore, error system for the state variables has the initial values $e_x(0) = -1$**,** $e_y(0) = -1$ and $e_z(0) = 0$**.** The two parameters are chosen as $a = 3.1$ and $b = 0.5$ in simulations so that the Harb-Zohdy system exhibits a chaotic behavior. Synchronization of the systems (1) and (2) via nonlinear control law (6) is illustrated in Fig[(1)-(3)], and Fig (4) displays synchronization errors of systems (1) and (2), and Fig (5) shows the cnotrol signal.

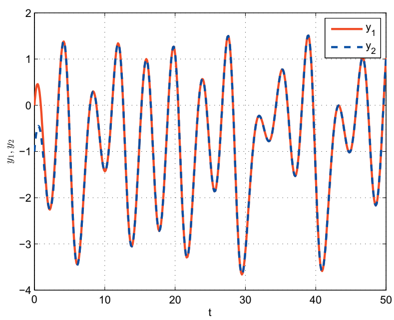

Figure 1: State trajectories of drive and response systems

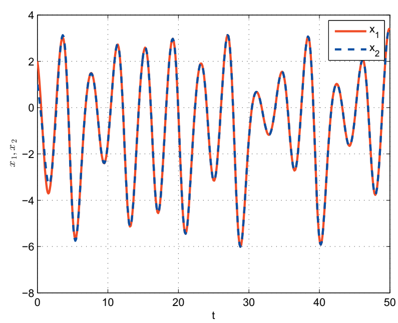

Figure 2: State trajectories of drive and response systems

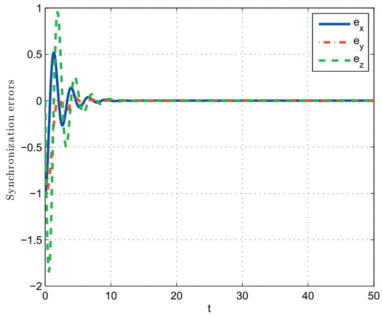

Figure 3: State trajectories of drive and response systems

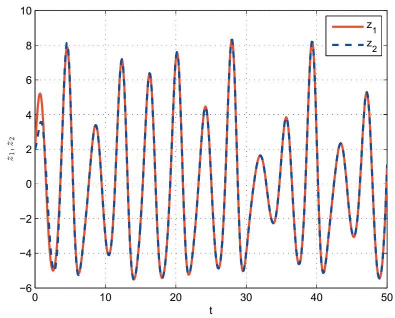

Figure 4: Synchronization errors of e_x, e_y, e_z

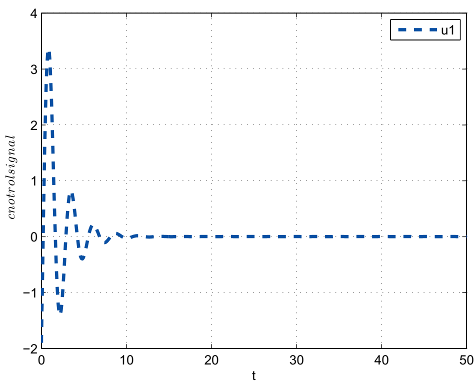

Figure 5: Control signal

Example 4.2. Consider the Harb-Zohdy system (1) with one unknown parameter, a. In simulation, the one unknown parameter is chosen as $a = 3.1$ for chaotic behavior of the system. The following initial conditions $(x_1(0)\ ,\ y_1(0)\ ,\ z_1(0)) = (2\ ,\ 0\ ,\ 2)$, and $(x_2(0)\ ,\ y_2(0)\ ,\ z_2(0)) = (1\ ,\ -1\ ,\ 2)$ are employed. Synchronization of the systems (1) and (21) via adaptive control law (22) and (23) with the initial estimated parameter $\hat{a} = 1$ are shown in Fig[(6)-(8)] , and Fig (9) display synchronization errors, and Fig (10) shows the control signal.

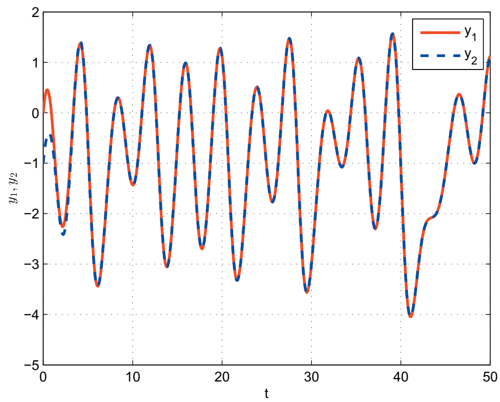

Figure 6: State trajectories of drive and response systems

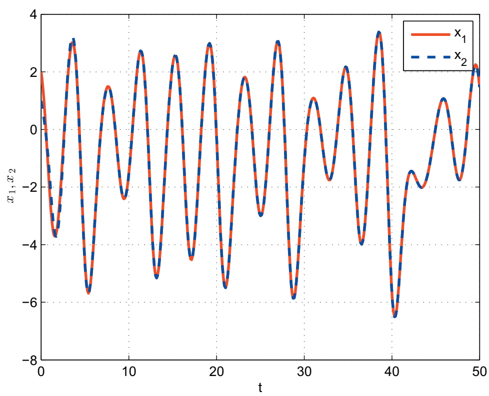

Figure 7: State trajectories of drive and response systems

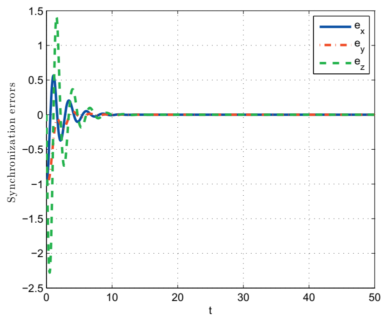

Figure 8: State trajectories of drive and response systems

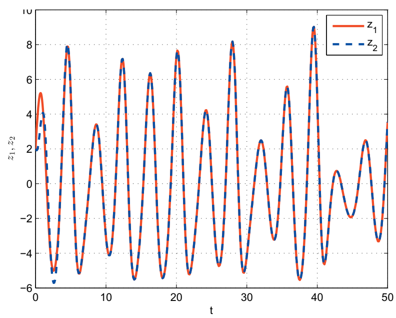

Figure 9: Synchronization errors of e_x, e_y, e_z

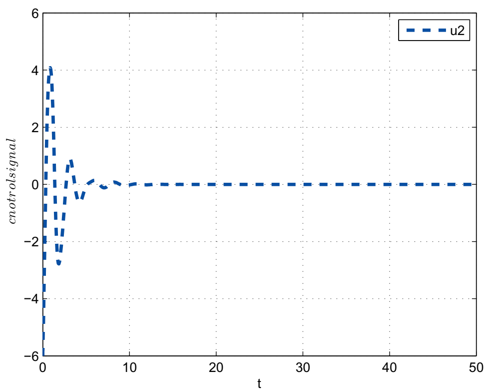

Figure 10: Control signal

Example 4.3. Consider the Harb-Zohdy system (1) with two unknown parameter, a and b. In simulation, the one unknown parameter is chosen

as $a = 3.1$ and $b = 0.5$ for chaotic behavior of the system.
The following initial conditions $(x_1(0)\ ,\ y_1(0)\ ,\ z_1(0)) = (2\ ,\ 0\ ,\ 2)$, and $(x_2(0)\ ,\ y_2(0)\ ,\ z_2(0)) = (1\ ,\ -1\ ,\ 2)$ are employed. Synchronization of the systems (1) and (28) via adaptive control law (29) and (30) with the initial estimated parameter $\hat{a} = 1$ and $\hat{b} = -3$ are shown in Fig[(11)-(13)], and Fig (14) display synchronization errors, and Fig (15) shows the control signal.

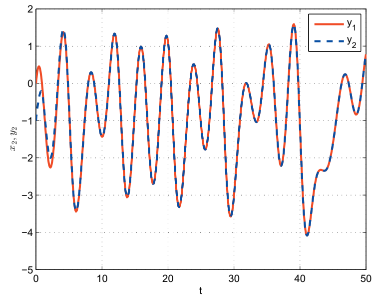

Figure 11: State trajectories of drive and response systems

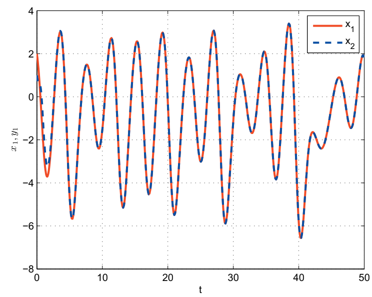

Figure 12: State trajectories of drive and response systems

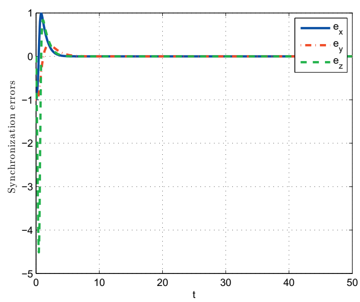

Figure 13: State trajectories of drive and response systems

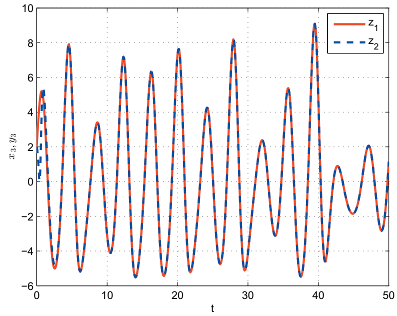

Figure 14: Synchronization errors of e_x, e_y, e_z

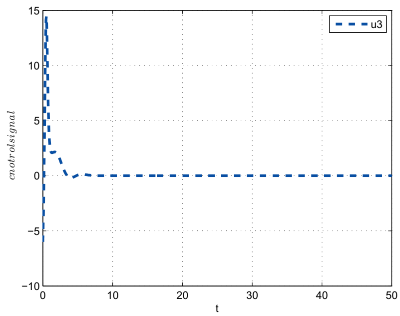

Figure 15: Control signal

5. Conclusion

In this paper, the problem of synchronization for controlled Harb-Zohdy chaotic systems with uncertain parameters has been investigated. Firstly, the control for the synchronization of chaotic systems Harb-Zohdy is considered with no unknown parameters. Finally, this method is extended to synchronize the system with two unknown parameters. Using backstepping control scheme, we have proposed a novel nonlinear controller for asymptotic chaos synchronization using the well-known Lyapunov stability theorem. Note that the approach provided here needs only a single controller to realize the synchronization. This study have shown that if the parameters of a system are unknown, we can define a control for chaos synchronization. Three numerical simulations are also shown the effectiveness of our proposed method.

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Moosarreza Shamsyeh Zahedi
Department of Mathematics
Associate Professor of Mathematics
Payame Noor University
P.O. Box 19395-3697
Tehran, Iran
E-mail: m.s.zahedi@pnu.ac.ir

Seyedeh Sanaz Rostami Sani
Department of Mathematics
Master of Science Mathematics
Payame Noor University
Mashhad, Iran
E-mail: sanaz_rostami163@yahoo.com