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Original Research Paper

On Neighborhood Dimension and Wiener Index of Prime Graph $PG_2(\mathbb{Z}_{2^n p^m})$

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Abstract. Suppose that $PG_2(\mathbb{Z}_{2^n p^m})$ is the prime graph with the vertex set of the finite ring $\mathbb{Z}_{2^n p^m}$, where p is a prime number greater than two and n, m are positive integers. In this paper, we decompose $PG_2(\mathbb{Z}_{2^n p^m})$ and obtain neighborhood metric dimension and Wiener index of the graph.

AMS Subject Classification: M05C12; 05C75; 05E30; 05C09
Keywords and Phrases: Distance, metric dimension, neighborhood metric dimension, Wiener index

1 Introduction

All graphs considered in this paper are connected, simple, undirected and finite. Let $G(V, E)$ be a graph with vertex set V and edge set E. For graph theoretic terminology we refer to [3]. We say that a vertex

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$u \in V$ distinguishes two vertices $x, y \in V$ if $d(u,x) \neq d(u,y)$, where $d(x,y)$ represents the length of a shortest path between x and y in G. A *metric generator* for G is a set $B \subseteq V$ with the property that for each pair of vertices $x, y \in V$, there exists a vertex $u \in B$ that distinguishes x and y. A set A is called a *metric basis* for G if $|A| = min\{|B| : B$ is a metric generator for $G\}$, and in this case, $dim(G) = |A|$ is the *metric dimension* of G.

Harary and Melter [4] in 1976 introduced the concept of *resolving set* of a graph and calculated the metric dimension of a tree graph. Since then it has been widely used in graph theory, chemistry, biology, robotics and many other disciplines. The concept of *neighborhood number* of a graph was introduced in 1985 by Sampathkumar [6]. After more than twenty years, in 2018, a group of authors [7] studied on one class of *neighborhood resolving set* of a graph. They continued by neighborhood resolving sets of a graph [9] and studied the graphs of neighborhood metric dimension two [8].

For a non-zero commutative ring R, let $Z(R)$ be the set of zero-divisors of R. In [5], Pirzada and Altaf introduced an extended zero-divisor graph whose vertices are the non-zero zero-divisors of a ring R and two distinct elements x and y in the set $Z^*(R) = Z(R) \setminus \{0\}$ are adjacent if and only if $xy = 0$ or $x + y \in Z(R)$. They characterized finite commutative rings whose extended zero-divisor graph have clique number 1 or 2. The total graph of R denoted by $T(\Gamma(R))$, is an undirected graph with all elements of R as vertices and two distinct vertices x and y are adjacent if and only if $x+y \in Z(R)$ [1]. We define the *prime graph* a simple undirected graph, denoted by $PG_2(R)$, with all non-zero elements of R as vertices, and two vertices x, y are adjacent if and only if either $xy = 0$ or $x + y \in Z(R)$. By definitions, it is clear that the total graph $T(\Gamma(R))$ is a spanning subgraph of $PG_2(R)$. Throughout, we assume that p is an odd prime and n, m are positive integers. We will use the results obtained on the metric dimension of the total graph $T(\Gamma(\mathbb{Z}_{2^n p^m}))$ in [2], and investigate the neighborhood metric dimension and the Wiener index of the prime graph $PG_2(\mathbb{Z}_{2^n p^m})$ according to a decomposition of it.

2 A Classification and Decomposition for $PG_2(\mathbb{Z}_{2^np^m})$

We know that $Z(\mathbb{Z}_{2^np^m})$ is not an ideal of $\mathbb{Z}_{2^np^m}$. So, by [1], $T(\Gamma(\mathbb{Z}_{2^np^m}))$ is a connected graph with $diam(T(\Gamma(\mathbb{Z}_{2^np^m}))) = 2$ and $girth(T(\Gamma(\mathbb{Z}_{2^np^m}))$ $= 3$. In [10], the authors decomposed the total graph $T(\Gamma(\mathbb{Z}_{2^np^m}))$ into some complete and complete bipartite graphs, as follows. In [2], the authors also studied the metric dimension of this total graph and showed that $dim(T(\Gamma(\mathbb{Z}_{2^np^m}))) = 2^np^m - 2p$.

Theorem 2.1. *(See [10]) For all $n, m \geq 1$ and $p \geq 3$, we have the following decompositions;*

(i) $T(\Gamma(\mathbb{Z}_{2p})) = 2K_p + pK_{1,1}$.

(ii) $T(\Gamma(\mathbb{Z}_{2^np})) = 2K_{2^{n-1}p} + pK_{2^{n-1},2^{n-1}}$.

(iii) $T(\Gamma(\mathbb{Z}_{2^np^m})) = 2K_{2^{n-1}p^m} + pK_{2^{n-1}p^{m-1},2^{n-1}p^{m-1}}$.

Remark 2.2. According to Theorem 2.1, $T(\Gamma(\mathbb{Z}_{2^np^m}))$ is decomposed into $p+2$ subgraphs; p complete bipartites $K_{2^{n-1}p^{m-1},2^{n-1}p^{m-1}}$, and two complete graphs $K_{2^{n-1}p^m}$. Let V_{even} and V_{odd} denote the even and odd vertices of G, respectively. We assume that $S_0 = \{2kp; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$, $T_0 = \{(2k+1)p; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$, $I = \{1, \ldots, [\frac{p-1}{2}]\}$ and $J = \{[\frac{p-1}{2}] + 1, \ldots, p-1\}$. For $i \in I$, let $S_i = \{2kp + 2^i; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$, $T_i = \{(2k+1)p - 2^i; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$ and for $j \in J$, set $S_j = \{2kp - 2^j; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$ and $T_j = \{(2k+1)p + 2^j; \quad k = 0, \ldots, 2^{n-1}p^{m-1} - 1\}$. Then, $V_{even} = S_0 \cup S_i \cup S_j$ and $V_{odd} = T_0 \cup T_i \cup T_j$. Further, $|V_{even}| = |V_{odd}| = 2^{n-1}p^m$ and for all $0 \leq i \leq p-1$, $|S_i| = |T_i| = 2^{n-1}p^{m-1}$.

By this classification, the subgraphs induced by V_{even} and V_{odd} are complete graphs $K_{2^{n-1}p^m}$. Also, for $0 \leq i \leq p-1$, (S_i, T_i) is a partition of the vertex set of each complete bipartite subgraph $K_{2^{n-1}p^{m-1},2^{n-1}p^{m-1}}$.

Throughout the paper, we use the notations of Remark 2.2.

Lemma 2.3. *For the prime graph $G = PG_2(\mathbb{Z}_{2^np^m})$ the followings hold.*

(i) $d(0, y) = 1$ *for any* $0 \neq y \in V(G)$.

(ii) *For* $0 \leq i, j \leq p-1$,

- *(a)* *If* $x \in S_i$, $y \in S_j$, *then* $d(x, y) = 1$;
- *(b)* *If* $x \in T_i$, $y \in T_j$, *then* $d(x, y) = 1$.

(iii) *For any* $x \in S_i$ *and* $y \in T_j$,

$$d(x, y) = \begin{cases} 1 & i = j, \\ 2 & i \neq j \end{cases}$$

where $0 \leq i, j \leq p-1$.

(iv) *For* $A = \{k2^n; \quad 1 \leq k \leq p^m - 1, p \nmid k\}$, $B = \{y \in T_0; \quad y = (2k+1)p^m, 0 \leq k \leq 2^{n-1} - 1\}$, $d(x, y) = 1$ *for any* $x \in A$, $y \in B$.

Proof.

(i) It is obvious.

(ii) By Remark 2.2, it is clear that for (a), the subgraph induced by S_i's is complete. Similarly, for (b), the subgraph induced by T_i's is complete.

(iii) Let $1 \leq i \leq p-1$, and $0 \neq x \in S_i$, $y \in T_i$. Then $x = 2kp + 2^i \in S_i$, $y = (2k+1)p - 2^i \in T_i$, so, $x + y = (4k+1)p \in Z(G)$ and $d(x, y) = 1$. Moreover, if $x \in S_i$ and $y \in T_j$, such that $i \in I$, $j \in J$ and $i \neq j$, then x is not adjacent to y and since $diam(G) = 2$, then $d(x, y) = 2$.

(iv) Let $x \in A$, $y \in B$, then $x = k2^n$, $y = (2k' + 1)p^m$. So, $xy = 0$ and $d(x, y) = 1$.

□

Theorem 2.4. *For all* $p \geq 3$, *we have*

$$PG_2(\mathbb{Z}_{2p}) = 2K_p + pK_{1,1} + 2K_{1,p-1}.$$

Proof. It is easy to see that $T(\Gamma(\mathbb{Z}_{2p}))$ is an spanning subgraph of $PG_2(\mathbb{Z}_{2p})$. So, according to part (i) of Theorem 2.1, there exist two complete graphs K_p with even and odd vertices and p distinct pairs of even-odd vertices in $T(\Gamma(\mathbb{Z}_{2p}))$. The rest of the edges of $PG_2(\mathbb{Z}_{2p})$ consist of the edges between zero and odd vertices, and the adjacencies between the vertex p and even vertices. These two class of edges form two star graphs $K_{1,p-1}$. □

Example 2.5. Consider the prime graph on $\mathbb{Z}_6$. By Theorem 2.4, $PG_2(\mathbb{Z}_6) = 2K_3+3K_{1,1}+2K_{1,2}$ represents the decomposition. In Figure 1, to avoid overcrowding, we ignore the drawing $2K_3$ and just show the $3K_{1,1} + 2K_{1,2}$ of the decomposition.

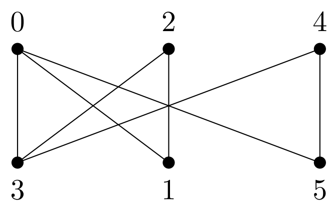

Figure 1: $PG_2(\mathbb{Z}_6) = 2K_3 + 3K_{1,1} + 2K_{1,2}$

Theorem 2.6. *For all $m \geq 1$ and $p \geq 3$, we have*

$$PG_2(\mathbb{Z}_{2p^m}) = 2K_{p^m} + pK_{p^{m-1},p^{m-1}} + 2K_{1,p^{m-1}(p-1)}.$$

Proof. According to part (iii) of Theorem 2.1, two complete graphs K_{p^m} and p complete bipartite graphs $K_{p^{m-1},p^{m-1}}$ appear in decomposition of $PG_2(\mathbb{Z}_{2p^m})$. Further, zero is adjacent to all odd vertices of T_i, $1 \leq i \leq p-1$, and p^m is adjacent to all even vertices of S_i, $1 \leq i \leq p-1$. Since $|S_i| = |T_i| = p^{m-1}$, these adjacencies form two star graphs $K_{1,p^{m-1}(p-1)}$. □

Example 2.7. Consider the prime graph on the ring $\mathbb{Z}_{18}$. By Theorem 2.6, $PG_2(\mathbb{Z}_{18}) = 2K_9 + 3K_{3,3} + 2K_{1,6}$. In Figure 2, to avoid overcrowding, we ignore the drawing $2K_9$ and just show the $3K_{3,3} + 2K_{1,6}$ of the decomposition.

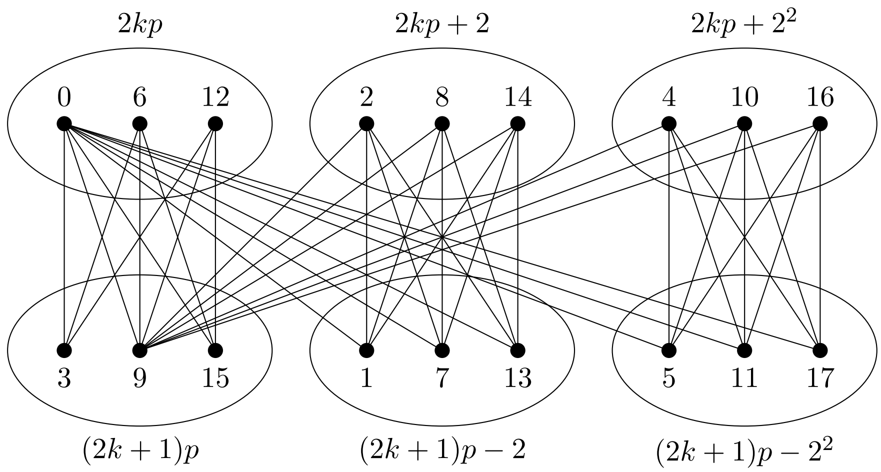

Figure 2: $PG_2(\mathbb{Z}_{18}) = 2K_9 + 3K_{3,3} + 2K_{1,6}$

Theorem 2.8. *For all $n \geq 1$ and $p \geq 3$, we have*

$$PG_2(\mathbb{Z}_{2^n p}) = 2K_{2^{n-1}p} + pK_{2^{n-1},2^{n-1}} + K_{2^{n-1},p-1} + K_{1,2^{n-1}(p-1)}.$$

Proof. As the proof of the above theorems, by Theorem 2.1, $PG_2(\mathbb{Z}_{2^n p})$ has p complete bipartite graphs $K_{2^{n-1},2^{n-1}}$ and two complete graphs $K_{2^{n-1}p}$ and as indused subgraphs. The edges between zero and odd vertices in T_i, $1 \leq i \leq p-1$, form the star graph $K_{1,2^{n-1}(p-1)}$. Also, the adjacencies between the vertices of T_0, with $|T_0| = 2^{n-1}$, and the even vertices of the form $x = 2^n k$, where $1 \leq k \leq p-1$, create $K_{2^{n-1},p-1}$. □

Example 2.9. Consider the prime graph on the ring $\mathbb{Z}_{20}$. By Theorem 2.8, $PG_2(\mathbb{Z}_{20}) = 2K_{10} + 5K_{2,2} + K_{2,4} + K_{1,8}$. In Figure 3, to avoid overcrowding, we ignore the drawing $2K_{10}$ and just show the $5K_{2,2} + K_{2,4} + K_{1,8}$ of the decomposition.

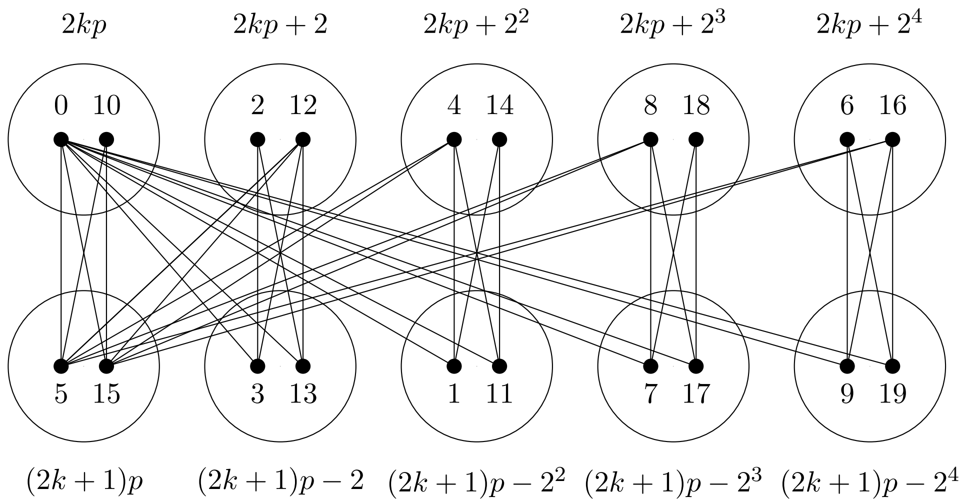

Figure 3: $PG_2(\mathbb{Z}_{20}) = 2K_{10} + 5K_{2,2} + K_{2,4} + K_{1,8}$

Theorem 2.10. *For all $n, m \geq 1$ and $p \geq 3$, we have*

$$PG_2(\mathbb{Z}_{2^n p^m}) =$$
$$2K_{2^{n-1}p^m} + pK_{2^{n-1}p^{m-1},2^{n-1}p^{m-1}} + K_{2^{n-1},p^{m-1}(p-1)} + K_{1,2^{n-1}p^{m-1}(p-1)}.$$

Proof. The adjacencies between zero and odd vertices in T_i, $1 \leq i \leq p-1$ form the star graph $K_{1,2^{n-1}p^{m-1}(p-1)}$. Also, the odd multiples of p^m in T_0 are adjacent to the vertices of the set $A = \{k2^n : 1 \leq k \leq p^{m-1}, p \nmid k\}$ such that $|A| = p^{m-1}(p-1)$. So, it forms $K_{2^{n-1},p^{m-1}(p-1)}$. Further, by Theorem 2.1, there exist two complete graphs $K_{2^{n-1}p^m}$ and p complete bipartite graphs $K_{2^{n-1}p^{m-1},2^{n-1}p^{m-1}}$ in the decomposition of $PG_2(\mathbb{Z}_{2^n p^m})$. $\square$

Example 2.11. Consider the prime graph on the ring $\mathbb{Z}_{36}$. By Theorem 2.8, $PG_2(\mathbb{Z}_{36}) = 2K_{18} + 3K_{6,6} + K_{2,6} + K_{1,12}$. In Figure 4, to avoid overcrowding, we ignore the drawing $2K_{18}$ and just show the $3K_{6,6} + K_{2,6} + K_{1,12}$ of the decomposition.

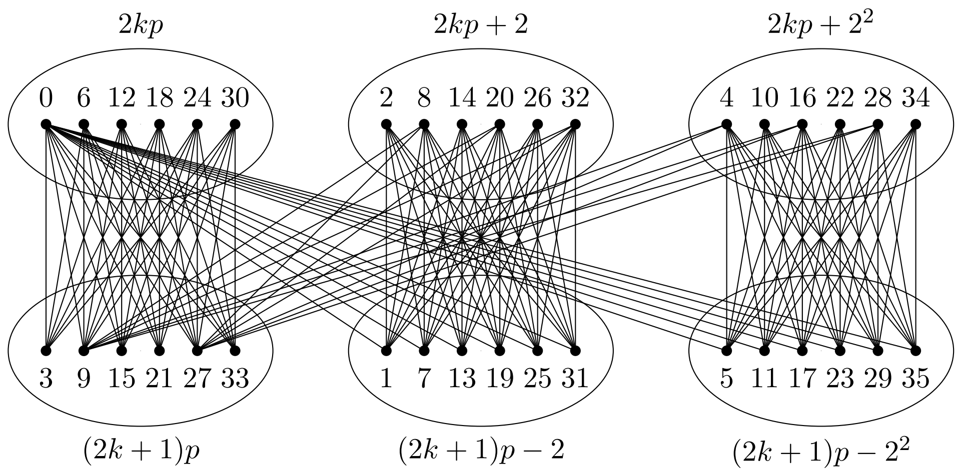

Figure 4: $PG_2(\mathbb{Z}_{36}) = 2K_{18} + 3K_{6,6} + K_{2,6} + K_{1,12}$

3 Neighborhood Metric Dimension of $PG_2(\mathbb{Z}_{2^n p^m})$

Let $N[v]$ denote the closed neighborhood of the vertex $v \in V$, i.e. $N[v] = \{x \in V : d(x, v) \leq 1\}$. A *neighborhood set* of G is a subset S of the vertex set of G such that $G = \bigcup_{v \in S} G_v$ where $G_x = \langle N[x] \rangle$. Further, a subset S of V is called a *resolving set* of G if for each pair u, v of vertices of G there is a vertex $t \in S$ with the property that $|d(v, t) - d(u, t)| > 0$. A neighboring set of G that also serves as a resolving set of G is called a *neighborhood resolving set* of G. In other words, neighborhood resolving set S is an ordered subset $S = (s_1, s_2, \ldots, s_k)$ of V such that $\Gamma(x/S) \neq \Gamma(y/S)$ for all $x, y \in V - S$ and $G = \bigcup_{i=1}^{k} \langle N[s_i] \rangle$, where $\Gamma(a/S) = (d(a, s_1), d(a, s_2), \ldots, d(a, s_k))$ is called the code of vertex a with respect to S. The minimum cardinality of a neighborhood resolving set of G is called *neighborhood metric dimension* of G and is denoted by $nmd(G)$.

Theorem 3.1. $nmd(PG_2(\mathbb{Z}_{2p})) = p + 1$.

Proof. We claim that $S = V_{even} \cup \{x\}$ such that $x \in V_{odd}$ is a neighborhood resolving set for $PG_2(\mathbb{Z}_{2p})$. According to Theorem 2.4, $G =$

$\bigcup_{v\in V_{even}}\langle N[v]\cup N[x]\rangle$ such that $\bigcup_{v\in V_{even}}\langle N[v]\rangle = K_p + pK_{1,1} + 2K_{1,p-1}$ and $N[x]$ contains the other K_p induced by odd vertices. Also, by Lemma 2.3, for all $x, y \in V - S$, there exists a non-zero vertex $s \in S$ such that $1 = d(x, s) \neq d(y, s) = 2$. Thus, $\Gamma(x/S) \neq \Gamma(y/S)$. So, S is a neighborhood resolving set. Note that if $S = V_{even}$ or $S = V_{odd}$ then $G \neq \bigcup_{s\in S}\langle N[s]\rangle$. Now, let $S = A \cup B$ such that $A \subset V_{even}$, $B \subset V_{odd}$, $|A| = k$ and $|B| = p - k$, $k < p$. There are two cases; If $0 \in A$, then there exists $u \in V_{even}$ and $v \in V_{odd}$ such that $uv \notin \bigcup_{s\in S}\langle N[s]\rangle$. If $0 \notin A$, then there exists $w \in V_{odd}$ such that $0w \notin \bigcup_{s\in S}\langle N[s]\rangle$. Therefore, S is a minimum neighborhood resolving set for $PG_2(\mathbb{Z}_{2p})$. □

Example 3.2. Consider $G = PG_2(\mathbb{Z}_{10})$. See Figure 5. Then $S = \{0, 2, 4, 6, 8, 1\}$ is a neighborhood resolving set for G since,
$\Gamma(3/S) = (d(3, 0), d(3, 2), d(3, 4), d(3, 6), d(3, 8), d(3, 1)) = (1, 1, 2, 2, 2, 1)$;
$\Gamma(5/S) = (d(5, 0), d(5, 2), d(5, 4), d(5, 6), d(5, 8), d(5, 1)) = (1, 1, 1, 1, 1, 1)$;
$\Gamma(7/S) = (d(7, 0), d(7, 2), d(7, 4), d(7, 6), d(7, 8), d(7, 1)) = (1, 2, 2, 2, 1, 1)$;
$\Gamma(9/S) = (d(9, 0), d(9, 2), d(9, 4), d(9, 6), d(9, 8), d(9, 1)) = (1, 2, 2, 1, 2, 1)$;
and $PG_2(\mathbb{Z}_{10}) = \bigcup_{v\in S}\langle N[v]\rangle$. So, $nmd(PG_2(\mathbb{Z}_{10})) = 6$.

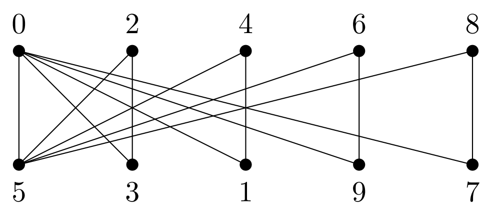

Figure 5: $PG_2(\mathbb{Z}_{10}) = 2K_5 + 5K_{1,1} + 2K_{1,4}$

Theorem 3.3. $nmd(PG_2(\mathbb{Z}_{2^n p^m})) = 2^n p^m - p - 1$.

Proof. Let $x \in B = \{y \in T_0;\quad y = (2k + 1)p^m, 0 \leq k \leq 2^{n-1} - 1\}$ and let $A = \{t_0, t_1, \ldots, t_{p-1}\}$ be a representative set for T_i's such that $t_i \in T_i$ and $t_0 \notin B$. Set $A' = A \cup \{x\}$. We claim that $S = V - A'$ is a neighborhood resolving set for $PG_2(\mathbb{Z}_{2^n p^m})$. According to Theorem 2.10, $G = \bigcup_{s\in S}\langle N[s]\rangle$. Also, by Lemma 2.3, for any $u, v \in A'$, there exists $s \in S$ such that $1 = d(u, s) \neq d(v, s) = 2$. Further, if we add a vertex y to A', then $y \in T_i$ for some $i \in \{0, \ldots, p-1\}$ and for any $s \in S$, $d(y, s) = d(t_i, s)$. Therefore, S is a minimum neighborhood resolving set for $PG_2(\mathbb{Z}_{2^n p^m})$ and $nmd(PG_2(\mathbb{Z}_{2^n p^m})) = 2^n p^m - (p+1)$. □

4 Wiener Index of $PG_2(\mathbb{Z}_{2^n p^m})$

The Wiener index of a graph, introduced by Wiener in [11], turns out to be among the most important of the graph indices. The Wiener index of a graph G is defined as $W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d(u, v)$.

Remark 4.1. Let $G = PG_2(\mathbb{Z}_{2^n p^m})$. For $i = 1, 2$, denote $D_i = |\{(a, b); a, b \in V(G), d(a, b) = i\}|$. By Lemma 2.3, for every pair of distinct vertices $x, y \in V(G)$, we either have $d(x, y) = 1$ or $d(x, y) = 2$, so $D_1 + D_2 = |V(G))|(|V(G)| - 1)$. Therefore,

$$W(G) = \frac{1}{2}(D_1 + 2D_2) = |V(G)|(|V(G)| - 1) - \frac{1}{2} D_1.$$

Theorem 4.2. *Wiener Index of $PG_2(\mathbb{Z}_{2p})$ is $3p^2 - 4p + 2$.*

Proof. By the decomposition of $PG_2(\mathbb{Z}_{2p})$ in Theorem 2.4, one can see that

$$D_1 = 2p(p-1) + 2p + 4(p-1) = 2p^2 + 4p - 4.$$

So,

$$\begin{aligned} W(PG_2(\mathbb{Z}_{2p})) &= 2p(2p-1) - \frac{1}{2}(2p^2 + 4p - 4) \\ &= 3p^2 - 4p + 2. \end{aligned}$$

□

Theorem 4.3. *Wiener Index of $PG_2(\mathbb{Z}_{2p^m})$ is $3p^{2m} - 3p^m - p^{2m-1} + 2p^{m-1}$.*

Proof. By the structure of $PG_2(\mathbb{Z}_{2p^m})$ in Theorem 2.6, we get

$$\begin{aligned} D_1 &= 2p^m(p^m - 1) + 2p(p^{m-1})^2 + 4p^{m-1}(p-1) \\ &= 2p^{2m} + 2p^m + 2p^{2m-1} - 4p^{m-1}. \end{aligned}$$

So,

$$W(PG_2(\mathbb{Z}_{2p^m})) = 2p^m(p^m-1) - \frac{1}{2}(2p^{2m}+2p^m+2p^{2m-1}-4p^{m-1})$$
$$= 3p^{2m}-3p^m-p^{2m-1}+2p^{m-1}.$$

□

Theorem 4.4. *Wiener Index of $PG_2(\mathbb{Z}_{2^np})$ is $2^{2n}p^2-2^{n+1}p-2^{2n-2}p^2+2^{n-1}p-2^{2n-2}p+2^n$.*

Proof. By the structure of $PG_2(\mathbb{Z}_{2^np})$ in Theorem 2.8, we have

$$D_1 = 2\times 2^{n-1}p(2^{n-1}p-1)+2p(2^{n-1})^2+2^{n-1}(p-1)\times 2+2\times 2^{n-1}(p-1)$$
$$= 2^np(2^{n-1}p-1)+2p\times 2^{2n-2}+2^n(p-1)+2^n(p-1).$$

So,

$$W(PG_2(\mathbb{Z}_{2^np})) = 2^np(2^np-1)-\frac{1}{2}(2^np(2^{n-1}p-1)$$
$$+2p\times 2^{2n-2}+2^n(p-1)+2^n(p-1))$$
$$= 2^{2n}p^2-2^{n+1}p-2^{2n-2}p^2+2^{n-1}p-2^{2n-2}p+2^n.$$

□

Theorem 4.5. *Wiener Index of $PG_2(\mathbb{Z}_{2^np^m})$ is $2^{2n}p^{2m}-2^{n+1}p^m-2^{2n-2}p^{2m}+2^{n-1}p^m-2^{2n-2}p^{2m-1}$.*

Proof. According to the decomposition of $PG_2(\mathbb{Z}_{2^np^m})$ in Theorem 2.10,

$$D_1 = 2\times 2^{n-1}p^m(2^{n-1}p^m-1)+2p(2^{n-1}p^{m-1})^2+2\times 2^{n-1}p^{m-1}(p-1)+2\times 2^{n-1}p^{m-1}(p-1).$$

So, we arrive at

$$W(PG_2(\mathbb{Z}_{2^np^m})) = 2^np^m(2^np^m-1)-\frac{1}{2}D_1$$
$$= 2^{2n}p^{2m}-2^{n+1}p^m-2^{2n-2}p^{2m}+2^{n-1}p^m-2^{2n-2}p^{2m-1}.$$

□

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