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Transforming Resource Allocation Strategies: Harnessing Pseudo-Returns To Scale For Optimal Performance

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Abstract. This study explores and extends the concept of returns to scale within the framework of centralized data envelopment analysis. Traditional resource allocation models primarily constrain the ratio of changes between input and output indicators, yet more specific constraints are needed to better reflect system characteristics and indicator properties. To address this gap, this study introduces a novel principle, termed "pseudo scale efficiency," which facilitates the categorization of indicators and regulates the expansion coefficients based on system requirements. To evaluate the effectiveness of this principle, a new model

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is developed and tested using a numerical example. The results indicate that the proposed model achieves substantial resource savings compared to the Lozano and Villa (2004) model. This innovation contributes to more efficient resource allocation and enhances the accuracy of performance assessments in both production and service systems. By aligning more closely with organizational needs and real-world conditions, the proposed approach improves the overall efficiency of organizational systems.

AMS Subject Classification: MSC 90B50
Keywords and Phrases: Centralized Data Envelopment Analysis, Pseudo-Returns to Scale Principle, Expansion Coefficient, Resource Savings

1 Introduction

Data Envelopment Analysis (DEA) is a robust and widely utilized non-parametric method in operations research and economics, designed to evaluate the relative efficiency of decision-making units (DMUs). Unlike parametric approaches, DEA does not impose predefined functional forms or assumptions about the relationships between inputs and outputs, offering a high degree of flexibility for performance evaluation across diverse fields. By distinguishing efficient DMUs from inefficient ones, DEA not only provides a benchmark but also offers actionable insights for performance improvement. The methodology has been enriched by the development of numerous models, addressing various practical and theoretical challenges. One significant advancement in this domain is the focus on centralized resource allocation, a critical issue in management science that has garnered extensive attention in the literature. These centralized models enable the optimization of resource distribution across multiple DMUs under a unified framework, ensuring that overall system efficiency is maximized. In the literature review section, we referred to them. Significant distinction between centralized resource allocation models and other DEA models. In centralized DEA examines the total consumption of inputs and the total production of outputs. In models related to centralized resource allocation (CRA), there exists a centralized decision-making unit who supervises all activities. Consequently, it takes into account all inputs and outputs of DMUs and allocates new inputs to these units based on the system's requirements and size. With the aim of improving or at least not worsening the system's efficiency. It is important to note that in resource allocation for each activity, the objective is to have a total consumption of inputs that is less than or equal to before the allocation, while having a total production of outputs that is equal to or greater than before. In other words, the goal of these models is to set appropriate objectives, save on total resources, and, if possible, increase the total production in the systems. These models are solved using a single linear programming problem. The illustration of all DMUs lies on the efficiency frontier defined by themselves. Charnes et al. [9] introduced a linear programming method called Data Envelopment Analysis, which measures the efficiency of DMUs using estimation of production functions. In this research, the development of measuring the efficiency of decision-making units with benchmarking has been investigated. It is commonly used in the evaluation of public programs. They proposed the constant returns to scale (CCR) model. Then, they defined efficiency based on the CCR model. Banker et al. [7] evaluate the performance of DMUs with multiple inputs and outputs by introducing and applying Variable Returns to Scale (VRS). From both technical and economic perspectives, they examine and develop the concept of variable returns to scale. Lozano and Villa [23] presented the concept of centralized resource allocation using DEA. With conventional centralized DEA models, set targets are achieved for each DMU. Two fundamental characteristics are examined in the proposed model. The first characteristic is the location of all DMUs relatives to the efficient frontier. The second characteristic demonstrates the optimal shaping of the overall consumed input and produced output by finding weights that maximize the relative efficiency of units, equivalent to taking the averages of inputs and outputs, using a multiplier model. In the literature review section, studies conducted in the field of centralized resource allocation based on DEA have been discussed. Thus far, models have been proposed in the field of centralized resource allocation under assumptions of constant returns to scale, variable returns to scale, trade-offs, two-stage network structures, and various other frameworks. (For example: Lozano and Villa [23]; Asmild [5]; Hosseinzadeh Lotfi et al. [19]; Davutyan et al. [11]; Hosseinzadeh Lotfi et al. [20]; Fang [14]; Fang [15]; Yu et al. [44]; Yang et al. [41];

Tao et al. [40]; and etc.) In the direction of system development, the ratio of changes in the sum of all input indicators to the sum of output indicators has been equal or less than one or greater than one.

In today's world, there exist systems in which the development coefficient for all input and output indicators is not uniform—meaning that some indicators change at different rates. Therefore, there is a need to modify the returns to scale (RTS) principle used for system development. Since DEA models are fundamentally designed based on axioms, it becomes necessary to address this issue by expanding the returns to scale principle and introducing a new axiom. However, several questions arise: How can the new axiom, which is capable of expanding returns to scale, be defined? Is it applicable to all systems across various domains? What approaches can control the variations in the development coefficients of non-uniform indicators, and how do these approaches contribute to improving system efficiency? In which fields can the proposed model address the challenges associated with non-uniform indicators, and what outcomes are expected? How can the aforementioned conditions be implemented in a centralized resource allocation model based on DEA? How is the proposed technology for such systems constructed within the framework of centralized resource allocation? What methods are available for evaluating efficiency in this context? Can the proposed model allocate inputs in such a manner that the total allocated inputs do not exceed- and even decrease from- the initial input amounts, while ensuring that the total outputs are at least equal to the initial output amounts?

The term "pseudo-returns to scale, (P-RTS)" is a new concept introduced for innovation in an article concerning the expansion of classical returns to scale. Since RTS are used for evaluating systems in terms of development, pseudo-returns to scale aim to implement the development of systems with flexibility in development coefficients among the indicators. If these development coefficients are equal to one, or if the ratio of input to output coefficients is less than or greater than one, it corresponds to classical returns to scale. However, if these coefficients are distinct among the indicators, the principle of P-RTS comes into play.

Therefore, it can be said that the principle of P-RTS is an extension of the principle of returns to scale.

To date, no research has been conducted in this area. In the literature review section, references are made to articles that have focused on centralized systems within the field of DEA. In this article, the introduction of the pseudo-returns to scale principle in centralized resource allocation (CRA) and the corresponding technology for allocation, as well as the design of a model based on the developed technology, is discussed. Additionally, the model derived from this innovation is compared with the foundational model of centralized resource allocation presented by Lozano et al. [23].

The innovation of this research stems from encountering systems in which the development coefficients of the indicators differ. Therefore, one of the essential requirements is to categorize the indicators based on their development coefficients. The categorization of indicators in evaluating organizations depends on the types of organizations being examined and managerial decisions. Service, manufacturing, and transportation systems can be cited as examples, in which indicators are defined based on the specific needs and characteristics of each of these systems. In general, each organization, depending on its type of activity, goals, and strategies, requires specific indicators that can help in a more accurate evaluation of its performance. For example, in a manufacturing organization, indicators such as production volume, product quality, and production costs may be of high importance. In this context, product quality generally holds greater significance than production volume, as high-quality products can help retain customers and increase sales. In a service organization, indicators such as customer satisfaction, response time, and service quality may take precedence. In this case, customer satisfaction, as a key indicator, usually holds more importance than response time, as a positive customer experience can aid in attracting and retaining them. In transportation systems, indicators such as the number of passengers transported, travel time, and transportation safety hold special significance. Transportation safety typically holds more importance than the number of passengers transported, as ensuring passenger safety not only contributes to the organization's credibility but also has a direct impact on customer satisfaction and loyalty.

Therefore, the categorization of indicators should be done considering the size of the system, the type of defined indicators, and the managerial decisions that the organization's managers have in mind. Managers should carefully examine what goals they expect from the system and, based on that, select appropriate indicators. This choice helps improve the organization's performance and can lead to optimal and strategic decision-making. Additionally, distinctive coefficients are also important in this regard, as they can indicate the varying priorities of different indicators in performance evaluation. Some indicators may hold greater importance in inputs or outputs compared to others, and this distinction can stem from specific market conditions, customer needs, or the strategic goals of the organization. Identifying and applying distinctive coefficients in the process of system development helps managers allocate resources optimally based on their actual priorities, thereby achieving improvements in performance and efficiency. This approach can lead to the establishment of a sustainable competitive advantage in today's complex and dynamic markets. The primary contributions of this study are as follows:

- Developing CRA production technology involves categorizing indicators and applying distinctive expansion coefficients to each group of indicators.
- Applying the principle of pseudo-returns to scale in the centralized data envelopment analysis (CDEA) model.
- Examining the allocated resources and their savings based on the newly developed principles corresponding to each index, and comparing them with the results of the CRA model by Lozano and Villa (2004).
- Analyzing the strengths of the proposed CDEA model in comparison to the Lozano et al. [23] CDEA model.

The subsequent sections of the paper are organized as follows: The second section presents a literature review. The third section describes the classical centralized Data Envelopment Analysis model. The fourth section defines the pseudo-returns to scale technology for systems with specific conditions, and then designs the proposed centralized Data Envelopment Analysis models that incorporate the principle of pseudo-returns to scale. The fifth section demonstrates the proposed models using a numerical example. The sixth section presents the results obtained from the proposed model and suggests directions for future research.

2 Literature Review

In this section, references to studies conducted in the field of CRA are cited. CRA has been applied in areas related to income efficiency, network structures, target setting, and centralized resource allocation models based on DEA principles, including constant returns to scale, variable returns to scale, and others. It has also been investigated in the context of deterministic, random, and fuzzy data. Lozano and Villa (2005) introduced an approach for efficiently allocating resources in a centralized decision-making unit that is close to the operational point. They discussed three specific models due to the simplicity of the input nature under BCC. The first model maximizes the reduction in total inputs while maintaining the overall output level, allowing the existing DMUs to approach efficiency. The second model maximizes the reduction in total inputs, maintains the overall output level, and adjusts the maximum remaining operational unit. The third model reduces some of the open operational points, ensuring the minimum reduction in the total input and maintaining the overall output level. Asmild et al. [5] have proposed a modified centralized resource allocation model under the oriented input of BCC on the model presented by Lozano et al. [23]. The model focuses exclusively on the allocation of inefficient units. The method is designed to generate optimal solutions. The proposed model is developed for non-discretionary and non-controllable input variables. The development of this model holds significant value in the field of data envelopment analysis theory, especially when analyzing decision-making processes is required. It is often applicable to organizations with centralized control over operational factors. Lozano et al. [25] introduced a DEA method for reallocating emission permits, applicable to both traditional regulation and market-based systems. The model assumes that firms produce both desirable (good) and undesirable (bad) outputs. The method aims to achieve three main objectives: increasing desirable outputs, reducing undesirable emissions, and minimizing resource use. These objectives are prioritized by the regulator. Importantly, the approach is independent of input and output prices and operates effectively regardless of the measurement units used. Lozano et al. [26] proposed a data envelopment analysis approach for target setting and resource allocation for the National Ports Organization in Spain. They introduced a non-radial centralized data envelopment analysis model with an oriented-output. Fang [14] offered a generalized CRA-BCC model based on the Lozano et al. [23] and Asmild et al. [5] models. Yu et al. [42] developed a centralized resource allocation model using a two-phase process. They also demonstrated that each DMU and the central unit are projected onto the efficiency frontier. In the model proposed by Yu et al. [42], Challenges arise due to model incompatibilities caused by the two-phase process. Therefore, a modified model by Yu and Hasyim was introduced. Mirsalehy et al.[31] introduced an alternative approach to centralized resource allocation for each DMU. The proposed approach provided a method for integrating the two fundamental models, Radial CRA-BCC and Non-radial CRA-SBM, into a unified framework termed connected CRA-SBM. In this model, adjusting the parameters enables shifting the analysis between CRA-BCC and CRA-SBM models, addressing inherent weaknesses of both models. They referred to it as the integrated CRA-SBM structure. In the proposed model, both inputs and outputs are simultaneously reduced and increased. With this proposed model, the image of all decision-making units is placed on the efficient frontier. Shamsi et al. [38] proposed centralized resource allocation using a multi-objective linear programming framework. They solve the multi-objective programming model using the entropy method and Zionts-Wallenius (Z-W) approach. Yu et al. [43] presented a slacks-based centralized DEA model for resource allocation in a single phase. They modified the single-phase slack-based CDEA model to incorporate transfer-in and transfer-out slacks, allowing for more effective reallocation and adjustment of resources. Lopez-Torres et al. [22] introduced an alternative model for reallocating human resources in a public education network, which they called centralized resource allocation. They made adjustments to preserve the additional education budget without compromising outputs, aiming to improve school performance while ensuring that the quality of education remained intact. Fang [15] expanded upon the centralized DEA models proposed by Lozano et al. [26] to allocate resources according to revenue efficiency across a set of DMUs within a centralized decision-making framework. His objective was to allocate resources in a manner that maximizes the total output revenue generated by all DMUs while operating under limited information. To clarify the factors contributing to the increase in total revenue resulting from the centralized resource allocation model, Fang further broke down aggregate revenue efficiency into three parts: aggregate output-oriented technical efficiency, aggregate output allocative efficiency, and aggregate revenue re-allocative efficiency. Hakim et al. [18] proposed a two-level data envelopment analysis model for centralized resource allocation, incorporating upper and lower bounds for decision-making units. The advantage of this model lies in its consideration of efficiency and effectiveness for resource allocation. In the upper-level model, input resources are allocated along the path that maximizes the effectiveness of organizations, while ensuring that the lower bound applies to the efficiency of all decision-making units. In the lower-level models, data envelopment analysis is used to determine the efficiency of decision-making units under the BCC model separately. Mottaghi et al. [34] applied resource allocation to systems with optional and non-discretionary inputs, taking into account environmental factors. They proposed a multi-objective linear programming model for resource allocation. Zhou et al. [45] presented a production possibility set for a two-stage network structure with random data. They applied data envelopment analysis to the two-stage network structure with random data under centralized control management. The presented model is a deterministic linear programming model. Ding and et. al. [12] introduced a new approach under a centralized decision-making environment to address fixed costs and resource allocation problems while considering technological heterogeneity. The proposed models are based on (CCR) assumption. They introduced the concepts of non-discretionary, border group, and meta-technology ratio. The level of technology reflects the characteristic decision-making units. Two centralized data envelopment analysis models under technological heterogeneity have been suggested. Yang et al. [41] proposed CRA and target setting approach based on data envelopment analysis. In this paper, they used the CCR model and extended the approach to other data envelopment analysis models. Resource allocation of inputs and goal setting is demonstrated based on variable returns to scale. Inputs and outputs are measured with precise values. Ma et al. [28] proposed Stackelberg and collective data envelopment analysis models for two-stage systems with shared resources. In this paper, they evaluated the efficiency of two-stage systems with shared inputs in both Stackelberg competition and cooperation situations. They presented the collective data envelopment analysis model with a two-stage network structure, including external inputs in the second stage. The overall system performance is intuitively demonstrated. Such systems are needed because the internal structure of a complex system is not only reflected in sub-stage organizations but also in the allocation of resources, with operational relationships between sub-stages. Sadeghi et al. [36] extended two centralized resource allocation methods based on the Lozano et al. method [23]. The main hypothesis of this research focuses on decision-making units under a central decision-making unit. It introduces all the targets related to the inputs and outputs of each unit in the next production period. They consider two ideas. The first one was to increase the outputs produced by designing resources and eliminating non-operational inputs as much as possible. Thereby bring the units to a strong efficient state. The second idea optimizes the income and cost functions so that they achieve the best performance. The proposed models examined both constant and variable returns to scale. It then demonstrates that the output targets and the allocation of input resources belong to the production possibility set (PPS). Momeni et al. [33] presented centralized data envelopment analysis based on emissions permits under environmental and trade regulations, taking into account the performance of countries. Ding et al. [13] introduced the fixed centralized resources allocation problem for a two-stage network production structure. Specifically, for collective two-stage models, they first evaluated the performance of each DMU with a two-stage collective model. Then, they introduced a cost allocation scheme that allows DMUs to operate efficiently under the assumption of CCR. Introducing the concepts of maximum satisfaction degree and fairness degree, they proposed a method for obtaining an optimal allocation scheme under centralized control. Kamyab et al. [21] proposed a centralized resource allocation model on a two-stage network structure using ratio data envelopment analysis. They applied this model to 13 world commercial banks. Ceasaroni [8] has proposed an integrated framework for analyzing and determining relationships between technical and centralized resource allocation of cost efficiency, along with output allocation for a number of companies. Furthermore, it delves into the interpretation of technical efficiency measures and their associations with cost analogs. The paper introduced an algorithm for solving nonlinear programming problems related to this subject. For decision-makers, a proper method for computing and comparing a combination of inputs, outputs, and the optimization of multiple units is presented. Tao et al. [40] have presented data envelopment analysis based on centralized resource allocation with network flows and the resource allocation profit function. They introduce quantitative analysis by examining the trade-off between production profits from resource allocation and the costs involved in the allocation process. Afsharian et al. [1] have reviewed data envelopment analysis approaches with the application of commonly used weights from the perspective of centralized management, where resource allocation costs are defined. They determine the optimal resource flows. Madadi et al. [29] have proposed a centralized resource allocation model for energy conservation and environmental pollution reduction. This model is designed based on multi-objective programming with the presence of undesirable outputs. The results obtained from the model show that the reduction in overall environmental pollution is proportionally greater than the reduction in total desirable outputs in energy savings. Fang [16] measured group performance under centralized management. The meta frontier shapes are identified in the decomposition of centralized performance indicators. A new decomposition method, which dominates over the shapes, has been proposed. Chu et al. [10] have suggested a healthcare resource allocation method for hospitals based on data envelopment analysis. They have designed a bi-objective model, where the first objective is to increase in output targets and the second objective being the allocation of resources proportional to the sizes of the units. To solve this bi-objective model, they have recommended using a trade-off model to obtain resource allocation results. Arocena et al. [4] have proposed a directional distance model for efficient resource allocation. A centralized decision-maker oversees all units. The designed model allocates financial aid from higher government layers to municipalities under judicial supervision. The aim of this model is to inform policymakers about achieving effectiveness, efficiency, and equitable resource utilization. This helps the decision-maker in several ways. First, it allows determining the overall optimal number of financial resources. The municipality needs to cover assumed public tasks and expected needs, so it allows estimating the potential reserves that could be derived from public resources in providing local services. The presented formula is based on the Russell directional distance function with weights. The decision-maker is allowed to simultaneously expand outputs and contract inputs, while facilitating priority setting. Soltanifar et al. [39] addressed a significant issue of resource allocation efficiency for various operational units. In this research, they introduce a novel approach to resource allocation and target setting. This method utilizes common-weight set and multi-objective optimization. Both of which align with a centralized decision-making character. Podinovski [35] proposed a resource allocation model for systems in which certain input and output components are shared among decision-making units (DMUs). In other words, for each unit, corresponding value of those components is unknown, and a general value is defined for all units. Therefore, inputs and outputs are categorized based on the system's conditions. These components are considered with the assumption of a union of independent and shared input indicators, as well as for outputs. Then, he discussed the principles of convexity and scalability under the given conditions and presented an appropriate resource allocation model. Mohammadi Nejad et al. [32] have proposed a model for centralized resource allocation based on data envelopment analysis with managerial feasibility. This model considers the presence of undesirable outputs with the objective of reducing these undesirable outputs. The proposed model is suitable from both environmental and economic perspectives and offers the benefits of resource allocation, target setting, and maximizing overall efficiency. (Models of resource allocation are considered under the assumption of managerial feasibility and target setting with the presence of undesirable outputs). Lozano et al. [27] have suggested a method for sum of fixed output using a multi-objective analysis based on centralized data envelopment analysis. The weighted Chebyshev method is utilized for this purpose. The goal of this approach is to adjust the sum of output objectives as close as possible to the ideal values. The model has been applied to the Tokyo 2020 Olympics. Amirteimoori et al. [2] introduced a stochastic resource allocation model that incorporates random data and undesirable outputs. Bai and Wang [6] introduced a Distributed-Optimization with Centralized-Refining (DO-CR) mechanism aimed at enhancing resource allocation efficiency by involving both access points and all devices. The DO-CR mechanism operates in two phases: Initially, it leverages the distributed processing capabilities of all devices, enabling them to optimize their resource allocation schemes using a novel resource reservation and reporting technique. Then, a centralized optimizer constructs a resource trading topology graph based on the individual optimization results and achieves the Pareto optimal solution through a graph-based algorithm. Madadi et al. [30] developed a model to allocate resources centrally, focusing on environmental technology. Their model deals with unwanted outputs using the weak disposability principle. An and et al. [3] proposed a novel fixed-sum DEA efficiency evaluation model for parallel structures and introduced both centralized and decentralized scenarios to construct the efficient frontier. The network DEA method can evaluate decision-making units (DMUs) with a network structure; however, in the real world, the total amount of some inputs and outputs is fixed, which is referred to as fixed-sum inputs/outputs. Finally, by evaluating the industrial performance of three major industries in each province of China for the year 2020, they compared the efficiency differences between provinces and provided insights for improving their industrial performance.

In the research conducted so far on centralized resource allocation, none of the studies have discussed or even mentioned the necessity and impact of having a distinct development coefficient for the indicators.

In the Lozano-Villa model [23], the principle of constant returns to scale is applied, meaning that the development coefficients of the indicators are considered identical. However, in this study, the proposed model assumes differentiated development coefficients for the indicators, which are designed based on the principle of P-RTS. Podinovski [35], categorization of the indicators was conducted based on shared and independent inputs. The author applied allocation to systems where the input and output components are shared among decision-making units. In contrast, the categorization of indicators in the current study is performed based on the system's requirements and interaction with the system manager. This categorization depends on the type of organizations for which the allocation is conducted, the selected indicators, and how their development coefficients vary. This study highlights a specific type of categorization.

3 Centralized Data Envelopment Analysis

This section addresses key research issues from a management perspective, including the optimal allocation of resources and setting appropriate goals. The objective is for systems to reach their maximum production potential within a production possibility set corresponding to available technologies. Therefore, DEA is introduced as an appropriate approach to solving this problem. In DEA models, all inputs and outputs are aggregated under the supervision of a central unit, and then resources are allocated based on constraints and the sizes of the DMUs. The goal is to reduce total inputs and increase or maintain total outputs. Ultimately, the aim is to maximize system performance. In traditional DEA models, the linear programming (LP) problem is solved independently for each DMU. However, in the CDEA model, the LP problem is solved simultaneously for all DMUs, where total inputs are reduced to be equal to or less than the total initial inputs of all decision-making units. Additionally, total outputs are either increased or maintained at a minimum level corresponding to the initial total input consumption. Centralized resource allocation models based on DEA have so far been presented under the assumptions of either CRS or VRS. In these models, it is assumed that the ratio of changes in the sum of each input indicator to the output indicator is either equal to, less than, or greater than one. In this section, a centralized resource allocation model based on traditional DEA is discussed.
The symbols used in this paper are listed in Table 1.

Table 1: Symbols and Definitions

Symbols	Definition	Symbols	Definition
$j = 1, \ldots, n$	Index corresponding to DMUs	α	Development coefficient of first category outputs and inputs
$k = 1, \ldots, n$	Index corresponding to DMUs after resource allocation	α'	Development coefficient of inputs and second category outputs
$i = 1, \ldots, m$	Specified index corresponding to inputs	μ_{jk}	The linear combination vector of DMU j^{th} corresponding to DMU_k
$r = 1, \ldots, p$; $(r \in O_1)$	Specified index corresponding to first category outputs	λ_{jk}	The intensity vector of DMU j^{th} corresponding to DMU_k, corresponding to inputs and the first category outputs
$r = h+1, \ldots, p+1$; $(r \in O_2)$	Specified index corresponding to second category outputs	γ_{jk}	The intensity vector of DMU j^{th} corresponding to DMU_k, corresponding to second category outputs
x_{ij}	Amount of input i^{th} for DMU_j	s_i^-	Surplus slack of the input i^{th}
y_{rj}	Amount of output r^{th} for DMU_j	s_r^+	Shortfall slack of output r^{th}
θ_i	Coefficient of variation corresponding to the i^{th}		

Lozano et al. [23] proposed a centralized data envelopment analysis model with n congruent decision-making units with m input and s output indicators as follows. The presented model allocates resources in two phases at oriented input. In the first phase, the radial oriented input resource allocation model is presented in a centralized manner under variable returns to scale.

$$
\begin{aligned}
&\min \theta \\
\text{s.t.}\quad & \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{jk} x_{ij} \leq \theta \sum_{j=1}^{n} x_{ij}, \quad i = 1, \ldots, m, \\
& \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_{jk} y_{rj} \geq \sum_{j=1}^{n} y_{rj}, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_{jk} = 1, \quad k = 1, \ldots, n, \\
& \lambda_{jk} \geq 0, \quad j, k = 1, \ldots, n, \quad \theta \text{ free.}
\end{aligned}
\tag{1}
$$

Model (1) is a linear programming model with $n^2 + 1$ variables and $m + S + k$ constraints and constraints. Assuming that the optimal value θ^* of Model (1) is obtained, the corresponding slacks for input and output indicators are calculated while preserving θ^* optimality in the second phase by solving the model. In the second phase, Model (2) is formulated to maximize the slack variables of surplus, shortfall, while preserving the optimality obtained from the first phase.

$$
\begin{aligned}
&\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
\text{s.t.}\quad & \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} x_{ij} = \theta^* \sum_{j=1}^{n} x_{ij} - s_i^-, \quad i = 1, \ldots, m, \\
& \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} y_{rj} = \sum_{j=1}^{n} y_{rj} + s_r^+, \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_{jk} = 1, \quad k = 1, \ldots, n, \\
& \lambda_{jk} \geq 0, \quad j, k = 1, \ldots, n, \quad s_r^+ \geq 0, \quad r = 1, \ldots, s, \\
& \quad s_i^- \geq 0, \quad i = 1, \ldots, m.
\end{aligned}
\tag{2}
$$

The non-radial, input-oriented CRA model of Lozano and Villa (2004) is constructed as (3); Assuming that w_i is considered as a weight coefficient for reducing the total of the inputs.

$$
\begin{aligned}
&\min \sum_{i=1}^{m} w_i\theta_i \\
\text{s.t.}\quad &\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} x_{ij} \leq \theta_i \sum_{j=1}^{n} x_{ij}, \quad i=1,\ldots,m, \\
&\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} y_{rj} \geq \sum_{j=1}^{n} y_{rj}, \quad r=1,\ldots,s, \\
&\sum_{j=1}^{n} \lambda_{jk} = 1, \quad k=1,\ldots,n, \\
&\lambda_{jk} \geq 0, \quad j,k=1,\ldots,n, \quad \theta_i \text{ free.}
\end{aligned} \tag{3}
$$

In the second phase, another model is derived by adding slack corresponding to the outputs while maintaining the optimal value θ^* obtained from the first phase of Model (3), by maximizing the total slack of the outputs (Lozano et al. [23]). Note, after solving the two- phase model, corresponding vectors $(\lambda^*_{j1}, ..., \lambda^*_{jk})$ for each decision-making unit k at the new point are obtained. The inputs and outputs at each new point are determined (4), by solving Lozano et al. model [23].

$$
\begin{pmatrix} \hat{x}_{ik}; & i=1,\ldots,m \\ \hat{y}_{rk}; & r=1,\ldots,s \end{pmatrix} = \begin{pmatrix} \hat{x}_{1k} \\ \vdots \\ \hat{x}_{mk} \\ \hat{y}_{1k} \\ \vdots \\ \hat{y}_{sk} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{n} \lambda^*_{jk} x_{1j} \\ \vdots \\ \sum_{j=1}^{n} \lambda^*_{jk} x_{mj} \\ \sum_{j=1}^{n} \lambda^*_{jk} y_{1j} \\ \vdots \\ \sum_{j=1}^{n} \lambda^*_{jk} y_{sj} \end{pmatrix}, \quad k=1,\ldots,n. \tag{4}
$$

In this section, the non-radial two-phase model of Lozano and Villa is presented under the given conditions, but it is designed with the assumption of constant returns to scale. Now, by adding slack corresponding to the outputs in Model (3) and applying the principle of constant returns to scale, a non-radial two-phase allocation model derived from the Lozano et al. [23] model is designed as model (5).

$$
\begin{aligned}
&\max \sum_{i=1}^{m} w_i\theta_i - \varepsilon\left(\sum_{r=1}^{s} s_r^+\right)\\
\text{s.t.}\quad &\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} = \theta_i\sum_{j=1}^{n}x_{ij}, \quad i=1,\dots,m,\\
&\sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} = \sum_{j=1}^{n}y_{rj} + s_r^+, \quad r=1,\dots,s,\\
&\lambda_{jk}\geq 0, \quad j,k=1,\dots,n, \quad \theta_i \text{ free}\\
&\quad s_r^+\geq 0, \quad r=1,\dots,s, \quad s_i^-\geq 0, \quad i=1,\dots,m.
\end{aligned}
\tag{5}
$$

In the next section, a developed centralized resource allocation model based on DEA is presented for cases where the expansion coefficient of variation in total input and output indicators is not the same. By developing returns to scale and constructing various technologies, the proposed models are designed.

4 Exploring Pseudo-Returns To Scale in CRA Development

Data Envelopment Analysis models are generally designed based on axiom principles. The BCC model, developed by Banker and et al. [7], is founded on the principles of observability, feasibility, convexity, and minimal interpolation. In this research, the principle of pseudo-returns to scale is added to these principles, and new technologies are designed that can help improve the efficiency and performance of organizations in resource allocation. Subsequently, based on the proposed technology, the model is developed. Before designing the technologies, the definition and characteristics of the pseudo-returns to scale principle are discussed. The pseudo-returns to scale principle means that different changes in input quantities can have varying effects on outputs, and these effects depend on the specific conditions of the organization being evaluated and the defined indicators. Two requirements are necessary

for the implementation of the pseudo-returns to scale principle: one is the classification of indicators based on distinctive development coefficients, and the other is assigning distinct coefficients to each category of indicators and specifying the range of development coefficients based on the needs of the systems and managerial decision-making. In this study, the indicators are categorized in such a way that inputs fall into one group, while outputs are divided into two separate groups. Accordingly, a fundamental condition is considered: the expansion coefficient of the input indicators must be equal to that of the first group of output indicators. Additionally, the significance of production in the second group of output indicators is assumed to be greater than in the first group; therefore, the expansion coefficient of the second output group must be larger than that of the first output group.
Therefore, the first aspect examined in the pseudo-return to scale principle is the categorization of indicators based on the judgment of system managers.

Suppose that $x_i = (x_1, x_2, \ldots, x_m)$ indicates the input components and $y_r = (y_1, y_2, \ldots, y_s)$ indicates output components , that are non-negative, and the index sets of input and output indicators are denoted by I and O, respectively, such that $I = \{1, \ldots, m\}$ and $O = \{1, \ldots, s\}$. Moreover, let O be the union of the sets O_1, O_2. In this case, we can write

$$O = O_1 \cup O_2 \neq \varnothing, \quad O_1 = \{1, \ldots, p\}, \quad O_2 = \{p+1, \ldots, s\}, \quad 1 \leq p \leq s.$$

Another aspect examined in the pseudo return to scale principle is the expansion coefficients of each category of constraints related to input and output indicators. In this regard, is the development coefficient of the input indicators and the first category of outputs. while is the development coefficient of the second category of outputs, such that $\alpha' \geq \alpha$. Moreover, the range of variation for these expansion coefficients is analyzed in three different statuses. The first status: $\alpha \geq 0$, the second status: $\alpha, \alpha' \in [0, 1]$ and the third status: $\alpha \geq 1$. Every status can exist a new technology. To create new technology, the pseudo-returns to scale principle is implemented on BCC technology. The BCC technology is presented below (6);

$$T_{BCC} = \left\{ (x, y) \;\middle|\; \begin{array}{c} \sum_{j=1}^{n} \mu_j x_{ij} \leq x_i, \quad i = 1, \ldots, m, \\ \sum_{j=1}^{n} \mu_j y_{rj} \geq y_r, \quad r = 1, \ldots, s, \\ \sum_{j=1}^{n} \mu_j = 1, \quad \mu_j \geq 0, \quad j = 1, \ldots, n \end{array} \right\} \tag{6}$$

The P-RTS principle is applied to the constraints related to the indicators. First, the output indicators are divided into two categories, and then applying development coefficients to the constraints category, the constraints are obtained as (7):

$$\begin{array}{c} \alpha \sum_{j=1}^{n} \mu_j x_{ij} \leq x_i, \quad i = 1, \ldots, m, \\ \alpha \sum_{j=1}^{n} \mu_j y_{rj} \geq y_r, \quad r = 1, \ldots, p, \\ \alpha' \sum_{j=1}^{n} \mu_j y_{rj} \geq y_r, \quad r = p+1, \ldots, s. \end{array} \tag{7}$$

By changing the variables $\alpha\mu_j = \lambda_j$, $\alpha'\mu_j = \gamma_j$ and substituting in (7), the constraints are obtained as (8):

$$\begin{array}{c} \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \quad i = 1, \ldots, m, \\ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \quad r = 1, \ldots, p, \\ \sum_{j=1}^{n} \gamma_j y_{rj} \geq y_r, \quad r = p+1, \ldots, s. \end{array} \tag{8}$$

On the other hand, if both sides of the constraint $\sum_{j=1}^{n} \mu_j = 1$ are multiplied by α and then multiplied by α' once more, while also considering each of the conditions related to the range of variation of the development coefficients, it transforms into new constraints. Constraints (9) illustrate the range of variations under different conditions.

$$\Lambda \begin{cases} \text{under status 1:} \quad \alpha \sum_{j=1}^{n} \mu_j = \alpha, \quad \alpha' \sum_{j=1}^{n} \mu_j = \alpha' \Rightarrow \lambda_j, \gamma_j \geq 0, \quad \gamma_j \geq \lambda_j. \\ \text{under status 2:} \quad \alpha \sum_{j=1}^{n} \mu_j = \alpha, \quad \alpha' \sum_{j=1}^{n} \mu_j = \alpha' \Rightarrow \sum_{j=1}^{n} \lambda_j \leq 1, \\ \sum_{j=1}^{n} \gamma_j \leq 1, \quad \sum_{j=1}^{n} \gamma_j \geq \sum_{j=1}^{n} \lambda_j. \\ \text{under status 3:} \quad \alpha \sum_{j=1}^{n} \mu_j = \alpha, \quad \alpha' \sum_{j=1}^{n} \mu_j = \alpha' \Rightarrow \sum_{j=1}^{n} \lambda_j \geq 1, \\ \sum_{j=1}^{n} \gamma_j \geq 1, \quad \sum_{j=1}^{n} \gamma_j \geq \sum_{j=1}^{n} \lambda_j. \end{cases} \tag{9}$$

Therefore, the proposed overall technology related to organizations with assumed conditions is constructed under the principle of pseudo-returns to scale (P-RTS) as (10).

$$T_{P-RTS} = \left\{ (x,y) \;\middle|\; \begin{array}{c} \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \quad i = 1, \ldots, m, \\ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \quad r = 1, \ldots, p, \\ \sum_{j=1}^{n} \gamma_j y_{rj} \geq y_r, \quad r = p+1, \ldots, s, \\ \lambda_j, \gamma_j \in \Lambda, \quad j = 1, \ldots, n \end{array} \right\} \tag{10}$$

If $\Lambda = (\lambda_j, \gamma_j \geq 0)$ is the case, then the technology is referred to as pseudo-constant returns to scale (P-CRS); if $\Lambda = \left(\sum_{j=1}^{n} \lambda_j \leq 1, \sum_{j=1}^{n} \gamma_j \leq 1\right)$ is the case, it is referred to as pseudo-decreasing returns to scale (P-DRS); and if $\Lambda = \left(\sum_{j=1}^{n} \lambda_j \geq 1, \sum_{j=1}^{n} \gamma_j \geq 1\right)$ is the case, it is referred to as pseudo-increasing returns to scale (P-IRS).

The technology based on the principle of P-RTS can vary for each system. There are two common features among technologies based on this principle. One of them is the classification of indicators based on distinctive development coefficients, and the other is the type of distinctive development coefficients and their application to the categorized indicators. In conventional DEA models, such as the Charnes-Cooper-Rhodes (CCR) model, the development coefficient is the same for all indicators. However, in the developed DEA model, the development coefficient of

some indicators is assumed to be distinctive from the development coefficients of other indicators. If the coefficient of development in all indicators is equal in the direction of system development, it is referred to as the principle of constant returns to scale (Charnes et al. [9]). If the ratio of changes in input indicators to output indicators is less than one, it is termed as the principle of increasing returns to scale (Seiford et al. [37]). In cases where this ratio exceeds one, it is called the principle of decreasing returns to scale (Färe and Grosskopf, [17]).
The developed DEA model corresponding to (P-RTS) is designed as (11):

$$
\begin{aligned}
&\min \frac{1-\frac{1}{m}\displaystyle\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\displaystyle\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}\\
&\text{s.t.}\\
&\sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=x_{io},\quad i=1,\ldots,m, && \text{(a)}\\
&\sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{ro},\quad r=1,\ldots,p, && \text{(b)}\\
&\sum_{j=1}^{n}\gamma_j y_{rj}-s_r^+=y_{ro},\quad r=p+1,\ldots,s, && \text{(c)}\\
&\sum_{j=1}^{n}\gamma_j \geq \sum_{j=1}^{n}\lambda_j, && \text{(d)}\\
&\lambda_j,\,\gamma_j\in\Lambda,\quad j=1,\ldots,n,\quad s_i^-\geq 0,\quad i=1,\ldots,m,\\
&s_r^+\geq 0,\quad r=1,\ldots,s.
\end{aligned}
\tag{11}
$$

The objective function is to minimize, and the efficiency value of the unit under evaluation DMU_o is equal to $\frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}$. By increasing the surplus variables corresponding to the inputs and the shortage variables corresponding to the outputs, the objective function is minimized.

This DEA model, formulated under P-RTS, includes the following constraints: Constraint (a) pertains to adjusting the inputs to the desired level. Constraint (b) involves adjusting the first classification of outputs to reach the required level, with the expansion coefficient of this classification considered to be the same as that of the inputs. Constraint (c) focuses on bringing the second classification of outputs to the required level, assuming an expansion coefficient different from that of the first classification of outputs. Constraint (d): The expansion coefficient of the second group of outputs is greater than that of the first group of outputs.

The aim of this research is to provide a different perspective on the allocation of centralized resources. Therefore, by applying the principle of pseudo-returns to scale in the technology of centralized resource allocation, models for the developed allocation of centralized resources are designed based on the proposed technologies. Before that, the differences between developed CDEA systems and developed DEA are discussed. There are four main differences between the developed DEA model and the proposed developed CRA model. In developed DEA models, LP models are solved independently for each DMU. However, in the developed CRA model, only one linear programming problem is solved. Simultaneously, it identifies the image of each DMU by applying distinct change coefficients to the indicators. In developed CDEA, instead of reducing every input indicator for each DMU, the sum of all inputs is reduced at once by considering the distinctive development coefficient across the sum of each indicator. Similarly, in developed CDEA, instead of increasing production for each DMU, the total of all outputs is simultaneously increased or maintained by applying the condition of the distinctive development coefficient between the sum of each indicator.

The developed technology of CDEA under the principle of pseudo-returns to scale is derived as (12):

$$T_{CDEA,P-RTS} = \left\{ (x,y) \;\middle|\; \begin{array}{c} \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} \leq \sum_{j=1}^{n} x_{ij}, \quad i=1,\dots,m, \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} \geq \sum_{j=1}^{n} y_{rj}, \quad r=1,\dots,p, \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\gamma_{jk}y_{rj} \geq \sum_{j=1}^{n} y_{rj}, \quad r=p+1,\dots,s, \\ \lambda_{jk},\gamma_{jk} \in \Omega, \quad k=1,\dots,n, \quad j=1,\dots,n. \end{array} \right\} \tag{12}$$

Such that Ω is defined as (13),

$$\Omega \begin{cases} \textit{under status 1:} \lambda_{jk},\gamma_{jk} \geq 0, \sum_{j=1}^{n}\gamma_{jk} \geq \sum_{j=1}^{n}\lambda_{jk}, k=1,\dots,n. \\ \textit{under status 2:} \sum_{j=1}^{n}\lambda_{jk} \leq 1, \sum_{j=1}^{n}\gamma_{jk} \leq 1, \sum_{j=1}^{n}\gamma_{jk} \geq \sum_{j=1}^{n}\lambda_{jk}, k=1,\dots,n. \\ \textit{under status 3:} \sum_{j=1}^{n}\lambda_{jk} \geq 1, \sum_{j=1}^{n}\gamma_{jk} \geq 1, \sum_{j=1}^{n}\gamma_{jk} \geq \sum_{j=1}^{n}\lambda_{jk}, k=1,\dots,n. \end{cases} \tag{13}$$

The model corresponding to the developed technology of Centralized Data Envelopment Analysis under the principle of P-CRS is designed as (14).

$$\begin{array}{lr} \min \quad \frac{1}{m}\sum_{i=1}^{m}\theta_i - \varepsilon\sum_{r=1}^{s}s_r^+ & \\ \text{s.t.} & \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} = \theta_i\sum_{j=1}^{n}x_{ij}, \quad i=1,\dots,m, & \text{(a)} \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} - s_r^+ = \sum_{j=1}^{n}y_{rj}, \quad r=1,\dots,p, & \text{(b)} \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\gamma_{jk}y_{rj} - s_r^+ = \sum_{j=1}^{n}y_{rj}, \quad r=p+1,\dots,s, & \text{(c)} \\ \sum_{k=1}^{n}\sum_{j=1}^{n}\gamma_{jk} \geq \sum_{j=1}^{n}\lambda_{jk}, \quad k=1,\dots,n, & \text{(d)} \\ \lambda_{jk},\gamma_{jk} \geq 0, \quad j=1,\dots,n, \quad k=1,\dots,n, & \\ s_r^+ \geq 0, \quad r=1,\dots,s, \quad \theta_i \text{ is free}, \quad i=1,\dots,m. & \end{array} \tag{14}$$

In such model (14), the objectives of this function can likely be interpreted as follows:
Model (14) is a multi-objective linear function of minimization type. It first minimizes the change coefficients of the inputs, then, based on the optimality of the first part of the objective function, increases the slack corresponding to the outputs. The optimal value of the objective function is equal to $\frac{1}{m}\sum_{i=1}^{m}\theta_i - \varepsilon\sum_{r=1}^{s}s_r^+$. Constraint category (a) relates to the allocation of resources for the input indicators. It allows to change in the sum of inputs across all units, based on the θ_i variable as non-radial. Constraint category (b) pertains to target setting for the first category of output indicators to reach the desired level. The development coefficient constraints category (a) and (b) are the equal. Constraint category (c) concerns setting the target for the second category of output indicators. While, the development coefficient (c) is more than (a) and (b). The constraint (d) indicates that the development coefficient of the total of the second category of outputs is greater than or equal to the development coefficient of the total of the first category of outputs and inputs. The model represents an extended centralized resource allocation framework within DEA that optimally distributes resources among DMUs while establishing a balance between efficiency improvement and equitable allocation. It takes into account resource-saving strategies and the preservation of outputs, ensuring feasibility and fairness across all units. Models (15) and (16) are designed under the centralized technology developed (P-DRS) and (P-IRS), respectively.

$$
\begin{aligned}
&\min \quad \tfrac{1}{m}\sum_{i=1}^{m}\theta_i - \varepsilon\sum_{r=1}^{s} s_r^{+} \\
&\text{s.t.} \\
&\text{Constraints ("a" to "d" from Model (14))} \\
&\sum_{j=1}^{n}\lambda_{jk} \leq 1, \quad k=1,\ldots,n, \\
&\sum_{j=1}^{n}\gamma_{jk} \leq 1, \quad k=1,\ldots,n, \\
&\lambda_{jk},\, \gamma_{jk} \geq 0, \quad j=1,\ldots,n, \quad k=1,\ldots,n, \\
&\quad s_r^{+} \geq 0, \quad r=1,\ldots,s, \quad \theta_i \text{ is free}, \quad i=1,\ldots,m.
\end{aligned}
\tag{15}
$$

The model (15) is considered for a scenario where resource allocation is accompanied by a contraction in the system's development. And, the model (16) is considered for a scenario where resource allocation is accompanied by an expansion in the system's development.

$$
\begin{aligned}
&\min \quad \tfrac{1}{m}\sum_{i=1}^{m}\theta_i - \varepsilon\sum_{r=1}^{s} s_r^{+} \\
&\text{Constraints ("a" to "d" from Model (14))} \\
&\sum_{j=1}^{n}\lambda_{jk} \geq 1, \quad k=1,\ldots,n, \\
&\sum_{j=1}^{n}\gamma_{jk} \geq 1, \quad k=1,\ldots,n, \\
&\lambda_{jk},\, \gamma_{jk} \geq 0, \quad j=1,\ldots,n, \quad k=1,\ldots,n, \\
&\quad s_r^{+} \geq 0, \quad r=1,\ldots,s, \quad \theta_i \text{ is free}, \quad i=1,\ldots,m.
\end{aligned}
\tag{16}
$$

Remark 4.1. *The extended models for centralized resource allocation are mathematically feasible, as established in Theorem 1. However, to ensure their practical applicability from a managerial perspective, additional constraints may need to be incorporated by carefully analyzing the system and engaging with its managers. For instance, when defining the* *allocation pattern for each DMU, it may be necessary to impose limits on the range of inputs that can be allocated or set boundaries on the outputs produced from these inputs. These constraints help ensure that resource allocation leads to the expected outcomes. Moreover, introducing such constraints may not affect the optimal value of the objective function, yet it can result in a different optimal allocation pattern. This indicates that the model may have multiple optimal solutions.*

Proposition 4.2. *Models 14, 15, 16 are feasible.*

Proof. To prove the feasibility of the given model, we need to show that there exists at least one set of values for the decision variables $(\gamma_{jk}, \lambda_{jk}, \theta_i, s_r^+)$ that satisfies all the constraints ("a" to "d").

Step 1: Assume the following initial values for the variables:

- $\gamma_{jk} = \frac{1}{n}$ for all k, j.
- $\lambda_{jk} = \frac{1}{n}$ for all k, j.
- $\theta_i = 1$ for all i.
- $s_r^+ = 0$ for all r.

Step 2: *Input Constraints (a):* since λ_{jk} are defined as averages $\frac{1}{n}$, the weighted sum of inputs $\sum_{j=1}^{n} \lambda_{jk} x_{ij}$ equals the original input values x_{ij}. Moreover, with $\theta_i = 1$, these constraints are satisfied.

Output Constraints (b and c): since $s_r^+ = 0$ and $\gamma_{jk} = \frac{1}{n}$, $\lambda_{jk} = \frac{1}{n}$ are defined as averages $\sum_{j=1}^{n} \lambda_{jk} y_{rj}$ and *Output Constraints (b and c):* since $s_r^+ = 0$ and $\gamma_{jk} = \frac{1}{n}$, $\lambda_{jk} = \frac{1}{n}$ are defined as averages $\sum_{j=1}^{n} \lambda_{jk} y_{rj}$ and

$\sum_{j=1}^{n} \gamma_{jk} y_{rj}$

are equal to or greater than the original output values y_{rj}. Thus, these constraints are satisfied.

Normalization Constraints in models (15) and (16): By construction, the sums of γ_{jk} and λ_{jk} are equal to one: $\sum_{j=1}^{n} \gamma_{jk} = \sum_{j=1}^{n} \lambda_{jk} = 1$.

Hence, these constraints are satisfied.

Therefore, by selecting the above initial values for the decision variables $(\gamma_{jk}, \lambda_{jk}, \theta_i, s_r^+)$, all constraints of the model are satisfied. Therefore, there exists at least one feasible solution for this model, proving that the model is feasible. □

Proposition 4.3. *The efficiency of targets obtained from resource allocation corresponding to each DMU_k of model (15) on the defined production possibility set improves or does not worsen compared to the efficiency of observable DMUs.*

Proof.

Let's assume that for every DMU_k target setting obtained from solving model (15),

$$(\tilde{x}_{ik}, \tilde{y}_{rk}) = (\tilde{x}_{1k}, \ldots, \tilde{x}_{mk}, \tilde{y}_{1k}, \ldots, \tilde{y}_{sk})$$

is explicitly determined. Using a proof by contradiction, suppose that the point $(\tilde{x}_{ik}, \tilde{y}_{rk})$ is not efficient or its efficiency does not improve. Therefore, by solving model (14), we obtain a vector

$$\lambda_{jk} = (\lambda_{1k}, \ldots, \lambda_{nk}), \gamma_{jk} = (\gamma_{1k}, \ldots, \gamma_{nk})$$

that satisfies

$$\sum_{j=1}^{n} \lambda_{jk} \leq 1, \quad \sum_{j=1}^{n} \gamma_{jk} \leq 1, \quad k = 1, \ldots, n.$$

The point of target setting corresponding to DMU_k is defined as follows:

$$\begin{aligned}
\tilde{x}_{ik} &= \sum_{j=1}^{n} \lambda_{jk} x_{ij} \leq \tilde{x}_{ik}, \quad i = 1, \ldots, m, \\
\tilde{y}_{rk} &= \sum_{j=1}^{n} \lambda_{jk} y_{rj} \geq \tilde{y}_{rk}, \quad r = 1, \ldots, p, \\
\tilde{y}_{rk} &= \sum_{j=1}^{n} \gamma_{jk} y_{rj} \geq \tilde{y}_{rk}, \quad r = p+1, \ldots, s.
\end{aligned}$$

So, at least one of the input or output components must exhibit strict inequality. Let's assume that on a specific input component i', which can be from inputs, strict inequality holds. Thus, we can write

$$\tilde{x}_{i'k} = \sum_{j=1}^{n} \lambda_{jk} x_{i'j} < \tilde{x}_{i'k}.$$

So, the optimal solution obtained from model (15) as oriented-input corresponding to DMU_k and the vector

$$\lambda_{jk} = (\lambda_{1k}, \dots, \lambda_{nk})$$

instead of the assumed optimum is equal to

$$\theta_{i'} = \frac{\sum_{j=1, j\neq k}^{n} \tilde{x}_{i'j} + \tilde{x}_{i'k}}{\sum_{j=1}^{n} x_{i'j}}, \quad \text{if } i' \in I.$$

So, we have:

$$\theta_i = \frac{\sum_{j=1, j\neq k}^{n} \tilde{x}_{ij} + \tilde{x}_{ik}}{\sum_{j=1}^{n} x_{ij}} < \frac{\sum_{k=1}^{n} \tilde{x}_{ik}}{\sum_{j=1}^{n} x_{ij}} = \theta_i^*, \quad \forall i \neq i'.$$

As a result, a feasible solution with a lower value for the objective function is obtained for model (15). On the other hand, it is possible that if at least one specific component like r' belongs to the first category of outputs, strict inequality is established such that

$$\bar{y}_{r'k} = \sum_{j=1}^{n} \lambda_{jk} y_{r'j} > \tilde{y}_{r'k}, \quad \text{if } r' \in O_1.$$

Alternatively, if the component r' belongs to the second category of output indicators, strict inequalities

$$\bar{y}_{r'k} = \sum_{j=1}^{n} \gamma_{jk} y_{r'j} > \tilde{y}_{r'k}, \quad \text{if } r' \in O_2$$

are established. Consequently, based on Model (15), corresponding to DMU_k vector

$$\lambda_{jk} = (\lambda_{1k}, \dots, \lambda_{nk}), \gamma_{jk} = (\gamma_{1k}, \dots, \gamma_{nk})$$

is directed towards a feasible solution

$$\lambda^*_{jk} = (\lambda^*_{1k}, \ldots, \lambda^*_{nk}), \gamma^*_{jk} = (\gamma^*_{1k}, \ldots, \gamma^*_{nk}).$$

So, the following inequalities corresponding to each category of constraints related to the outputs are resulted:

$$\sum_{r=1}^{p} s_r^{+*} + \sum_{r=1}^{p} (\bar{y}_{rk} - \tilde{y}_{rk}) > \sum_{r=1}^{p} s_r^*, \quad \text{if } r' \in O_1, r \in O_1.$$

$$\sum_{r=p+1}^{s} s_r^{+*} + \sum_{r=p+1}^{s} (\bar{y}_{rk} - \tilde{y}_{rk}) > \sum_{r=p+1}^{s} s_r^*, \quad \text{if } r' \in O_2, r \in O_2.$$

which is better than the previous optimum. Therefore, we encounter a contradiction. Thus, the target settings corresponding to each DMU obtained from solving Model (15) may dominate the assumed observable data of each DMU. Consequently, the unit's efficiency values improve with the data obtained from resource allocation.

□

Note 1 *The proposition 1 and 2 is similarly provable for models 14 and 16.*

5 Example

Let's consider an example where we are evaluating the efficiency of multiple systems and then allocating centralized resources under a single supervisory unit. Centralized resource allocation is one of the recommendations aimed at improving system performance. The objective of CRA development is to minimize the total input costs of systems and enhance system performance. Consequently, it seeks to maximize or maintain the total production from the allocated resources. In this section, we examine the results obtained from centralized resource allocation under the P-RTS principles using a numerical example.

The numerical example is presented for six DMUs with two inputs and two outputs. Where, the production of the second output is more important than the first output. In other words, the expansion coefficient of the second output is greater than that of the first output. The observed data for the DMUs were selected arbitrarily, without any specific objective. Table 2 displays the numerical data. Additionally, the last six columns present the efficiency scores of the DMUs under different technological.

Table 2: Data set and the efficiency value DMUs under RTS and P-RTS

	Input1	Input2	Output1	Output2	T_{CRS}	T_{P-CRS}	T_{DRS}	T_{P-DRS}	T_{IRS}	T_{P-IRS}
DMU1	4	7	5	12	1.00	1.00	1.00	1.00	1.00	1.00
DMU2	6	8	7	16	0.76	0.76	0.76	0.76	0.87	0.82
DMU3	5	6	5	18	1.00	0.87	1.00	0.87	1.00	1.00
DMU4	7	6	8	20	0.92	0.89	0.92	0.89	0.96	0.95
DMU5	8	9	9	23	0.81	0.78	0.81	0.78	0.81	0.81
DMU6	9	7	11	26	1.00	1.00	1.00	1.00	1.00	1.00
Summation	39	42	46	115	-	-	-	-	-	-

The efficiency values obtained by solving the extended DEA model (11) under pseudo-constant, pseudo-decreasing, and pseudo-increasing returns-to-scale technologies are shown in the seventh, ninth, and eleventh columns from the left, respectively. Meanwhile, the efficiency scores derived from solving the classical DEA model under standard RTS assumptions are presented in the sixth, eighth, and tenth columns. By comparing the efficiency scores between the standard RTS and the P-RTS technologies, it can be observed that the efficiency values obtained from the extended DEA model are less than or equal to those derived from the classical DEA model. This result is expected, as adding constraints on the expansion coefficients restricts the production possibility set, potentially leading to lower efficiency scores for the units. However, if the expansion coefficient for the second output increases significantly under the P-IRS and P-CRS assumptions, the production possibility set expands. In this case, there is a possibility that the efficiency scores of the units will improve.
Table 3 presents the results of applying Model (14) to the six DMUs. The obtained values represent benchmark for each DMU. Analyzing the efficiency of these benchmark reveals that the allocation pattern of each

unit lies on the efficient frontier of the production possibility set under the extended technology with P-CRS.

Table 3: Outcome of Resource Allocation Using Model 14

DMUs	**Input1**	**Input2**	**Output1**	**Output2**	**Efficiency on** T_{P-CRS}
DMU1	10.64	8.27	13	14.18	1
DMU2	6.17	4.80	7.54	8.23	1
DMU3	3.86	3.00	4.71	5.14	1
DMU4	4.63	3.60	5.66	6.17	1
DMU5	6.94	5.40	8.49	74.08	1
DMU6	5.40	4.20	6.60	7.20	1
Summation	37.64	29.27	46	115	-

The last line indicates the total consumed resources from each input indicator and the total production from each output indicator. Table 4 presents the results of applying Model (15) to the six DMUs. The obtained values represent benchmark for each DMU. Analyzing the efficiency of these benchmark reveals that the allocation pattern of each unit lies on the efficient frontier of the production possibility set under the extended technology with P-DRS.

Table 5, similar to Tables 3 and 4, illustrates the benchmark of six decision-making units. These results are obtained from the solution of Model (16) under the assumption of P-IRS.

As can be seen from Table 5, the benchmarking of the six decision-making units is represented at three points.
Table 6 presents the results obtained from solving Model Lozano and Villa (Model (5)) under CRS.

Table 4: Outcome of Resource Allocation Using Model 15

DMUs	**Input1**	**Input2**	**Output1**	**Output2**	**Efficiency on** T_{P-DRS}
DMU1	7.81	6.07	9.54	12.00	0.999
DMU2	9.00	7.00	11.00	26.00	1.000
DMU3	3.86	3.00	4.71	26.00	0.998
DMU4	4.63	3.60	5.66	25.80	1.000
DMU5	6.94	5.40	8.49	18.00	1.000
DMU6	5.40	4.20	6.60	7.20	0.999
Summation	37.64	29.27	46	115	-

Table 5: Outcome of Resource Allocation Using Model 16

DMUs	**Input1**	**Input2**	**Output1**	**Output2**	**Efficiency on** T_{P-IRS}
DMU1	5	5	6	12	1
DMU2	5	5	6	12	1
DMU3	5	5	6	12	1
DMU4	5	5	6	12	1
DMU5	9	7	11	55	1
DMU6	9	7	11	12	1
Summation	38	34	46	115	-

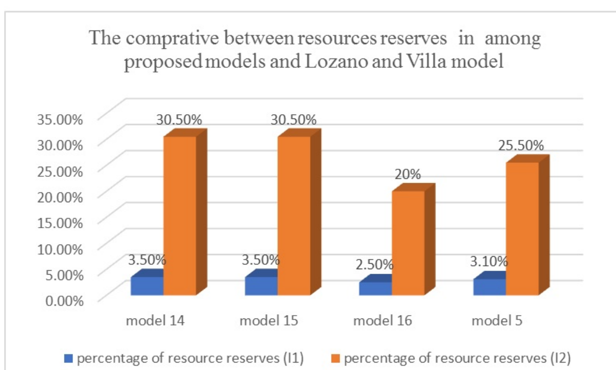

Figure 1: Comparative diagram of resource savings among models 14, 15, 16 and 5

Table 6: Outcome of Resource Allocation Using Model 5

DMUs	**Input1**	**Input2**	**Output1**	**Output2**
DMU1	10.79	10.21	13	37.00
DMU2	6.17	4.80	7.54	17.83
DMU3	3.86	3.00	4.71	11.14
DMU4	4.63	3.60	5.66	13.37
DMU5	6.94	5.40	8.49	20.06
DMU6	5.40	4.20	6.60	15.60
Summation	37.79	31.21	46	115

By comparing the last row of Tables 3, 4, 5, and 6 with Table 2, it can be concluded that with the allocation of resources under the developed models and the Lozano and Villa model (model 5) in this numerical example, the total of the productions has been maintained. However, there has been a total resource saving. Figure 1 compares the percentage of resource savings for each index related to Models 14, 15, 16, and 5.

The development coefficient of the sum of each input indicator is equal to the development coefficient of the sum of each indicator of the first category of outputs, while the development coefficient of the sum of each indicator of the second category of output indicators is greater than that of the other indicators. Under these conditions, technologies corresponding to centralized resource allocation are developed. The results obtained from resource allocation with the implementation of proposed models are as follows:

• By implementing Model 14, it resulted 3.50% and 30.50% in the sum of each input indicator, respectively. With these resource savings, the same total primary production is achieved.

• Results from Model 15 show that, in total, resource savings exist, indicating that consumable costs have decreased based on the allocated resources. The percentage of resource savings from the sum of each input indicator is the same as in Model 14.

• By implementing Model 16, the results related to resource savings are as follows: a 2.50% resource saving and a 20% resource saving from the sum of each input indicator.

• By solving Model 5, resource saving is 3.10% and 25.50% from the sum of each input index, respectively

By applying the developed resource allocation models and the Lozano and Villa model to the numerical example, we observe resource savings. Furthermore, with this resource saving, the same level of production was achieved. Comparing the percentage of resource savings between the models, Models 14 and 15, which operate under P-CRS and P-DRS, respectively, demonstrated greater resource savings compared to the Lozano model, which operates under constant returns to scale. Moreover, in Models 14, 15, and 16, the development coefficients among the indicators were distinct. This condition is a strength of the developed centralized resource allocation models, as the development coefficients in the indicators have a flexible capacity that aligns with the actual conditions of organizations. On the other hand, the efficiency values obtained for these organizations using the developed models are based on the actual performance of the system. Therefore, under the conditions of the organizations, it can also suggest appropriate benchmarking or an ideal target point, contributing to more effective resource management.

6 Results and Suggestions

In this article, based on the categorization of indicators and the assumptions made for their development coefficients, the production technology is proposed under the principle of pseudo returns to scale. Accordingly, a Data Envelopment Analysis model is designed to evaluate the efficiency of units that are subjected to such conditions. Then, it addresses the development of a centralized resource allocation model. By presenting this approach, it has been able to allocate systems in a way that controls resource wastage.

In response to the questions raised in the introduction, it can be stated that in this research, the principles of scale return have been developed in accordance with the conditions of the assumed system. The DEA model has been designed for a specific type of classification of indicators assumed in this study, and its efficiency values have been calculated based on the developed model. On the other hand, the defined principles have been applied to resource allocation, and the corresponding technology and model have been created. The proposed models have the capability to be implemented in various service and production sectors under such conditions of indicator classification, aiming for the development of the system in both contraction and expansion modes.

In fifth section, by applying the models to a numerical example, the results obtained from these models and the extent of resource savings have been examined in detail.

This study presents an innovative approach to centralized resource allocation based on DEA, which not only improves resource allocation but also guides system development along a logical path. The proposed models, designed based on production possibility sets, yield mathematically feasible results and were solved using GAMS software, leading to significant resource savings. The challenges posed by educational, service, and production systems, as well as the emergence of infectious diseases such as COVID-19, have made it necessary to shift towards the logical development of systems. This reality underscores the importance of defining appropriate principles in Data Envelopment Analysis to design corresponding production technologies. In these organizations, indicators used for evaluating and allocating resources are examined, and their expansion coefficients differ under specific and even normal conditions. Therefore, it is essential to introduce principles that align with the real world. Unlike previous studies that primarily focused on the principle of returns to scale (RTS), this research introduces a new principle called pseudo-returns to scale. This principle allows distinct coefficients to be applied to indicators, optimizing resource allocation according to the system's needs and managerial expectations. This approach enables managers to allocate resources more precisely and prevent wastage. From a managerial perspective, engaging with managers is crucial to ensure the correct allocation of resources to each decision-making unit. The proposed models particularly emphasize allocating resources in proportion to the system's needs and size, setting efficient and near-efficient targets in the corresponding production possibility sets. The concept of pseudo returns to scale emphasizes the potential for systems to enhance their efficiency by adjusting development coefficients, which is vital for optimizing overall system performance. By utilizing distinct coefficients in performance evaluation, organizations can better identify the varying priorities of different indicators. This differentiation allows for more informed managerial decision-making, as certain inputs or outputs may carry more weight due to specific market conditions or customer needs. Consequently, recognizing and applying these distinct coefficients enables managers to allocate resources more effectively, leading to improved system performance and efficiency.

Furthermore, innovative approaches in system development not only enhance operational efficiency but also contribute to establishing a sustainable competitive advantage in today's complex and dynamic markets. By leveraging the insights gained from distinct performance indicators and pseudo returns to scale, organizations can adapt more swiftly to changing market demands and optimize their resource management strategies. This adaptability is crucial for long-term success and resilience in an increasingly competitive landscape.

This approach can be applied to various types of data, including fuzzy, ratio, and random data, as well as to systems with undesirable inputs and outputs. Additionally, the models can be implemented in systems with a serial or parallel network structure with independent inputs and outputs and so on.

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