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Alternative Neutral Secondary Goals for Cross-Efficiency Evaluation

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Abstract. Cross-efficiency evaluation in data envelopment analysis is a useful tool for evaluating the performance of decision-making units (DMUs). Using secondary goals is one way to overcome the issue of the existence of multiple optimal solutions in the cross-efficiency evaluation method. This paper proposes two secondary goals with a neutral aspect that focuses only on the interests of the DMU being evaluated and is indifferent to other DMUs. In the proposed models, unlike many of the existing neutral models, the weights are selected without defining virtual DMUs and by defining the range of changes for the efficiency of each output of the DMU being evaluated. In the first proposed model, the efficiency of all the outputs of the unit under evaluation becomes as close as possible to each other. In the second model, the efficiency of all outputs of the evaluated unit moves closer to the upper limit,

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away from the lower limit. Finally, using two numerical examples, the effectiveness of the proposed models is shown by comparing them with the previously proposed models. Also, the TOPSIS model is used to integrate the efficiency scores obtained from different models, resulting in a unique score for ranking the units.

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1 Introduction

Data envelopment analysis (DEA) is a non-parametric mathematical optimization technique to measure the relative efficiency of DMUs, such as banks, industries, police stations, hospitals, tax offices, schools, and university departments [3, 9, 10, 19, 24]. The traditional models in DEA divide DMUs into two groups, efficient and inefficient, so efficient DMUs give an efficiency score of one, and inefficient DMUs get an efficiency score of less than one. Identical efficiency scores among DMUs result in an inability to rank them all. To resolve this difficulty, many researchers have done a lot of work to achieve a reasonable ranking of one. Sexton, Silkman and Hogan [21] suggested the cross-evaluation method as a ranking method in DEA that involves self-evaluation and peer-evaluation efficiency. In the self-evaluation section, the standard cross-efficiency method typically employs the CCR or BCC models, which are based on linear programming. These models often yield multiple optimal solutions, leading to potential changes in the ranking of each DMU based on their cross-efficiency scores, as each DMU receives several peer-evaluation efficiency scores [21]. To address this issue,Sexton, Silkman, and Hogan [21] proposed incorporating a secondary goal into the cross-efficiency method by introducing two attitudes: aggressive and benevolent.

Doyle and Green [8] proposed two models with aggressive and benevolent attitudes, the ideas of which are widely used in cross-efficiency evaluation. Both models aim to maximize the efficiency of the DMU under evaluation. However, the benevolent model simultaneously maximizes the average efficiency of other DMUs, while the aggressive model minimizes the average efficiency of other DMUs. Liang, Wu, Cook and Zhu [15] proposed benevolent game cross efficiency. This model determines a unique set of weights based on the Nash equilibrium and the benevolent strategy. Using the symmetric weight assignment technique, Jahanshahloo, Lotfi, Jafari and Maddahi [11] suggested a new secondary goal for the evaluation cross-efficiency score. Li, Wu, Zhu, Liang and Kou [14] explored reciprocal behaviors among DMUs in their cross-efficiency evaluations, introducing a threshold value to identify positive or negative reciprocal behaviors by comparing the peer-evaluated efficiency with the threshold value-based efficiency. Zhuang and Luo [29] introduced two concepts: task conflict cross-efficiency and relationship conflict cross-efficiency, to address the effects of conflict behavior and beneficial relationships among DMUs during cross-efficiency evaluation. Chen, Huang, Li and Wang [6] incorporated a meta-frontier analysis framework into the cross-efficiency method to create a novel efficiency evaluation approach. Additionally, Chen and Wang [4] provided an innovative definition of cross-efficiency and created two new target-setting approaches for individual DMUs and global optimization to enhance the cross-efficiency of DMUs in different decision-making situations. Chen, Wang and Huang [5] utilized prospect theory to capture how decision-makers' subjective preferences influence the aggregation process when evaluating gains and losses. They developed a new approach for aggregating cross-efficiency based on this theory. Zhang and Liao [28] proposed a stochastic cross-efficiency approach based on the prospect theory to determine the winner in public procurement tenders. Firstly, two cross-efficiency DEA models based on the prospect theory are developed to derive the cross-efficiencies of bidders. Next, they used a stochastic Benefit-of-the-Doubt model based on Monte Carlo simulation to aggregate the diverse cross-efficiencies derived from the evaluations of different experts. Contreras, Lozano and Hinojosa [7] designed a novel cross-efficiency model based on bargaining problems and the Kalai-Smorodinsky solution. Wu, Wang, Liu and Wu [27] created an innovative composite method for ranking DMUs by analyzing the Shannon entropy of cross-efficiency scores, considering both satisfaction and consensus perspectives. Liu, Zhang, Huang, Wang and Xiao [17] proposed a novel combined method consisting of dynamic network DEA, cross-efficiency evaluation, and Shannon entropy aggregation to evaluate the benefits of bus transit systems in 33 key Chinese cities from 2016 to 2019.

The neutral strategy for cross-efficiency evaluation was first introduced by Wang and Chin [25]. This approach differs from aggressive or benevolent models by aiming to determine a set of weights for each DMU's inputs and outputs from its own profit viewpoint. Wang, Chin and Luo [26] proposed a neutral cross-efficiency evaluation method based on the distance of each DMU from either the best DMU (IDMU) or the worst DMU (ADMU). Based upon the method in Wang, Chin and Luo [26], Carrillo and Jorge [2] proposed a neutral model that determines an optimal set of weights that maximize the efficiency score of the IDMU and minimize the efficiency score of the ADMU simultaneously while maintaining the evaluated unit's efficiency. Shi, Wang and Chen [23] utilized an ideal and anti-ideal frontier as benchmarks and proposed a new method for evaluating cross-efficiency scores. Using IDMU and ADMU, Kao and Hung [12] incorporated prospect theory to create a new secondary objective based on the neutral strategy for evaluating cross-efficiency scores. Kao and Liu [13] explored two fundamental network systems, series and parallel, and created a relational model to evaluate the cross-efficiencies of these systems and their divisions. Based on this model, Örkcü, Özsoy, Örkcü and Bal [20] introduced a new neutral model for cross-efficiency evaluation of basic two-stage network systems. This model ranks each DMU based on the efficiency scores of sub-stages and the overall efficiency score, with the overall efficiency being the product of the individual stage efficiencies. Shi, Chen, Chen and Wang [22] proposed a neutral cross-efficiency evaluation method grounded in prospect theory, which considers the bounded rationality of DMUs when dealing with gains and losses, as secondary objectives. Liu, Zhang and Xu [18] proposed the neutral cross-efficiency evaluation method for general parallel systems.

Given that most neutral models use virtual units (which do not exist in the real world) to evaluate units with block box structure, in this article we aim to present a neutral model based on the input and output values of the units under study.

This paper presents two secondary goals from a neutral perspective,

drawing inspiration from the model by Wang and Chin [25]. The proposed models emphasize the efficiency of the outputs of the DMU under evaluation. The first model focuses on maximizing the equality of the evaluated unit's output efficiencies. In the second model, we aim for the performance of unit outputs under evaluation to be as close to their best performance as possible while maximizing the distance from their worst-performance. We compare the performance of the proposed models against neutral models using two practical examples. Additionally, we consolidate the efficiency scores for each unit across different models utilizing the TAPSIS model.
The structure of this paper is organized as follows. Section 2 examines cross-efficiency models, focusing on well-known neutral models. New models are introduced in Section 3. The examples in Section 4 illustrate the effectiveness of the proposed models. Conclusions are given in Section 5.

2 Cross-Efficiency Evaluation

Suppose we have a set of n DMUs, and each $DMU_j (j = 1, 2, ..., n)$ produces s different outputs indexes $Y_j = (y_{1j}, y_{2j}, ..., y_{sj}) \in \mathbb{R}^s_+$ from m different inputs indexes $X_j = (x_{1j}, x_{2j}, ..., x_{mj}) \in \mathbb{R}^m_+$, where $\mathbb{R}^s_+$ and $\mathbb{R}^m_+$ are two sets of non-negative numbers. As mentioned in the previous section, cross-efficiency evaluation includes two stages; in the self-evaluation stage, the multiplier form of the CCR model is often used to evaluate DMU under evaluation, DMU_p, which is as follows [3]:

$$
\begin{aligned}
E^*_{pp} = Max \sum_{r=1}^{s} u_{rp} y_{rp} & \\
s.t. \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leq 0, \quad j = 1, 2, ..., n & \\
\sum_{i=1}^{m} v_{ip} x_{ip} = 1, & \\
u_{rp} \geq 0, \quad v_{ip} \geq 0, r = 1, 2, ..., s, \quad i = 1, 2, ..., m &
\end{aligned}
\tag{1}
$$

Where v_{ip} and u_{rp}represent the i^{th} input and r^{th} output weights for evaluation DMU_p. E^*_{pp} is referred to as the CCR-efficiency of DMU_pwhich also reflects the self-evaluated efficiency of DMU_p. The peer evaluation efficiency of DMU_p

to $DMU_j(j = 1, 2, ..., n)$, using the optimal weights that DMU_p has chosen in the model (1), is then [21]:

$$E_{pj} = \frac{\sum_{r=1}^{s} u_{rp}^{*} y_{rj}}{\sum_{i=1}^{m} v_{ip}^{*} x_{ij}}, (p, j = 1, 2..., n) \tag{2}$$

Where (*) denotes optimal values in model (1). The cross-evaluation matrix can then be obtained as follows:

$$\begin{bmatrix} E_{11} & E_{12} & ... & E_{1n} \\ E_{21} & E_{22} & ... & E_{2n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ E_{n1} & E_{n2} & ... & E_{nn} \end{bmatrix}$$

The cross-efficiency score for DMU_j $(j = 1, 2, ..., n)$ can be obtained by the following equation [21]:

$$\bar{E}_j = \frac{1}{n} \sum_{p=1}^{n} E_{pj} \quad (j = 1, 2, ..., n) \tag{3}$$

The optimal weights derived from model (1) are typically not unique, leading to the arbitrary creation of the efficiency defined in (2) and (3). Sexton, Silkman and Hogan [21] proposed improving cross-efficiency evaluation by adding a secondary goal to optimize input and output weights while maintaining CCR-efficiency. They introduced aggressive and benevolent formulations, resulting in two weighted goal programming models. Based on the idea of adding secondary goals, several models have been proposed by researchers to reduce the problem of multiple optimal solutions in the CCR model for cross-efficiency evaluation. These models can be considered in three categories: aggressive, benevolent, and neutral goals. In what follows, We will review some of the most popular of these models.

2.1 The aggressive and benevolent models

Based on the seminal work of Sexton, Silkman and Hogan [21], introducing a secondary goal based on aggressive and benevolent attitudes can reduce the influence of multiple optimal solutions in model (1)for cross-efficiency evaluation. Building on this idea, Doyle and Green [8]introduced two following models as secondary goals based on benevolent and aggressive attitudes, whose ideas are widely used in cross-efficiency evaluation:

$$\begin{array}{l}\underset{(4-i)}{Min} \; Or \; \underset{(4-ii)}{Max} \; \sum_{r=1}^{s} u_{rp} \left(\sum_{j=1 \; j \neq p}^{n} y_{rj} \right) \\ s.t. \; \sum_{i=1}^{m} v_{ip} \left(\sum_{j=1 \; j \neq p}^{n} x_{ij} \right) = 1 \\ \sum_{r=1}^{s} u_{rp} y_{rp} - E_{pp}^{*} \sum_{i=1}^{m} v_{ip} x_{ip} = 0 \\ \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leq 0, \; j = 1, 2, ..., n; \; j \neq p \\ u_{rp} \geq 0, \; v_{ip} \geq 0, \; i = 1, 2, ..., s \, r = 1, 2, ..., m \end{array} \tag{4}$$

Model (4) aims to find optimal weights that not only maximize the efficiency of DMU_pbut also minimize (4-i), the aggressive model) or maximize (4-ii), in the benevolent model) the average efficiency of other DMUs[8]. Since the two formulations optimize the input and output weights in different ways, there is no guarantee that they lead to the same efficiency ranking or conclusion for the n DMUs.

2.2 The neutral models

When a DMU can determine a set unique of input and output weights, its primary goal is to optimize these weights in its favor. In practice, a DMU need not be concerned with being aggressive or benevolent toward others. This idea has attracted the attention of researchers in recent years.

2.2.1 The wang and chin [25]'s model

Wang and Chin [25] introduced a new cross-efficiency model, termed the neutral model, which avoids both aggressive and benevolent stances. Their model is as follows:

$$\begin{array}{ll}\max \min_{r=1,2,...,s} \left\{ \frac{u_{rp} y_{rp}}{\sum_{i=1}^{m} v_{ip} x_{ip}} \right\} & \\ s.t. \quad E_{pj} = \sum_{r=1}^{s} u_{rp} y_{rj} / \sum_{i=1}^{m} v_{ip} x_{ij} \leq 1, \; j = 1, 2, ..., n \, j \neq p & \\ \quad E_{pp}^{*} = \sum_{r=1}^{s} u_{rp} y_{rp} / \sum_{i=1}^{m} v_{ip} x_{ip} & \\ \quad u_{rp} \geq 0, \;\; v_{ip} \geq 0, r = 1, 2, ..., m \, i = 1, 2, ..., s & \end{array} \tag{5}$$

Where $\frac{u_{rp}y_{rp}}{\sum_{i=1}^{m} v_{ip}x_{ip}}$ is the efficiency of the r^{th}output of DMU_p.
Model (5) for DMUP seeks input and output weights that maximize overall efficiency while ensuring each of its outputs is individually as efficient as possible.

2.2.2 The carrillo and jorge [2]'s model

Another of the well-known secondary goahel for cross-efficiency evaluation from a neutral perspective introduced by Carrillo and Jorge [2]. They used the two following hypothetical DMUs, IDMU and ADMU, are used that represent the best and worst possible performers, respectively, within the given production context.

$$IDMU = \left\{ (x^{\min}, y^{\max}) | x_i^{\min} = \min_j \{x_{ij}\},\ y_r^{\max} = \max_j \{y_{rj}\} \right\} \tag{6}$$

$$ADMU = \left\{ (x^{\max}, y^{\min}) | x_i^{\max} = \max_j \{x_{ij}\},\ y_r^{\min} = \min_j \{y_{rj}\} \right\} \tag{7}$$

Proposed model by Carrillo and Jorge [2] is as follows:

$$\begin{aligned}
& \min\ \sum_{i=1}^{m} v_{ip}(x_i^{\max} - x_i^{\min}) + \sum_{r=1}^{s} u_{rp}(y_r^{\max} - y_r^{\min}) \\
& s.t.\ \sum_{r=1}^{s} u_{rp}y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq\ 0,\ j = 1, 2, ..., n \\
& \quad \sum_{i=1}^{m} v_{ip}x_{ip} = 1, \\
& \quad E_{pp}^* - \sum_{r=1}^{s} u_{rp}y_{rp} = 0, \\
& \quad u_{rp} \geq 0,\ \ v_{ip} \geq 0,\ \ r = 1, 2, ..., s\ ,\ i = 1, 2, ..., m
\end{aligned} \tag{8}$$

The efficiency of the ideal unit establishes an upper bound for any DMU, while the efficiency of the anti-ideal unit sets a lower bound. Thus, minimizing the efficiency of the ideal unit helps approximate the minimization of the maximum cross-efficiency, and maximizing the efficiency of the anti-ideal unit aligns with the maximization of the minimum cross-efficiency. Model (8) explores both approaches within a bi-objective framework.

2.2.3 The liu, song an yang [16]'s model

Standard cross-efficiency evaluation models typically assume that decision makers (DMs) are entirely rational, overlooking the significant influence of their risk attitudes on the evaluation process. To fill this gap,Liu, Song an Yang [16] introduced a neutral model based on prospect theory, introducing a prospect value that uses IDMU and ADMU as reference points (RPs) to address the non-rational psychological factors of a DMU under risk. They by taking ADMU as RP, constructed the following model, in which DMU_p tries to choose a set of weights to maximize its gain.

$$\begin{aligned} &Max \quad \sum_{i=1}^{m} v_{ip}(x_i^{\max} - x_{ip})^{\alpha} + \sum_{r=1}^{s} u_{rp}(y_{rp} - y_r^{\min})^{\alpha} \\ &s.t. \quad \text{model (8) costraints.} \end{aligned} \tag{9}$$

In the next model, Liu, Song and Yang [16] used the IDMU as the RP. In this model, DMU_p attempts to select a set of weights to minimize its losses.

$$\begin{aligned} &Min \quad \sum_{i=1}^{m} v_{ip}\lambda(x_{ip} - x_i^{\min})^{\beta} + \sum_{r=1}^{s} u_{rp}\lambda(y_r^{\max} - y_{rp})^{\beta} \\ &s.t. \quad \text{model (8) costraints.} \end{aligned} \tag{10}$$

2.2.4 The shi, chen, chen and wang [22]'s model

Shi, Chen, Chen and Wang [22] assumed that there is a production possibility set which can use m interval inputs $[a_i x_i^{min}, b_i x_i^{max}](i = 1, 2, ..., m)$ to generate s interval outputs $[c_r y_r^{min}, d_r y_r^{max}](r = 1, 2, ..., s)$ where $a = (a_1, a_2, ..., a_m), b = (b_1, b_2, ..., b_m), c = (c_1, c_2, ..., c_s)$ and $d = (d_1, d_2, ..., d_s)$ are all vectors, and $1 \leq a_i \leq \frac{x_i^{max}}{x_i^{min}}, \frac{x_i^{min}}{x_i^{max}} \leq b_i \leq 1 (i = 1, 2, ..., m), 1 \leq c_r \leq \frac{y_r^{max}}{y_r^{min}}, \frac{y_r^{min}}{y_r^{max}} \leq d_r \leq 1 (r = 1, 2, ..., s)$ where this DMU is called an interval-DMU.
Based on interval reference points (IRPs) to consider bounded rational behavior, Shi, Chen, Chen and Wang [22] proposed the following neutral cross-efficiency evaluation model.

$$\begin{aligned} &Min \quad \sum_{i=1}^{m} v_{ip} W_{ip}^{interval-in} + \sum_{r=1}^{s} u_{rp} W_{rp}^{interval-out} \\ &s.t. \quad \text{model (8) costraints.} \end{aligned} \tag{11}$$

where $W_{ip}^{interval-in}$ and $W_{rp}^{interval-out}$ are defined as follows [22]:

$W_{ip}^{interval-in} =$

$$\begin{cases} -\lambda(-b_i x_i^{max} + x_{ip})^{\beta}, & x_{ip} > b_i x_i^{max} \\ -\lambda(-a_i x_i^{min} + x_{ip})^{\beta} + (b_i x_i^{max} - x_{ip})^{\alpha}, & a_i x_i^{max} \leq x_{ip} \leq b_i x_i^{max} \\ (a_i x_i^{max} - x_{ip})^{\alpha}, & x_{ip} < a_i x_i^{min} \end{cases}$$

$W_{rp}^{interval-out} =$

$$\begin{cases} -\lambda(-y_{rp} + c_r y_r^{min})^{\beta}, & y_{rp} < c_r y_r^{min} \\ -\lambda(-y_{rp} + d_r y_r^{max})^{\beta} + (y_{rp} - c_r y_r^{min})^{\alpha}, & c_r y_r^{min} \leq y_{rp} \leq d_r y_r^{max} \\ (y_{rp} - d_r y_r^{max})^{\alpha}, & y_{rp} > d_r y_r^{max} \end{cases}$$

Model (11) differs from models (9-10) in that the RPs can be any precise number or interval DMU inferior to ADMU and superior to IDMU, rather than being restricted to just ADMU and IDMU. In this case, the RPs illustrate the bounded rationality of decision-makers regarding gains and losses. Consequently, model (11) accommodates irrational behavior of DMs while overcoming the limitations of the precise reference point in application.

Remark 2.1. *Model(9) is special case of model(11) when* $a_i = \dfrac{x_i^{max}}{x_i^{min}}, b_i = 1, c_r = 1, d_r = \dfrac{y_r^{min}}{y_{r^{max}}}$.

Remark 2.2. *Model(10) is special case of model(11) when* $a_i = 1, b_i = \dfrac{x_i^{min}}{x_i^{max}}, c_r = \dfrac{y_r^{max}}{y_r^{min}}, d_r = 1$.

Remark 2.3. *In the numerical examples section of this article, the interval-DMU uses interval inputs* $[x^{min}, x^{max}]$ *to produce interval outputs*$[y^{min}, y^{max}]$.

The values of parameters α, β, and λ in (9), (10) and (11) are constants and reflect the bounded rationality of the decision-maker and vary from person to person. The research findings indicate that $\alpha = \beta = 0.88$ and $\lambda = 2.25$ effectively represent the bounded rational behavior of most decision-makers.
In the sections that follow two new models will be suggested and their performance will be compared with the above models.

3 Proposed Model

As mentioned in the last section, in neutral cross-efficiency models, DMUs typically focus on maximizing gains and minimizing losses without considering the interests of their peers. In this section, we introduce two new neutral secondary goal models for cross-efficiency evaluation. The first proposed model is inspired by the model (5). This model sets input and output weights to minimize the gap between the minimum and maximum efficiency of the evaluated DMU outputs. Let $DMU_p(p = 1, 2, ..., n)$ be DMU under evaluation. The first proposed model is as follows:

$$\begin{aligned} \min \quad & \max_{r=1,2,\ldots,s} \left\{E_p^r\right\} - \min_{r=1,2,\ldots,s} \left\{E_p^r\right\} \\ s.t. \quad & E_{pj} = \sum_{r=1}^{s} u_{rp} y_{rj} / \sum_{i=1}^{m} v_{ip} x_{ij} \leq 1, \; j = 1, 2, ..., n \,, \; j \neq p \\ & E_{pp}^* = \sum_{r=1}^{s} u_{rp} y_{rp} / \sum_{i=1}^{m} v_{ip} x_{ip} \\ & u_{rp} \geq 0, \; r = 1, 2, ..., m \\ & v_{ip} \geq 0, \; i = 1, 2, ..., s \end{aligned} \tag{12}$$

Where $E_p^r = \frac{u_{rp} y_{rp}}{\sum_{i=1}^{m} v_{ip} x_{ip}}$ is defined in (5). The above model searches for a set of input and output weights for DMU_p to maximize its efficiency as a whole while trying to make $E_p^r(r = 1, 2, ..., s)$ as close as possible to being equal. In other words, the proposed model aims to make the evaluated DMU outputs as similar as possible in efficiency. In contrast to model (5), which focuses solely on improving the worst-performing DMU_p outputs, the model (12) aims to equalize the efficiency scores of all DMU_p outputs. Via Charnes and Cooper's transformation [3], model (12) can be transformed into the next linear programming model as follows:

$$\begin{aligned} & \lambda^* = Min \, \lambda_2 - \lambda_1 \\ & s.t. \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leq 0, \; j = 1, 2, ..., n \\ & \sum_{i=1}^{m} v_{ip} x_{ip} = 1, \\ & \quad E_{pp}^* - \sum_{r=1}^{s} u_{rp} y_{rp} = 0, \\ & \quad \lambda_1 \leq u_{rp} y_{rp} \leq \lambda_2, \; r = 1, 2, ..., s \\ & \quad u_{rp} \geq 0, \; r = 1, 2, ..., m \\ & \quad v_{ip} \geq 0, \; , i = 1, 2, ..., s \\ & \lambda_1 \geq 0 \end{aligned} \tag{13}$$

Where $u_{rp} \, (r = 1, 2, ..., m)$, $v_{ip}(i = 1, 2, ..., s)$,λ_1 and λ_2 are decision variables. From the perspective of multi-criteria decision analysis, DMU under evaluation can choose a set of input and output weights to make each of its outputs' efficiency distance from λ_2 as small as possible, and each of outputs' efficiency distance from λ_1 as large as possible. Based on this idea, we propose another neutral cross-efficiency model in the following form:

$$\begin{aligned}
&Min \sum_{r=1}^{s}(\lambda_2 - u_{rp}y_{rp}) - \sum_{r=1}^{s}(u_{rp}y_{rp} - \lambda_1)\\
&s.t. \sum_{r=1}^{s} u_{rp}y_{rj} - \sum_{i=1}^{m} v_{ip}x_{ij} \leq 0, \quad j = 1, 2, ..., n\\
&\sum_{i=1}^{m} v_{ip}x_{ip} = 1,\\
&E^*_{pp} - \sum_{r=1}^{s} u_{rp}y_{rp} = 0,\\
&\lambda_1 \leq u_{rp}y_{rp} \leq \lambda_2, \ r = 1, 2, ..., s\\
&u_{rp} \geq 0, \ \ r = 1, 2, ..., m\\
&v_{ip} \geq 0, \ i = 1, 2, ..., s\\
&\lambda_1 \geq 0
\end{aligned} \tag{14}$$

In model (14),the variation between $E^r_p(r = 1, 2, ..., s)$ and λ_2 is minimized while that between $E^r_p(r = 1, 2, ..., s)$ and λ_1 is maximized.
Models (12) and (14) determine the input and output weights from the angle of the DMU_p itself and have nothing to do with the efficiency of other DMUs. Thus, the proposed model is a neutral model rather than an aggressive or benevolent model.
As with all DEA models for cross-efficiency evaluation, models (13) and (14) need to be solved n times for each DMU. Therefore, n efficiency scores are generated for each DMU, and its cross-efficiency score can be calculated using (3).

4 Numerical Example

In this section, we evaluate the performance of our proposed models using two numerical examples from previous DEA studies.
Example 4.1. In this example, 17 forest districts with four inputs (x_1 :Budget, x_2 :Initial stocking, x_3 :Labor, x_4 :Land) and three outputs (y_1 :Main product, y_2 :Soil conservation, y_3 :Recreation) are considered for measuring efficiency. The real value is taken from [12] and it is shown in Table 1.

The last column of Table 1 shows the efficiency scores of the 17 forest districts

Table 1: Input and output data of the 17 forest districts in Taiwan.

DMU	Inputs				Outputs			CCR Efficiency
	x_1 (dollars)	x_2 (m^3)	x_3 (persons)	x_4 (ha)	y_1 (m^3)	y_2 (m^3)	y_3 ($visits$)	
1	51.62	11.23	49.22	33.52	40.49	14.89	3166.71	**1**
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45	**1**
3	66.65	104.18	257.09	13.65	139.74	115.96	0	**1**
4	27.87	107.6	14	146.43	25.47	131.79	0	**1**
5	51.28	117.51	32.07	84.5	46.2	144.99	0	**1**
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92	**1**
7	25.83	105.8	9.51	227.2	19.4	120.09	0	**1**
8	123.02	82.44	87.35	98.8	43.33	125.84	404.69	**1**
9	61.95	99.77	33	86.37	45.43	79.6	1252.62	**1**
10	80.33	104.65	53.3	79.06	27.28	132.49	42.67	0.94047
11	205.92	183.49	144.16	59.66	14.09	196.29	16.15	0.93462
12	82.09	104.94	46.51	127.28	44.87	108.53	0	0.82896
13	202.21	187.74	149.39	93.65	44.97	184.77	0	0.79983
14	67.55	82.83	44.37	60.85	26.04	85	23.95	0.77316
15	72.6	132.73	44.67	173.48	5.55	135.65	24.13	0.76247
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09	0.74348
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14	0.68679

Table 2: Cross efficiency score for 17 forest districts by different models.

DMU	Aggressive Model(4-i)	Benevolent Model(4-ii)	wang2010 Model(5)	call2018 Model(8)	Liu2019AD Model(9)	Liu2019ID Model(10)	ShiChen2021 Model(11)	Model(13)	Model(14)
1	0.651 (5)	0.886 (4)	0.7361 (8)	0.6898 (9)	0.5885 (9)	0.7425 (3)	0.5475 (9)	0.8337 (5)	0.8471 (5)
2	0.6579 (4)	0.9448 (2)	0.8498 (4)	0.8611 (2)	0.7325 (4)	0.681 (5)	0.6781 (4)	0.853 (3)	0.8587 (3)
3	0.5807 (8)	0.8692 (6)	0.7702 (5)	0.7733 (6)	0.6818 (5)	0.631 (7)	0.5802 (8)	0.8128 (6)	0.7709(7)
4	**0.7170** (1)	0.9147 (3)	0.857 (3)	0.8342 (3)	0.7757 (2)	0.7376 (4)	0.7287 (2)	0.8406 (4)	0.9180(2)
5	0.7139 (2)	**0.9638** (1)	0.9038 (2)	**0.8924** (1)	0.7743 (3)	**0.7649** (1)	0.7242 (3)	**0.9173** (1)	**0.9565**(1)
6	0.7012 (3)	0.8776 (5)	**0.9321** (1)	0.8083 (4)	**0.8366** (1)	0.7467 (2)	**0.7301** (1)	0.8812 (2)	0.8550(4)
7	0.6505 (6)	0.7942 (9)	0.7169 (9)	0.7231 (8)	0.6735 (6)	0.6606 (6)	0.6559 (5)	0.707 (9)	0.7945(6)
8	0.5839 (7)	0.8379 (7)	0.7397 (7)	0.7752 (5)	0.6386 (8)	0.6079 (9)	0.5868 (7)	0.758 (7)	0.7457(8)
9	0.5399 (10)	0.691 (12)	0.6486 (10)	0.6449 (11)	0.532 (12)	0.6175 (8)	0.515 (10)	0.6877(10)	0.7426(9)
10	0.5688 (9)	0.8261 (8)	0.7447 (6)	0.7495 (7)	0.6407 (7)	0.5794 (10)	0.5919 (6)	0.7376 (8)	0.7269(10)
11	0.4377 (13)	0.635 (13)	0.5857 (13)	0.5802 (13)	0.5036 (13)	0.4305 (15)	0.4665 (13)	0.5589 (13)	0.5151(15)
12	0.5145 (11)	0.7131 (10)	0.6327 (11)	0.6641 (10)	0.5441 (10)	0.5606 (11)	0.5118 (11)	0.6658 (11)	0.6961(11)
13	0.4255 (14)	0.6122 (15)	0.5579 (14)	0.5638 (14)	0.4767 (15)	0.4356 (13)	0.44 (14)	0.5566 (14)	0.5308(14)
14	0.4863 (12)	0.6923 (11)	0.6266 (12)	0.6364 (12)	0.5349 (11)	0.5098 (12)	0.4973 (12)	0.6374 (12)	0.6347(12)
15	0.4053 (15)	0.6187 (14)	0.5385 (15)	0.5388 (15)	0.4837 (14)	0.3798 (16)	0.4389 (15)	0.5067 (16)	0.5072(16)
16	0.3181 (17)	0.536 (17)	0.4381 (17)	0.4698 (17)	0.4049 (17)	0.3108 (17)	0.3252 (17)	0.4029 (17)	0.3930(17)
17	0.3906 (16)	0.5714 (16)	0.4994 (16)	0.5334 (16)	0.4328 (16)	0.4314 (14)	0.3827 (16)	0.5372 (15)	0.5590(13)
Sum	9.3428	12.9840	11.7778	11.7382	10.2550	9.8274	9.4010	11.8942	12.0517
Average	0.5496	0.7638	0.6928	0.6905	0.6032	0.5781	0.5530	0.6997	0.7089
STD	0.1202	0.1345	0.1398	0.1221	0.1251	0.1367	0.1192	0.1448	0.1571
Max	0.7170	0.9638	0.9321	0.8924	0.8366	0.7649	0.7301	0.9173	0.9565
Min	0.3181	0.5360	0.4381	0.4698	0.4049	0.3108	0.3252	0.4029	0.3930

calculated from the Model (1). These scores are the highest values that the districts can attain. As can be seen, the CCR model identifies DMU1 through DMU9 as DEA-efficient units. To re-rank all units, we use models (4-i,ii), (5), (8-11) and (13-14). The cross-efficiency scores obtained from different models for each forest district are shown in the second through tenth columns of Table

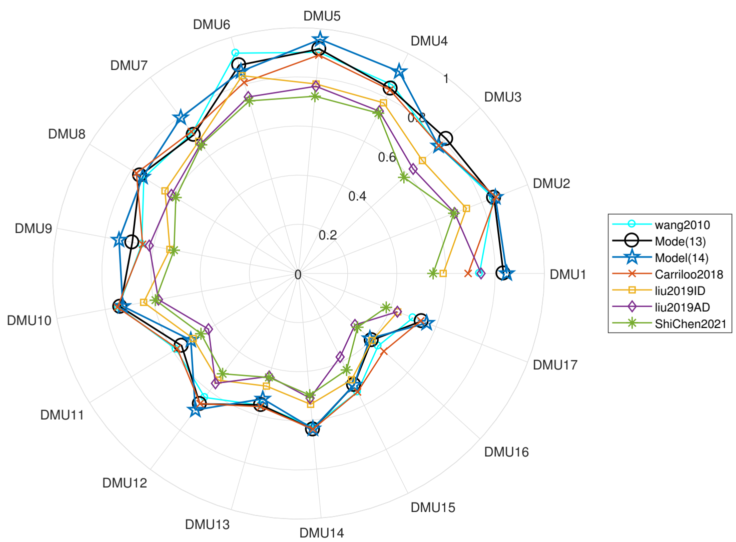

Figure 1: Illustrative comparison between the cross-efficiency score of neutral models for each DMU in example 4.1.

2. Also, the highest score for each model is bolded and underlined. It is seen that the proposed models(13-14) and models (4-ii), (8) and (10) select DMU 5 as the best DMU.

The nineteenth row of Table 2 shows that the sum of efficiencies in models (13-14) is higher than in model (4-i) (the aggressive model) but lower than in model (4-ii) (the benevolent model). So, these models have neutral behavior in the evaluation of each unit. Furthermore, the sum of efficiencies in each of the model (13-14) is higher than that of each of the neutral model (5, 8, 9-11). All the above results are reasonable regarding the structures of the proposed models.

The rank of each DMU in each model next to its efficiency score (in parentheses in blue) is given in Table 2. As can be seen, DMU5 took first place in 5 out of 9 methods. Note that DMU16 has the worst performance of all methods. Efficient DMUs 1 to 8 have been ranked 1 to 9 in all methods, while efficient DMU9 has obtained a worse ranking than inefficient DMU10 in most methods.

Figure 1 compares the results of models (4-i,ii), (5), (8-11), and (13-14) in Example 4.1. It uses the efficiency scores in Table 2. As can be seen, Model (14) generally produces the highest efficiency score, while Model (11) tends to have the lowest.

The last four rows of Table 2 present the statistical characteristics of the efficiency scores from the compared models. Figure 2 illustrates the normal distribution of the efficiency scores produced by the proposed models. As can be seen in Figure 2, the efficiency of DMUs produced by the first proposed model follows an almost symmetric distribution, while the efficiency of DMUs produced by the second proposed model follows an asymmetric distribution with right-skewness.

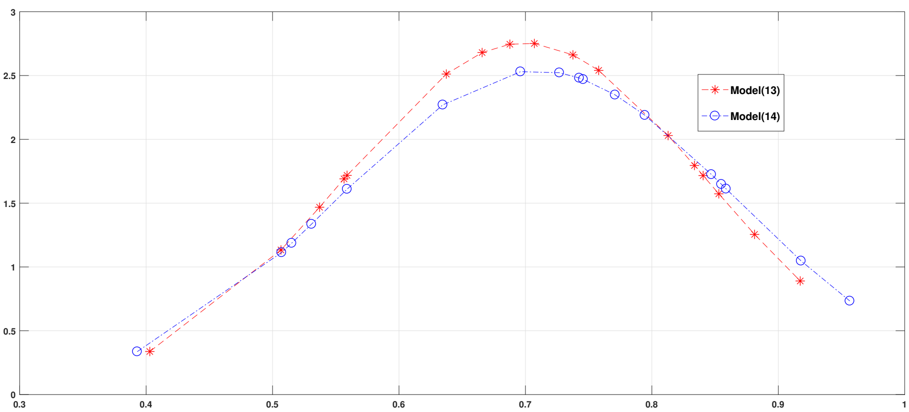

Figure 2: The normal distribution of the efficiency scores produced by the proposed models in example 4.1.

Example 4.2. By inspiration from the empirical example in [2], this example looks at the performance of some top worldwide research and development (R& D) investment companies in software and computer services. R& D includes activities that companies undertake to innovate and introduce new products and services. It is often the first stage in the development process. The goal is typically to bring new products and services to market and add to the company's bottom line. The EU Industrial R&D Investment Scoreboard provides economic and financial data and analysis of the top corporate R&D investors from the EU and abroad. This is based on company data extracted directly from each company's Annual Report. The Scoreboard is published annually to offer a reliable benchmarking tool for comparing companies, sectors, and regions, and to analyze emerging investment trends. It also aims to increase public awareness and support for R&D investments among companies and policymakers, encouraging them to disclose their R&D funding. The 2022 edition of the Scoreboard analyzes the 2500 companies that invested the largest sums in R&D worldwide in 2021. These companies, with headquarters in 41 countries and more than 900k subsidiaries all over the world, each invested over EUR 48.5 million in R&D in 2021. The total investment across all 2500 com-

Table 3: Data of the 20 software & computer companies in among the top corporate R& D according the EU Industrial R&D Investment Scoreboard in 2021.

DMU	Companies	Country	Inputs			Outputs		CCR
			x_1 (€million)	x_2 (€million)	x_3 (€million)	y_1 (€million)	y_2 (€million)	Efficiency
1	ALPHABET	US	156500	21755.2	27866.8	227473.9	769312.7	1
2	META	US	71970	16393.2	21768.5	104122.3	798490.6	1
3	MICROSOFT	US	221000	21089.5	21642.2	175057.3	2002997.3	1
4	ALIBABA GROUP HOLDING	China	254941	7388.5	7687.3	118232.6	400766.9	1
5	TENCENT	China	112771	4061.2	7190.5	77631.2	523059.9	1
6	ORACLE	US	143000	3982.9	6373.8	37471.3	219716.5	0.499
7	IBM	US	307600	1820.6	5248.1	50635.7	111062.6	0.9459
8	SAP	Germany	107415	800.0	5168.0	27842.0	156634.3	0.64801
9	SALESFORCE	US	73541	633.1	3942.3	23390.4	229294.8	0.7747
10	BAIDU	China	45500	1510.2	3456.4	17254.5	39254.2	0.54187
11	VMWARE	US	37500	340.8	2754.7	11346.5	14659.6	0.54363
12	ADOBE	US	25988	307.3	2242.6	13937.0	279168.8	1
13	NETEASE	China	32064	222.0	1950.9	12142.0	55433.5	0.81015
14	ELECTRONIC ARTS	US	12900	166.0	1930.1	6172.5	36486.1	0.69635
15	UBER TECHNOLOGIES	US	29300	263.1	1813.5	15411.4	65118.6	1
16	HEWLETT -PACKARD ENTERPRISE	US	60400	2209.1	1747.3	24531.2	17854.9	0.8327
17	WORKDAY	US	15200	384.8	1659.2	4537.2	46065.3	0.40895
18	SOFTBANK	Japan	59721	6455.7	1551.4	48096.9	82328.8	1
19	INTUIT	US	13500	46.8	1545.1	8505.2	136500.1	1
20	TWITTER	US	7500	893.1	1335.0	4483.0	45443.1	0.59784

panies was EUR 1093.9 billion - an amount equivalent to 86% of the world's business-funded R&D and passing the trillion Euro mark for the first time [1]. We will be particularly focused on 20 companies among the top 337 corporate R&D investors within the Software and Computer Services industrial sector. Based on the information received from the scoreboard, the information of the investigated companies was based on three inputs (x_1: Number of employees, x_2: Capital expenditure, x_3: R& D investment) and two outputs (y_1: Net sales, y_2: Market capitalization) is shown in Table 3.

As can be seen in the last column in Table 3, 9 of 20 DMUs according to the efficiency score of the CCR model are efficient DMUs, so these DMUs cannot be ranked. To reevaluate the performance of efficient and inefficient DMUs in this example, we used the cross-efficiency models from section two and our proposed models. The rank of each company within each model alongside its efficiency score (in parentheses in blue) is shown in the second to tenth columns of Table 4. Table 4 indicates that DMU17 performs the worst, while DMU19 achieved 6 first and 3 second places. Also, of the 9 efficient DMUs (their numbers are in red in Table 4), all except DMU2 ranked from 1 to 9. DMU13 and DMU9(inefficient DMUs) have achieved ninth place five and three times, respectively, while DMU2 has done so only once. Additionally, DMU15 has the smallest change in earned ranks (fourth rank).

Figure 3 illustrates a comparison of the results of the models (13-14) with the

Table 4: Results of models in Example 4.2

DMU	Aggressive Model(4-i)	Benevolent Model(4-ii)	wang2010 Model(5)	call2018 Model(8)	Liu2019AD Model(9)	Liu2019ID Model(10)	ShiChen2021 Model(11)	Model(13)	Model(14)
1	0.6244 (6)	0.8473 (5)	0.7374 (8)	0.7972 (5)	0.7013 (7)	0.6332 (6)	0.6576 (6)	0.6997 (8)	0.7547 (7)
2	0.4988 (12)	0.6187 (11)	0.5715 (12)	0.6293 (9)	0.518 (12)	0.5128 (11)	0.5007 (12)	0.5341 (13)	0.5888 (11)
3	0.5715 (8)	0.758 (8)	0.7391 (7)	0.7335 (8)	0.6443 (8)	0.6114 (8)	0.5573 (8)	0.7069 (7)	0.7502 (8)
4	0.6518 (5)	0.7923 (7)	0.7844 (6)	0.7414 (7)	0.726 (6)	0.6638 (5)	0.6606 (5)	0.7841 (5)	0.7686 (6)
5	0.7209 (3)	0.9256 (2)	0.8771 (3)	0.8718 (3)	0.8072 (2)	0.7488 (3)	0.7238 (4)	0.8645 (3)	0.875 (3)
6	0.3364 (18)	0.4178 (18)	0.4079 (16)	0.3927 (19)	0.3746 (18)	0.3486 (17)	0.338 (18)	0.4065 (16)	0.403 (16)
7	0.4759 (13)	0.4811 (15)	0.5237 (13)	0.4603 (15)	0.4846 (13)	0.4745 (13)	0.4744 (13)	0.5492 (12)	0.5053 (13)
8	0.4318 (14)	0.4938 (13)	0.4809 (14)	0.4682 (14)	0.4532 (14)	0.4417 (14)	0.4322 (14)	0.4914 (14)	0.474 (14)
9	0.5266 (10)	0.6036 (12)	0.6032 (9)	0.575 (12)	0.5504 (11)	0.5515 (10)	0.5192 (10)	0.6141 (9)	0.5959 (9)
10	0.3451 (17)	0.4504 (16)	0.398 (17)	0.4195 (16)	0.3883 (16)	0.3463 (18)	0.3566 (17)	0.3926 (17)	0.3984 (18)
11	0.3532 (16)	0.4326 (17)	0.3822 (19)	0.4061 (18)	0.382 (17)	0.3512 (16)	0.3637 (16)	0.3848 (18)	0.3808 (19)
12	0.7802 (2)	0.9051 (3)	**0.9228** (1)	0.8835 (2)	0.7993 (3)	0.8407 (2)	0.7402 (2)	**0.9228** (1)	**0.9228** (1)
13	0.5465 (9)	0.6397 (9)	0.5988 (10)	0.6055 (10)	0.5765 (9)	0.5545 (9)	0.5513 (9)	0.6082 (10)	0.5937 (10)
14	0.3974 (15)	0.493 (14)	0.4361 (15)	0.4732 (13)	0.4227 (15)	0.4041 (15)	0.4023 (15)	0.4294 (15)	0.4405 (15)
15	0.7199 (4)	0.8566 (4)	0.7964 (4)	0.8094 (4)	0.7674 (4)	0.7293 (4)	0.728 (3)	0.8059 (4)	0.7902 (4)
16	0.5031 (11)	0.6238 (10)	0.5898 (11)	0.5786 (11)	0.562 (10)	0.4957 (12)	0.5185 (11)	0.586 (11)	0.579 (12)
17	0.2874 (20)	0.3605 (20)	0.3372 (20)	0.3472 (20)	0.3093 (20)	0.2993 (20)	0.2834 (20)	0.3315 (20)	0.3395 (20)
18	0.6205 (7)	0.8218 (6)	0.7856 (5)	0.7489 (6)	0.7263 (5)	0.6264 (7)	0.6479 (7)	0.757 (6)	0.7856 (5)
19	**0.8449** (1)	**0.9267** (1)	0.8976 (2)	**0.9081** (1)	**0.8445** (1)	**0.8864** (1)	**0.8147** (1)	0.8976 (2)	0.902 (2)
20	0.3213 (19)	0.4161 (19)	0.3902 (18)	0.4125 (17)	0.3485 (19)	0.3355 (19)	0.315 (19)	0.3696 (19)	0.3992 (17)
Sum	10.5573	12.8645	12.2599	12.2618	11.3864	10.8556	10.5853	12.1357	12.2472
Average	0.5279	0.6432	0.6130	0.6131	0.5693	0.5428	0.5293	0.6068	0.6124
STD	0.1589	0.1898	0.1867	0.1811	0.1679	0.1690	0.1552	0.1850	0.1871
Max	0.8449	0.9267	0.9228	0.9081	0.8445	0.8864	0.8147	0.9228	0.9228
Min	0.2874	0.3605	0.3372	0.3472	0.3093	0.2993	0.2834	0.3315	0.3395

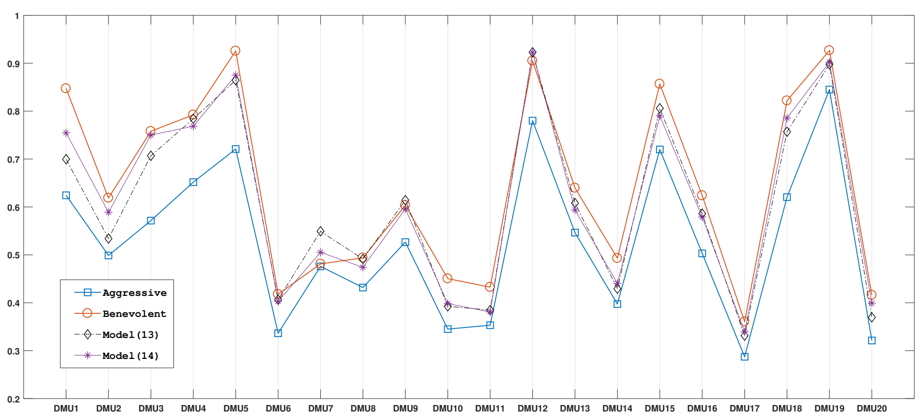

Figure 3: Illustrative comparison between the cross-efficiency score of the proposed, aggressive, and benevolent models for each DMU in Example 4.2.

benevolent and aggressive models in Table 4. Figure 3 shows that the efficiency values from models (13-14) mostly fall between benevolent and aggressive efficiency. This means that the proposed models have neutral behavior.

Table 5: Ranking models correlation test in example 4.2.

		Model(4-i)	Model(4-ii)	Model(5)	Model(8)	Model(9)	Model(10)	Model(11)	Model(13)	Model(14)
Model(4-i)	Correlation	1.0000	0.9835	0.9790	0.9714	0.9910	0.9970	0.9985	0.9850	0.97744
	Sig.(bilateral)		0	0	0	0	0	0	0	0
Model(4-ii)	Correlation	0.9835	1.0000	0.9654	0.9880	0.9895	0.9790	0.9805	0.9609	0.96391
	Sig.(bilateral)	0		0	0	0	0	0	0	0
Model(5)	Correlation	0.9790	0.9654	1.0000	0.9609	0.9820	0.9790	0.9774	0.9955	0.99549
	Sig.(bilateral)	0	0		0	0	0	0	0	0
Model(8)	Correlation	0.9714	0.9880	0.9609	1.0000	0.9744	0.9699	0.9699	0.9489	0.96692
	Sig.(bilateral)	0	0	0		0	0	0	0	0
Model(9)	Correlation	0.9910	0.9895	0.9820	0.9744	1.0000	0.9850	0.9880	0.9820	0.97594
	Sig.(bilateral)	0	0	0	0		0	0	0	0
Model(10)	Correlation	0.9970	0.9790	0.9790	0.9699	0.9850	1.0000	0.9955	0.9835	0.98195
	Sig.(bilateral)	0	0	0	0	0		0	0	0
Model(11)	Correlation	0.9985	0.9805	0.9774	0.9699	0.9880	0.9955	1.0000	0.9835	0.97594
	Sig.(bilateral)	0	0	0	0	0	0		0	
Model(13)	Correlation	0.9850	0.9609	0.9955	0.9489	0.9820	0.9835	0.9835	1.0000	0.98797
	Sig.(bilateral)	0	0	0	0	0	0	0		0
Model(14)	Correlation	0.9774	0.9639	0.9955	0.9669	0.9759	0.9820	0.9759	0.9880	1
	Sig.(bilateral)	0	0	0	0	0	0	0	0	

The rank correlation coefficient can be used to evaluate the significance of the relationship between the models mentioned previously. Spearman is a commonly used non-parametric method that use rank correlation. Spearman's rank coefficient calculations are based on the deviation of ranks. Table 5 shows the value of Spearman's rank correlation coefficients of the eight models in Table 4 to assess the similarities between the rankings induced from the corresponding values. In all the cases, the values are statistically significant at the 0.0001 level. The test values correlations among all of the models are all above 0.9. Therefore, it can be concluded that the proposed models are reasonable.

We use the TOPSIS method to rank each DMU uniquely based on the models mentioned earlier. First, we define $IDpoint = [1, 1, ...1]_{1\times 9}$ and $ADpoint = [20, 20, ..., 20]_{1\times 9}$ as ideal and anti-ideal points respectively. According to TOPSIS ranking criteria, $c_j = \dfrac{d_j^-}{d_j^- + d_j^+}$, we calculate d_j^+ and d_j^- as the distance of the rank vector of each DMU_j from the ideal point and anti-ideal point respectively. Table 6 presents the TOPSIS ranking criteria for evaluating rank of companies (DMUs) based on the results outlined in Table 4. As can be seen in Table 6, DMU_{19}, DMU_{12} and DMU_5 rank first to third, while DMU_{17} ranks last.

Table 6: Ranking of Companies (DMUs) based on TOPSIS method in example 4.2.

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
TOPSIS Score	0.7102	0.4509	0.643	0.7463	0.8967	0.1491	0.3525	0.3219	0.5144	0.1674	0.1507	0.9401	0.5554	0.282	0.8472	0.4738	0	0.7346	0.97	0.0917
Rank	7	12	8	5	3	18	13	14	10	16	17	2	9	15	4	11	20	6	1	19

5 Conclusions

The cross-efficiency score reflects the efficiency scores from the basic DEA models, specifically the CCR and BCC models. Due to the non-unique weights generated by these models, the final scores for each DMU in the cross-efficiency method may vary. To address this, researchers have proposed various secondary goal models based on different attitudes. In this paper, we introduced two secondary goal models based on a neutral perspective. Accordingly, in the first model, the DMU under evaluation seeks a set of input and output weights to align the efficiency of the evaluated DMU outputs as closely as possible. In the second model, which is based on multiple criteria decision analysis, the weights are assigned in a way that the efficiency of each output of the DMU under evaluation is as close as possible to the maximum efficiency while also distancing it from the minimum efficiency. Two practical examples were used to demonstrate the validity and efficiency of the proposed models. Spearman's method was employed to assess the similarity between the performances of the previous models and the proposed models.
Additionally, the TOPSIS method was utilized to combine the ranking results obtained from different models, and each DMU was assigned a score for ranking. Future research directions should focus on incorporating negative data and multi-stage systems into these models.

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