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Original Research Paper

Soft Crossed Hypermodules And Soft HG-Hypergroupoids

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Abstract. In this article, we introduce the soft subhypergroupoid and soft action hypergroupoid and study their properties. We consider the category of soft hypergroupoids whose objects are soft hypergroupoids and morphisms are soft hypergroupoid homomorphisms. Also we consider the concept of soft crossed hypermodules and the category of soft crossed hypermodules. We show that a soft hg-hypergroupoid, can be obtained from each soft crossed hypermodule and a soft crossed hypermodule, can be obtained from each soft hg-hypergroupoid. In addition, we show that soft hg-hypergroupoids and soft crossed hypermodules, are equivalent categories.

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1 Introduction

There are many issues of different sciences, including chemistry, physics, medicine, economics, social and environmental sciences, and many other sciences, are not defined with definite and completely clear data. In other

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words, since we are not faced with completely specific data, we cannot solve these problems with classical and usual mathematical methods. To solve uncertainty problems, various methods have been stated and studied. Fuzzy set theory [43] and rough set theory [34], are among the famous theories that can be mentioned. For further reading, refer to [18, 29, 36, 40, 42]. These methods, despite their usefulness, also have some problems. To improve the previous methods and solve some of the problems, in 1999, the concept of soft sets was presented by Molodtsov [33]. Since then, many studies have been carried out by different people on this theory, and these studies continue at a rapid pace [2, 30, 31, 35]. The application of soft sets in decision-making problems has been studied by Maji et al. [30,31]. In this study, they described some operations on soft set. The relationship between information systems and soft sets has been studied by Pei et al. [35]. Some notions such as the restricted intersection, union, and restricted difference, were studied by Ali et al. in [2]. A comparison of rough sets and soft sets was done by Akta et al. in [1]. In addition, they expressed the concept of soft groups and their properties.

There are a large number of fields in which crossed modules are used in their study. Therefore, studying crossed modules and all kinds of automorphisms is very important. Crossed modules were defined by Whitehead [39]. There are many interesting applications of crossed modules, such as Actor, Pullback, Pushout, and induced crossed modules [3–5]. n-Complete, and representations of crossed modules were studied by Dehghanizadeh and Davvaz [24–26]. Polygroups were studied by Comer [15], also see in [19]. Comer and Davvaz extended the algebraic theory to polygroups. Alp and Davvaz [6], expressed the concept of crossed polymodule of polygroups along with some of its properties. Moreover, they introduced new important classes by the fundamental relations. The pushout and pullback in crossed polymodules theory have been introduced by Alp and Davvaz, and they described the structure of these two concepts in crossed polymodules [7]. Arvasi et al. [10–13], introduced the notion of a 2-crossed module, which is a generalization of crossed modules. In [27, 28], Dehghanizadeh et al. introduced the notion of crossed polysquare. Yamak et al. applied the theory of soft sets to a hyperstructure, the so-called hypergroupoid [41]. They introduced the notions of soft hypergroupoids, soft subhypergroupoids, and homomorphism of soft hypergroupoids. In addition, the two main connections between the class of L-fuzzy hypergroupoids and the class of soft hypergroupoids were established. Many related properties of extended union, restricted union, extended intersection, restricted intersection, $\vee$-union, and $\wedge$-intersection of the family of soft hypergroupoids were also surveyed. A lot of studies have been done about the hyperstructures and the soft theory in hyperstructures. For example, refer to [8, 9, 16, 17, 20–23, 32, 37, 38, 44].

In this article, we introduce the category of soft crossed hypermodules.

2 Preamble

We state some definitions and necessary theorems of soft sets [31, 33]. Let U be an initial universe set and E be a set of parameters. As usual, $\mathcal{P}(U)$ denotes the power set of U and $R \subseteq E$.

Definition 2.1. A pair $(\mathcal{F}, R)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : R \longrightarrow \mathcal{P}(U)$. In fact, a soft set over U is a parameterized family of subsets of the universe U. For $\kappa \in R$, $\mathcal{F}(\kappa)$ may be considered as the set of κ-approximate elements of the soft set $(\mathcal{F}, R)$.

Definition 2.2. For two soft sets $(\mathcal{F}, R)$ and $(\mathcal{G}, T)$ over U, we say that $(\mathcal{F}, R)$ is a soft subset of $(\mathcal{G}, T)$, denoted by $(\mathcal{F}, R) \subseteq (\mathcal{G}, T)$, if the following conditions hold:

1. $R \subseteq T$,
2. for all $\kappa \in R, \mathcal{F}(\kappa)$ and $\mathcal{G}(\kappa)$, are identical approximations.

Two soft sets $(\mathcal{F}, R)$ and $(\mathcal{G}, T)$ over U are called soft equal, if $(\mathcal{F}, R) \subseteq (\mathcal{G}, T)$ and $(\mathcal{G}, T) \subseteq (\mathcal{F}, R)$.

Definition 2.3. Let $(\mathcal{F}, R)$ be a soft set. The set $Supp(\mathcal{F}, R) = \{x \in R | \mathcal{F}(x) \neq \phi\}$ is called the support of the soft set $(\mathcal{F}, R)$. A soft set $(\mathcal{F}, R)$ is non-null, if $Supp(\mathcal{F}, R) \neq \phi$.

We recall that one of the several natural generalizations of group theory, is hypergroup. Regarding the action on their elements, in any group, the combination of two elements is one element, but in any hypergroup, is a set. We point out that hypergroups have important applications in many fields, such as lattices, geometry, color scheme, and combinatorics [19]. Applications of hypergroups studied by Comer [15], also see [19, 20]. In fact, they extended the algebraic theory to polygroups. According [15], a polygroup is a multi-valued system $\mathcal{M} =< P, \circ, e, ^{-1} >$, with $e \in P$, $^{-1} : P \longrightarrow P$, $\circ : P \times P \longrightarrow \mathcal{P}^*(P)$, where the following axioms hold, for all $r, s, t \in P$:

1. $(r \circ s) \circ t = r \circ (s \circ t)$
2. $e \circ r = r \circ e = r$
3. $r \in s \circ t$ implies $s \in r \circ t^{-1}$ and $t \in s^{-1} \circ r$.

$\mathcal{P}^*(P)$ is the set of all the non-empty subsets of P, and also if $x \in P$ and R, T are non-empty subsets of P, then we have $R \circ T = \bigcup\limits_{\substack{b \in T \\ a \in R}} a \circ b$, $x \circ T = \{x\} \circ T$ and $R \circ x = R \circ \{x\}$.

The following, are the facts that are clearly concluded from the principles of the polygroups: $e \in r \circ r^{-1} \cap r^{-1} \circ r$, $e^{-1} = e$ and $(r^{-1})^{-1} = r$.

Example 2.4. If we consider the set P as $P = \{e, r, s\}$, then $P =< P, \circ, e, ^{-1} >$ along with polyaction which have shown in the Table 2.1. is a polygroup.

$\circ$	e	r	s
e	e	r	s
r	r	$\{e, s\}$	$\{r, s\}$
s	s	$\{r, s\}$	$\{e, r\}$

Table 2.1.

Definition 2.5. A crossed hypermodule $\chi = (C, P, \partial, \kappa)$ is consists of hypergroups $< C, *, e, ^{-1} >$ and $< P, \circ, e, ^{-1} >$ together with a strong homomorphism $\partial : C \longrightarrow P$ and a (left) action $\kappa : P \times C \longrightarrow \mathcal{P}^*(C)$ on C, satisfying the following conditions:

1. $\partial({}^{p}c) = p \circ \partial(c) \circ p^{-1}$, for all $c \in C$ and $p \in P$,
2. ${}^{\partial(c)}c' = c * c' * c^{-1}$, for all $c, c' \in C$.

Example 2.6. (i) In every hyperygroup, the set containing only the identity member is always a subhypergroup, and this subhypergroup is normal in the hypergroup. Therefore, we have crossed hypermodule $(1, P) = (1, P, c_1, id\mid_{c_1})$.

(ii) Every hypergroup P contains the whole hypergroup P as a normal subhypergroup. So, we always have crossed hypermodule $(P, P) = (P, P, c, id_P)$.

(iii) Consider the following hypergroup morphisms of an abelian hypergroup P, written multiplicatively,

$$l : 1 \to Aut(P) \qquad i \to id_P \qquad k : P \to 1 \qquad p \to 1$$

So, we have a crossed hypermodule $(P, 1) = (P, 1, l, k)$.

Example 2.7. (i) [8] A conjugation crossed hypermodule is an inclusion of a normal subhypergroup N of P, with action given by conjugation. In fact, for any hypergroup P, the identity map $\mathrm{id}_P : P \longrightarrow P$ is a crossed hypermodule with the action of P on itself by conjugation. Indeed, there are two canonical ways a hypergroup P may be regarded as a crossed hypermodule: via the identity map or the inclusion of the trivial subhypergroup.

(ii) If C is a P-hypermodule, then there is a well defined action κ of P on C. This, together with the zero homomorphisms, creates a crossed hypermodule (C, P, ∂, κ).

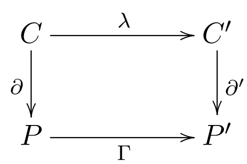
Example 2.8. Let P be a hypergroup and $N \trianglelefteq P$ be a normal subhypergroup. Consider the hypergroup morphism

$$\begin{array}{rcl} C_N : P & \longrightarrow & Aut(N) \\ p & \longmapsto & (c_p\mid_N^N : n \to n^p) \end{array}$$

So the following crossed hypermodule exists:

$$(N, P) = (N, P, C_N, id_P\mid_N)$$

Definition 2.9. Consider the crossed hypermodules $\chi = (C, P, \partial, \kappa)$ and $\chi' = (C', P', \partial', \kappa')$. A crossed hypermodule morphism $f = (\lambda, \Gamma) : \chi \to \chi'$ is a tuple of strong homomorphism, such that the diagram



commutes, and $\lambda(p\kappa c) = \Gamma(p)\kappa'\lambda(c)$, for all $p \in P, c \in C$.

To continue the study, we state some definitions and necessary theorems of soft hypergroups. In what follows, let P be a hypergroup and R be a non-empty set. The notation $\mathcal{R}$ is an arbitrary binary relation between an element of R and an element of P. A set-valued function $\mathcal{F} : R \longrightarrow \mathcal{P}(P)$, can be defined as $\mathcal{F}(x) = \{y \in P | (x, y) \in \mathcal{R}\}$ for all $x \in R$. So the pair $(\mathcal{F}, R)$ is a soft set over P.

Definition 2.10. Let $(\mathcal{F}, R)$ be a non-null soft set over P. Then $(\mathcal{F}, R)$ is called a (normal) soft hypergroup over P if $\mathcal{F}(x)$ is a (normal) subhypergroup of P for all $x \in Supp(\mathcal{F}, R)$.

Definition 2.11. Let P_1 and P_2 be two hypergroups, $(\mathcal{F}, R)$ and $(\mathcal{G}, T)$ be soft hypergroups over P_1 and P_2, respectively. If $f : P_1 \longrightarrow P_2$ and $g : R \longrightarrow T$ are two mappings, then (f, g) is called a soft homomorphism if the following conditions hold:

1. f is a strong epimorphism;
2. g is a surjective mapping;
3. for all $x \in R$, $f(\mathcal{F}(x)) = \mathcal{G}(g(x))$.

Definition 2.12. If there is a soft homomorphism (f, g) between $(\mathcal{F}, R)$ and $(\mathcal{G}, T)$, we say that $(\mathcal{F}, R)$ is soft homomorphic to $(\mathcal{G}, T)$, denoted by $(\mathcal{F}, R) \sim (\mathcal{G}, T)$. Furthermore, if f is a strong isomorphism and g is a bijective mapping, then (f, g) is called a soft isomorphism, and $(\mathcal{F}, R)$ is soft isomorphic to $(\mathcal{G}, T)$, denoted by $(\mathcal{F}, R) \simeq (\mathcal{G}, T)$.

3 Category of soft hg-hypergroupoids

In this section, using the concept of soft hypergroup and soft hypergroupoid, we introduce and study the concept of soft hg-hypergroupoids. In addition, we present the category of soft hypergroup-hypergroupoids.

Definition 3.1. A hypergroup-hypergroupoid (or HG-hypergroupoid), is a hypergroup object in the category of hypergroupiods.

Definition 3.2. Suppose that H is a hg-hypergroupoid, $\mathcal{P}(H)$ is all subhg-hypergroupoids of H, and $\mathcal{F}$ is mapping $\mathcal{F} : R \longrightarrow \mathcal{P}(H)$, where R is a set of parameters, such that the set $\mathcal{F}(\kappa)$ is a subhg-hypergroupoid of H, for all $\kappa \in R$. Then the pair $(\mathcal{F}, R)$ is called a soft hg-hypergroupoid on H, and it is represented by the $(H, \mathcal{F}, R)$.

Remark 3.3. (i) A soft hg-hypergroupoid on H hg-hypergroupoid can be defined as a parametrized family of subhg-hypergroupoid of hg-hypergroupoid H.

(ii) Every hg-groupoid has a hypergroupoid structure. So every soft hg-hypergroupoid, has a soft hypergroupoid structure.

Now, we state and prove the following proposition:

Proposition 3.4. *The soft hypergroup $(\mathcal{F}, R)$, on the abelian hypergroup H, is a soft hg-hypergroupoid.*

Proof. Let $(\mathcal{F}, R)$ be a soft hypergroup on an abelian hypergroup H. So, $\mathcal{F}(\kappa)$ is a subhypergroup of H, for all $\kappa \in R$. Since H is abelian, so each $f(\kappa)$ is abelian, and so, is a hg-hypergroupoid. Moreover $f(\kappa)$ is a subhg-hypergroupoid of the hg-hypergroupoid H, for all $\kappa \in R$. Therefore, the soft hypegroup $(\mathcal{F}, R)$, is a soft hg-hypergroupoid. □

Definition 3.5. If $(H, \mathcal{F}, R)$ and $(H', \mathcal{F}', R')$ are two soft hg-hypergroupoids on their hg-hypergroupoids H and H', respectively. Then $(f, g) : (H, \mathcal{F}, R) \longrightarrow (H', \mathcal{F}', R')$ is called hg-hypergroupoid homomorphism, when (f, g) is a soft homomorphism.

Remark 3.6. A new category is obtained by taking the objects as soft hg-hypergroupoids and their morphisms as soft hg-hypergroupoid homomorphisms between them. We call this category, the category of soft hg-hypergroupoids and we show that with SHG-HGC.

Example 3.7. Let $(H, \mathcal{F}, R)$ be a soft hg-hypergroupoid and $(H', \mathcal{F}', R')$ be a soft subhypergroupoid of $(H, \mathcal{F}, R)$. If $Ob(\mathcal{F}'(\kappa)) \leq Ob(\mathcal{F}(\kappa))$ and $Mor(\mathcal{F}'(\kappa)) \leq Mor(\mathcal{F}(\kappa))$, for all $\kappa \in R'$, then $(H', \mathcal{F}', R')$ is called a soft subhg-hypergroupoid $(H, \mathcal{F}, R)$.

Definition 3.8. Let $(H', \mathcal{F}', R')$ be a soft hg-hypergroupoid of soft hg-hypergroupoid $(H, \mathcal{F}, R)$, such that, $Ob(\mathcal{F}'(\kappa)) \trianglerighteq Ob(\mathcal{F}(\kappa))$ and $Mor(\mathcal{F}'(\kappa)) \trianglerighteq Mor(\mathcal{F}(\kappa))$ for all $\kappa \in R'$. So $(H', \mathcal{F}', R')$ is called a normal soft subhg-hypergroupoid of $(H, \mathcal{F}, R)$.

4 Category of soft crossed hypergroupoids

In this section, we introduce soft crossed hypermodules.

Definition 4.1. Let P_1 and P_2 be two hypergroups, and $(P_1, \mathcal{F}, R)$, $(P_2, \mathcal{F}', T)$ are soft hypergroups over P_1 and P_2, respectively. In addition if $f : P_1 \to P_2$, $g : R \to T$ are two mapping, then (f, g) is called a soft homomorphism, if the following conditions hold:

1. f is a strong epimorphism,
2. g is a surjective mapping,
3. $f(\mathcal{F}(a)) = \mathcal{F}'(g(a))$, for all $a \in R$.

Definition 4.2. Suppose that P_1 and P_2 are two hypergroups, and $(P_1, \mathcal{F}, R)$, $(P_2, \mathcal{F}', R)$ are soft hypergroups over P_1 and P_2, respectively. Also $\mu = (f, g)$ is a soft homomorphism between $(P_1, \mathcal{F}, R)$ and $(P_2, \mathcal{F}', R)$, and

$$\begin{aligned} \Pi : P_2 \times P_1 &\longrightarrow \mathcal{P}^*(P_1) \\ (p_2, p_1) &\longmapsto \Pi(p_2, p_1) = p_1^{p_2}, \end{aligned}$$

is a (left) soft hyperaction P_2 on P_1, such that for all $\kappa \in R$,

$$\begin{aligned} \Pi_\kappa : \mathcal{F}(\kappa) \times \mathcal{F}'(\kappa) &\longrightarrow \mathcal{P}^*(\mathcal{F}'(\kappa)) \\ (H, K) &\longmapsto \Pi_\kappa(H, K) = H^K = \bigcup_{\substack{h \in H \\ k \in K}} h^k, \end{aligned}$$

and for all $\kappa \in R$, the following conditions are satisfied,

1. $f(K_1^{H_1}) = K_1 f(H_1) K_1^{-1}$, for all $K_1 \subseteq \mathcal{F}'(\kappa)$, and all $H_1 \subseteq \mathcal{F}(g(\kappa))$,
2. $f(K_1)K_2 = K_1 K_2 K_1^{-1}$, for all $K_1, K_2 \subseteq \mathcal{F}'(\kappa)$,

then (P_1, P_2, μ, R), is called a soft crossed hypermodule.

Example 4.3. If P_1 and P_2 are two hypergroups, $\mathcal{F}(\kappa) = P_1$ and $\mathcal{F}'(\kappa) = P_2$, for $\kappa \in R$, then (P_1, P_2, μ, R), is a soft crossed hypermodule.

Remark 4.4. In example 4.3, if P_1 and P_2 are two groups, then soft crossed hypermodule structure given, returns to the crossed module.

Example 4.5. If P_1 and P_2 are two hypergroups, and $(P_1, \mathcal{F}, R)$, $(P_2, \mathcal{F}', R)$ are soft hypergroups over P_1 and P_2 respectively. Also $\mu = (f, g)$ is a soft homomorphism between $(P_1, \mathcal{F}, R)$ and $(P_2, \mathcal{F}', R)$, and

$$\begin{array}{rcl} \Pi_\kappa : \mathcal{F}(\kappa) \times \mathcal{F}'(\kappa) & \longrightarrow & \mathcal{P}^*(\mathcal{F}'(\kappa)) \\ (H, K) & \longmapsto & \Pi_\kappa(H, K) = H^K = H, \end{array}$$

for all $\kappa \in R$, then (P_1, P_2, μ, R), is a soft crossed hypermodule.

Example 4.6. Suppose that P is a hypergroup, soft hypergroup $(P, \mathcal{F}, R)$, hyperact on itself with conjugate action. Also, $I = (I_P, I_R) : (P, \mathcal{F}, R) \to (P, \mathcal{F}, R)$, is a soft homomorphism, and so (P, P, I, R) has a structure soft crossed hypermodule.

Definition 4.7. Suppose that P_1, P_2, P_1' and P_2' are soft hypergroups, (P_1, P_2, μ, R) and (P_1', P_2', μ', T) are two soft crossed hypermodules,

$$\delta = (\delta_1, \mu_1) : (P_1, Q, R) \to (P_1', Q', T),$$

and

$$\delta^* = (\delta_2, \mu_2) : (P_2, \mathcal{F}, R) \to (P_2', \mathcal{F}', T),$$

are two soft homomorphisms. If for all $\kappa \in R$, the following conditions are met,

1. $\delta_2 \mu = \mu' \delta_1$,
2. $\delta_1(HK) = \delta_2(H)\delta_1(K)$, for all $H \subseteq \mathcal{F}(\kappa)$, and for all $K \subseteq Q(\kappa)$,
3. $(\delta_2 \times \delta_1)(\mathcal{F}(\kappa), Q(\kappa)) = (\mathcal{F}' \times Q')(\mu_2(\kappa), \mu_1(\kappa))$,

then, (δ, δ^*) is called a soft crossed hypermodule homomorphism, that means,

$$(\delta, \delta^*) : (P_1, P_2, \mu, R) \to (P_1', P_2', \mu', T).$$

Remark 4.8. If the objects are soft crossed hypermodules, and morphisms are soft crossed hypermodule morphisms between them, then we have a new category which we call it the category of soft crossed hypermodules and denote it by *SCHM*.

5 Categorys of soft crossed hypergroupoids and soft hg-hypergroupoids

In 1976, Brown and Spenccer proved that the category of crossed modules and the category of group-groupoids are equivalent [14]. In this section, we prove that soft crossed hypermodules and soft hg-hypergroupoids, are equivalent categories.

Theorem 5.1. *A soft hg-hypergroupoid, can be obtained from each soft crossed hypermodule.*

Proof. Let (H_1, H_2, δ, R) be a soft crossed hypermodule. So, H_1 and H_2 are hypergroups, and $(H_1, \mathcal{F}_1, R)$, $(H_2, \mathcal{F}_2, R)$ are soft hypergroups. By the set of objects H_2, the set of morphisms H_2, and the semi-direct product of $H_2 \ltimes H_1$, we construct $H = (H_2, H_2 \ltimes H_1)$, in the form of Hg-hypergroupoid. In addition, $\mathcal{F}_2(\kappa) \ltimes \mathcal{F}_1(\kappa)$ is a subhypergroup of $H_2 \ltimes H_1$, for all $\kappa \in R$. We can consider;

$$\begin{aligned} \mathcal{F}'' : R &\longrightarrow \mathcal{P}(H) \\ \kappa &\longmapsto \mathcal{F}''(\kappa) = (\mathcal{F}_2(\kappa), \mathcal{F}_2(\kappa) \ltimes \mathcal{F}_1(\kappa)) \end{aligned}$$

for all $\kappa \in R$.
But, $H_\kappa = (\mathcal{F}_2(\kappa), \mathcal{F}_2(\kappa) \ltimes \mathcal{F}_1(\kappa))$ for all $\kappa \in R$, has a Hg-hypergroupoid structure, and $H = (H_2, H_2 \ltimes H_1)$ is a subhg-hypergroupid. Hence $(H, \mathcal{F}'', R)$ is a soft hg-hypergroupoid. $\square$

Theorem 5.2. *A soft crossed hypermodule, can be obtained from each soft hg-hypergroupoid.*

Proof. Suppose that $(H_2, \mathcal{F}_2, R)$ is a soft hg-hypergroupoid. So, in $\mathcal{F}_2 : R \longrightarrow \mathcal{P}(H_2)$, $\mathcal{F}_2(\kappa)$ is a subhg-hypergroupoid of H_2, for all $\kappa \in R$. But H_2 is a subhg-hypergroupoid, therefore $Ob(H_2)$ has a hypergroup structure, and by

$$\begin{array}{rcl} \mathcal{F}_1 : R & \longrightarrow & \mathcal{P}(Ob(H_2)) \\ \kappa & \longmapsto & \mathcal{F}_1(\kappa) = Ob(\mathcal{F}_2(\kappa)) \end{array}$$

in the transform $\mathcal{F}_1$, defined as $Ob(\mathcal{F}_2(\kappa)) \leq Ob(H_2)$, for all $\kappa \in R$, the structure $(Ob(H_2), \mathcal{F}_1, R)$ is a soft hypergroup.
Also, H_2 and $\mathcal{F}_2(\kappa)$ are hg-hypergroupoids, and in transformations, $S : Mor(H_2) \longrightarrow Ob(H_2)$, and $S_\kappa : Mor(\mathcal{F}_2(\kappa)) \longrightarrow Ob(\mathcal{F}_2(\kappa))$, $KerS$ and $KerS_\kappa$ are hypergroups. Hence, the set of all subhypergroups of the $KerS$, is $\mathcal{P}(KerS)$.
Now, we define the $\mathcal{F}_3$ as

$$\begin{array}{rcl} \mathcal{F}_3 : R & \longrightarrow & \mathcal{P}(KerS) \\ \kappa & \longmapsto & \mathcal{F}_3(\kappa) = KerS_\kappa \end{array}$$

We have, $KerS_\kappa \leq KerS$, for all $\kappa \in R$, and the $(KreS, \mathcal{F}_3, R)$ is a soft hypergroup.
If r is a transformation as

$$r : Mor(H_2) \longrightarrow Ob(H_2), \quad \text{and} \quad (r \mid_{KerS}) = \rho_0 : KerS \longrightarrow Ob(H_2),$$

then, the transformation ρ_0 is a hypergroup homomorphism, and if $\beta : R \longrightarrow R$, is a subsequent transformation, then in diagram

$$\begin{array}{ccc} R & \xrightarrow{\mathcal{F}_3} & \mathcal{P}(KerS) \\ {\scriptstyle\beta}\downarrow & & \downarrow{\scriptstyle\rho_0} \\ R & \xrightarrow[\mathcal{F}_1]{} & \mathcal{P}(Ob(H_2)) \end{array}$$

we have, $\rho_0 \mathcal{F}_3 = \mathcal{F}_1 \beta$, and $\rho = (\rho_0, \beta) : (KerS, \mathcal{F}_3, R) \longrightarrow (Ob(H_2), \mathcal{F}_2, R)$ is a soft hypergroup homomorphism.
On the other hand, transform of

$$\begin{array}{rcl} \Pi_\kappa : Ob(\mathcal{F}_2(\kappa)) \times KerS_\kappa & \longrightarrow & KerS_\kappa \\ (x, y) & \longmapsto & \Pi_\kappa(x, y) = xy = I_x y I_x^{-1}, \end{array}$$

is an action, and therefore, soft hypergroup $(Ob(H_2), \mathcal{F}_1, R)$ has an action on the soft hypergroup $(KerS, \mathcal{F}_3, R)$.
But for all $\kappa \in R$ and $r(y) = x$,

$$\rho_0(xy) = \rho_0(I_x y I_x^{-1}) = \rho_0(I_x)\rho_0(y)\rho_0(I_x^{-1}) = x\rho_0(y)x^{-1},$$

and

$$\rho_0(y)y_1 = I_{\rho_0(y)} y_1 I_{\rho_0(y)}^{-1} = I_{r(y)} y_1 I_{r(y)}^{-1} = I_x y_1 I_x^{-1} = y y_1 y^{-1},$$

such that, the $(KerS, Ob(H_2), \rho, R)$, is a soft crossed hypermodule. □
Now, we prove that the category of soft hg-hypergroupoids is equivalent to the category of soft crossed hypermodules.

Theorem 5.3. *The category of soft hg-hypergroupoids is equivalent to the category of soft crossed hypermodules.*

Proof. Suppose that $\chi_1 = (H_1, \mathcal{G}_1, \delta_1, R_1)$ and $\chi_2 = (H_2, \mathcal{G}_2, \delta_2, R_2)$ are two soft crossed hypermodules,

$$h = (h_1, h_2) : \quad H_1 \to H_2$$

,

$$h^* = (h'_1, h'_2) : \quad \mathcal{G}_1 \to \mathcal{G}_2$$

and (h, h^*) is soft crossed hypermodule homomorphism. By Theorem 5.1, $\tau = (h'_1, h'_1 \times h_1)$, is functor and so, μ from soft crossed hypermoules to soft hg-hypergroupoids defined by $\mu(h, h^*) = (\tau, h'_2)$ is functor. But, if $(P_1, \mathcal{F}_1, R_1)$ and $(P_2, \mathcal{F}_2, R_2)$ are two hg-hypergroupoids and $\tau = (\tau_1, \tau_2)$ is a functor, and $(\tau, \eta) : (P_1, \mathcal{F}_1, R_1) \longrightarrow (P_2, \mathcal{F}_2, R_2)$, is soft hg-hypergroupoid homomorphism, then ν from soft hg-hypergroupoids to soft crossed hypermoules, defined by $\nu(\tau, \eta) = (h, h^*)$ is functor, where that is $h = (\tau_2 \mid_{KerS}, \eta)$ and $h^* = (\tau_1, \eta)$. We have considered ν and μ as functors, so $\nu\mu$ and $\mu\nu$ are functors. To complete the proof, it suffices to prove that $\nu\mu$, is identity in soft crossed hypermodules, and $\mu\nu$, is identity in soft hg-hypergroupoids. I and $\nu\mu$ are functor, for soft crossed hypermoules. The transformation of $\varsigma_{(\chi_1)} : \nu\mu(\chi_1) \longrightarrow I(\chi_1)$, is definable. Therefore, for all soft crossed hypermoule homomorphism of

$h = (h_1, h_2) : \chi_1 \longrightarrow \chi_2$, the diagram below is commutatively,

$$\begin{array}{ccc} \nu\mu(\chi_1) & \xrightarrow{\nu\mu(h)} & \nu\mu(\chi_2) \\ \downarrow{\scriptstyle \varsigma_{\chi_1}} & & \downarrow{\scriptstyle \varsigma_{\chi_2}} \\ I(\chi_1) & \xrightarrow[I(h)]{} & I(\chi_2) \end{array}$$

Hence, the transformation $\varsigma : \nu\mu \longrightarrow I$, is a natural transformation, and therefore $\nu\mu \simeq I$. I and $\mu\nu$ are functor, for soft hg-hypergroupoids. The transformation of $\varsigma_{(P_1,\mathcal{F}_1,R_1)} : \mu\nu(P_1, \mathcal{F}_1, R_1) \longrightarrow I(P_1, \mathcal{F}_1, R_1)$, is definable. So, for all soft hg-hypergroupoid homomorphism of

$$\zeta = (\tau, \eta) : (P_1, \mathcal{F}_1, R_1) \longrightarrow (P_2, \mathcal{F}_2, R_2),$$

the following diagram is commutatively,

$$\begin{array}{ccc} \mu\nu(P_1, \mathcal{F}_1, R_1) & \xrightarrow{\mu\nu(\zeta)} & \mu\nu(P_2, \mathcal{F}_2, R_2) \\ \downarrow{\scriptstyle \varsigma_{(P_1,\mathcal{F}_1,R_1)}} & & \downarrow{\scriptstyle \varsigma_{(P_2,\mathcal{F}_2,R_2)}} \\ I(P_1, \mathcal{F}_1, R_1) & \xrightarrow[I(\zeta)]{} & I(P_2, \mathcal{F}_2, R_2) \end{array}$$

Hence, the transformation $\varsigma : \mu\nu \longrightarrow I$, is a natural transformation, and therefore $\mu\nu \simeq I$. Therefore, the category of soft hg-hypergroupoids is equivalent to the category of soft crossed hypermodules.

□

6 Conclusion

In this article, after stating some definitions and necessary theorems of soft sets, according to the definition of soft hypergrouoid, we studied some of their properties. After that, by introducing the soft subhypergroupoid, we checked its properties. In addition, we defined soft action hypergroupoid, and obtained some results. In this manner, we established the category of soft hypergroupoids whose objects are soft hypergroupoids and, whose morphisms are soft hypergroupoid homomorphisms. Also, the concept of the soft crossed hypermodules was defined.

By defining the category of soft crossed hypermodules, it has been shown that a soft hg-hypergroupoid, can be obtained from each soft crossed hypermodule and a soft crossed hypermodule, can be obtained from each soft hg-hypergroupoid. Also we shown that soft hg-hypergroupoids and soft crossed hypermodules, are equivalent categories. In the continuation of the studies, it is possible to define and investigate the properties of soft 2-crossed modules and expand its concepts and properties to soft 2-crossed hypermodules.

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