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Evaluation of Geometry Learning Criteria and Selection of the Best Teaching Method using the Combined FTOPSIS - FBWM Technique

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Abstract. The aim of this research is to identify geometry learning indicators and determine the best teaching method using a combined fuzzy Delphi and FBWM-FTOPSIS technique The statistical population of this study includes 14 mathematics education experts specializing in geometry. Three types of questionnaires were used to collect data: a fuzzy Delphi questionnaire to identify indicators, a fuzzy best-worst method (FBWM) questionnaire to weight indicators, and a fuzzy TOPSIS questionnaire to prioritize the best teaching method. Initially, through literature review and expert opinions gathered from the questionnaires, primary and secondary factors influencing geometry learning were identified. Ultimately, 146 factors related to teacher, student, and space/facilities indicators were identified, with 110 factors accepted for further analysis. Subsequently, using the FBWM technique, final

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weights for the primary indicators were calculated, followed by determining the best teaching method in geometry using the fuzzy TOPSIS technique.

Teaching methods ranked included cooperative group learning, computer based instruction, exploratory learning, and other conventional methods. The results indicated that cooperative group learning emerged as the best method for teaching geometry, showing significant effectiveness and meaningful correlation with geometric concepts. These findings suggest that implementing interactive and collaborative methods can enhance student learning and deepen their understanding of geometric concepts.

AMS Subject Classification: 97G40

Keywords : Geometry, Teaching Method, Fuzzy Delphi Technique, FBWM, FTOPSIS

1 Introduction

Mathematics, as one of the fundamental subjects in educational systems, holds special importance in assessing students' skills. Despite its pivotal role, mathematics is often perceived as challenging and intimidating, and its learning process comes with various challenges (Arciosa, 2022). In many cases, students tend to memorize and reproduce information rather than achieving deep understanding, which does not enhance their logical thinking and creativity. Geometry, due to its specific nature and ability to connect abstract concepts with the real world, is of significant importance. This branch of mathematics not only aids in visual conceptualization but also engages students in intriguing reasoning (Kuzle, 2023). Therefore, effective geometry education can foster problem-solving skills and critical thinking among students. In recent years, geometric ideas have garnered much attention due to their new applications in mathematics and other fields such as sciences and arts. With new definitions and interpretations emerging, geometry encompasses various visual phenomena, making it an intriguing and substantial area for many educators. Moreover, possessing geometric knowledge appears essential for solving mathematical problems and everyday life issues (Pintrich et al, 2018). The National Mathematics Teachers' Association emphasizes the importance of geometry in school mathematics, acknowledging that teaching geometry provides an opportunity to enhance students' reasoning and logical skills. Geometry is a crucial topic in the discussion of spatial visualization in school mathematics, occupying a considerable portion of the mathematics curriculum. Furthermore, as geometry constitutes a valuable part of human culture, civilization, and history, it can effectively illustrate the relationship between mathematics and the real world for students (Ryan, 2020).

Fuzzy Logic in Geometry Education

Given the inherent uncertainty and subjectivity in educational assessment and teaching methodologies, fuzzy logic provides an effective approach to model such complexities. Fuzzy logic, introduced by Zadeh (1965), allows for handling imprecise information and provides a framework to evaluate subjective judgments systematically. This is particularly useful in geometry education, where multiple qualitative factors such as students' spatial reasoning, visual perception, and abstract understanding play a critical role. The fuzzy Delphi method (Ishikawa et al., 1993), a structured expert consensus approach, is employed to identify and validate key geometry learning indicators. By incorporating expert opinions and accommodating uncertainty in their judgments, the fuzzy Delphi method ensures a comprehensive and reliable set of indicators. Similarly, the fuzzy Best-Worst Method (FBWM) (Rezaei, 2015) facilitates the weighting and prioritization of these indicators by comparing their relative importance in a pairwise manner, while minimizing inconsistency. Lastly, the fuzzy TOPSIS technique (Hwang and Yoon, 1981) is used to rank and select the most effective teaching methods by considering multiple criteria, balancing both positive and negative aspects.These fuzzy methodologies address the inherent vagueness in human reasoning, allowing educators and researchers to design adaptive and effective teaching frameworks. By leveraging fuzzy logic, this study not only enhances the precision of identifying learning indicators but also provides a robust decision-making process for selecting optimal teaching methods.

Research Objectives

Based on these considerations, the research objectives are as follows:

1. Identification of geometry learning indicators using the fuzzy Delphi method.

2. Weighting and prioritizing geometry learning indicators using the fuzzy FBWM method.

3. Selection of the best teaching method based on the identified indicators using the fuzzy FTOPSIS technique.

2 Theoretical Foundations and Research Background A Review of Geometry

Geometry may be the oldest branch of mathematics. The Egyptians were the first to discover its principles, and their geometric problems often stemmed from their everyday needs. Egyptian geometry can be considered more as a repository of calculation rules without any substantial or justificatory basis (Greenberg, 2008). Among them, the Greeks also played an unparalleled role in advancing geometric knowledge. The foundation of ancient geometry relied primarily on experimentation, conjecture, similarities, and intuition, focusing more on relationships between lengths, surfaces, and volumes of physical shapes during that era (Razzak, 2020). The first systematic geometry, namely the one that derived its propositions through reasoning, was established by a Greek educator named Thales. Indeed, extracting regular laws through proofs is a prominent characteristic of Greek mathematics (Palatnik, 2022). Pythagoras and his disciples continued Thales' method of organizing geometry for two centuries. About 300 years before the birth of Christ, Euclid published his masterpiece, the 13-volume "Elements," compiling all known geometric results and consolidating the previous works and experiences. In the evolution of geometry, which has led to the emergence of new geometries and spaces, Iranian mathematicians have played an important role. Hakim Omar Khayyam was the first to discuss and address the issues of equations in terms of unknowns in order of grade, analyzing and examining. Khayyam is the first mathematician to find the roots of the third-degree equation geometrically and to prepare the ground for the application of algebra in geometry.
Importance of Geometry in the Curriculum Mathematics Program Geometry, as an important branch of mathematics, plays a very important role in the curriculum of mathematics. This role includes the devel-

opment of problem-solving skills, strengthening logical reasoning, increasing creativity and innovation, developing geometric thinking, and improving students' cognitive abilities.

The importance of geometry in mathematics curriculum programs includes presenting various problem-solving challenges, transferring mathematical concepts in visual and geometric ways, developing geometric and spatial thinking, connecting with other curriculum subjects, and connecting with the real world.

Geometry in mathematics curriculum programs helps students present mathematical concepts in a more realistic and visual way. It also helps them improve their ability to solve complex mathematical problems and have more logical arguments. In general, geometry has a very important role in mathematics curriculum programs and helps students focus on their mathematical thinking and skills. Geometry, as one of the main branches of mathematics, has a very important role in mathematics curriculum programs. This importance is based on various reasons, including the development of reasoning abilities, strengthening spatial visualization skills, and extensive applications in daily life and other sciences (Ajai, 2023).

Thom et al. (2024) explored the role of visual geometry and spatial reasoning in STEM education (Science, Technology, Engineering, and Mathematics), highlighting their significance in enhancing spatial skills and their impact on effective learning in these disciplines. The study demonstrates that concepts of visual geometry and spatial reasoning are powerful tools for better understanding complex concepts in science and engineering, helping students improve their 3D visualization and geometric analysis skills. Moreover, instruction in visual geometry can enhance students' spatial abilities and lead to better learning outcomes across various STEM fields. The article provides recommendations for integrating these concepts into STEM educational programs, including the use of design and simulation software, hands-on activities, and practical projects. Research findings indicate that students receiving instruction in visual geometry perform better in spatial and geometric problems, underscoring the importance of incorporating these concepts into STEM curricula.

Ozdemir et al. (2024) examined the effects of ACE cycle-based instruction (Activity, Class, Exercise) on students' self-efficacy beliefs in learning polygons in their study titled "The Impact of ACE Cycle-Based Instruction on Geometric Self-Efficacy Beliefs in Polygon Learning". The results indicate that using the ACE cycle in teaching polygons significantly enhances students' self-efficacy beliefs in geometry. Specifically, this method assists students in better understanding geometric concepts and gaining more confidence in solving geometric problems by providing collaborative activities and targeted exercises. These findings demonstrate that active and interaction-based teaching approaches can have a positive impact on academic achievement and self-efficacy beliefs across various educational domains.

Puechmorel (2023) explored the role of differential geometry and category theory in understanding learning processes in their study titled "The Role of Differential Geometry and Category Theory in Understanding Learning Processes". This research particularly focuses on the concept of feedback bundles and their application in modeling focused learning. The findings of this study indicate that using mathematical structures such as feedback bundles can lead to a better understanding of the dynamics of learning and complex interactions within educational systems. This mathematical approach facilitates a more precise analysis of learning processes and provides new tools for designing and improving educational methods. Consequently, employing differential geometry and category theory in the study of learning can contribute to the development of stronger and more practical theories in this field.

Fey et al, (2008), in their study titled "Challenges and Strategies for Improving Learning in Mathematics, Especially in Geometry", examines various topics including mathematical conceptualization, attention and concentration, enhancing mathematical thinking, and effective learning behaviors. The research aims to identify the challenges present in the learning process of mathematics and proposes solutions to enhance teaching and learning in this domain. The author investigates innovative teaching methods, develops cognitive educational programs, and explores the use of technology in mathematics education. By providing strategies to address challenges in learning mathematics, this article contributes to the development of theories and practical approaches in the field of mathematics education.

3 Research Methodology

The research, designed with a descriptive and applied objective, employs a hierarchical and rigorous approach to identify and select the best teaching methods for improving geometry learning, emphasizing learning indicators. The main stages of this methodology are as follows:
Utilization of Fuzzy Delphi Method for assessing and synthesizing experts' opinions: In this stage, experts' opinions on the importance of geometry learning indicators are gathered and evaluated. The Fuzzy Delphi method allows hierarchical aggregation and evaluation of opinions, considering uncertainty and fuzziness in the process.
Employment of Fuzzy Pairwise Comparisons (Fuzzy BWM): Following the consolidation of experts' opinions, weights are assigned to geometry learning indicators using the Fuzzy Pairwise Comparisons (Fuzzy BWM) method. This approach facilitates comparison and prioritization among different variables, taking into account uncertainty in the information.
Application of Fuzzy Topsis Method for ranking teaching methods: In this phase, based on data collected from questionnaires, different teaching methods are ranked according to geometry learning indicators using the Fuzzy Topsis method. This method comprehensively analyzes and evaluates the impact of each teaching method on geometry learning, considering various aspects of each method.

4 Research Stages

At this stage, after determining the theoretical framework of the research, the data are collected and processed as follows:

4.1 Identification of Geometry Learning using Fuzzy Delphi Technique

To identify the learning indicators in geometry, relevant indicators from previous studies were selected using a semi-structured questionnaire designed in the Delphi method and distributed to experts in the field of education. In order to identify the three-level indicators, the perspectives of 14 experts in the field of education were used. In terms of gender, 6 were male and 8 were female. In terms of age, 1 was under 30 years old, 6 were between 31 and 40 years old, 5 were between 41 and 50 years old, and 2 were over 51 years old. In terms of education, 7 of the experts had a bachelor's degree and 7 had a master's degree. In terms of work experience, 1 had less than 5 years, 1 had between 5 and 10 years, 2 had between 11 and 15 years of work experience, and 10 had more than 15 years of experience. The questionnaire was tailored to experts' responses, asking them to indicate the importance of each desired factor through a spectrum of opinions. If necessary, factors not initially included in the questionnaire were added to the list. Ultimately, after calculating the importance of criteria, factors scoring above 0.7 were selected as effective factors. Subsequently, second and third-level sub-indicator questionnaires were presented to experts and completed and reviewed similarly to the initial questionnaire. The use of a minimum score of 0.7 for confirming indicators was based on past research literature. The learning indicators have been categorized into four seasons, each comprising two levels of indicators. The first level, which is the primary level common across all seasons, includes teachers, students, and space and equipment. The second level, which is the sub-level, has been identified separately for each season. This will identify indicators in a more specialized and relevant way. If the previous two levels are ignored, general factors will be identified. And the accuracy of the work will be lower. A summary of the results of the fuzzy Delphi method is presented in Table 1:

Table 1: Summary of Delphi Technique Results for Primary Level Indicators

primary Indicator	**Average round 1**	**Average round 2**	**Coclusion**
Teacher	0.86	0.86	Accepted
Student	0.91	0.91	Accepted
Space and equipment	0.84	0.84	Accepted

Based on the results of the fuzzy Delphi technique, indicators with an average score of 0.7 or higher are considered important and very important, and they are accepted for further consideration. The identified indicators for weighting are presented in Table 2:

Table 2: Geometry Learning Indicators Based on Four Chapters

Chapter1: Geometric Drawing and Reasoning	**Chapter2: Thales' Theorem, Similarity**	**Chapter3: Polygons**	**Chapter4: Spatial Visualization**
	Teacher		
Teacher's analytical breakdown capabilities	Establishing relevance to daily life	Teacher's analytical breakdown capabilities	Establishing relevance to daily life
Sequencing and structuring of content	Sequencing and structuring of content	Logical reasoning in addressing issues by the teacher	Sequencing and structuring of content
Fostering student inquiry, research, and creativity	Teacher's analytical breakdown capabilities	Accuracy in addressing misconceptions and common student errors	Teacher's analytical breakdown capabilities
Teacher's analytical breakdown capabilities	Logical organization	Using different teaching methods based on the lesson topic	Logical organization

Chapter 1	**Chapter 2**	**Chapter 3**	**Chapter 4**
Logical organization	Logical reasoning in addressing issues by the teacher	Sequencing and structuring of content	Logical reasoning in addressing issues by the teacher
Logical reasoning in addressing issues by the teacher	Creating intellectual order	Establishing logical organization	Creating intellectual order
Creating intellectual order	Recognizing logical relationships between concepts	Recognizing logical relationships between concepts	Recognizing logical relationships between concepts
Recognizing logical relationships between concepts	Formulating practical questions to create motivation	Articulating geometric ideas precisely	Formulating practical questions to create motivation
Formulating practical questions to create motivation	Accuracy in addressing misconceptions and common student errors	Discussing the importance of reasoning and fair judgment	Using different teaching methods based on the lesson topic
Sequencing and structuring of content	Using different teaching methods based on the lesson topic		Accuracy in addressing misconceptions and common student errors

Chapter 1	**Chapter 2**	**Chapter 3**	**Chapter 4**
Logical organization	Articulating geometric ideas precisely		Strengthening mathematical discourse
Fostering student inquiry, research, and creativity	Discussing the importance of reasoning and fair judgment		
	Strengthening mathematical discourse		
Student			
Solving exercises and problems	Solving exercises and problems	Solving exercises and problems	Solving exercises and problems
Regular attendance in class	Regular attendance in class	Regular attendance in class	Regular attendance in class
Familiarity with points, lines, and planes	Precise understanding of proportion and its properties	Familiarity with polygons and their identification	Familiarity with points, lines, and planes
Recognizing coincident points	Understanding the basic theorem of triangle similarity	Identifying convex and concave polygons	Recognizing coincident points

Chapter 1	**Chapter 2**	**Chapter 3**	**Chapter 4**
Understanding the concept of perpendicularity, line perpendicular to a plane, and two perpendicular planes	Average intelligence and learning ability of students	Recognizing important quadrilaterals and defining them	Average intelligence and learning ability of students
Average intelligence and learning ability of students	Understanding the equality of areas of two triangles with a common base	Using triangles for reasoning	Understanding different perspectives
Understanding the concept of perpendicularity, line perpendicular to a plane, and two perpendicular planes	Familiarity with Thales' theorem and its proof	Average intelligence and learning ability of students	Understanding cross-sectional surfaces created in sphere, cylinder, prism, and cone
Understanding cross-sectional surfaces created in sphere, cylinder, prism, and cone	Understanding the relationship between angles and sides	Method for calculating the area of polygons	Understanding rotation around an axis and visualization of the shape created by it

Chapter 1	**Chapter 2**	**Chapter 3**	**Chapter 4**
Understanding rotation around an axis and visualization of the shape created by it	Proving the Pythagorean theorem		Understanding different positions of two lines in a plane and space
	Understanding the relationships of the sides of a right triangle		amiliarity with cross-sections of a spatial body
	Equipment and Space		
Textbook	Textbook	Textbook	Textbook
Suitable space for group work	Smart classroom	Creating environments for practical use of mathematical applications	Various three-dimensional objects and tools
Using handmade geometric structures and real-world objects in teaching spatial visualization	Creating environments for practical use of mathematical applications	Sponge-like geometric shapes	Creating environments for practical use of mathematical applications
Smart classroom	Drawing tool		Smart classroom

Chapter 1	**Chapter 2**	**Chapter 3**	**Chapter 4**
Various three-dimensional objects and tools	Suitable space for group work		Various three-dimensional objects and tools
Understanding rotation around an axis and visualization of the shape created by it	Various three-dimensional objects and tools		Visiting buildings and seeing geometric shapes in architecture
			Using fruits shaped like a cylinder, sphere, prism, and cone

According to the identification of the third level of indicators affecting geometry learning by the Delphi technique, in the first chapter of the teacher index, the three sub-indices of recognizing logical relationships between concepts, the teacher's analytical power, and logical reasoning in dealing with problems by the teacher, and in the student section, the highest scores are obtained by familiarity with points, lines, and planes, regular attendance in the classroom, and solving exercises and problems in the space and equipment section, respectively, textbooks, using geometric artifacts and real-world objects in teaching visual thinking, and appropriate space for group work. In the second chapter of the teacher index, the three sub-indices of stating practical questions to create motivation, using different teaching methods according to the subject of the lesson, and the teacher's analytical power, and in the student section, the highest scores are obtained by regular attendance in the classroom, solving exercises and problems, and familiarity with the basic theorem of similarity of triangles in the space and equipment section, respectively, creating environments for using practical examples of mathematical applications, textbooks, and smart classrooms. In the third chapter of the teacher index, the three sub-indices of accuracy in common misunderstandings and mistakes of students, the statement of practical questions to create motivation and logical reasoning in dealing with problems by the teacher, and in the student section, respectively, solving exercises and problems, regular attendance in the classroom and using triangles for reasoning in the space and equipment section, respectively, the textbook, creating environments for using practical examples of mathematical applications, and sponge artifacts in the shape of polygons have the highest scores. In the fourth chapter of the teacher index, the three sub-indices of teacher's analytical power, strengthening and developing the spirit of inquiry, research, and creativity of students, and observing the logical order of the materials, and in the student section, respectively, regular attendance in the classroom, solving exercises and problems, and recognizing the rotation around the axis and visualizing the shape created by it in the space and equipment section, respectively, the textbook, using geometric artifacts and real-world objects in teaching visual thinking, and various three-dimensional tools and objects have the highest scores.

4.2 Prioritization of Curriculum Chapters Using AHP Technique

For prioritizing curriculum chapters, the Analytic Hierarchy Process (AHP) method has been employed. In this method, the primary or first-level indicators include Chapter 1, Chapter 2, Chapter 3, and Chapter 4, while the second-level indicators consist of Teacher, Students, and Space and Equipment. Table 3 displays the weights of the first and second-level indicators influential in geometry learning.

Table 3: Weights of Primary and Secondary Indicators Influential in Geometry Learning

Primary Indicators	Weight	Secondary Indicators	Weight
Chapter 1	0.21	Teacher	0.11
		Student	0.06
		Space and Equipment	0.04
Chapter 2	0.40	Teacher	0.22
		Student	0.11
		Space and Equipment	0.08
Chapter 3	0.25	Teacher	0.13
		Student	0.07
		Space and Equipment	0.05
Chapter 4	0.15	Teacher	0.08
		Student	0.04
		Space and Equipment	0.03

The results indicate that Chapter 2 holds the highest importance among the other chapters in the process of geometry learning, followed by Chapter 3, Chapter 1, and Chapter 4, respectively. Additionally, the role of the teacher in facilitating student learning has been confirmed as a highly significant indicator.

4.3 Ranking Learning Indicators Using FBWM Technique

In the process of determining the best and worst weights of geometry learning criteria using the FBWM technique, first, the results of previous stages identifying indicators related to each of the four chapters were distributed among experts via a questionnaire. Experts selected the best and worst criteria from each chapter, focusing on teacher, student, and space and equipment in their evaluations. In the next step, the best and worst indicators were compared with all other indicators using a scale from 1 to 9. Subsequently, through implementation in the LINGO software, the weights of sub-criteria related to the teacher in Chapter 1 were extracted. In the final stage, by multiplying each of the sub-criteria by their main factor, the final weights of each desired factor were determined and specified. This process continued similarly for other factors. This method not only helps identify the most important factors in the geometry learning process but also provides a more precise quantitative weighting to these factors, facilitating necessary improvements in teaching and learning. Finally, the final weights of factors for each of the chapters are presented in the following table:

Table 4: Final Weights of Chapter 1 Learning Indicators

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
Chapter 1	0.21	Teacher	0.11	Establishing daily life connections for this chapter	0.112	0.01232
				Observing sequence and order of content	0.064	0.00704
				Enhancing students' inquiry, research, and creativity spirit	0.112	0.01232
				Teacher's analytical power	0.292	0.03212
				Logical presentation of materials	0.112	0.01232
				Logical reasoning in addressing issues by the teacher	0.045	0.00495

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Creating intellectual order	0.075	0.00825
				Recognizing logical relationships between concepts	0.075	0.00825
				Formulating practical questions to motivate	0.112	0.01232
		Student	0.06	Solving exercises and problems	0.076	0.00456
				Regular class attendance	0.091	0.00546
				Average intelligence and learning ability of students	0.054	0.00324
				Understanding points, lines, and planes	0.076	0.00456
				Recognizing collinear points	0.114	0.00684
				Understanding perpendicularity, line perpendicularity to a plane, and two planes perpendicular to each other	0.282	0.01692

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Understanding views from different directions	0.091	0.00546
				Recognizing cross-sectional shapes created in sphere, cylinder, prism, and cone	0.065	0.0039
				Understanding rotation around an axis and visualizing shapes created by it	0.152	0.00912
		Space & Equipment	0.04	Smart classroom	0.093	0.00372
				Suitable space for group work	0.124	0.00496
				Textbook	0.039	0.00156
				Creating environments for practical examples of mathematical applications	0.107	0.00428
				Various 3D tools and objects	0.529	0.02116

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Using handmade geometric structures and real-world objects in spatial thinking education	0.107	0.00428

In the teacher factor of Chapter 1, "Teacher's analytical power" with a weight of 0.292 holds the highest priority, followed by "Establishing daily life connections and enhancing inquiry spirit" with a weight of 0.112 in second priority, and "Creating intellectual order" with a weight of 0.075 in third priority. In the student factor, "Understanding perpendicularity" with a weight of 0.282, "Understanding rotation around an axis" with a weight of 0.152, and "Recognizing collinear points" with a weight of 0.114 rank as first to third priorities respectively. In the space and equipment factor, "Various 3D tools and objects" with a weight of 0.529, "Suitable space for group work" with a weight of 0.124, and "Using handmade geometric structures" with a weight of 0.107 rank as first to third priorities respectively.

Table 5: Final Weights of Chapter 2 Learning Indicators

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
Chapter 2	0.40	Teacher	0.22	Establishing daily life connections for this chapter	0.112	0.01232
				Observing sequence and order of content	0.044	0.00968
				Enhancing students' inquiry, research, and creativity spirit	0.044	0.00968
				Teacher's analytical power	0.057	0.01254
				Logical presentation of materials	0.079	0.01738
				Logical reasoning in addressing issues by the teacher	0.057	0.01254
				Creating intellectual order	0.044	0.00968
				Recognizing logical relationships between concepts	0.044	0.00968

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
				Formulating practical questions to motivate	0.057	0.01254
				Attention to misconceptions and common student errors	0.050	0.011
				Using various teaching methods based on lesson topic	0.225	0.0495
				Strengthening mathematical discourse	0.027	0.00594
				Precise expression of geometric ideas	0.080	0.0176
				Introducing geometry as a science based on reasoning and logic	0.099	0.02178
				Speaking about the importance of reasoning and fair judgment	0.050	0.011
		Student	0.11	Solving exercises and problems	0.078	0.00858
				Regular class attendance	0.078	0.00858

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Average intelligence and learning ability of students	0.047	0.00517
				Precise and clear understanding of proportion and its properties	0.058	0.00638
				Understanding equality of areas of two triangles with a common base	0.093	0.01023
				Familiarity with the Thales' theorem and its proof	0.093	0.01023
				Understanding the concept of similarity of two triangles and the relationship between angles and sides	0.280	0.0308
				Understanding the fundamental theorem of similarity of triangles	0.078	0.00858
				Proof of the Pythagorean theoremt	0.078	0.00858

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Understanding the relationships of lengths in a right-angled triangle	0.117	0.01287
		Space & Equipment	0.08	Smart classroom	0.104	0.00832
				Ruler	0.173	0.01384
				Suitable space for group work	0.173	0.01384
				Textbook	0.058	0.00464
				Creating environments for practical examples of mathematical applications	0.318	0.02544
				Various 3D tools and objects	0.173	0.01384

Data analysis shows that in Chapter 2, the primary priority in the teacher factor is assigned to ”Using various teaching methods based on lesson topic” with a weight of 0.225. This indicates the importance of flexibility and adapting teaching methods to the educational content, which can significantly impact student learning. In the student factor, "Understanding the concept of similarity of two triangles and the relationship between angles and sides" with a weight of 0.280 has been identified as the most critical factor, emphasizing the need for a deep understanding of fundamental geometry concepts to enhance students' analytical abilities. Additionally, in the space and equipment factor, "Creating environments for practical examples of mathematical applications" with a weight of 0.318 has been prioritized as the first priority, demonstrating that providing practical and tangible learning environments can contribute to improving the understanding of geometric concepts. These findings underscore that a combination of diverse teaching methods, focus on fundamental concepts, and providing suitable learning environments are key factors in improving geometry education in schools.

Table 6: Final Weights of Chapter 3 Learning Indicators

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
Chapter 3	0.25	Teacher	0.13	Observing sequence and order of content	0.086	0.01118
				Teacher's analytical power	0.072	0.00936
				Logical presentation of materials	0.108	0.01404
				Logical reasoning in addressing issues by the teacher	0.108	0.01404

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
				Recognizing logical relationships between concepts	0.290	0.0377
				Formulating practical questions to motivate	0.054	0.00702
				Attention to misconceptions and common student errors	0.086	0.01118
				Using various teaching methods based on lesson topic	0.061	0.00793
				Precise expression of geometric ideas	0.086	0.01118
				Speaking about the importance of reasoning and fair judgment	0.050	0.0065
		Student	0.07	Solving exercises and problems	0.099	0.00693
				Regular class attendance	0.083	0.00581
				Average intelligence and learning ability of students	0.062	0.00434

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Familiarity with polygons and their identification	0.099	0.00693
				Recognizing convex and concave polygons	0.124	0.00868
				Identifying important quadrilaterals and their definitions	0.310	0.0217
				Methods of calculating the area of polygons	0.099	0.00693
				Using triangles for reasoning	0.124	0.00868
		Space & Equipment	0.05	Textbook	0.105	0.00525
				Creating environments for practical examples of mathematical applications	0.159	0.00795
				Handmade sponge constructions in polygon shapes	0.737	0.03685

Analysis of the results from the assessment tables of Chapter 3 indicates that prioritizing indicators significantly impacts the improvement of geometry learning processes. In the teacher factor, "Recognizing logical relationships between concepts" with a weight of 0.290 as the first priority emphasizes the teacher's role in facilitating understanding of logical connections between various geometric topics. This can help students better comprehend complex concepts and apply them in practical scenarios. In the student factor, "Identifying important quadrilaterals and their definitions" with a weight of 0.310 as the first priority underscores the need to focus on fundamental principles and geometric basics in the learning process. This prioritization assists students in establishing stronger foundations in geometry. Lastly, in the space and equipment factor, "Handmade sponge constructions in polygon shapes" with a weight of 0.737 highlights the importance of using interactive and practical tools in geometry education. Utilizing these tools can enhance students' visualization of geometric concepts and increase their engagement with the course materials. Together, these findings emphasize the importance of a comprehensive and balanced approach to geometry education, focusing on logical teaching methods, solid foundations, and interactive tools.

Table 7: Final Weights of Chapter 4 Learning Indicators

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
Chapter 4	0.15	Teacher	0.08	Establishing connection of this chapter with daily life	0.284	0.02272

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
				Observing sequence and order of content	0.060	0.0048
				Strengthening and developing students' inquiry, research, and creativity	0.053	0.00424
				Teacher's analytical power	0.070	0.0056
				Logical presentation of materials	0.084	0.00672
				Logical reasoning in addressing issues by the teacher	0.084	0.00672
				Creating intellectual order	0.070	0.0056
				Recognizing logical relationships between concepts	0.105	0.0084
				Formulating practical questions to motivate	0.047	0.00376
				Using various teaching methods based on lesson topic	0.047	0.00376

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
				Strengthening mathematical discourse	0.047	0.00376
				Speaking about the importance of reasoning and fair judgment	0.049	0.00392
		Student	0.04	Solving exercises and problems	0.098	0.00392
				Regular class attendance	0.078	0.00312
				Average intelligence and learning ability of students	0.034	0.00136
				Familiarity with point, line, and plane	0.056	0.00224
				Recognizing collinear points	0.078	0.00312
				Recognizing different configurations of two lines in a plane and space	0.098	0.00392
				Understanding the concept of perspectives from different directions	0.230	0.0092

Level 1 Factor	**Level 1 Weight**	**Level 2 Factor**	**Level 2 Weight**	**Sub-factor**	**Sub-factor Weight**	**Final Weight**
				Familiarity with cross-sections of a spatial body	0.098	0.00392
				Recognizing sectional surfaces created in sphere, cylinder, prism, and cone	0.131	0.00524
				Understanding rotation about an axis and visualizing the resulting shape	0.098	0.00392
		Space & Equipment	0.03	Smart classroom	0.125	0.00375
				Textbook	0.042	0.00126
				Creating environments for practical examples of mathematical applications	0.104	0.00312
				Various three-dimensional tools and objects	0.104	0.00312

Level 1 Factor	Level 1 Weight	Level 2 Factor	Level 2 Weight	Sub-factor	Sub-factor Weight	Final Weight
				Visiting buildings and observing geometric shapes in architecture	0.125	0.00375
				Using fruits shaped as cylinder, sphere, prism, and cone	0.104	0.00312
				Using handmade geometric models and real-world objects in spatial thinking education	0.396	0.01188

Analysis of the results indicates that in the process of learning geometry, linking educational concepts with daily life and their practical applications for students is highly important. "Establishing connection of this chapter with daily life" has been selected as the top priority with a weight of 0.284 in the Teacher factor of Chapter 4. This approach helps students grasp geometric concepts more concretely and find greater motivation for learning. Additionally, "Understanding the concept of perspectives from different directions" with a weight of 0.230 as the top priority in the Student factor of Chapter 4 highlights the importance of enhancing spatial visualization abilities in geometry learning. The use of "Handmade geometric models and real-world objects" with a weight of 0.396 as the top priority in the Space and Equipment factor also emphasizes the role of interactive and practical teaching methods in improving the learning process of geometry and enhancing students' understanding of complex concepts. These findings underscore the importance of using interactive and practical teaching methods to improve the geometry learning process and increase students' understanding of complex concepts.

4.4 Ranking Teaching Methods in Geometry Using Topsis Technique

In this section of the research, the Topsis method has been employed to rank teaching methods in geometry. In the first step, the examined options, including exploratory teaching methods, computer-based methods, group and collaborative methods, problem-solving based methods, lecturing, preparatory methods, and scientific circulation-based methods, were evaluated using indicators relevant to geometry chapters. In the second step, collected data was gathered and valued to calculate fuzzy values, which were then entered into the corresponding table in Excel software. The next stage involved non-dimensional scaling of the fuzzy decision matrix using linear scale transformation. Finally,by calculating the distance of each option from the positive and negative ideal options, a final ranking was conducted, aiding in identifying options closer to the ideal solution and providing optimal performance. The use of the Topsis method in this study not only helps determine the best teaching method for each geometry chapter but also contributes to improving educational and learning processes in this field through precise and systematic evaluation. The results of the teaching methods ranking are presented in the following table:

Table 8: Results of Geometry Teaching Methods Ranking with Topsis Method

Options	**Abbreviation Symbol**	**Closeness Measure**	**Rank**
Exploratory Teaching Method	A1	0.29	3rd
Computer-Based Teaching Method	A2	0.55	2nd
Group and Collaborative Teaching Method	A3	0.72	1st
Problem-Solving Teaching Method	A4	0.34	6th
Lecture-Based Teaching Method	A5	0.21	7th
Preparatory Teaching Method	A6	0.49	4th
Scientific Circulation-Based Teaching Method	A7	0.37	5th

5 Discussion and Conclusion

In this research, using the fuzzy Delphi method, primary and secondary factors influencing geometry learning were identified. For this purpose, questionnaires related to the fuzzy Delphi technique were administered in three stages to 14 experts to gather their opinions on the factors under investigation. Each factor was assigned a qualitative word using fuzzy spectrum, and additional factors were introduced as needed based on the research objectives. Only factors with an average score above 0.7 were considered. Based on this, experts' opinions led to the confirmation of three main factors. Using the fuzzy Delphi method, initially, 60 teacher secondary indices from chapters one to four were examined, out of which 51 indices were confirmed by the research experts. Subsequently, 50 student secondary indices from chapters one to four were reviewed, with 37 of these indices being accepted. Finally, in the area of facility and equipment secondary indices, 36 indices were reviewed, with 22 being confirmed by the experts. Analysis of the factors influencing geometry learning through the fuzzy Delphi method indicates that the most influential factors can be categorized into three main groups: teacher, student, and facility and equipment. In the role of the teacher, key factors such as analytical power, maintaining content sequence, and relating content to daily life in different chapters are of particular importance. In the first chapter, fostering questioning spirit and creativity among students is also crucial. In the second chapter, emphasis on logical reasoning and using diverse teaching methods contributes to enhancing learning. In the third chapter, teacher's analytical skills and logical reasoning, along with posing practical questions, are significant, while in the fourth chapter, using diverse teaching methods and analytical approaches for spatial visualization learning play a critical role. Regarding the student's role, regular class attendance, solving exercises and problems, and understanding basic geometry concepts such as point, line, and plane are essential for success in the first chapter. In the second chapter, solving exercises and problems, regular class attendance, and precise understanding of proportion and its characteristics are crucial. In the third chapter, familiarity with the concept of polygons and identifying their types is effective. In the fourth chapter, understanding perspectives from different angles and recognizing cross-sectional areas and rotation around axes are key skills. In the role of facilities and equipment, the use of geometric models and suitable environments for group work in the first chapter, creating environments for practical examples and smart classrooms in the second chapter, environments with practical examples of mathematical applications in the third chapter, and using three-dimensional tools and objects to teach spatial thinking in the fourth chapter are important. Therefore, this research demonstrates that geometry learning requires attention to multiple factors categorized into three main sections: teacher, student, and facility and equipment. Teachers can significantly impact learning with diverse teaching methods and by relating content to daily life. Students will achieve greater success through regular class attendance, solving exercises, and grasping fundamental concepts. Additionally, the use of suitable facilities and equipment, including educational tools and practical environments, can aid in improving the learning process. Overall, a comprehensive and coordinated approach across all these areas can lead to effective and sustainable geometry learning. In the next stage, using the FBMW technique, the influential indices in geometry learning were weighted and prioritized. Analysis of weights indicates that the second chapter of geometry has the greatest impact on student learning, with a weight of 0.40. This chapter holds special importance, requiring greater attention in teaching and educational planning. The third chapter follows with a weight of 0.25, and the first chapter with 0.21, indicating both chapters also have significant impacts. The fourth chapter, with a weight of 0.15, holds the least importance but should still not be disregarded. In the examination of secondary level indices, the teacher has the most significant impact across all chapters, especially in the second chapter with a weight of 0.22. This underscores the critical role of the teacher in the learning process. Student indices also play an important role across all chapters, particularly in the second chapter. Although facilities and equipment have a lower weight compared to teachers and students, they still have a considerable impact on student learning. These results indicate that to improve geometry learning, special focus should be placed on the second chapter and the role of teachers, as well as improving educational facilities and equipment. The second chapter, due to its high weight (0.40), requires more attention in teaching methods and educational resources. Teachers play a vital role across all chapters and should enhance their teaching abilities and methods. Additionally, students should effectively grasp concepts through solving exercises, problems, and regular class attendance. Improving educational facilities and equipment, despite having a lower weight, contributes positively to learning and should not be overlooked. Overall, a comprehensive and coordinated approach across all these areas can lead to effective and sustainable geometry learning. In the next stage, the fuzzy TOPSIS method was used to select the best teaching method for geometry. TOPSIS prioritizes and ranks options based on predetermined criteria. This research used primary criteria such as chapters for weighting, which were weighted using the Analytic Hierarchy Process (AHP) method. Then, different options were evaluated and ranked based on these criteria. In this analysis, results indicated that collaborative group teaching using TOPSIS as the best option for teaching geometry has been selected. This choice has been validated due to its potential to enhance active student participation. With this method, students have the opportunity to present their ideas for solving geometric problems in group activities, engage in logical reasoning, and benefit from each other's experiences. On the other hand, other methods such as computer-based teaching, exploratory teaching, and pre-structured teaching have also been ranked accordingly, each possessing their own unique features and advantages. For example, computer-based teaching can enhance the learning process through the use of new technologies, facilitating greater interaction with geometric concepts. Similarly, exploratory teaching allows students to autonomously discover concepts and utilize experimental approaches to problem-solving. However, selecting the best teaching method depends on the specific conditions of each educational environment and the needs of students, which, according to TOPSIS analysis results, can significantly improve the geometry teaching process.

6 Practical Recommendations

The aim of this study is to review past research in the field of geometry and identify key factors influencing it, in order to select an appropriate method for teaching aimed at improving geometry learning. Making sound decisions requires a coherent understanding of various influential factors on decision-making environments. Based on the findings and analysis conducted, the practical recommendations for improving the geometry education process include:

6.1 Enhancing Collaborative and Participatory Teaching Methodsn

Using methods that encourage students to engage in group activities and collaboration in class can accelerate the learning and understanding of geometric concepts. This approach allows students to collaborate with each other actively and enhance their understanding of concepts.

6.2 Optimal Use of Computer Technologies

Developing and upgrading technology-based educational systems, such as interactive software, video systems, and virtual learning tools, can have a positive impact on student learning. These technologies can be employed as interactive and engaging tools during instructional sessions.

6.3 Connecting Geometric Concepts to Everyday Life

Efforts to directly relate educational concepts to students' everyday realities can improve motivation and enhance the application of geometric concepts in real life. Using practical and applicable examples relevant to daily life can strengthen this connection.

6.4 Facilitating the Use of Active Learning Environments

Creating active learning environments that include using three-dimensional geometric tools, modeling and simulation, and employing geometric objects in virtual reality, among others, can ensure better learning of geometric concepts.

6.5 Emphasis on Developing Logical Reasoning and Problem-Solving Skills

Strengthening logical reasoning, fair judgment, and problem-solving skills in students through interactive geometric examples and problems can enhance their analytical and reasoning abilities. These practical recommendations are aimed at fostering effective and sustainable improvements in the process of teaching geometry, aligning with the identified influential factors and findings of the study.

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