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Original Research Paper

An Integrated Model for Classifying Flexible Measures in Inverse DEA: Application to Banking Industry

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Abstract. In the traditional data envelopment analysis (DEA) models, the role of measures from input and output aspects is known. However, in many cases, we face a situation where some measures can play the role of input or output. The role of these measures is determined as input or output with the aim of maximizing the efficiency of the decision making unit (DMU) under evaluation. In this paper, we present a novel inverse DEA model to classify these inputs and outputs. We determine the new level of inputs and outputs and flexible measures by choosing the target efficiency for the DMUs. In this regard, the new model may choose flexible measures as input or output, but the main goal is to reach the target efficiency level. In the following, we will illustrate the presented approach with a simple numerical example. Finally, a numerical real example propose in the banking industry in Indonesia to clarify and demonstrate the suggested approach. We also bring the results of the models.

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1 Introduction

Original DEA models capture the performance of the DMUs by assuming that the state of each measure is clearly defined as an input or output variable. However, in the real world, certain measures can play the role of input for some DMUs and the role of output for others. These variables are known as flexible measures. For example, the measure of "research income" in a higher education program, or the measures of "high value customers" and "deposit" in a bank branch, can be considered as a flexible measure. These measures can have the role of input or output. (Beasley, [7]). Bala and Cook [8] evaluated the performance of branches in the banking industry in the presence of flexible measures. Cook and Zhou [9] presented a model in the form of a fractional programming problem to determine whether a measurement is input or output. Toloo[22] showed that their model may not calculate efficiency scores correctly and presented a mixed integer linear programming model to deal with flexible measures. Amirteimoori and Emrouznejad [6], Toloo[23] stated that one of the drawbacks of this method presented by Cook and Zhou[9] is the requirement to enter additional information to decide on the role of each variable, and also if the model has a different optimal solution, the results of choosing a flexible measure as input or output are the same for some DMUs and it is reasonable not to consider it for classifying inputs and outputs. Amirteimoori et al.[5] presented a model based on flexible slacks to calculate the relative efficiency of DMUs in the presence of flexible measures. Kordrostami et al.[18] proposed DEA models by considering integer-valued DEA for evaluation efficiency in presence of flexible measures. They classified inputs and outputs. Kordrostami and Noveiri[19] developed a novel model when flexible and negative data are in dataset. Azizi and Amirteimoori [7] proposed models for classifying inputs and outputs in the presence of imprecise data and presented efficiency evaluation models in the simultaneous presence of imprecise and flexible data. They consider imprecise data as intervals. Tohidi and Matroud [20] proposed a non-oriented model to classify inputs and outputs when we have flexible measures. Toloo et al. [24] presented a non-radial directional distance-based DEA model for determine role of inputs and outputs as flexible measures. Their approach be including two models that were pessimistic and optimistic, from both individual

and summative points of view. Kiyadeh et al.[17] proposed a slacks-based classification DEA model to evaluate the efficiency of the DMUs in the presence of flexible measures. They showed that their model is more suitable for achieving the desired objectives in DEA than the previous approaches. Ghiyasi and Cook [16] presented a new DEA model in the presence of flexible measures as dual role variables. They showed that the model of Cook and Zhu[9] in the variable returns to scale technology for the aggregate unit may have an unbounded optimal solution. They revised the model of Cook and Zhu[9].
The traditional DEA models aim to calculate the efficiency score of a DMU. However, inverse DEA models suppose that the amount of efficiency of a DMU is predetermined and the levels of inputs or outputs determined. Zhang and Cui [28] was first suggested by inverse DEA model and in the following Wei et al.[27] developed these models. Gattoufi et al. [13] proposed a new inverse DEA model in mergers and acquisitions for estimating the optimal level of inputs and outputs for a given efficiency target. They proposed a new model that combined the level of inputs and outputs of two DMUs to provide a new DMU with a certain level of efficiency target. Amin et al. [2] proposed a general model for firms' restructuring. The restructuring scenarios, namely consolidation and split. Emrouznejad et al. [12] developed a new application of inverse DEA in environmental efficiency to determine the optimal allocation of CO_2 emissions reduction in Chinese manufacturing industries. Wegener and Amin [25] proposed an inverse DEA model for minimizing greenhouse gas emissions in the gas and oil industries. Ghiyasi [15] introduced novel criterion models in the inverse DEA. Amin and Al-Muharrami [3] proposed inverse DEA models in the mergers and acquisitions of firms in the presence of negative data. Amin et al. [4] presented a combined inverse DEA and goal programming approach for target setting. Gerami et al. [14] proposed a generalized inverse DEA model for firm restructuring based on value efficiency. Toloo et al.[24] proposed new DEA models for classifying flexible measures, they used of the role of non-Archimedean epsilon. Ebrahimi and Hajizadeh [10] proposed a novel DEA model for solving performance measurement problems with flexible measures, they applied it tehran stock exchange. Abolghasem et al.[1] proposed a new model for evaluating cross-efficiency of to healthcare systems in

the presence of flexible measures. Toloo et al.[25] developed a non-radial directional distance method on classifying inputs and outputs in DEA and they applied their approaches to banking industry.
Ebrahimi and Toloo[11] developed a new approach in the classifications of flexible measures in presence of imprecise DEA.
It can be said that the main contribution of this paper is as follows. We present a new model with the structure of inverse DEA. It determines the role of flexible measures as input and output in the models based on the target efficiency level. Also, the model determines the optimal level of inputs and outputs and the flexible measures of the DMUs that are selected as merger DMUs. The target efficiency score predetermines by decision maker. The presented models were presented to carry out the merger process based on the inverse DEA in order to create new units with a certain target efficiency level. New models offer the ability to perform inverse DEA in the presence of flexible measures.
The remainder of the paper organized as follows. The second section presents the DEA model for dealing with flexible measures. The third section present a new inverse DEA in presence of flexible measures. The fourth section illustrate models with a numerical example. The fifth section proposed an application in banking and at the end we present the results of the research.

2 Inverse DEA for Merger

In the inverse DEA process, we obtain the most suitable inputs (outputs) of the DMU under evaluation while keeping the relative efficiency score without change. Thus, we can propose strengths and weaknesses of organizations. To determine the optimal level for input and output of DMUs, we can apply inverse DEA models. In this study, we present a process in the inverse DEA process. We obtained the corresponding efficiency scores for each of the DMUs using DEA model.
Let to create a new unit $DMU_T = (X_T, Y_T)$ with a certain amount of efficiency target in the merging process. We use the input and output levels of $DMU_k = (X_k, Y_k)$ and $DMU_h = (X_h, Y_h)$ as two observed units. Suppose x_{ij} and y_{rj} are the ith input and the rth output of the DMU_j, for each $r = 1, \cdots, s$, $i = 1, \cdots, m$, $j = 1, \cdots, n$. Consider

there are two pre-merger DMUs, DMU_k and DMU_h, they are merged to produce a new post-merger entity, DMU_T. The inverse DEA technical efficiency model in the input oriented based on Gattoufi et al. [13] can be expressed as:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m}(\varphi_{ik}+\varphi_{ih}) \\
s.t. \quad & \sum_{j\in F}\lambda_j x_{ij}+(x_{ik}+x_{ih})\lambda_T \le \bar{\rho}(\varphi_{ik}+\varphi_{ih}), \quad i=1,\cdots,m, \\
& \sum_{j\in F}\lambda_j y_{rj}+(y_{rk}+y_{rh})\lambda_T \ge (y_{rk}+y_{rh}), \qquad r=1,\cdots,s, \\
& \sum_{j\in F}\lambda_j+\lambda_T=1, \\
& \quad 0\le\varphi_{ik}\le x_{ik}, \quad i=1,\cdots,m, \\
& \quad 0\le\varphi_{ih}\le x_{ih}, \quad i=1,\cdots,m, \\
& \qquad \lambda_j\ge 0, \lambda_T\ge 0, \ j\in F.
\end{aligned} \tag{1}
$$

Model(1) propose a approach for the inverse DEA process with respect to the target efficiency. We obtain the new level of input components. We maintain relative efficiency of under evaluation DMU unchanged and obtain a new combination of input components to output components of under evaluation DMU in such a way that new inputs and outputs can be feasible.

Suppose F shows a set of existing counterparts in the post-merger evaluation process, and for each $j \in F$, λ_j is the intensity variable, and λ_T shows the intensity variable corresponding to the new unit, namely,DMU_T in the merging process. Assume that $(\varphi^*_{1k}, \varphi^*_{2k}, ..., \varphi^*_{mk})$, $(\varphi^*_{1h}, \varphi^*_{2h}, ..., \varphi^*_{mh})$, are an optimal solution of model (3). We define the input and output level and the technical efficiency score of the new unit resulting from the merging process as follows.

$X_T = (\varphi^*_{1k} + \varphi^*_{1h}, \varphi^*_{2k} + \varphi^*_{2h}, ..., \varphi^*_{mk} + \varphi^*_{mh})$, $Y_T = (y_{1k} + y_{1h}, y_{2k} + y_{2h}, ..., y_{sk} + y_{sh})$, $\bar{\rho}$. It should be noted that in model (1), we predetermine the efficiency score corresponding to the post-merger entity, namely DMU_T, as the given efficiency target and consider the output level of the post-merger entity equal to the sum of the outputs of the pre-merger DMUs, and we obtain the minimum input level of the post-merger entity. Similarly, the inverse DEA technical efficiency model in the output oriented can be expressed as follows.

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} \psi_r \\
s.t. \quad & \sum_{j\in F} \lambda_j x_{ij} + (x_{ik}+x_{ih})\lambda_T \le (x_{ik}+x_{ih}), \quad i=1,\cdots,m, \\
& \sum_{j\in F} \lambda_j y_{rj} + (y_{rk}+y_{rh}+\psi_r)\lambda_T \ge \bar{\gamma}(y_{rk}+y_{rh}+\psi_r),\ r=1,\cdots,s, \\
& \sum_{j\in F} \lambda_j + \lambda_T = 1. \\
& \quad 0 \le \psi_r, \quad r=1,\cdots,s, \\
& \quad \lambda_j \ge 0, \lambda_T \ge 0,\ j\in F.
\end{aligned} \tag{2}$$

Assume that $(\psi_1^*, \psi_2^*, ..., \psi_s^*)$ is an optimal solution of model (2). We define the input and output level and the technical efficiency score of the new unit resulting from the merging process as follows. $X_T = (x_{1k} + x_{1h}, x_{2k}+x_{2h}, ..., x_{mk}+x_{mh})$, $Y_T = (y_{1k}+y_{1h}+\psi_1^*, y_{2k}+y_{2h}+\psi_2^*, ..., y_{sk}+y_{sh}+\psi_s^*)$, $\bar{\gamma}$. It should be noted that in model (2), we predetermine the efficiency score corresponding to the post-merger entity, namely DMU_T, as the given efficiency target and consider the output level of the post-merger entity equal to the sum of the inputs of the pre-merger DMUs, and we obtain the maximum output level of the post-merger entity.

3 Inputs and Outputs Classification in DEA

Consider n DMUs as $DMU_j = (X_j, Z_j, Y_j)$, where the input vector $X_j = (x_{1j}, ..., x_{mj}) \in R^m$ are used to produce the output vector $Y_j = (y_{1j}, ..., y_{sj}) \in R^s$. Also assume that each of these DMUs has a flexible measure as $Z_j = (z_{1j}, ..., z_{Dj}) \in R^D$. This measure is considered as an input for some DMUs and as an output for others. Also, this measure can play the role of input or output for the unit under evaluation. Flexible measures will play an input or output role in the evaluation of DMUs. Therefore, the values of flexible measures will be effective in evaluating efficiency. If they have the role of input, they decrease and if they have the role of output, they increase the efficiency of the DMU under evaluation. DEA models for dealing with flexible sizes are different compared to inverse DEA models. The former models obtain the efficiency scores from the DMUs, while in the later models, the optimal level of inputs and outputs from new units is determined based on the target efficiency scores. In both models, the role of flexibility measures is determined as input and output. The model (1) proposed for measuring the efficiency of $DMU_o = (X_o, Z_o, Y_o)$ in presence of flexible measures by Amirteimoori and Emrouznejad [6] as follows.

$$\begin{aligned}
\rho^{CL*} = \min \ & \rho^{CL} \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \rho^{CL} x_{io}, \quad i = 1, \cdots, m, \\
& \sum_{j=1}^{n} \lambda_j z_{fj} \leq \rho^{CL} z_{fo} + M\sigma_f, \ f = 1, \cdots, l, \\
& \sum_{j=1}^{n} \lambda_j z_{fj} \geq z_{fo} - M\eta_f, \ f = 1, \cdots, l, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \cdots, s, \\
& \quad 0 \leq \lambda_j, \ j = 1, \cdots, n, \\
& \quad \sigma_f + \eta_f = 1, \qquad f = 1, \cdots, l, \\
& \quad \sigma_f, \eta_f \in \{0, 1\}, \qquad f = 1, \cdots, l.
\end{aligned} \tag{3}$$

In model (3), by choosing $\sigma_f = 0$ and $\eta_f = 1$ the fourth constraint is redundant and the third constraint is satisfied. In this way, z_{fo} is considered as input for the unit under evaluation, i.e. $DMU_o = (X_o, Z_o, Y_o)$. Similarly, in model (3) by choosing $\sigma_f = 1$ and $\eta_f = 0$ the third constraint is redundant and the fourthconstraint is established. In this way, z_{fo} for the unit under evaluation, i.e. $DMU_o = (X_o, Z_o, Y_o)$ is considered as output. In model (3), only one of the the third and the fourthconstraint will be satisfied, and these two constraints are not simultaneously satisfied. M is a large number. In the model (3),λ_j is the intensity variable, the variable ρ^{CL} is corresponding variable to reduce input components.

Definition 3.1. $DMU_o = (X_o, Z_o, Y_o)$ is efficient DMU if and only if $\rho^{CL*} = 1$, else it is inefficient.

4 Inputs and Outputs Classification Based on the Inverse DEA

Now, we propose a novel inverse DEA for classifying of inputs and outputs in DEA. This model obtain the optimal level of inputs and flexible measures based on the target efficiency score of the new unit created in the merger process. Let $\bar{\rho}^{CL}$ is the target efficiency score for the new unit created (the merged entity T) in the merger process by selecting DMU_k and DMU_h as units to merge in the inverse DEA process. λ_j is the intensity variable. DMUs k and h are consolidating their activities. Let's T show the merged entity generated by the consolidation and also F is the set of indices of all DMUs except k and h. Let φ_{ik} and φ_{ih} be the levels of the i-th input from the merging DMU_k and DMU_h, respectively, also, let τ_{fk} and τ_{fh} be the levels of the f-th flexible measure (in input role) from the merging DMU_k and DMU_h, respectively that is kept by the merged entity T. In this consolidation, we proposed the following input oriented inverse DEA model for classifying of inputs and outputs.

$$
\begin{array}{ll}
\rho^{CL*} = \min \ (\sum_{i=1}^{m}(\varphi_{ik}+\varphi_{ih}) + \sum_{f=1}^{l}(\tau_{fk}+\tau_{fh})) & \\
s.t. \quad \sum_{j\in F}\lambda_j x_{ij} + \lambda_T(x_{ik}+x_{ih}) \leq \bar{\rho}^{CL}(\varphi_{ik}+\varphi_{ih}), & i=1,\cdots,m, \\
\sum_{j\in F}\lambda_j z_{fj} + \lambda_T(z_{fk}+z_{fh}) \leq \bar{\rho}^{CL}(\tau_{fk}+\tau_{fh}) + M\sigma_f, & f=1,\cdots,l, \\
\sum_{j\in F}\lambda_j z_{fj} + \lambda_T(z_{fk}+z_{fh}) \geq (z_{fk}+z_{fh}) - M\eta_f, & f=1,\cdots,l, \\
\sum_{j\in F}\lambda_j y_{rj} + \lambda_T(y_{rk}+y_{rh}) \geq (y_{rk}+y_{rh}), & r=1,\cdots,s, \\
0 \leq \lambda_j, \ j\in F & \\
\sigma_f + \eta_f = 1, & f=1,\cdots,l, \\
\sigma_f, \eta_f \in \{0,1\}, & f=1,\cdots,l.
\end{array}
\tag{4}
$$

In model (4), by choosing $\sigma_f = 0$ and $\eta_f = 1$ third constraint is redundant and fourth constraint is satisfied. In this way, z_{fo} is considered as input for the DMU under evaluation, i.e. $DMU_o = (X_o, Z_o, Y_o)$ in the

inverse DEA process. Similarly, in model (4) by choosing $\sigma_f = 1$ and $\eta_f = 0$ fourth constraint is redundant and third constraint is established in the inverse DEA process. In this way, z_{fo} for the DMU under evaluation, i.e. $DMU_o = (X_o, Z_o, Y_o)$ is considered as output. In model (4), only one of the third and fourth constraints will be satisfied, and these two constraints are not simultaneously satisfied.M is a large number. Assume that $(\varphi^*_{1k}, \varphi^*_{2k}, \cdots, \varphi^*_{mk}, \tau^*_{1k}, \tau^*_{2k}, \cdots, \tau^*_{lk})$, $(\varphi^*_{1h}, \varphi^*_{2h}, \cdots, \varphi^*_{mh}, \tau^*_{1h}, \tau^*_{2h}, \cdots, \tau^*_{lh})$ are an optimal solution of model (4). We define the input and output level and the technical efficiency score of the new DMU resulting from the merging process as follows.
$X_T = (\varphi^*_{1k} + \varphi^*_{1h}, \varphi^*_{2k} + \varphi^*_{2h}, \cdots, \varphi^*_{mk} + \varphi^*_{mh})$, $Z_T = (\tau^*_{1k} + \tau^*_{1h}, \tau^*_{2k} + \tau^*_{2h}, \cdots, \tau^*_{lk} + \tau^*_{lh})$, $Y_T = (y_{1k} + y_{1h}, y_{2k} + y_{2h}, \cdots, y_{sk} + y_{sh})$, $\bar{\rho}^{CL}$.

Theorem 4.1. *The model (4) is always feasible.*

Proof. Put $\lambda_j = 0$, $\lambda_T = 1$, $\varphi_{ik} = ((x_{ik} + x_{ih})/\bar{\rho}^{CL})$, $i = 1, \cdots, m$, $\sigma_f = 1$, $f = 1, \cdots, l$, $\tau_{fk} = \tau_{fh} = 0$, $f = 1, \cdots, l$, $\eta_f = 0$, $f = 1, \cdots, l$. It can be easily shown that this is a feasible solution for model (4) and the proof is complete. □

Similarly, we propose the inverse DEA efficiency model in the output oriented for classifying of inputs and outputs can as following.

$$
\begin{aligned}
\max\ & (\sum_{r=1}^{s} \psi_r + \sum_{f=1}^{l} \omega_f) \\
s.t.\ & \sum_{j \in F} \lambda_j x_{ij} + \lambda_T (x_{ik} + x_{ih}) \leq (x_{ik} + x_{ih}), \quad i = 1, \cdots, m, \\
& \sum_{j \in F} \lambda_j z_{fj} + \lambda_T (z_{fk} + z_{fh}) \leq (\tau_{fk} + \tau_{fh}) + M\sigma_f, \ f = 1, \cdots, l, \\
& \sum_{j \in F} \lambda_j z_{fj} + \lambda_T (z_{fk} + z_{fh}) \geq \bar{\gamma}^{CL} (z_{fk} + z_{fh} + \omega_f) - M\eta_f, \ f = 1, \cdots, l, \\
& \sum_{j \in F} \lambda_j y_{rj} + \lambda_T (y_{rk} + y_{rh}) \geq \bar{\gamma}^{CL} (y_{rk} + y_{rh} + \psi_r), \qquad r = 1, \cdots, s, \\
& \lambda_j \geq 0, \ j \in F \\
& \sigma_f + \eta_f = 1, \qquad f = 1, \cdots, l, \\
& \sigma_f, \eta_f \in \{0, 1\}, \qquad f = 1, \cdots, l.
\end{aligned}
\tag{5}
$$

Table 1: Input–output data.

DMU	Input1	Flexible	Output
1	1	3	6
2	2	5	6
3	1	6	5
4	1	7	6
5	1	8	5
6	2	5	6
7	7	7	2
8	9	3	2
Aggregated DMU	24	44	38

5 Numerical Examples

In this section, we use a numerical example to illustrate the proposed approach. We use of a data in Cook and Zhu's [9] for illustrating the proposed inverse DEA model in this paper in merger process. Let eight DMUs that consume one input to produce one output and has one flexible measure. The flexible measure can play the role of either an input or an output. At first, First, we obtain the efficiency score of the DMUs in the absence of flexible measure. The results are in the second column of Table (2). DMUs 1 and 4 are efficient and the other DMUs are inefficient. In the following, we solve model (3) in the two state. In the first state, we consider the aggregated DMU in the evaluation efficiency, the results are in the third to fifth columns of Table (2). Also. In the second state, we do not consider the aggregated DMU in the evaluation efficiency, the results are in the sixth to eighth columns of Table (2). DMUs 1 and 4 are efficient and the other DMUs are inefficient. As can be seen, in the evaluation of DMUs based on the model (3), DMUs 1, 2, 6, 7 and 8 select flexible measure as output and DMUs 3, 4 and 5 select flexible measure as input. Aggregated DMU select flexible measure as output. The second columns of Table2 show the results of model (3) without the flexible measure. The third to eighth columns of Table 2 show the results of model (3) by considering the flexible measure. The third to fifth columns of Table 2 show the results of model (3) by considering the flexible measure and the aggregated DMU. Also, The fifth to

Table 2: Classification results of model (3).

DMU	Efficiency	Efficiency	σ_1	η_1	Efficiency	σ_1	η_1
1	1	1	1	0	1	1	0
2	0.5	0.5	1	0	0.5	1	0
3	0.8333	0.8333	0	1	0.8333	0	1
4	1	1	0	1	1	0	1
5	0.8333	0.8333	0	1	0.8333	0	1
6	0.5	0.5	1	0	0.5	1	0
7	0.0476	0.125	1	0	0.125	1	0
8	0.037	0.0427	1	0	0.0427	1	0
AD	0.2639	0.2639	1	0	-	-	-

eighth columns of Table 2 show the results of model (3) by considering the flexible measure and without the aggregated DMU. We show the merged DMU by T. In the input-orientation, model keeps the amount of output of both DMUs, that is $Y_T = Y_k + Y_h$, and the model (4) find the minimum amount of the input and flexible measure (in the input role) of DMU_k and DMU_h in order to reach the desired given efficiency target. Table (3) shows the levels of the input and flexible measure from the merging DMU k and DMU h, for predetermined target efficiency score of DMU T. The results for different selection of DMU_k and DMU_h as units to merge in the inverse DEA process and different target efficiency score are shown in Table (3). For example, by putting $DMU_k = 5$ and $DMU_h = 6$, and the target efficiency equal to 1, we obtain merged DMU as follows. $X_T = (\varphi^*_{15} + \varphi^*_{16}) = (1 + 0.8571) = 1.8571$, $Z_T = (\tau^*_{15} + \tau^*_{16}) = (0 + 0) = 0$, $Y_T = (y_{15} + y_{16}) = (5 + 6) = 11$.

6 Application for Commercial Banks in Indonesia

Commercial banks play an important role in the development plans of a country. The proper performance of banks can also improve the performance of other related industries. Because these banks can provide facilities to other industries, such as factories, and create the dynamics of these industries. One of the most important issues in the banking

Table 3: The results of inverse DEA process based on the model (4) in numerical example.

Merge DMU	Target Efficiency	φ_{1k}	φ_{1h}
$DMU_k = 2$ and $DMU_h = 3$	1		0
$DMU_k = 2$ and $DMU_h = 3$	0.9	1.8333	0.037
$DMU_k = 2$ and $DMU_h = 3$	0.7	2	0.619
$DMU_k = 7$ and $DMU_h = 8$	1	2	0
$DMU_k = 7$ and $DMU_h = 8$	0.9	1.25	0
$DMU_k = 7$ and $DMU_h = 8$	0.7	1.3889	0
$DMU_k = 5$ and $DMU_h = 6$	1	1.7857	0.8571
$DMU_k = 5$ and $DMU_h = 6$	0.9	1	1.0635
$DMU_k = 5$ and $DMU_h = 6$	0.8	1	1.3214
τ_{1k}	τ_{1h}	σ_1	η_1
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0
0	0	1	0

system is the evaluation of the performance of banks over a period of time. Using an accurate tool for performance evaluation is very important. One of the appropriate techniques to evaluate the performance and measure the efficiency of banks is DEA. In addition to determining the efficiency score of banks, DEA can also rank them based on the efficiency scores and provide their strengths and weaknesses as DMUs. It can also provide appropriate targets corresponding to the inputs and outputs of banks. But the information about the inputs and outputs of a bank may be uncertain and have a degree of change during the evaluation period. For example, data may have a degree of fluctuation that cannot be determined accurately. However, it can be determined that this data is in a special interval. The proposed approach in this paper can be used to evaluate the performance of DMUs in uncertain conditions and to evaluate banks in the same condition in the presence

of uncertain data. In this section, we use our approach to evaluate the performance of commercial banks in Indonesia. The dataset mainly covers the variables from the balance sheet and income statement. The data for commercial banks is taken from the Fitch Solutions database (https://www.fitchsolutions.com/fitch-connect).
The information on banks is proposed in Table 4. It can be said that the reason for choosing these banks for evaluation is that the authors tried to use real data to show the applicability of their approach. Also, these results can help bank managers make appropriate decisions to improve the performance of banks. The input and output data are shown in Tables (5) and (6).

In this evaluation, we choose three inputs, three outputs, and one flexible measure for each bank. Inputs include personnel expenses, total interest expenses, and non-interest expenses.
Personnel expenses: include the costs that the bank pays for its personnel during this evaluation period. These costs include salaries, insurance, benefits and bonuses, overtime, insurance, and medical treatment.
Total interest expenses: They are the amount of interest paid to bank customers. Customers place their deposits with the bank based on a specific contract. The bank charges interest on each deposit. The total amount paid to customers during the evaluation period for these deposits is called the net interest expense.
Non-interest expenses: These costs include costs that are not directly related to attracting and maintaining deposit funds. These costs include the bank's costs in various cases, including building rent, costs related to the maintenance of bank properties, current costs of the bank, costs of creating and maintaining software and hardware facilities, service costs such as water and electricity, gas, and energy costs.
Also, in this evaluation, a flexible measure for banks was considered. This measure includes facilities and deferred loans. Overdue facilities are facilities where the payment of principal and interest is delayed for a period of time. If the borrower does not pay the principal or interest on his loan within a certain period, this loan is placed in the group of deferred assets. Non-payment of these loans can lead to a decrease in the bank's income, and the lower the amount of these arrears, the more appropriate it is from the point of view of management. These over-

Table 4: Name banks.

Bank Name	Bank
P.T. Bank Pan Indonesia Tbk	B01
PT Bank ANZ Indonesia	B02
PT Bank Artha Graha Internasional, Tbk	B03
PT Bank Bumi Arta Tbk	B04
PT Bank Central Asia Tbk	B05
PT Bank CIMB Niaga Tbk	B06
PT Bank CTBC Indonesia	B07
PT Bank Danamon Indonesia Tbk	B08
PT Bank DBS Indonesia	B09
PT Bank HSBC Indonesia	B10
PT Bank ICBC Indonesia	B11
PT Bank KB Bukopin, Tbk	B12
PT Bank KEB Hana Indonesia	B13
PT Bank Mandiri (Persero) Tbk	B14
PT Bank Mayapada Internasional Tbk	B15
PT Bank Maybank Indonesia Tbk	B16
PT Bank Mega Tbk	B17
PT Bank Mizuho Indonesia	B18
PT Bank Negara Indonesia (Persero) Tbk	B19
PT Bank Oke Indonesia Tbk	B20
PT Bank Pembangunan Daerah Banten, Tbk	B21
PT Bank QNB Indonesia Tbk	B22
PT Bank Rakyat Indonesia (Persero) Tbk	B23
PT Bank Rakyat Indonesia Agroniaga Tbk	B24
PT Bank Resona Perdania	B25
PT Bank Sahabat Sampoerna	B26
PT Bank Tabungan Negara (Persero) Tbk	B27
PT Bank Victoria International Tbk	B28
PT Bank Woori Saudara Indonesia 1906, Tbk	B29

due claims can be considered an undesirable output in the evaluation of banks' performance.

Three desirable outputs were also considered in this evaluation. Desired outputs include net interest income, non-interest income, and total deposits.

Net interest income: These incomes include the income that the bank

Table 5: Inputs of banks.

Bank	Input1	Input2	Input3
B01	166.32	523.1	380.74
B02	15.62	9.42	22.69
B03	22.54	74.88	73.24
B04	10.17	24.59	18.65
B05	946.46	797.01	2124.25
B06	302.79	583.66	577.45
B07	20.23	37.17	31.03
B08	357.16	452.9	621.59
B09	100.16	137.76	236.82
B10	122.69	101.6	236.68
B11	21.79	124.16	35.56
B12	61.83	337.21	196.61
B13	27.32	93.85	58.76
B14	1279.51	2184.54	2640.76
B15	56.91	356.2	102.04
B16	179.82	394.68	407.69
B17	89.95	293.01	210.81
B18	19.5	53.49	38.94
B19	691.3	1348.51	1716.69
B20	8.4	13.48	16.27
B21	8.44	24	20.89
B22	21.83	68.88	34.84
B23	1865.99	2674.41	3545.33
B24	14	92.65	22.25
B25	9.97	30.7	23.78
B26	17.62	45.59	29.76
B27	211.07	1149.13	478.32
B28	12.43	113.5	34.95
B29	18.48	83.53	52.45

earns from providing loan facilities to customers. This interest rate is determined by the bank based on this contract with customers. These incomes are the result of subtracting the total amount received from customers from the loan amount given to them. The total amount of net interest income for each of the banks is considered a desirable output. The bigger the amount of these revenues, the more income the bank can

earn.
Non-interest incomes: These incomes include bank incomes other than bank interest. These incomes include the income earned from customers from various services, including various fees, income from the transfer of various funds by customers, ATM machines, income from interbank transfers, income from Internet services, fees related to sending SMS to customers, etc.
Total deposits: These deposits include current deposits, short-term deposits, and long-term deposits. The larger the total amount of deposits, the higher the liquidity of the bank, and the bank can pay facilities to its customers, and as a result, it can receive higher interest from the place of payment of facilities. The bank pays a small interest rate for short-term deposits but pays more interest for long-term deposits. But they do not pay interest on current deposits. The more time the deposits are available to the bank and the larger their amount, the greater the bank's liquidity will be, and the bank can pay facilities to its customers from the deposits and earn a larger profit from the interest on the facilities. At first, First, we obtain the efficiency score of the banks in the absence of flexible measure. The results are in the second column of Table (7). Banks B02, B05, B10, B11, B17, B18, B24, B28 and B29 are efficient and the other banks are inefficient. In the following, we solve model (3). The results are in the third to fifth columns of Table (7). Banks B02, B05, B10, B11, B17, B18, B24, B28 and B29 are efficient and the other banks are inefficient. As can be seen, in the evaluation of DMUs based on the model (3), all banks except banks B18 and B29 select flexible measure as output and these two banks select flexible measure as input. The first column show the results of model (3) without the flexible measure.Also, the other columns of Table 7 show the results of model (3) with the flexible measure. Tables (8) and (9) gives the minimum amount of inputs and flexible measure (in the input role) from each banks DMU_k and DMU_h that should be kept in order to reach the predetermined target efficiency. The second column shows the target efficiency scores. Based on the last two columns, show class flexible measure as input or output. In the first column, we select merging banks in a different way to show the results of model (4). As we know, the role of flexible variables as input or output can be different based on different approaches (see

Table 6: Outputs of banks.

Bank	output1	output2	output3	Flexible measure
B01	596.76	226.63	10237.69	9238.85
B02	46.81	21.13	524.33	533.06
B03	44.79	8.21	1816.51	882.13
B04	20.35	1.27	423.77	324.43
B05	3867.18	1461.25	60327.25	41734.99
B06	884.12	275.65	14918.88	12389.54
B07	39.51	14.41	963.13	829.89
B08	971.42	274.85	8947.95	9511.59
B09	307.45	131.15	4438.47	3504.13
B10	274.64	209.61	5434.35	4139.96
B11	67.03	19.52	2903.91	2278.2
B12	39.14	65.39	4058.25	4322.48
B13	109.23	17.77	1987.98	2033.89
B14	4072.26	1506.22	74763.09	63297.05
B15	12.09	4.81	5326.29	3991.08
B16	514.7	158.4	8737.26	7463.4
B17	277.45	206.27	5752.46	3437.54
B18	87.89	38.75	1820.9	3058.51
B19	2633.96	945.42	48835.45	41560.19
B20	17.65	2.03	257.95	304.85
B21	2.39	1.82	230.96	268.69
B22	18.58	6.99	848.84	845.4
B23	5615.73	2151.41	81168.9	66784.3
B24	44.28	4.23	1645.4	1381.92
B25	33.05	5.36	783.35	782.61
B26	47.05	1.34	749.2	579.39
B27	630.79	153.59	19781.92	18441.26
B28	13.45	26.62	1543.54	1052.35
B29	88.41	18.18	1322.01	2127.75

Amirteimoori and Emrouznejad [6]).

For example consider $DMU_k = B4$ and $DMU_h = B5$ as merging banks. At first, we consider target efficiency scores equal to one. By this selection, the merged entity T will be efficient, the optimal level of inputs, flexible measure, and output corresponding to the merged entity T will be as follows. $DMU_T = (\varphi^*_{1B4} + \varphi^*_{1B5}, \varphi^*_{2B4} + \varphi^*_{2B5}, \varphi^*_{3B4} +$

Table 7: The efficiency scores of Indonesian banks.

Bank	Efficiency	Efficiency	σ_1	η_1
B01	0.7852	0.7852	1	0
B02	1	1	1	0
B03	0.7964	0.7964	1	0
B04	0.4967	0.4967	1	0
B05	1	1	1	0
B06	0.7157	0.7157	1	0
B07	0.7131	0.7131	1	0
B08	0.7523	0.7523	1	0
B09	0.7721	0.7721	1	0
B10	1	1	1	0
B11	1	1	1	0
B12	0.5863	0.5863	1	0
B13	0.8772	0.8772	1	0
B14	0.7712	0.7712	1	0
B15	0.7023	0.7023	1	0
B16	0.6526	0.6526	1	0
B17	1	1	1	0
B18	1	1	0	1
B19	0.8796	0.8796	1	0
B20	0.5263	0.5263	1	0
B21	0.2889	0.2889	1	0
B22	0.4297	0.4297	1	0
B23	0.7756	0.7756	1	0
B24	1	1	1	0
B25	0.8024	0.8024	1	0
B26	0.7005	0.7005	1	0
B27	0.8083	0.8083	1	0
B28	1	1	1	0
B29	1	1	0	1

$\varphi^*_{3B5}, \tau^*_{1B5} + \tau^*_{1B6}, y_{1B4} + y_{1B5}, y_{2B4} + y_{2B5}) = (10.17+, 946.46, 24.59 + 797.01, 18.65 + 2124.25, 0 + 0, 20.35 + 3867.18, 1.27 + 1461.25, 423.77 + 60327.25) = (956.63, 821.6, 2142.9, 0, 3887.53, 1462.52, 60751.02)$. The merged entity T selected the flexible measure as output factor by considering $DMU_k = B4$ and $DMU_h = B5$ as merging banks. The results show that the merged bank T can reach any predetermined target level

Table 8: The results of inverse DEA process based on the model (4) in the case study.

Merge DMU	TE	φ_{1k}	φ_{2k}	φ_{3k}
$DMU_k = 12$ and $DMU_h = 13$	1	61.83	83.7615	129.2988
$DMU_k = 12$ and $DMU_h = 13$	0.9	61.83	103.4961	143.6653
$DMU_k = 12$ and $DMU_h = 13$	0.7	61.83	173.8595	175.1646
$DMU_k = 21$ and $DMU_h = 22$	1	8.44	0	20.89
$DMU_k = 21$ and $DMU_h = 22$	0.8	8.44	0	20.89
$DMU_k = 21$ and $DMU_h = 22$	0.7	8.44	0	20.89
$DMU_k = 4$ and $DMU_h = 5$	1	10.17	24.59	18.65
$DMU_k = 14$ and $DMU_h = 15$	1	926.4323	1773.2934	1903.6248
$DMU_k = 14$ and $DMU_h = 15$	0.9	1029.3692	2009.9038	2115.1387
$DMU_k = 14$ and $DMU_h = 15$	0.8	1195.3565	2184.54	2483.1449

Table 9: The results of inverse DEA process based on the model (4) in the case study.

φ_{1h}	φ_{2h}	φ_{3h}	τ_{1k}	τ_{1h}	σ_1	η_1
2.919	93.85	0	0	0	1	0
10.1134	93.85	0	0	0	1	0
27.32	93.85	0	0	0	1	0
3.1236	31.7198	2.2016	0	0	1	0
6.0145	39.6497	7.9744	0	0	1	0
8.0794	45.3139	12.0979	0	0	1	0
946.46	797.01	2124.25	0	0	1	0
0	356.2	0	0	0	1	0
0	356.2	0	0	0	1	0
0	356.2	0	0	0	1	0

(even efficient), if the new inverse DEA model (2) is solved. We can determine the role of flexible measures as input or output.

7 Conclusion

One of the key issues in DEA is classifying inputs and outputs is one of the key issues in order to maximize the efficiency score of DMUs. There is some of studies have investigated the efficiency of DMUs when flexible measures are present. In DEA models, the efficiency score of DMUs is determined based on input and output variables. The role of these variables as input or output is initially determined by the decision maker. However, some of these variables can play the role of input and output in the evaluation model. These variables are called flexible measures. In this paper, a different strategy was proposed to determine the role of these variables. In this regard, we presented a new model with the structure of inverse DEA models for the classification of inputs and outputs. The model presented in this paper, we solved the model by choosing two DMUs as merging units and determining the target efficiency score for the newly created unit, and the new level of inputs and flexible measures in the input role are determined in such a way that the new unit should have the target efficiency score. The new model provides a different view for the classification of inputs and outputs in DEA. As future works, the model presented in this paper can be developed to perform a general process by considering more than two DMUs a merging units in the merger process. As another development, we can also develop the above models for the two-stage network structure in DEA. We can also develop the above models for the partnership process of firms. Also, further research may investigate the flexible measures for centralized allocation problem where both flexible measures and environmental factors be considered.

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