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Some Results on Complex Intuitionistic Fuzzy Soft ℓ- Groups

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Abstract. In this paper, we develop more properties on complex intuitionistic fuzzy soft ℓ-group ($\mathcal{CIFSL}$-$\mathcal{G}$) structure. We initiate the concept of complex intuitionistic fuzzy soft ℓ - homomorphism and present its pertinent properties. We extend this ideology to define the concept of invariant complex intuitionistic fuzzy soft function and develop some new results. Moreover, we establish the notion of normal complex intuitionistic fuzzy soft ℓ-group ($\mathcal{NCIFSL}$-$\mathcal{G}$) and discuss its various fundamental algebraic characteristics.

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1 Introduction

In recent times, fuzzy lattice-ordered algebraic structures have intensified the attention of many researchers [24, 25]. Consequently, numerous applications and methodologies have been developed under the fuzzy lattice ordered structures [13, 14]. Vimala [27] proposed the theory of fuzzy lattice-ordered group which combines the characteristics of fuzzy sets [28] and lattice-ordered groups [5]. Likewise, many concepts were developed [3, 11, 12] based on [27].

1.1 Review of related literature

In 2007, Aktas et al. [1] introduced the concept of soft groups, which includes the idea of soft sets [10] in group structures. Sezgin and Atagun [26] presented the normalistic soft groups and interpreted its homomorphism as an expansion to soft groups. In 2009, the concept of fuzzy soft groups is originated by Aygunoglu et al. [4]. Also, Aygunoglu et al. defined the fuzzy soft function, fuzzy soft homomorphism, homomorphic image and pre-image. Then, many researches are done in extensions of fuzzy sets with group structure [3, 20, 15].

In the field of electrical engineering, complex numbers has the ability to solve many problems that cannot be solved by using the real numbers. Complex numbers can be used to model the periodicity of the elements by its phase term. In 2002, Ramot et al. [21] proposed the new idea called complex fuzzy set (CFS), which is a natural extension of fuzzy set theory. CFS is characterized by a membership function whose range extends from $[0, 1]$ to the unit circle in the complex plane, which includes the amplitude and phase terms. The CFSs can able to solve the two-dimensional uncertainty problems by its phase term, these CFSs have received extensive attention from researchers in the decision-making field.

Alkouri et al. [2] pioneered the concept of complex intuitionistic fuzzy sets (CIFSs), which are distinguished by belongingness and non- belongingness. Based on this idea, the notion of complex intuitionistic fuzzy soft sets [8] have been developed. Since then, the extensions of CFSs have become a vigorous area of research in different disciplines such as engineering, medical science, social science, physics, statistics, graph theory, artificial intelligence, signal processing, multiagent systems, pattern recognition, computer networks, decision making and so on [9, 7, 22, 23]. In 2019, S. G. Quek et al. [15] introduced the concept of complex intuitionistic fuzzy soft group and investigated its algebraic structures. In continuation, the concept of complex intuitionistic fuzzy soft lattice ordered group ($\mathcal{CIFSL}$-$\mathcal{G}$) is bestowed in [19, 18]. Further, some new operations on $\mathcal{CIFSL}$-$\mathcal{G}$ such as sum, product, bounded product, bounded difference and disjoint sum, and also CIFS-COPRAS method are presented in [17] and then, CIFS SWARA-COPRAS method is developed in [6]. Followed by that, here we introduce the concept of lattice ordered group homomorphism (ℓ- homomorphism) and normality on $\mathcal{CIFSL}$-$\mathcal{G}$.

1.2 Main Contribution

Many researchers have developed the complex intuitionistic fuzzy soft set theory. In the existing works, the theory of complex intuitionistic fuzzy soft set over the lattice ordered group have yet to be investigated. Thus, motivated by this, we bestowed the concept of complex intuitionistic fuzzy soft lattice ordered group ($\mathcal{CIFSL}$-$\mathcal{G}$) and studied its pertinent properties in [18]. Since then, $\mathcal{CIFSL}$-$\mathcal{G}$ have been widely developed in [6, 16, 17, 18, 19]. This paper is a continuation of the study in $\mathcal{CIFSL}$-$\mathcal{G}$ [19, 18, 17, 16] and we want to get more properties in this structure. In this paper, we develop homomorphic image (inverse image) of $\mathcal{CIFSL}$-$\mathcal{G}$ under classical $\ell-$homomorphism. Also, we have discussed various fundamental properties. These properties are based on both classical ℓ- homomorphism and complex intuitionistic fuzzy soft set structure. Moreover, we define the concept of normal complex intuitionistic fuzzy soft lattice ordered group and verify some of its properties.

1.3 Paper outline

The rest of this paper is organized as follows: Section 2 contains some relevant basic concepts. Section 3 provides the concept of complex intuitionistic fuzzy soft ℓ- homomorphism and its several characteristics. Section 4 presents the concept of normality on $\mathcal{CIFSL}$-$\mathcal{G}$ and interprets its properties.

2 Preliminaries

In this section, we review the basic concepts of lattice-ordered groups and complex intuitionistic fuzzy soft lattice-ordered groups.

Throughout this paper $\vee$ and $\wedge$ are the maximum and minimum operators, respectively and

(i) $\mu \geq \nu$, if both $r \geq \tau$ and $w \geq \psi$ and

(ii) $\mu \leq \nu$, if both $r \leq \tau$ and $w \leq \psi$, where $\mu = re^{iw}$ and $\nu = \tau e^{i\psi}$, with $r, \tau \in [0,1]$ and $w, \psi \in (0, 2\pi]$.

Definition 2.1. [5] The system $G = (G, *, \leq)$ is called a lattice ordered group (ℓ- group), if

(i) $(G, *)$ is a group

(ii) $(G, \leq)$ is a lattice

(iii) $a \leq b \Rightarrow x * a * y \leq x * b * y$, for all $a, b, x, y \in G$.

Definition 2.2. [5] Let H and K be ℓ - groups and $\phi : H \longrightarrow K$ be a group homomorphism from H into H such that $(a \vee b)\phi = a\phi \vee b\phi$, $(a \wedge b)\phi = a\phi \wedge b\phi$, for all $a, b \in H$, then ϕ is known as an ℓ - homomorphism.

Definition 2.3. [8] Let E be a set of parameters, $CIFS(U)$ denote the set of all complex intuitionistic fuzzy sets on U and $\widetilde{F}$ be a function from E to $CIFS(U)$. Then $(\widetilde{F}, E)$, is a complex intuitionistic fuzzy soft set $(CIFSS)$ on U. For all $\varepsilon \in E$,

$$\begin{aligned}
(\widetilde{F}, E) &= \{(\varepsilon, \widetilde{F}(\varepsilon)) : \varepsilon \in E, \widetilde{F}(\varepsilon) \in CIFS(U)\}, \textit{ in which} \\
\widetilde{F}(\varepsilon) &= \{(x, \mu_{\widetilde{F}(\varepsilon)}(x), \nu_{\widetilde{F}(\varepsilon)}(x)) \ : \ x \in U\} \\
&= \{(x, r_{\widetilde{F}(\varepsilon)}(x)e^{iw_{\widetilde{F}(\varepsilon)}(x)}, \tau_{\widetilde{F}(\varepsilon)}(x)e^{i\psi_{\widetilde{F}(\varepsilon)}(x)}) \ : \ x \in U\}
\end{aligned}$$

where $\mu_{\widetilde{F}(\varepsilon)} : U \longrightarrow \{a : a \in \mathbb{C}, |a| \leq 1\}$ and $\nu_{\widetilde{F}(\varepsilon)} : U \longrightarrow \{a' : a' \in \mathbb{C}, |a'| \leq 1\}$, $|\mu_{\widetilde{F}(\varepsilon)}(x) + \nu_{\widetilde{F}(\varepsilon)}(x)| \leq 1$.

Definition 2.4. [8] Let $(\widetilde{F}_1, A_1)$ and $(\widetilde{F}_2, A_2)$ be $CIFSS$s over U. Then $(\widetilde{F}_1, A_1) \subseteq (\widetilde{F}_2, A_2)$, if

(i) $A_1 \subseteq A_2$,

(ii) $\widetilde{F}_1(a_1) \subseteq \widetilde{F}_2(a_1)$, that is $\mu_{\widetilde{F}_1(a_1)}(x) \leq \mu_{\widetilde{F}_2(a_1)}(x)$ and $\nu_{\widetilde{F}_2(a_1)}(x) \leq \nu_{\widetilde{F}_1(a_1)}(x)$, for all $x \in U,\ a_1 \in A_1$.

Definition 2.5. citere1 Let $(\widetilde{F}, E) \in CIFSS$ over the ℓ-group G. Then $(\widetilde{F}, E)$ is said to be a complex intuitionistic fuzzy soft lattice ordered group ($\mathcal{CIFSL}$-$\mathcal{G}$) over G if for all $a \in \rho(\widetilde{F}, E)$ and $x, y, e_G \in G$,

(i) $\mu_{\widetilde{F}(a)}(xy^{-1}) \geq \mu_{\widetilde{F}(a)}(x) \wedge \mu_{\widetilde{F}(a)}(y)$,

(ii) $\nu_{\widetilde{F}(a)}(xy^{-1}) \leq \nu_{\widetilde{F}(a)}(x) \vee \nu_{\widetilde{F}(a)}(y)$,

(iii) $\mu_{\widetilde{F}(a)}(x \vee e_G) \geq \mu_{\widetilde{F}(a)}(x)$,

(iv) $\nu_{\widetilde{F}(a)}(x \vee e_G) \leq \nu_{\widetilde{F}(a)}(x)$,

where $\rho(\widetilde{F}, E) = \{a \in E : \widetilde{F}(a) \textit{ is non null}\}$ is the support set of $(\widetilde{F}, E)$ and e_G is the identity element of G.

3 l - Homomorphism on CIFSL- G

In this section, first we define the image and pre-image of a complex intuitionistic fuzzy soft function. Furthermore, we propose the complex intuitionistic fuzzy soft ℓ- homomorphism ($\mathcal{CIFS}$ *ℓ-homomorphism*) and interpret the characteristics of $\mathcal{CIFS}$ *ℓ-homomorphism*.
Throughout this paper G_1 and G_2 denotes the ℓ - groups.

Definition 3.1. Let A_1 and A_2 be the parameter sets for $CIFSS$s over the universal sets U_1 and U_2, respectively. If $\phi : U_1 \longrightarrow U_2$ and $\varphi : A_1 \longrightarrow A_2$ are two functions, then the pair (ϕ, φ) is known a complex intuitionistic fuzzy soft function ($CIFS - Function$).

Definition 3.2. Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ $\in CIFSS$ over U_1 and U_2, respectively and let (ϕ, φ) be a CIFS-Function. Then

(i) The image of $(\widetilde{F}_1, A_1)$ under (ϕ, φ), denoted by $(\phi, \varphi)(\widetilde{F}_1, A_1)$, is $CIFSS$ over U_2, defined by $(\phi, \varphi)(\widetilde{F}_1, A_1) = (\phi(\widetilde{F}_1), \varphi(A_1))$ such that

$$\mu_{\phi(\widetilde{F}_1)(a_2)}(y) = \begin{cases} \bigvee\limits_{\phi(x)=y} \bigvee\limits_{\varphi(a_1)=a_2} \mu_{\widetilde{F}_1(a_1)}(x), & \text{if } \phi^{-1}(y) \neq \emptyset \\ 0, & \text{Otherwise} \end{cases},$$

$$\nu_{\phi(\widetilde{F}_1)(a_2)}(y) = \begin{cases} \bigwedge\limits_{\phi(x)=y} \bigwedge\limits_{\varphi(a_1)=a_2} \nu_{\widetilde{F}_1(a_1)}(x), & \text{if } \phi^{-1}(y) \neq \emptyset \\ 0, & \text{Otherwise,} \end{cases}$$

where $a_2 \in \varphi(A_1)$ and $y \in U_2$.

(ii) The pre-image of $(\widetilde{F}_2, A_2)$ under (ϕ, φ), denoted by $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2)$, is $CIFSS$ over U_1, defined by $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2) = (\phi^{-1}(\widetilde{F}_2), \varphi^{-1}(A_2))$ such that

$$\mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \mu_{\widetilde{F}_2(\varphi(a_1))}\phi(x) \text{ and } \nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \nu_{\widetilde{F}_2(\varphi(a_1))}\phi(x),$$

where $a_1 \in \varphi^{-1}(A_2)$ and $x \in U_1$.

Definition 3.3. The $CIFS - Function$ (ϕ, φ) is known as a complex intuitionistic fuzzy soft ℓ- homomorphism ($\mathcal{CIFS}$ *ℓ-homomorphism*), if ϕ be a homomorphism of ℓ- group from G_1 to G_2, where G_1 and G_2 are ℓ- groups. If ϕ and φ are injective (surjective), then (ϕ, φ) is said to be injective (surjective).

Example 3.4. Consider the ℓ- group G $= (\mathbb{Z}, +, \vee, \wedge)$ and the parameter sets $A_1 = \{a_1, b_1, c_1\}$, $A_2 = \{a_2, b_2\}$. Now, we define the $\mathcal{CIFSL}$-$\mathcal{G}s$ $(\widetilde{F}_1, A_1)$ and $(\widetilde{F}_2, A_2)$ over the ℓ- group $\mathbb{Z}$ by

$$\widetilde{F}_1(a_1) = \begin{cases} (x, 0.8e^{i(1.7\pi)}, 0.2e^{i(0.3\pi)}), & x = 0 \\ (x, 0.45e^{i\pi}, 0.55e^{i(0.8\pi)}), & \text{otherwise} \end{cases},$$

$$\widetilde{F}_1(b_1) = \begin{cases} (x, 0.9e^{i(1.5\pi)}, 0.1e^{i(0.5\pi)}), & \text{if } x = 0 \\ (x, 0.2e^{i(0.4\pi)}, 0.8e^{i(1.6\pi)}), & \text{otherwise} \end{cases}$$

$$\widetilde{F}_1(c_1) = \begin{cases} (x, 0.7e^{i(1.2\pi)}, 0.2e^{i(0.7\pi)}), & \text{if } x = 0 \\ (x, 0.65e^{i(0.1\pi)}, 0.3e^{i(1.8\pi)}), & \text{otherwise} \end{cases}, \text{ and}$$

$$\widetilde{F}_2(a_2) = \begin{cases} (x, 0.6e^{i(0.5\pi)}, 0.35e^{i(0.9\pi)}), & x = 0 \\ (x, 0.2e^{i(0.2\pi)}, 0.75e^{i(1.8\pi)}), & \text{otherwise} \end{cases},$$

$$\widetilde{F}_2(b_2)=\begin{cases}(x, 0.7e^{i\pi}, 0.2e^{i\pi}), & \text{if } x=0\\ (x, 0.6e^{i(0.7\pi)}, 0.4e^{i(1.3\pi)}), & \text{otherwise.}\end{cases}$$

Define $\phi : \mathbb{Z} \longrightarrow \mathbb{Z}$ and $\varphi : A_1 \longrightarrow A_2$ such that $\phi(x) = nx$ and $\varphi(a_1) = \varphi(b_1) = a_2$, $\varphi(c_1) = b_2$, where ϕ is a ℓ- homomorphisim. Then $(\phi, \varphi)(\widetilde{F}_1, A_1)$ is given by

$\phi(\widetilde{F}_1)(a_2) = \begin{cases}(y, 0.9e^{i(1.7\pi)}, 0.1e^{i(0.3\pi)}), & y=0\\ (y, 0.45e^{i\pi}, 0.55e^{i(0.8\pi)}), & \text{otherwise}\end{cases}$, and $\phi(\widetilde{F}_1)(b_2) = \widetilde{F}_1(c_1)$.

Also $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2)$ is given by,

$\phi^{-1}(\widetilde{F}_2)(a_1) = \phi^{-1}(\widetilde{F}_2)(b_1) = \widetilde{F}_2(a_2)$ and $\phi^{-1}(\widetilde{F}_2)(c_1) = \widetilde{F}_2(b_2)$, for $a_1, b_1, c_1 \in \varphi^{-1}(A_2)$.

Theorem 3.5. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}\text{-}\mathcal{G}$ over G_1 and G_2, respectively. Let (ϕ, φ) be complex intuitionistic fuzzy soft function from G_1 and G_2. Then*

(i) $\mu_{\phi(\widetilde{F}_1)(a_2)}(y) = r_{\phi(\widetilde{F}_1)(a_2)}(y)e^{iw_{\phi(\widetilde{F}_1)(a_2)}(y)}$ *and*
$\nu_{\phi(\widetilde{F}_1)(a_2)}(y) = \tau_{\phi(\widetilde{F}_1)(a_2)}(y)e^{i\psi_{\phi(\widetilde{F}_1)(a_2)}(y)}$

(ii) $\mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = r_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x)e^{iw_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x)}$ *and*
$\nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \tau_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x)e^{i\psi_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x)}$,
where $a_1 \in \varphi^{-1}(A_2)$, $a_2 \in \varphi(A_1)$, $x \in G_1$ and $y \in G_2$.

Proof. Let $x \in G_1$, $y \in G_2, a_1 \in A_1$ and $a_2 \in A_2$.

(i)
$$\begin{aligned}\mu_{\phi(\widetilde{F}_1)(a_2)}(y) &= \bigvee_{\phi(x)=y}\ \bigvee_{\varphi(a_1)=a_2} \mu_{\widetilde{F}_1(a_1)}(x)\\ &= \bigvee_{\phi(x)=y}\ \bigvee_{\varphi(a_1)=a_2} r_{\widetilde{F}_1(a_1)}(x)e^{iw_{\widetilde{F}_1(a_1)}(x)}\\ &= \Big[\bigvee_{\phi(x)=y}\ \bigvee_{\varphi(a_1)=a_2} r_{\widetilde{F}_1(a_1)}(x)\Big]e^{i\Big[\bigvee_{\phi(x)=y}\bigvee_{\varphi(a_1)=a_2} w_{\widetilde{F}_1(a_1)}(x)\Big]}\\ &= r_{\phi(\widetilde{F}_1)(a_2)}(y)e^{iw_{\phi(\widetilde{F}_1)(a_2)}(y)}.\end{aligned}$$

Similarly, we can get $\nu_{\phi(\widetilde{F}_1)(a_2)}(y) = \tau_{\phi(\widetilde{F}_1)(a_2)}(y)e^{i\psi_{\phi(\widetilde{F}_1)(a_2)}(y)}$ and also we can prove (ii). $\square$

Proposition 3.6. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ $\in \mathcal{CIFSL}$-$\mathcal{G}$ over G_1 and G_2, respectively. Let $\phi : G_1 \longrightarrow G_2$ be a ℓ - homomorphism and $\varphi : A_1 \longrightarrow A_2$ be a onto function. Then*

(a) the image of $(\widetilde{F}_1, A_1)$ under (ϕ, φ) is $\mathcal{CIFSL}$-$\mathcal{G}$ over G_2,

(b) the pre-image of $(\widetilde{F}_2, A_2)$ under (ϕ, φ) is $\mathcal{CIFSL}$-$\mathcal{G}$ over G_1.

Proof. (a) Since, $\phi : G_1 \longrightarrow G_2$ is ℓ - homomorphism i.e., ϕ is a group homomorphism from G_1 into G_2 such that for all $x, y \in G_1$,

$$\phi(x \vee y) = \phi(x) \vee \phi(y), \ \phi(x \wedge y) = \phi(x) \wedge \phi(y).$$

Let $x, y, e_{G_2} \in G_2$, where e_{G_2} is the identity element of G_2. $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ $\in \mathcal{CIFSL}$-$\mathcal{G}$,

(i) $\mu_{\phi(\widetilde{F}_1)(a_2)}(x \vee e_{G_2})$

$= \bigvee_{\phi(z)=x\vee e_{G_2}} \bigvee_{\varphi(a_1)=a_2} \{\mu_{\widetilde{F}_1(a_1)}(z) | z \in \phi^{-1}(x \vee e_{G_2})\}$

$\geq \bigvee_{\phi(u\vee e_{G_1})=x\vee e_{G_2}} \bigvee_{\varphi(a_1)=a_2} \{\mu_{\widetilde{F}_1(a_1)}(u \vee e_{G_1}) | u \in \phi^{-1}(x), \ e_{G_1} \in \phi^{-1}(e_{G_2})\}$

$\geq \bigvee_{\phi(u)=x} \bigvee_{\varphi(a_1)=a_2} \{\mu_{\widetilde{F}_1(a_1)}(u) | u \in \phi^{-1}(x)\}$

$= \mu_{\phi(\widetilde{F}_1)(a_2)}(x)$.

Similarly, we can get (ii) $\nu_{\phi(\widetilde{F}_1)(a_2)}(x \vee e_{G_2}) \leq \nu_{\phi(\widetilde{F}_1)(a_2)}(x)$, (iii) $\mu_{\phi(\widetilde{F}_1)(a_2)}(xy^{-1}) \geq \mu_{\phi(\widetilde{F}_1)(a_2)}(x) \wedge \mu_{\phi(\widetilde{F}_1)(a_2)}(y)$ and (iv) $\nu_{\phi(\widetilde{F}_1)(a_2)}(xy^{-1}) \leq \nu_{\phi(\widetilde{F}_1)(a_2)}(x) \vee \nu_{\phi(\widetilde{F}_1)(a_2)}(y)$. Hence, $(\phi(\widetilde{F}_1), \varphi(A_1)) \in \mathcal{CIFSL}$-$\mathcal{G}(G_2)$. $\square$

Proposition 3.7. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ $\in \mathcal{CIFSL}$-$\mathcal{G}$ over G_1 and G_2, respectively and (ϕ, φ) is a $\mathcal{CIFS}$ ℓ-homomorphism from G_1 to G_2. Then*

$$\mu_{\phi(\widetilde{F}_1)(a_2)}(e_{G_2}) = \mu_{\widetilde{F}_1(a_1)}(e_{G_1}) \ \text{ and } \ \nu_{\phi(\widetilde{F}_1)(a_2)}(e_{G_2}) = \nu_{\widetilde{F}_1(a_1)}(e_{G_1}),$$

where $a_2 \in \varphi(A_1)$ and e_{G_1} and e_{G_2} are the identity elements of G_1 and G_2, respectively.

Proof. Let e_{G_1} and e_{G_2} are the identity elements of G_1 and G_2, respectively. For $a_2 \in \varphi(A_1)$,

$$\mu_{\phi(\widetilde{F}_1)(a_2)}(e_{G_2}) = \bigvee_{\phi(e_{G_1})=e_{G_2}} \bigvee_{\varphi(a_1)=a_2} \mu_{\widetilde{F}_1(a_1)}(e_{G_1}) = \mu_{\widetilde{F}_1(a_1)}(e_{G_1}) \text{ and}$$
$$\nu_{\phi(\widetilde{F}_1)(a_2)}(e_{G_2}) = \bigwedge_{\phi(e_{G_1})=e_{G_2}} \bigwedge_{\varphi(a_1)=a_2} \nu_{\widetilde{F}_1(a_1)}(e_{G_1}) = \nu_{\widetilde{F}_1(a_1)}(e_{G_1}).$$

Since, $\mu_{\widetilde{F}_1(a_1)}(e_{G_1}) \geq \mu_{\widetilde{F}_1(a_1)}(x)$ and $\nu_{\widetilde{F}_1(a_1)}(e_{G_1}) \leq \nu_{\widetilde{F}_1(a_1)}(x)$, for all $x, e_{G_1} \in G_1$ and $\phi(e_{G_1}) = e_{G_2}$. Hence the proof. $\square$

Proposition 3.8. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ $\in \mathcal{CIFSL}$-$\mathcal{G}$ over G_1 and G_2, respectively and (ϕ, φ) is a onto $\mathcal{CIFS}$ ℓ-homomorphism from G_1 to G_2. Then the inverse image of (ϕ, φ) is a $\mathcal{CIFSL}$-$\mathcal{G}$ of G_1, which is a constant on ker ϕ.*

Proof. Let $x \in ker\ \phi$ and $\varphi(a_1) = a_2 \in \varphi(A_1)$. Then

$$\mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \mu_{\widetilde{F}_2(\varphi(a_1))}(e_{G_2}) = \mu_{\widetilde{F}_2(\varphi(a_1))}(\phi(e_{G_1})) = \mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(e_{G_1})$$
$$\nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \nu_{\widetilde{F}_2(\varphi(a_1))}(e_{G_2}) = \nu_{\widetilde{F}_2(\varphi(a_1))}(\phi(e_{G_1})) = \nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(e_{G_1}),$$

where e_{G_1} and e_{G_2} are the identity elements of G_1 and G_2, respectively. By Proposition 3.6, $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G_1)$. This completes the proof. $\square$

Proposition 3.9. *Let $(\widetilde{F}_1, A_1), (\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}$ over the ℓ-group G and $(\widetilde{F}'_1, A'_1)$,$(\widetilde{F}'_2, A'_2) \in \mathcal{CIFSL}$-$\mathcal{G}$ over the ℓ-group G' and let (ϕ, φ) is a complex intuitionistic fuzzy soft function from G to G'. Then*

(i) $(\widetilde{F}_1, A_1) \subseteq (\widetilde{F}_2, A_2) \Rightarrow (\phi(\widetilde{F}_1), \varphi(A_1)) \subseteq (\phi(\widetilde{F}_2), \varphi(A_2))$

(ii) $(\widetilde{F}'_1, A'_1) \subseteq (\widetilde{F}'_2, A'_2) \Rightarrow (\phi^{-1}(\widetilde{F}'_1), \varphi^{-1}(A'_1)) \subseteq (\phi^{-1}(\widetilde{F}'_2), \varphi^{-1}(A'_2))$.

Definition 3.10. Let $(\widetilde{F}_1, A_1) \in \mathcal{CIFSL}$-$\mathcal{G}$ of G_1 and (ϕ, φ) be $CIFS-Function$ from G_1 to G_2. Then $(\widetilde{F}_1, A_1)$ is called (ϕ, φ)- invariant if $\phi(u) = \phi(v)$ implies that

$$\mu_{\widetilde{F}_1(a_1)}(u) = \mu_{\widetilde{F}_1(a_1)}(v) \text{ i.e., } r_{\widetilde{F}_1(a_1)}(u) = r_{\widetilde{F}_1(a_1)}(v), w_{\widetilde{F}_1(a_1)}(u) = w_{\widetilde{F}_1(a_1)}(v)$$
$$\nu_{\widetilde{F}_1(a_1)}(u) = \nu_{\widetilde{F}_1(a_1)}(v), \text{ i.e., } \tau_{\widetilde{F}_1(a_1)}(u) = \tau_{\widetilde{F}_1(a_1)}(v),\ \psi_{\widetilde{F}_1(a_1)}(u) = \psi_{\widetilde{F}_1(a_1)}(v),$$

for all $u, v \in G_1$ and $a_1 \in A_1$.

Proposition 3.11. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}$ over G_1 and G_2, respectively and (ϕ, φ) is a complex intuitionistic fuzzy soft function from G_1 to G_2. Then $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2)$ is an invariant complex intuitionistic fuzzy soft function with respect to (ϕ, φ).*

Proof. Let $x, y \in G_1$ such that $\phi(x) = \phi(y)$. Then for all $a_1 \in \varphi^{-1}(A_2)$, $\mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \mu_{\widetilde{F}_1(\phi(a_1))}(\phi(x)) = \mu_{\widetilde{F}_1(\phi(a_1))}(\phi(y)) = \mu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(y)$. Similarly, we get $\nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(x) = \nu_{\phi^{-1}(\widetilde{F}_2)(a_1)}(y)$. Hence, $(\phi, \varphi)^{-1}(\widetilde{F}_2, A_2)$ is an invariant complex intuitionistic fuzzy soft function with respect to (ϕ, φ). $\square$

Proposition 3.12. *Let $(\widetilde{F}_1, A_1) \in \mathcal{CIFSL}$-$\mathcal{G}$ of G_1 and (ϕ, φ) is a onto $\mathcal{CIFS}\ell$-homomorphism from G_1 to G_2. If $(\widetilde{F}_1, A_1)$ be (ϕ, φ)- invariant, then $(\phi, \varphi)^{-1}\big[(\phi, \varphi)(\widetilde{F}_1, A_1)\big] = (\widetilde{F}_1, A_1)$.*

Theorem 3.13. *Let $(\widetilde{F}, E)$, $(\widetilde{F}', E') \in \mathcal{CIFSL}$-$\mathcal{G}$ over G and G', respectively and (ϕ, φ) is a onto $\mathcal{CIFS}$ ℓ-homomorphism. Then there is a one to one order preserving correspondence between the $\mathcal{CIFSL}$-$\mathcal{G}$ of G and those of G' which are (ϕ, φ)- invariant.*

Proof. Let $\chi(G)$ denote the collection of all $\mathcal{CIFSL}$-$\mathcal{G}$ of G and $\chi(G')$ denote the collection of all $\mathcal{CIFSL}$-$\mathcal{G}$ of G'. Define $f : \chi(G) \longrightarrow \chi(G')$ and $g : \chi(G') \longrightarrow \chi(G)$ such that $f(\widetilde{F}, E) = (\phi, \varphi)(\widetilde{F}, E)$ and $g(\widetilde{F}', E') = (\phi^{-1}, \varphi^{-1})\ (\widetilde{F}', E')$. By Proposition 3.6, f and g are well-defined. Since (ϕ, φ) is invariant, then $(\phi, \varphi)[(\phi^{-1}, \varphi^{-1})(\widetilde{F}', E')] = (\widetilde{F}', E')$ and $(\phi^{-1}, \varphi^{-1})[(\phi, \varphi)(\widetilde{F}, E)] = (\widetilde{F}, E)$. Therefore, f and g are inverse to each other, hence there is one to one correspondence. Moreover, by Proposition 3.9 we have, $(\widetilde{F}, E) \subseteq (\widetilde{F}', E')$ implies $(\phi(\widetilde{F}), \varphi(E)) \subseteq (\phi(\widetilde{F}'), \varphi(E'))$. Hence the correspondence is order preserving. $\square$

Theorem 3.14. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}$ over G_1, respectively and (ϕ, φ) is a onto $\mathcal{CIFS}$ ℓ-homomorphism. Then*

$$(\phi, \varphi)((\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)) \subseteq \big[(\phi, \varphi)(\widetilde{F}_1, A_1)\big]\widetilde{\cap}\big[(\phi, \varphi)(\widetilde{F}_2, A_2)\big]$$

and the equality holds if at least of $(\widetilde{F}_1, A_1)$ or $(\widetilde{F}_2, A_2)$ is (ϕ, φ)- invariant.

Proof. Let $(\widetilde{F}_1, A_1)$ and $(\widetilde{F}_2, A_2)$ be two $\mathcal{CIFSL}$-$\mathcal{G}$ over G_1. Then the intersection of $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2)$ and is denoted as $(\widetilde{F}_1, A_1) \widetilde{\cap} (\widetilde{F}_2, A_2)$, where $\mu_{(\widetilde{F}_1 \widetilde{\cap} \widetilde{F}_2)(a)}(x) = \mu_{\widetilde{F}_1(a)}(x) \wedge \mu_{\widetilde{F}_2(a)}(x)$ and $\nu_{(\widetilde{F}_1 \widetilde{\cap} \widetilde{F}_2)(a)}(x) = \nu_{\widetilde{F}_1(a)}(x) \vee \nu_{\widetilde{F}_2(a)}(x)$, for all $a \in A_1 \cap A_2$ and $x \in G_1$.

Since, $(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\widetilde{\subseteq}(\widetilde{F}_1, A_1)$ and $(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\widetilde{\subseteq}(\widetilde{F}_2, A_2)$.

By Proposition 3.9,

$$\begin{aligned} &(\phi, \varphi)\big[(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\big]\widetilde{\subseteq}(\phi, \varphi)(\widetilde{F}_1, A_1) \text{ and} \\ &(\phi, \varphi)\big[(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\big]\widetilde{\subseteq}(\phi, \varphi)(\widetilde{F}_2, A_2) \\ \Rightarrow &(\phi, \varphi)\big[(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\big] \subseteq (\phi, \varphi)(\widetilde{F}_1, A_1)\widetilde{\cap}(\phi, \varphi)(\widetilde{F}_2, A_2). \end{aligned}$$

Next, if $(\widetilde{F}_2, A_2)$ is (ϕ, φ)- invariant, then we have to prove that:

$$(\phi, \varphi)(\widetilde{F}_1, A_1)\widetilde{\cap}(\phi, \varphi)(\widetilde{F}_2, A_2) \subseteq (\phi, \varphi)\big[(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2)\big].$$

Let $h = \mu_{\phi(\widetilde{F}_1)(a_1)}(y) \wedge \mu_{\phi(\widetilde{F}_2)(a_1)}(y)$ and $k = \mu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y)$, for $e_1 \in A_1 \cap A_2$, $x \in G_1$.

$$\Rightarrow h = \bigvee_{\phi(x)=y} \bigvee_{\varphi(e_1)=a_1} \mu_{\widetilde{F}_1(e_1)}(x) \wedge \mu_{\phi(\widetilde{F}_2)(a_1)}(y), \quad \text{by Definition 3.2}$$

$$\Rightarrow h \leq \bigvee_{\phi(x)=y} \bigvee_{\varphi(e_1)=a_1} \mu_{\widetilde{F}_1(e_1)}(x) \text{ and } h \leq \mu_{\phi(\widetilde{F}_2)(a_1)}(y).$$

Then, for $\delta > 0$, $\exists\, x \in \phi^{-1}(y)$ such that $h - \delta < \mu_{\widetilde{F}_1(e_1)}(x)$ and $h - \delta < \mu_{\phi(\widetilde{F}_2)(a_1)}(y)$.

$$\therefore h - \delta < \mu_{\phi(\widetilde{F}_2)(a_1)}(y)$$

$$\Rightarrow h - \delta < \mu_{\phi(\widetilde{F}_2)(e_1)}(\phi(x)) = \mu_{\phi^{-1}(\phi(\widetilde{F}_2))(e_1)}(x) = \mu_{\widetilde{F}_2(e_1)}(x),$$

since $(\widetilde{F}_2, A_2)$ is (ϕ, φ)- invariant.

$$\Rightarrow h - \delta < \mu_{\widetilde{F}_1(e_1)}(x) \wedge \mu_{\widetilde{F}_2(e_1)}(x) = \mu_{(\widetilde{F}_1 \cap \widetilde{F}_2)(e_1)}(x)$$

$$h - \delta < \bigvee_{\phi(x)=y} \bigvee_{\varphi(e_1)=a_1} \mu_{(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(x) = \mu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y) = k.$$

Since $\delta > 0$ is arbitrary, $h \leq k$.

Hence, $\mu_{\phi(\widetilde{F}_1)(a_1)}(y) \wedge \mu_{\phi(\widetilde{F}_2)(a_1)}(y) \leq \mu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y)$ (1)

Next, let $i = \nu_{\phi(\widetilde{F}_1)(a_1)}(y) \vee \nu_{\phi(\widetilde{F}_2)(a_1)}(y)$ and $j = \nu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y)$, for $a_1 \in A_1 \cap A_2,\ y \in G_1$.

$$\Rightarrow i = \bigwedge_{\phi(x)=y} \bigwedge_{\varphi(e_1)=a_1} \nu_{\widetilde{F}_1(e_1)}(x) \vee \nu_{\phi(\widetilde{F}_2)(a_1)}(y), \quad \text{by Definition 3.2}$$

$$\Rightarrow i \geq \bigwedge_{\phi(x)=y} \bigwedge_{\varphi(e_1)=a_1} \nu_{\widetilde{F}_1(e_1)}(x) \text{ and } i \geq \nu_{\phi(\widetilde{F}_2)(a_1)}(y).$$

Then, for $\delta > 0,\ \exists\ x \in \phi^{-1}(y)$ such that $i - \delta > \nu_{\widetilde{F}_1(e_1)}(x)$ and $i - \delta > \nu_{\phi(\widetilde{F}_2)(a_1)}(y)$.

$$\therefore i - \delta > \nu_{\phi(\widetilde{F}_2)(a_1)}(y)$$

$$\Rightarrow i - \delta > \nu_{\phi(\widetilde{F}_2)(e_1)}(\phi(x)) = \nu_{\phi^{-1}(\phi(\widetilde{F}_2))(e_1)}(x) = \nu_{\widetilde{F}_2(e_1)}(x),$$

since $(\widetilde{F}_2, A_2)$ is (ϕ, φ)- invariant.

$$\Rightarrow i - \delta > \nu_{\widetilde{F}_1(e_1)}(x) \vee \nu_{\widetilde{F}_2(e_1)}(x) = \nu_{(\widetilde{F}_1 \cap \widetilde{F}_2)(e_1)}(x)$$

$$i - \delta > \bigwedge_{\phi(x)=y} \bigwedge_{\varphi(e_1)=a_1} \nu_{(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(x) = \nu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y) = j.$$

Since $\delta > 0$ is arbitrary, $i \geq j$.

$$\text{Hence, } \nu_{\phi(\widetilde{F}_1)(a_1)}(y) \vee \nu_{\phi(\widetilde{F}_2)(a_1)}(y) \geq \nu_{\phi(\widetilde{F}_1 \cap \widetilde{F}_2)(a_1)}(y) \tag{2}$$

From (1) and (2), $(\phi, \varphi)(\widetilde{F}_1, A_1) \widetilde{\cap} (\phi, \varphi)(\widetilde{F}_2, A_2) \subseteq (\phi, \varphi)\big[(\widetilde{F}_1, A_1) \widetilde{\cap} (\widetilde{F}_2, A_2)\big]$. This completes the proof. □

4 Normality on CIFSL-G

In this section, we define the notion of normal complex intuitionistic fuzzy soft lattice ordered group and verify its algebraic properties.

Definition 4.1. Let $(\widetilde{F}, E) \in \mathcal{CIFSL}\text{-}\mathcal{G}(G)$. Then $(\widetilde{F}, E)$ is said to be normal complex intuitionistic fuzzy soft lattice ordered group on G, if for all $a \in E$, and $x, y \in G$,

$$\mu_{\widetilde{F}(a)}(xyx^{-1}) \geq \mu_{\widetilde{F}(a)}(y) \text{ and } \nu_{\widetilde{F}(a)}(xyx^{-1}) \leq \nu_{\widetilde{F}(a)}(y)$$

and it is denoted by $(\widetilde{F}, E) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G)$.

Example 4.2. Consider the ℓ- group $\mathbb{R} \times \mathbb{R}$ under addition with the ordered relation $\leq$ is defined as $(x, y) \leq (u, v) \iff x = u, y = v$ or

$x \leq u, y < v$.
Now, we define the CIFSS $(\widetilde{F}, E)$ of the ℓ- group $\mathbb{R} \times \mathbb{R}$ by

$$\mu_{\widetilde{F}(a)}(x,y) = \begin{cases} 0.5e^{i2\pi(0.7)}, & \text{if } y = 0 \\ 0, & \text{otherwise} \end{cases}, \nu_{\widetilde{F}(a)}(x,y) = \begin{cases} 0.5e^{i2\pi(0.3)}, & \text{if } y = 0 \\ 1e^{i2\pi(1)}, & \text{otherwise} \end{cases}$$

$$\mu_{\widetilde{F}(b)}(x,y) = \begin{cases} 0.3e^{i2\pi(0.4)}, & \text{if } y = 0 \\ 0.1e^{i2\pi(0.2)}, & \text{otherwise} \end{cases}, \nu_{\widetilde{F}(b)}(x,y) = \begin{cases} 0.7e^{i2\pi(0.6)}, & \text{if } y = 0 \\ 0.9e^{i2\pi(0.8)}, & \text{otherwise,} \end{cases}$$

for $E = \{a, b\}$ and $(x, y) \in \mathbb{R} \times \mathbb{R}$.
It is easy to verify that $(\widetilde{F}, E) \in \mathcal{NCIFSL}$-$\mathcal{G}$.

The proof of the following theorems are easy to verify from Definition 4.1.

Proposition 4.3. *Let $(\widetilde{F}, E) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. Then the following are equivalent, for all $a \in E$, and $x, y \in G$,*

(i) $(\widetilde{F}, E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$
(ii) $\mu_{\widetilde{F}(a)}(xyx^{-1}) = \mu_{\widetilde{F}(a)}(y)$ *and* $\nu_{\widetilde{F}(a)}(xyx^{-1}) = \nu_{\widetilde{F}(a)}(y)$
(iii) $\mu_{\widetilde{F}(a)}(xy) = \mu_{\widetilde{F}(a)}(yx)$ *and* $\nu_{\widetilde{F}(a)}(xy) = \mu_{\widetilde{F}(a)}(yx)$.

Proposition 4.4. *Let $(\widetilde{F}_1, A_1), (\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$. Then*

(i) $(\widetilde{F}_1, A_1)\widetilde{\cap}(\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$,
(ii) if $A_1 \cap A_2 = \emptyset$, then $(\widetilde{F}_1, A_1)\widetilde{\cup}(\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$.

Definition 4.5. [17] Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. Then their sum $(\widetilde{F}_1, A_1) \oplus (\widetilde{F}_2, A_2)$ is defined as $(\widetilde{F}_1, A_1) \oplus (\widetilde{F}_2, A_2) = \{(c, (\widetilde{F}_1 \oplus \widetilde{F}_2)(c)) : c = (a, b) \in A_1 \times A_2\}$, where

$$\mu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(x) = \sup_{x = p \vee q} \{min\{r_{\widetilde{F}_1(a)}(p), r_{\widetilde{F}_2(b)}(q)\}e^{i\ min\{w_{\widetilde{F}_1(a)}(p), w_{\widetilde{F}_2(b)}(q)\}}\} \text{ and}$$

$$\nu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(x) = \inf_{x = p \vee q} \{max\{\tau_{\widetilde{F}_1(a)}(p), \tau_{\widetilde{F}_2(b)}(q)\}e^{i\ max\{\psi_{\widetilde{F}_1(a)}(p), \psi_{\widetilde{F}_2(b)}(q)\}}\},$$

for all $x \in G$.

Theorem 4.6. *The sum of two $\mathcal{NCIFSL}$-$\mathcal{G}$ of G is again a $\mathcal{NCIFSL}$-$\mathcal{G}$ of G.*

Proof. Let $(\widetilde{F}_1, A_1)$ and $(\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$. By Theorem 3.2 in [17], $(\widetilde{F}_1, A_1) \oplus (\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. Let $x = p \vee q$ and $y = r \vee s$ be two elements in G and $c = (a, b) \in A_1 \times A_2$. By using Definition 4.5,

$$
\begin{aligned}
&\mu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(xy) \\
&= \sup_{xy=(p\vee q)(r\vee s)=(p\vee q)r\vee(p\vee q)s} \{min\{r_{\widetilde{F}_1(a)}((p\vee q)r), r_{\widetilde{F}_2(b)}((p\vee q)s)\} \\
&\quad e^{i\ min\{w_{\widetilde{F}_1(a)}((p\vee q)r), w_{\widetilde{F}_2(b)}((p\vee q)s)\}}\} \\
&= \sup_{xy=(p\vee q)r\vee(p\vee q)s} \{min\{\mu_{\widetilde{F}_1(a)}((p\vee q)r), \mu_{\widetilde{F}_2(b)}((p\vee q)s)\}\} \\
&= \sup_{yx=r(p\vee q)\vee s(p\vee q)} \{min\{\mu_{\widetilde{F}_1(a)}(r(p\vee q)), \mu_{\widetilde{F}_2(b)}(s(p\vee q))\}\} \\
&= \mu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(yx) \qquad (3)
\end{aligned}
$$

Similarly we can get, $\nu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(xy) = \nu_{(\widetilde{F}_1 \oplus \widetilde{F}_2)(c)}(yx)$ (4)

From (3) and (4), $(\widetilde{F}_1, A_1) \oplus (\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$. $\square$

Definition 4.7. [17] Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. Then their product $(\widetilde{F}_1, A_1) \otimes (\widetilde{F}_2, A_2)$ is defined as: $(\widetilde{F}_1, A_1) \otimes (\widetilde{F}_2, A_2) = \{(c, (\widetilde{F}_1 \otimes \widetilde{F}_2)(c)) : c = (a, b) \in A_1 \times A_2\}$, where

$\mu_{(\widetilde{F}_1 \otimes \widetilde{F}_2)(c)}(x) = \sup_{x \leq p \wedge q} \{min\{r_{\widetilde{F}_1(a)}(p), r_{\widetilde{F}_2(b)}(q)\} e^{i\ min\{w_{\widetilde{F}_1(a)}(p), w_{\widetilde{F}_2(b)}(q)\}}\}$ and

$\nu_{(\widetilde{F}_1 \otimes \widetilde{F}_2)(c)}(x) = \inf_{x \leq p \wedge q} \{max\{\tau_{\widetilde{F}_1(a)}(p), \tau_{\widetilde{F}_2(b)}(q)\} e^{i\ max\{\psi_{\widetilde{F}_1(a)}(p), \psi_{\widetilde{F}_2(b)}(q)\}}\}$,

for all $x \in G$.

Theorem 4.8. *The product of two $\mathcal{NCIFSL}$-$\mathcal{G}$ of G is again a $\mathcal{NCIFSL}$-$\mathcal{G}$ of G.*

Proposition 4.9. *Let $(\widetilde{F}, E) \in CIFSS(G)$ where $(\widetilde{F}, E)$ is non- null. Then the following are equivalent*

(i) $(\widetilde{F}, E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$.

(ii) For all $a \in E$ and for arbitrary $\alpha, \beta \in \mathcal{O}_1$ with $\widetilde{F}_{(\alpha,\beta)}(a) \neq \emptyset$, $\widetilde{F}_{(\alpha,\beta)}(a)$ is normal ℓ-group.

Proposition 4.10. *Let $(\widetilde{F}_1, A_1)$, $(\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}$-$\mathcal{G}$ over G_1 and G_2, respectively. Let $\phi : G_1 \longrightarrow G_2$ be a ℓ - homomorphism and $\varphi : A_1 \longrightarrow A_2$ be a onto function. Then* *(i) the image of* $(\widetilde{F}_1, A_1)$ *under* (ϕ, φ) *is* $\mathcal{NCIFSL}$-$\mathcal{G}$ *over* G_2.

(ii) the inverse image of $(\widetilde{F}_2, A_2)$ *under* (ϕ, φ) *is* $\mathcal{NCIFSL}$-$\mathcal{G}$ *over* G_1.

Definition 4.11. [15] Let U_1 and U_2 be two universal sets, $\varphi : U_1 \longrightarrow U_2$ be a function, $A_1,\ A_2$ be two set of parameters, $(\widetilde{F}_1, A_1) \in CIFSS(U_1)$ and $(\widetilde{F}_2, A_2) \in CIFSS(U_2)$. Define $(\varphi(\widetilde{F}_1), A_1) \in CIFSS(U_2)$ and $(\varphi^{-1}(\widetilde{F}_2), A_2) \in CIFSS(U_1)$ as follows:

(i) $(\varphi(\widetilde{F}_1), A_1)$ is such that for all $y \in U_2$ and $a_1 \in A_1$,
$\mu_{\varphi(\widetilde{F}_1)(a_1)}(y) = \bigvee\{\{\mu_{\widetilde{F}_1(a_1)}(x) : x \in U_1, \varphi(x) = y\} \cup \{0\}\}$ and
$\nu_{\varphi(\widetilde{F}_1)(a_1)}(y) = \bigwedge\{\{\nu_{\widetilde{F}_1(a_1)}(x) : x \in U_1, \varphi(x) = y\} \cup \{1\}\}$

(ii) $(\varphi^{-1}(\widetilde{F}_2), A_2)$ is such that for all $x \in U_1$ and $a_2 \in A_2$,
$\mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) = \mu_{\widetilde{F}_2(a_2)}(\varphi(x))$ and $\nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) = \nu_{\widetilde{F}_2(a_2)}(\varphi(x))$.

Proposition 4.12. *Let* $\varphi : G \longrightarrow G'$ *be surjective* ℓ *- homomorphism. Let* $(\widetilde{F}_1, A_1) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$ *and* $(\widetilde{F}_2, A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G')$. *Then*
(a) $(\varphi(\widetilde{F}_1), A_1) \in \mathcal{CIFSL}$-$\mathcal{G}(G')$ *provided that*

$$\bigvee\left\{\mu_{\widetilde{F}_1(a_1)}(p) \wedge \mu_{\widetilde{F}_1(a_1)}(q) : \begin{array}{l} p, q \in G \\ \varphi(p) = x, \varphi(q) = y \end{array}\right\} \geq \mu_{\widetilde{F}_1(a_1)}(x) \wedge \mu_{\widetilde{F}_1(a_1)}(y),$$

$$\bigwedge\left\{\nu_{\widetilde{F}_1(a_1)}(p) \vee \nu_{\widetilde{F}_1(a_1)}(q) : \begin{array}{l} p, q \in G \\ \varphi(p) = x, \varphi(q) = y \end{array}\right\} \leq \nu_{\widetilde{F}_1(a_1)}(x) \vee \nu_{\widetilde{F}_1(a_1)}(y),$$

for all $x, y \in G'$
(b) $(\varphi^{-1}(\widetilde{F}_2), A_2) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$.

Proof. For (a): By Definition 4.11, we have $(\varphi(\widetilde{F}_1), A_1) \in CIFSS(G')$. For all $a_1 \in A_1$ and $x, e_{G'} \in G'$, where $e_{G'}$ is the identity element of G'.

(i) $\mu_{\varphi(\widetilde{F}_1)(a_1)}(x \vee e_{G'})$

$= \bigvee\{\{\mu_{\widetilde{F}_1(a_1)}(y) : y \in G, \varphi(y) = x \vee e_{G'}\} \cup \{0\}\}$
$= \bigvee\{\mu_{\widetilde{F}_1(a_1)}(y) : y \in G, \varphi(y) = x \vee e_{G'}\}$
$[\because\ \varphi\ is\ surjective]$
$\geq \bigvee\{\mu_{\widetilde{F}_1(a_1)}(u \vee e_G) : u, e_G \in G, \varphi(u) = x, \varphi(e_G) = e_{G'}\}$
$[\because\ \varphi\ is\ \ell - homomorphism]$
$\geq \bigvee\{\mu_{\widetilde{F}_1(a_1)}(u) \wedge \mu_{\widetilde{F}_1(a_1)}(e_G) : u, e_G \in G, \varphi(u) = x, \varphi(e_G) = e_{G'}\}$
$[\because\ (\widetilde{F}_1, A_1) \in \mathcal{CIFSL}\text{-}\mathcal{G}(G)]$
$= \mu_{\widetilde{F}_1(a_1)}(x) \wedge \mu_{\widetilde{F}_1(a_1)}(e_{G'})$

(ii) $\nu_{\varphi(\widetilde{F}_1)(a_1)}(x \vee e_{G'})$
$= \bigwedge\{\{\nu_{\widetilde{F}_1(a_1)}(y) : y \in G, \varphi(y) = x \vee e_{G'}\} \cup \{1\}\}$
$= \bigwedge\{\nu_{\widetilde{F}_1(a_1)}(y) : y \in G, \varphi(y) = x \vee e_{G'}\}$
$[\because\ \varphi\ is\ surjective]$
$\leq \bigwedge\{\nu_{\widetilde{F}_1(a_1)}(u \vee e_G) : u, e_G \in G, \varphi(u) = x, \varphi(e_G) = e_{G'}\}$
$[\because\ \varphi\ is\ \ell - homomorphism]$
$\leq \bigwedge\{\nu_{\widetilde{F}_1(a_1)}(u) \vee \nu_{\widetilde{F}_1(a_1)}(e_G) : u, e_G \in G, \varphi(u) = x, \varphi(e_G) = e_{G'}\}$
$[\because\ (\widetilde{F}_1, A_1) \in \mathcal{CIFSL}\text{-}\mathcal{G}(G)]$
$= \nu_{\widetilde{F}_1(a_1)}(x) \vee \nu_{\widetilde{F}_1(a_1)}(e_{G'})$

For (b): By Definition 4.11, $(\varphi^{-1}(\widetilde{F}_2), A_2) \in CIFSS(G)$ and also by using Definition 4.11 we obtain the following:

(i) $\mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x \vee e_G)$
$= \mu_{\widetilde{F}_2(a_2)}(\varphi(x \vee e_G))$
$= \mu_{\widetilde{F}_2(a_2)}(\varphi(x) \vee \varphi(e_G))$ $[\because\ \varphi\ is\ \ell -\ homomorphism]$
$\geq \mu_{\widetilde{F}_2(a_2)}(\varphi(x)) \wedge \mu_{\widetilde{F}_2(a_2)}(\varphi(e_G))$ $[\because\ (\widetilde{F}_2, A_2)\ \in\ \mathcal{CIFSL}\text{-}\mathcal{G}(G)]$
$= \mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) \wedge \mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(e_G)$ $[By\ Definition\ 4.11]$

(ii) $\nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x \vee e_G)$

$$
\begin{aligned}
&= \nu_{\widetilde{F}_2(a_2)}(\varphi(x \vee e_G)) \\
&= \nu_{\widetilde{F}_2(a_2)}(\varphi(x) \vee \varphi(e_G)) && [\because\ \varphi \ is\ \ell - homomorphism] \\
&\leq \nu_{\widetilde{F}_2(a_2)}(\varphi(x)) \vee \nu_{\widetilde{F}_2(a_2)}(\varphi(e_G)) && [\because\ (\widetilde{F}_2, A_2) \in \mathcal{CIFSL}\text{-}\mathcal{G}(G)] \\
&= \nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) \vee \nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(e_G) && [By\ Definition\ 4.11]
\end{aligned}
$$

Similarly, we can verify that (iii) $\mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(xy^{-1}) \geq \mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) \wedge \mu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(y)$ and (iv) $\nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(xy^{-1}) \leq \nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(x) \vee \nu_{\varphi^{-1}(\widetilde{F}_2)(a_2)}(y)$.
$\therefore$ $(\varphi^{-1}(\widetilde{F}_2), A_2) \in \mathcal{CIFSL}\text{-}\mathcal{G}(G)$. □

Proposition 4.13. *Let $\varphi : G \longrightarrow G'$ be surjective ℓ - homomorphism. Let $(\widetilde{F}_1, A_1) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G)$ and $(\widetilde{F}_2, A_2) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G')$. Then (a) $(\varphi(\widetilde{F}_1), A_1) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G')$ provided that*

$$
\bigvee \left\{ \mu_{\widetilde{F}_1(a_1)}(p) \wedge \mu_{\widetilde{F}_1(a_1)}(q) : \begin{array}{l} p, q \in G \\ \varphi(p) = x, \varphi(q) = y \end{array} \right\} \geq \mu_{\widetilde{F}_1(a_1)}(x) \wedge \mu_{\widetilde{F}_1(a_1)}(y) \ and
$$

$$
\bigwedge \left\{ \nu_{\widetilde{F}_1(a_1)}(p) \vee \nu_{\widetilde{F}_1(a_1)}(q) : \begin{array}{l} p, q \in G \\ \varphi(p) = x, \varphi(q) = y \end{array} \right\} \leq \nu_{\widetilde{F}_1(a_1)}(x) \vee \nu_{\widetilde{F}_1(a_1)}(y),
$$

for all $x, y \in G'$
(b) $(\varphi^{-1}(\widetilde{F}_2), A_2) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G)$.

We recall that the elements x and y of G are conjugates if there exists $z \in G$ such that $x = zyz^{-1}$. From this point of view, we can extend the notion to $\mathcal{CIFSL}\text{-}\mathcal{G}$.

Proposition 4.14. *Let $(\widetilde{F}, E) \in \mathcal{CIFSL}\text{-}\mathcal{G}$ of a ℓ-group G. Then $(\widetilde{F}, E) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G)$ if and only if $(\widetilde{F}, E)$ is constant on the conjugacy classes of G.*

Proof. Suppose that $(\widetilde{F}, E)$ is normal. Then $\mu_{\widetilde{F}(a)}(y^{-1}xy) = \mu_{\widetilde{F}(a)}(x)$ and $\nu_{\widetilde{F}(a)}(y^{-1}xy) = \nu_{\widetilde{F}(a)}(x)$, for all $x, y \in G$, and $a \in E$. Hence $(\widetilde{F}, E)$ is constant on the conjugacy classes of G. Conversely, Suppose $(\widetilde{F}, E)$ is constant on each conjugacy classes of G. Then, $\mu_{\widetilde{F}(a)}(xy) = \mu_{\widetilde{F}(a)}(xyxx^{-1}) = \mu_{\widetilde{F}(a)}(yx)$ and $\nu_{\widetilde{F}(a)}(xy) = \nu_{\widetilde{F}(a)}(xyxx^{-1}) = \nu_{\widetilde{F}(a)}(yx)$, for all $x, y \in G,\ a \in E$. Hence $(\widetilde{F}, E) \in \mathcal{NCIFSL}\text{-}\mathcal{G}(G)$. □ Next, we define the commutators of $\mathcal{CIFSL}$-$\mathcal{G}(G)$. In group theory, the commutators of x and y of G (is denoted by $[x,y]$) is the element $x^{-1}y^{-1}xy$.

Proposition 4.15. *Let $(\widetilde{F},E) \in \mathcal{CIFSL}$-$\mathcal{G}$ of a ℓ-group G. Then $(\widetilde{F},E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$ if and only if $\mu_{\widetilde{F}(a)}([x,y]) \geq \mu_{\widetilde{F}(a)}(x)$ and $\nu_{\widetilde{F}(a)}([x,y]) \leq \nu_{\widetilde{F}(a)}(x)$, for all $x,y \in G$ and $a \in E$.*

Proof. Suppose that $(\widetilde{F},E)$ is normal. Let $x,y \in G$. Then for all $a \in E$, $\mu_{\widetilde{F}(a)}([x,y]) = \mu_{\widetilde{F}(a)}(x^{-1}y^{-1}xy) \geq \mu_{\widetilde{F}(a)}(x^{-1}) \wedge \mu_{\widetilde{F}(a)}(y^{-1}xy) = \mu_{\widetilde{F}(a)}(x)$. Similarly, we get $\nu_{\widetilde{F}(a)}([x,y]) \leq \nu_{\widetilde{F}(a)}(x)$. Conversely, assume that $(\widetilde{F},E)$ satisfies the inequalities, then for all $a \in E$ and $x,y \in G$, $\mu_{\widetilde{F}(a)}(x^{-1}zx) = \mu_{\widetilde{F}(a)}(zz^{-1}x^{-1}zx) \geq \mu_{\widetilde{F}(a)}(z) \wedge \mu_{\widetilde{F}(a)}([z,x]) = \mu_{\widetilde{F}(a)}(z)$ and similarly, $\nu_{\widetilde{F}(a)}(x^{-1}zx) \leq \nu_{\widetilde{F}(a)}(z)$. Then $(\widetilde{F},E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$. $\square$

Proposition 4.16. *Let $(\widetilde{F},E) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. If $\mu_{\widetilde{F}(a)}([x,y]) = \mu_{\widetilde{F}(a)}(e_G)$ and $\nu_{\widetilde{F}(a)}([x,y]) = \nu_{\widetilde{F}(a)}(e_G)$, for all $x,y,e_G \in G$ and $a \in E$. Then $(\widetilde{F},E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$.*

Definition 4.17. Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E) \in \mathcal{CIFSL}$-$\mathcal{G}(G)$. Then the commutator of $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E)$ is the $\mathcal{CIFSS}(G)$ denoted by $\big[(\widetilde{F}_1,E),(\widetilde{F}_2,E)\big]$ and is defined as follows
$\big[(\widetilde{F}_1,E),(\widetilde{F}_2,E)\big] = \{< a, \mu_{\big[\widetilde{F}_1,\widetilde{F}_2\big](a)}(z), \nu_{\big[\widetilde{F}_1,\widetilde{F}_2\big](a)}(z) > \ \mid a \in E,\ z \in G\}$, where

$$\begin{aligned}
\mu_{[\widetilde{F}_1,\widetilde{F}_2](a)}(z) &= \sup_{z=xyx^{-1}y^{-1}} \big\{min\{r_{\widetilde{F}_1(a)}(x), r_{\widetilde{F}_2(a)}(y)\}e^{i\, min\{w_{\widetilde{F}_1(a)}(x), w_{\widetilde{F}_2(a)}(y)\}}\big\},\\
\nu_{[\widetilde{F}_1,\widetilde{F}_2](a)}(z) &= \inf_{z=xyx^{-1}y^{-1}} \big\{max\{\tau_{\widetilde{F}_1(a)}(x), \tau_{\widetilde{F}_2(a)}(y)\}e^{imax\{\psi_{\widetilde{F}_1(a)}(x), \psi_{\widetilde{F}_2(a)}(y)\}}\big\},
\end{aligned}$$

for all $x,y \in G$.

Theorem 4.18. *Let $(\widetilde{F}_1,E)$ and $(\widetilde{F}_2,E) \in \mathcal{NCIFSL}$-$\mathcal{G}(G)$. Then $\big[(\widetilde{F}_1,E),(\widetilde{F}_2,E)\big] \subseteq (\widetilde{F}_1,E)\widetilde{\cap}(\widetilde{F}_2,E)$.*

Proof. Suppose that $z = xyx^{-1}y^{-1}$, for $x, y \in G$. For all $a \in E$,

$$\begin{aligned}
& \mu_{(\widetilde{F}_1 \widetilde{\cap} \widetilde{F}_2)(a)}(z) \\
= \;& \mu_{\widetilde{F}_1(a)}(z) \wedge \mu_{\widetilde{F}_2(a)}(z) \\
= \;& \mu_{\widetilde{F}_1(a)}(xyx^{-1}y^{-1}) \wedge \mu_{\widetilde{F}_2(a)}(xyx^{-1}y^{-1}) \\
\geq \;& \big(\mu_{\widetilde{F}_1(a)}(x) \wedge \mu_{\widetilde{F}_1(a)}(yx^{-1}y^{-1})\big) \wedge \big(\mu_{\widetilde{F}_2(a)}(xyx^{-1}) \wedge \mu_{\widetilde{F}_2(a)}(y^{-1})\big) \\
= \;& \big(\mu_{\widetilde{F}_1(a)}(x) \wedge \mu_{\widetilde{F}_1(a)}(x^{-1})\big) \wedge \big(\mu_{\widetilde{F}_2(a)}(y) \wedge \mu_{\widetilde{F}_2(a)}(y^{-1})\big) \\
= \;& \mu_{\widetilde{F}_1(a)}(x) \wedge \mu_{\widetilde{F}_2(a)}(y) \\
\geq \;& \sup_{z=xyx^{-1}y^{-1}} \mu_{\widetilde{F}_1(a)}(x) \wedge \mu_{\widetilde{F}_2(a)}(y) \\
= \;& \sup_{z=xyx^{-1}y^{-1}} (r_{\widetilde{F}_1(a)}(x) \wedge r_{\widetilde{F}_2(a)}(y)) e^{i(w_{\widetilde{F}_1(a)}(x) \wedge w_{\widetilde{F}_2(a)})} \\
= \;& \mu_{\left[\widetilde{F}_1, \widetilde{F}_2\right](a)}(z)
\end{aligned}$$

Similarly, $\nu_{(\widetilde{F}_1 \widetilde{\cap} \widetilde{F}_2)(a)}(z) \leq \nu_{\left[\widetilde{F}_1, \widetilde{F}_2\right](a)}(z)$, for all $a \in E$. Then by Definition 2.4, $\big[(\widetilde{F}_1, E), (\widetilde{F}_2, E)\big] \subseteq (\widetilde{F}_1, E) \widetilde{\cap} (\widetilde{F}_2, E)$. $\square$

5 Conclusion

In this present work, we proposed the complex intuitionistic fuzzy soft ℓ - homomorphism on $\mathcal{CIFSL}$-$\mathcal{G}$. Also, the concept of invariant complex intuitionistic fuzzy soft function is constructed. Some of the desirable characteristics of invariant complex intuitionistic fuzzy soft function are investigated in detail. In addition, this work focused on $\mathcal{NCIFSL}$-$\mathcal{G}$ and its related properties. To extend this work, we can study the algebraic structure of $\mathcal{CIFSL}$-$\mathcal{G}$ associated with ℓ - congruence relation.

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