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t-Best Proximity Pair in Fuzzy Normed Spaces

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Abstract. This study has considered the problem of finding best proximity pair in fuzzy metric spaces and uniformly convex fuzzy Banach spaces for fuzzy cyclic contraction map. We prove the uniqueness of this point in uniformly fuzzy Banach spaces. We also give an algorithm to find a best proximity point for the map S in setting of a uniformly convex fuzzy Banach spaces.

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1. Introduction

Let A and B be subsets of a fuzzy metric space $(X, M, *)$ and $S : A \cup B \to A \cup B$ be a map such that $S(A) \subset B$ and $S(B) \subset A$. The set of $x \in A \cup B$ such that $M(x, Sx, t) = M(A, B, t)$ is called the set of t-best proximity pair of (A, B) and denoted by $P_S^t(A, B)$. The problem of finding t-best proximity pair is the main object of this paper. We study this problem in fuzzy metric space and uniformly convex fuzzy Banach spaces and compare this problem in two spaces.
We give conditions which prove the uniqueness of proximity pair in uniformly convex fuzzy Banach spaces.

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In the first part we prove some theorems in fuzzy metric spaces and in the second part we discuss about this problem in uniformly convex fuzzy Banach spaces and state main result and give an algorithm to find best proximity pair.
t-Best proximity pair also evolves a generalization of the concept of fuzzy fixed point of mapping. Indeed every best proximity pair is a fixed point of S, whenever $A \cap B \neq \emptyset$. (See [2, 3, 6, 8])

2. Preliminaries

In this section we recall some definitions and lemmas that we use to proof of some theorems in other sections.

Definition 2.1. *A binary operation* $* : [0;1] \times [0;1] \to [0;1]$ *is a continuous t-norm if* $*$ *satisfying conditions:*
(1) $*$ *is commutative and associative;*
(2) $*$ *is continuous;*
(3) $a * 1 = a$ *for all* $a \in [0;1]$*;*
(4) $a * b \leqslant c * d$ *whenever* $a \leqslant c$ *and* $b \leqslant d$*, and* $a; b; c; d \in [0;1]$.

Definition 2.2. *The 3-tuple* $(X, M, *)$ *is said to be a fuzzy metric space if* X *is an arbitrary set,* $*$ *is a continuous t-norm and* M *is a fuzzy set on* $X \times X \times (0, \infty)$ *satisfying the following conditions for all* $x, y, z \in X$ *and* $t, s > 0$
$(M1)$ $M(x, y, t) > 0$,
$(M2)$ $M(x, y, t) = 1$ *if and only if* $x = y$,
$(M3)$ $M(x, y, t) = M(y, x, t)$,
$(M4)$ $M(x, y, s + t) \geqslant M(x, z, s) * M(z, y, t)$,
$(M5)$ *the function* $M(x, y, .) : (0, \infty) \to [0, 1]$ *is*

Definition 2.3. *Let* $(X, M, *)$ *be a fuzzy metric space and* $A, B \neq \emptyset$ *are two subsets of* X*.* $M(A, B, t)$ *is defined as follows:*

$$M(A, B, t) = sup\{M(x, y, t) : (x, y) \in A \times B\}.$$

Definition 2.4. *Let* X *be a linear space on* $\mathbb{R}$*. A function* $N : X \times \mathbb{R} \to [0, 1]$ *is called a fuzzy norm if and only if for every* $x, y \in X$ *and every*

$c \in \mathbb{R}$ the following properties are satisfy
$(N1)$ $N(x,t) = 0$ for every $t \in \mathbb{R}^- \cup \{0\}$,
$(N2)$ $N(x,t) = 1$ if and only if $x = 0$ for every $t \in \mathbb{R}^+$,
$(N3)$ $N(cx,t) = N(x, \frac{t}{|c|})$ for every $c \neq 0$ and $t \in \mathbb{R}^+$,
$(N4)$ $N(x+y, s+t) \geqslant min\{N(x,s), N(y,t)\}$ for every $s,t \in \mathbb{R}$ and $x,y \in X$,
$(N5)$ the function $N(x,.)$ is nondecreasing on $\mathbb{R}$, and $\lim_{t\to\infty} N(x,t) = 1$. A pair (X,N) is called a fuzzy normed space.

We say that (X,N) satisfies conditions N6 and N7 if:
$(N6)$ for each $t > 0$, $N(x,t) > 0$ implies $x = 0$.
$(N7)$ for $x \neq 0$, $N(x,.)$ is a continuous function of $\mathbb{R}$ and strictly increasing on the subset $\{t : 0 < N(x,t) < 1\}$ of $\mathbb{R}$.

Definition 2.5. *Let A and B be nonempty subset of fuzzy metric space $(X,M,*)$. A map $S : A\cup B \to A\cup B$ is a fuzzy cyclic contraction map if it satisfies*
i) $S(A) \subset B$ and $S(B) \subset A$.
ii) For some $0 < k < 1$ we have $M(Sx,Sy,t) \geqslant kM(x,y,t) + (1-k)M(A,B,t)$.

Definition 2.6. *Let (X,N) be a fuzzy normed linear space. A sequence $\{x_n\}$ in X is said to be a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t \in (0,\infty)$ there exits N_0 such that $N(x_m - x_n, t) > 1-\varepsilon$ for each $m > n \geqslant N_0$.*

Definition 2.7. *Let (X,N) be a fuzzy normed linear space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ is said convergent to $x \in X$ if for each $0 < \varepsilon < 1$ and $t \in (0,\infty)$ there exits N_0 such that $N(x_n - x, t) > 1-\varepsilon$ for each $n \geqslant N_0$.*

Definition 2.8. *Let (X,N) be a fuzzy normed linear space. A subset A of X is said to be fuzzy bounded (f-bounded) if there exits $0 < \alpha < 1$ and $t > 0$ such that $N(x,t) > 1-\alpha$ for each $x \in A$.*

Definition 2.9. *Let (X,N) be a fuzzy normed linear space also A and B are two nonempty subsets of X. Then $N(A-B,t)$, for $t > 0$ is defined* *as follows*

$$N(A-B,t)=sup\{N(x-y,t):(x,y)\in A\times B\}.$$

3. Best Proximity Pair in Fuzzy Metric Spaces

The purpose of this section is to state some conditions such that under these conditions the existence of proximity pair guaranteed.

Proposition 3.1. *Let A and B be nonempty subsets of a fuzzy metric space $(X,M,*)$ and $S:A\cup B\to A\cup B$ a cyclic contraction map. For $x_0\in A\cup B$ and $x_{n+1}=Sx_n$, $n=0,1,2,...$ we have $M(x_n,Sx_n,t)\to M(A,B,t)$.*

Proof. Since S is a cyclic contraction map then for each $n\geqslant 0$, we have

$$\begin{aligned}
M(x_n,Sx_n,t) &= M(Sx_{n-1},Sx_n,t)\\
&\geqslant kM(x_{n-1},x_n,t)+(1-k)M(A,B,t)\\
&= kM(Sx_{n-2},Sx_{n-1},t)+(1-k)M(A,B,t)\\
&\geqslant k^2M(x_{n-2},x_{n-1},t)+(1-k^2)M(A,B,t).
\end{aligned}$$

So by induction

$$M(x_n,Sx_n,t)\geqslant k^nM(x_0,x_1,t)+(1-k^n)M(A,B,t).$$

Therefore $M(x_n,Sx_n,t)\to M(A,B,t)$. $\square$

Lemma 3.2. *Let A and B be nonempty subsets of a fuzzy metric space $(X,M,*)$ with $a*b=min\{a,b\}$ or $a*b=a.b$ and $S:A\cup B\to A\cup B$ a cyclic contraction map. For $x_0\in A\cup B$ and $x_{n+1}=Sx_n$, $n=0,1,2,...$ the sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are bounded.*

Proof. Suppose $x_0\in A$. Since $M(x_{2n},x_{2n+1},t)\to M(A,B,t)$, it is enough to prove that $\{x_{2n+1}\}$ is bounded. Suppose that $\{x_{2n+1}\}$ is not bounded, then for each $t\in(0,\infty)$ and $0<r<1$ there exists an N_0 such that

$$M(S^2x_0,S^{2N_0-1}x_0,t)>1-r, M(S^2x_0,S^{2N_0+1}x_0,t)\leqslant 1-r,$$

where $1-r < min\{M(x_0, Sx_0, t/2), M(A, B, 2t)\}$.
By the cyclic contraction property of S

$$\begin{aligned}\frac{(1-r)-M(A,B,2t)}{k^2}+M(A,B,2t) &> M(x_0,S^{2N_0-1}x_0,2t)\\ &\geqslant M(x_0,S^2x_0,t)*M(S^2x_0,S^{2N_0-1}x_0,t)\\ &\geqslant M(x_0,S^2x_0,t)*(1-r)\\ &\geqslant M(x_0,Sx_0,t/2)*M(x_0,Sx_0,t/2)*(1-r)\\ &=1-r.\end{aligned}$$

Thus $1-r > M(A, B, 2t)$ which is a contradiction.
In the case $a*b = a.b$ we need to consider $(1-r) < \frac{(1-k^2)M(A,B,2t)}{1-k^2M(x_0,Sx_0,t/2)}$ for contradiction. □

Theorem 3.3. *Let A and B be nonempty closed subsets of a complete fuzzy metric space $(X, M, *)$ and $a*b = min\{a, b\}$ (or $a*b = ab$). Suppose that the mapping $S : A\cup B \to A\cup B$ satisfying $S(A) \subset B, S(B) \subset A$, and*

$$M(Sx, Sy, t) \geqslant kM(x, y, t)+l[M(x, Sx, t)+M(y, Sy, t)]+mM(A, B, t),$$

for all $x, y \in A \cup B$, where $k + 2l + m < 1$ and $k, l, m \geqslant 0$. If A (or B) is boundedly compact, then there exists a $x \in A \cup B$ with $M(x, Sx, t) = M(A, B, t)$.

Proof. Suppose x_0 is an arbitrary point of $A\cup B$ and define $x_{n+1} = Sx_n$ for each $n \geqslant 0$. Now

$$\begin{aligned} M(x_{n+1},x_{n+2},t) &= M(Sx_n,Sx_{n+1},t)\\ &\geqslant kM(x_n,x_{n+1},t)+l[M(x_n,Sx_n,t)\\ &+ M(x_{n+1},Sx_{n+1},t)]+mM(A,B,t).\end{aligned}$$

So

$$M(x_{n+1},x_{n+2},t) \geqslant \frac{k+l}{1-l}M(x_n,x_{n+1},t)+\frac{m}{1-l}M(A,B,t),$$

and

$$M(x_{n+1},x_{n+2},t) \geqslant \alpha M(x_n,x_{n+1},t)+(1-\alpha)M(A,B,t),$$

where $\alpha = \frac{k+l}{1-l} < 1$. Hence, by induction:

$$M(x_{n+1}, x_n, t) \geqslant \alpha^n M(x_0, x_1, t) + (1 - \alpha^n)M(A, B, t),$$

and

$$M(x_{n+1}, x_n, t) \to M(A, B, t).$$

Therefore, by Lemma 3.2. both sequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are fuzzy bounded. Now, since A (or B) is boundedly compact, then $\{x_{2n}\}$ has a convergent subsequence, and so there exists a $x \in A$ such that $M(x, Sx, t) = M(A, B, t)$. □

4. Best Proximity Pair in Uniformly Convex Fuzzy Banach Spaces

In this section we want to give some contraction condition which under these the problem of proximity pair has a solution.

Definition 4.1. *A fuzzy Banach space (X, N) is said to be uniformly convex fuzzy Banach space if for each $\varepsilon \in (0, 2]$ there exist a $\delta \in (0, 1)$ such that for $x, y \in X$ and $k_x \leqslant 1, k_y \leqslant 1, k_{x-y} > \varepsilon$ implies $k_{(x+y)/2} \leqslant \delta$ where*

$$k_x = \sup_{0<\alpha<1} \{inf\{t > 0 : N(x, t) \geqslant \alpha\}\}.$$

Example 4.2. Let $(X, \|.\|)$ be a uniformly convex Banach space. We define

$$N(x, t) = \begin{cases} 1 & t > \|x\| \\ \frac{1}{\|x\|}t & 0 < t \leqslant \|x\| \\ 0 & t \leqslant 0 \end{cases}$$

An easy verification shows that (X, N) is a uniformly convex fuzzy Banach space and $N(x, .)$ is continuous.

Theorem 4.3. *Let (X, N) be a fuzzy normed linear space. Assume further that,*
(N6) If $t > 0$, $N(x; t) > 0$ implies $x = 0$.
Define $\|x\|_\alpha = \inf\{t > 0 : N(x, t) \geqslant \alpha\}, \alpha \in (0; 1)$. Then $\{\|.\|_\alpha : \alpha \in$

$(0,1)\}$ is ascending family of norms on X and they are called α-norms on X corresponding to the fuzzy norm N on X. If the condition N6 dropped from above theorem then $\{\|.\|_\alpha : \alpha \in (0,1)\}$ is ascending family of seminorms on X.

Definition 4.4. *Let (X, N) be a fuzzy normed linear space. For each $0 \leqslant \varepsilon \leqslant 2$ we define,*

$$S_\varepsilon = \{(x,y) : x, y \in X \quad and \quad K_x \leqslant 1, K_y \leqslant 1, K_{x-y} > \varepsilon\}.$$

Definition 4.5. *The modulus of convexity of a fuzzy normed linear space (X, N) is a function $\delta : [0,2] \to [0,1)$ defined by*

$$\delta(\varepsilon) = inf\{1 - \sup_{0<\alpha<1}\{inf\{t > 0 : N(\frac{x+y}{2}, t) \geqslant \alpha\}\} : \forall (x,y) \in S_\varepsilon\}.$$

Definition 4.6. *Let (X, N) be a fuzzy normed linear space, also A and B are two closed convex subsets of X. We define K_{A-B} as follows:*

$$K_{A-B} = \sup\{k_{x-y} : (x,y) \in A \times B \quad and \quad k_x \leqslant 1, k_y \leqslant 1\}.$$

From Theorem 4.3, $K_{A-B} = \sup\{\|x-y\|_1 : (x,y) \in A \times B$ and $k_x \leqslant 1, k_y \leqslant 1\}$, so $K_{A-B} \leqslant 2$.

Proposition 4.7. *Let (X, N) be a fuzzy normed linear space and $N(x, .)$ is upper semicontinuous. Then (X, N) is uniformly convex if and only for each $\varepsilon \in (0,2]$, there exists a $\delta \in (0,1)$ such that for $x, y \in X$,*

$$\left.\begin{array}{c} \mathrm{N(x,1) = 1} \\ \mathrm{N(y,1) = 1} \\ \mathrm{N(x-y,}\varepsilon\mathrm{) < 1} \end{array}\right\} \Rightarrow N(\frac{x+y}{2}, \delta) = 1.$$

Proposition 4.8. *Let (X, N) be a fuzzy normed linear space. Then $k_x \leqslant d$ iff $k_{\frac{x}{d}} \leqslant 1$ provided $d > 0$.*

Remark 4.9. *In Proposition 4.7, Bag and Samanta showed that $N(x,1) = 1$ iff $k_x \leqslant 1$ and $N(x-y,\varepsilon) < 1$ iff $k_{x-y} > \varepsilon$. If we combine Proposition*

4.7, and Proposition 4.8, then (X, N) is uniformly convex fuzzy Banach space if and only for each $\varepsilon \in (0, 2]$, there exists a $\delta \in (0, 1)$ such that for $x, y \in X$,

$$\left.\begin{array}{c} \mathrm{N(x,d)} = 1 \\ \mathrm{N(y,d)} = 1 \\ \mathrm{N(x-y,}d\varepsilon) < 1 \end{array}\right\} \Rightarrow N(\frac{x+y}{2}, d\delta) = 1.$$

Proposition 4.10. *Let (X, N) be a fuzzy normed linear space. Then (X, N) is uniformly convex if and only if $\delta(\varepsilon) > 0$ for $\varepsilon > 0$.*

Theorem 4.11. *Let A be a nonempty closed convex subset and B a nonempty closed subset of a uniformly convex fuzzy Banach space (X, N), $N(x, .)$ is continuous and strictly increasing for each $t \leqslant K_{A-B}$. Let $\{x_n\}$ and $\{z_n\}$ be sequences in A and $\{y_n\}$ a sequence in B satisfying:*

i) *$N(z_n - y_n, t) \to N(A - B, t)$.*

ii) *For every $0 < \varepsilon < 1$ and $t \in (0, \infty)$, there exists N_0 such that for all $m > n \geqslant N_0, N(x_m - y_n, t) \geqslant N(A - B, t) - \varepsilon$*

Then for every $0 < \varepsilon < 1$ there exists an N_1 such that for all $m > n \geqslant N_1, N(x_m - z_n, t) \geqslant 1 - \varepsilon$.

Proof. Assume the contrary, then there exist $0 < \varepsilon_0 < 1$ and $t_0 \in (0, \infty)$ such that $N(x_{m_k} - z_{n_k}, t_0) < 1 - \varepsilon_0$ for some $m_k > n_k \geqslant k$ and all $k \in \mathbb{N}$. For this $\varepsilon_0 > 0$ and each $t \in (0, \infty)$ there exists N_0 such that for all $m_k > n_k \geqslant N_0, N(x_{m_k} - y_{n_k}, t) \geqslant N(A - B, t) - \varepsilon_0$.
Also there exists N_2 such that for all $n_k \geqslant N_2, N(z_{n_k} - y_{n_k}, t) \geqslant N(A - B, t) - \varepsilon_0$ for all $t \in (0, \infty)$. Put $N_1 = max\{N_0, N_2\}$. we have $K_{x_{m_k} - y_{n_k}} \leqslant K_{A-B}$ and $K_{z_{n_k} - y_{n_k}} \leqslant K_{A-B}$. By uniform convexity there exists $0 < \delta < 1$ such that for all $m_k > n_k \geqslant N_1$

$$K_{(\frac{x_{m_k}+z_{n_k}}{2} - y_{n_k})} \leqslant \delta K_{A-B}.$$

$K_{(\frac{x_{m_k}+z_{n_k}}{2} - y_{n_k})} \leqslant \delta K_{A-B}$ is equivalent to $N(\frac{x_{m_k}+z_{n_k}}{2} - y_{n_k}, \delta K_{A-B}) = 1$, since $N(x, .)$ is strictly increasing for each $t \leqslant K_{A-B}$ and $\delta < 1$, so

$N(\frac{x_{m_k}+z_{n_k}}{2} - y_{n_k}, K_{A-B}) > 1$. This is a contradiction, hence the proof is complete. □

Corollary 4.12. *Let A be a nonempty closed convex subset and B a nonempty closed subset of a uniformly convex fuzzy Banach space (X, N), $N(x, .)$ is continuous and strictly increasing for each $t \leqslant K_{A-B}$. Let $\{x_n\}$ and $\{z_n\}$ be sequences in A and $\{y_n\}$ a sequence in B satisfying:*

i) $N(x_n - y_n, t) \to N(A - B, t)$,

ii) $N(z_n - y_n, t) \to N(A - B, t)$.

Then, $N(x_n - z_n, t) \to 1$.

Proof. It is a consequence of Theorem 4.11. □

Theorem 4.13. *Let (X, N) be a uniformly convex fuzzy Banach space such that $N(x, .)$ is continuous and strictly increasing for each $t \leqslant K_{A-B}$. Let A, B be nonempty closed convex subsets of X. $S : A \cup B \to A \cup B$ satisfying $S(A) \subset B$, $S(B) \subset A$ and*

$$N(Sx - Sy, t) \geqslant kN(x - y, t) + (1 - k)N(A - B, t),$$

for each $t \in (0, K_{A-B})$, $0 < k < 1$. For each $0 < t \leqslant K_{A-B}$ there exists a unique element $x \in A$ such that $N(x - Sx, t) = N(A - B, t)$. Further, if $x_0 \in A$ and $Sx_n = x_{n+1}$ then $\{x_{2n}\}$ converges to the above unique element.

Proof. Suppose that $x_0 \in A$ and define $Sx_n = x_{n+1}$ by Proposition 3.1, $N(x_{2n} - Sx_{2n}, t)$, $N(S^2x_{2n} - Sx_{2n}, t)$, converge to $N(A - B, t)$, then by Corollary 4.12, $N(x_{2n} - x_{2(n+1)}, t) \to 1$. Similarly $N(Sx_{2n} - Sx_{2(n+1)}, t) \to 1$. We now show that for each $0 < \varepsilon < 1$ and $t \in (0, \infty)$, there exists $N_0 \in \mathbb{N}$ such that for each $m > n > N_0$, $N(x_{2m} - Sx_{2n}, t) > N(A - B, t) - \varepsilon$.
Suppose there exists $0 < \varepsilon_0 < 1$ and $t_0 \in (0, \infty)$, such that for each $k \in \mathbb{N}$ there exist $m_k > n_k \geqslant k$ such that:

$$N(x_{2m_k} - Sx_{2n_k}, t_0) \leqslant N(A - B, t_0) - \varepsilon_0.$$

Let m_k be the least integer greater than n_k to satisfy the above inequality.
For each $\varepsilon > 0$

$$\begin{aligned}N(A-B,t_0)-\varepsilon_0 &\geqslant N(x_{2m_k}-Sx_{2n_k},t_0)\\ &\geqslant min\{N(x_{2m_k}-x_{2(m_k-1)},\varepsilon),N(x_{2(m_k-1)}-x_{2n_k},t_0-\varepsilon)\}.\end{aligned}$$

Since $N(x_{2m_k}-x_{2(m_k-1)},\varepsilon)\to 1$ and $N(x,.)$ is continuous

$$N(A-B,t_0)-\varepsilon_0 = N(x_{2m_k}-Sx_{2n_k},t_0).$$

Consequently

$$\begin{aligned}N(x_{2m_k}-Sx_{2n_k},t_0) \geqslant min\{&N(x_{2m_k}-x_{2(m_k+1)},\varepsilon/2),N(x_{2(m_k+1)}-\\ &Sx_{2(n_k+1)},t_0-\varepsilon),N(Sx_{2(n_k+1)}-Sx_{2n_k},\varepsilon/2)\}.\end{aligned}$$

By the same reason as above

$$\begin{aligned}N(x_{2m_k}-Sx_{2n_k},t_0) &\geqslant N(x_{2(m_k+1)}-Sx_{2(n_k+1)},t_0)\\ &\geqslant k^2N(x_{2m_k}-Sx_{2n_k},t_0)+(1-k^2)N(A-B,t_0).\end{aligned}$$

Therefore

$$N(A-B,t_0)-\varepsilon_0 \geqslant k^2(N(A-B,t_0)-\varepsilon_0)+(1-k^2)N(A-B,t_0).$$

So $N(A-B,t_0)-\varepsilon_0 \geqslant N(A-B,t_0)-k^2\varepsilon_0$, which is a contradiction. Therefore $N(x_{2m}-Sx_{2n},t)\to N(A-B,t)$. By Corollary 4.12, $N(x_{2m}-x_{2n},t)\to 1$, so $\{x_{2n}\}$ is a fuzzy cauchy sequence, converges to (say) x and $N(x-Sx,t)=N(A-B,t)$. $\square$

Example 4.14. If the convexity assumption is dropped from Theorem 4.13, then the convergence and uniqueness is not guaranteed even in finite dimensional spaces. Consider $X=\mathbb{R}^4$, $A=\{e_1,e_3\}$ and $B=\{e_2,e_4\}$. Then X with the norm defined in Example 4.2 is uniformly convex fuzzy Banach space, define $Se_i=e_{i+1}$ for $i=1,2,3$ and $Se_4=e_1$.

Theorem 4.15. *Let A, B be nonempty closed convex subsets of a uniformly convex fuzzy Banach space (X,N) and $N(x,.)$ is continuous and* *strictly increasing for each* $t \leqslant K_{A-B}$. *Let* $S : A \cup B \to A \cup B$ *satisfying* $S(A) \subset B$, $S(B) \subset A$ *and*

$$N(Sx-Sy,t) \geqslant kN(x-y,t)+l(N(x-Sx,t)+N(y-Sy,t))+mN(A-B,t),$$

for each $t \in (0,\infty)$, $k, l, m \geqslant 0$ *and* $k + 2l + m < 1$. *For each* $0 < t \leqslant K_{A-B}$ *there exists a unique element* $x \in A$ *such that* $N(x - Sx, t) = N(A - B, t)$. *Furthermore, if* $x_0 \in A$ *and* $Sx_n = x_{n+1}$ *then* $\{x_{2n}\}$ *converges to the above unique element.*

Proof. The proof is similar to Theorem 4.13. □

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