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# Soft Spectrum and Soft Multiplicative Linear Functionals

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**Abstract.** The purpose of this paper is to introduce the concept of soft multiplicative linear functional in soft Banach algebras. Then the notions of soft spectrum and soft spectral radius of an element in soft Banach algebras are introduced. Furthermore we discuss some important characterizations and theorems of them.

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**Keywords and Phrases:** Soft sets, Soft Banach algebra, Soft linear functional, Soft multiplicative linear functional, Soft spectrum.

## 1 Introduction

Molodtsov [15] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modelling the problems with incomplete information in engineering, physics, computer science, economics,

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social sciences and medical sciences. Das and Samanta [6] presented the idea of soft linear functional over soft linear spaces and studied some basic properties of those operators. Thakur and Samanta [19] introduced the new concept of soft convergence of sequence of soft elements and deduced some of its basic properties. They also defined the concept of soft Banach algebras. In this paper we introduce the concept of soft multiplicative linear functional in soft Banach algebras and deduce some of its properties. Then we introduce the concept of soft spectrum of an element in soft Banach algebras. Also we discuss some characterizations and theorems about these notions. In the next section we recall some definitions, theorems and basic notions in a soft set normed space. For more information about soft set theory, soft normed linear spaces, soft linear functional, and their applications we refer to [2], [4], [5], [8], [11]-[19].

## 2 Preliminaries

In this paper, we assume that the reader is familiar with the basic concepts and symbols of soft set theory, soft metric, soft normed spaces, and convergent sequences of soft elements in these spaces. Also, $X$ refers to an initial universe and $A$ is the set of all parameters for $X$. Absolute soft set $X$ is denoted by $\check{X}$ and the set of all soft elements of $\check{X}$ is denoted by SE($\check{X}$) [3].

**Proposition 2.1.** (Decomposition theorem)([19]) *In a soft normed linear space* $(\check{X}, \|.\|, A)$*, if we define for each* $\lambda \in A$*,* $\|.\|_\lambda : X \to \mathcal{R}^+$ *be a mapping such that for each* $\xi \in X$*,* $\|\xi\|_\lambda = \|\tilde{x}\|(\lambda)$*, where* $\tilde{x}\tilde{\in}\check{X}$ *is such that* $\tilde{x}(\lambda) = \xi$*. Then for each* $\lambda \in A$*,* $(X, \|.\|_\lambda)$ *is normed linear space.*

**Proposition 2.2.** ([19]) *A sequence* $\{\tilde{x}_n\}$ *of soft elements in a soft normed linear space* $(\check{X}, \|.\|, A)$ *is soft convergent to* $\tilde{x}$ if and only if $\{\tilde{x}_n(\lambda)\}$ *is convergent to* $\tilde{x}(\lambda)$ *in* $(X, \|.\|_\lambda)$ $\forall \lambda \in A$*, where* $\|.\|_\lambda$ *defined as in Decomposition theorem.*

**Definition 2.3.** ([3]) *A soft normed linear space* $(\check{X}, \|.\|, A)$ *is said to be soft complete if every Cauchy sequence in* $(\check{X}, \|.\|, A)$ *is soft convergent* *in* $(\check{X}, \|.\|, A)$*. Every complete soft normed linear space is called a soft Banach space.*

**Proposition 2.4.** ([19]) *The set of all soft real numbers,* $\mathcal{R}(A)$ *and the set of all soft complex numbers,* $\mathcal{C}(A)$ *are soft complete.*
The definitions of *compact*, *sequential compact* and *totally bounded* for a soft metric space $(\check{X}, d, A)$ are similar to the same definitions for a metric space and we refer the reader to [10] and [13].

**Theorem 2.5.** ([10]) *Let* $(\check{X}, d, A)$ *be a soft metric space.* $(\check{X}, d, A)$ *is soft sequential compact metric space if and only if* $(\check{X}, d, A)$ *is both complete and totally bounded soft metric space.*

**Theorem 2.6.** ([10]) *Let* $(\check{X}, d, A)$ *be a soft metric space. Then the following statements are equivalent:*

(1). $(\check{X}, d, A)$ *is soft compact metric space,*
(2). $(\check{X}, d, A)$ *is soft sequential compact metric space.*

**Definition 2.7.** ([7])*Let* $\check{X}$*,* $\check{Y}$ *be two soft metric space and* $T : \mathcal{S}E(\check{X}) \to \mathcal{S}E(\check{Y})$ *be a soft operator. Then* $T$ *is said to be soft linear, if*

(L1). $T$ *is additive, i.e.,* $T(\tilde{x}_1 + \tilde{x}_2) = T(\tilde{x}_1) + T(\tilde{x}_2)$*, for every soft elements* $\tilde{x}_1, \tilde{x}_2 \tilde{\in} \check{X}$*,*
(L2). $T$ *is homogeneous, i.e., for every soft scalar* $\tilde{c}$*,* $T(\tilde{c}\tilde{x}) = \tilde{c}T(\tilde{x})$*, for every soft elements* $\tilde{x} \tilde{\in} \check{X}$*.*

**Definition 2.8.** ([7]) *Let* $\check{X}$*,* $\check{Y}$ *be two soft normed space. The operator* $T : \mathcal{S}E(\check{X}) \to \mathcal{S}E(\check{Y})$ *is said to be continuous at* $\tilde{x}_0 \tilde{\in} \check{X}$*, if for every sequence* $\{\tilde{x}_n\}$ *of soft elements of* $\check{X}$ *with* $\tilde{x}_n \hookrightarrow \tilde{x}_0$ *as* $n \to \infty$ *we have* $T(\tilde{x}_n) \hookrightarrow T(\tilde{x}_0)$ *as* $n \to \infty$.

**Definition 2.9.** ([7]) *Let* $T : \mathcal{S}E(\check{X}) \to \mathcal{S}E(\check{Y})$ *be a soft linear operator, where* $\check{X}, \check{Y}$ *are soft normed linear spaces. The operator* $T$ *is called bounded, if there exist some positive soft real number* $\tilde{M}$ *such that for each* $\tilde{x} \tilde{\in} \check{X}$*,* $\|T(\tilde{x})\| \tilde{\leq} \tilde{M} \|\tilde{x}\|$.

**Theorem 2.10.** ([7]) *Let* $T : \mathcal{S}E(\check{X}) \to \mathcal{S}E(\check{Y})$ *be a soft linear operator, where* $\check{X}, \check{Y}$ *are soft normed linear spaces. If* $T$ *is bounded then* $T$ *is continuous.*

**Definition 2.11.** ([19]) *Let $V$ be an algebra over field* C *of complex numbers and let $A$ be the parameter set and $(G, A)$ be a soft set over $V$. Now $(G, A)$ is said to be a soft algebra of $V$ over* C *if $G(\lambda)$ is a subalgebra of $V$, $\forall\lambda \in A$. If $(G, A)$ is also a soft Banach space with respect to a soft norm that satisfies the inequality $\|\tilde{x}\tilde{y}\| \leq \|\tilde{x}\|\|\tilde{y}\|$, for all $\tilde{x}, \tilde{y}\tilde{\in}(G, A)$ and if $(G, A)$ contains an identity $\bar{e}$ such that $\tilde{x}\bar{e} = \bar{e}\tilde{x} = \tilde{x}$ with $\|\tilde{e}\| = \bar{1}$, then $(G, A)$ is called a soft Banach algebra. In addition, if in a soft Banach algebra $(G, A)$, $\tilde{x}\tilde{y} = \tilde{y}\tilde{x}$, $\forall\tilde{x}, \tilde{y} \in (G, A)$ then $(G, A)$ is called a commutative soft Banach algebra.*

From now on, $(G, A)$ is a soft set over complex algebra $V$.

**Proposition 2.12.** ([19]) *$(G, A)$ is a soft Banach algebra if and only if $G(\lambda)$ is a Banach algebra, $\forall\lambda \in A$.*

**Definition 2.13.** ([19]) *Let $(G, A)$ be a soft Banach algebra. A soft element $\tilde{x}\tilde{\in}(G, A)$ is said to be invertible if it has an inverse in $(G, A)$, i.e., if there exists a soft element $\tilde{y}\tilde{\in}(G, A)$ such that $\tilde{x}\tilde{y} = \tilde{y}\tilde{x} = \bar{e}$ and then $\tilde{y}$ is called the inverse of $\tilde{x}$, denoted by $\tilde{x}^{-1}$. Otherwise, $\tilde{x}$ is said to be non-invertible soft element of $(G, A)$ .*

**Proposition 2.14.** ([19]) *Let $(G, A)$ be a soft Banach algebra. If $\tilde{x}\tilde{\in}(G, A)$ satisfies $\|\tilde{x}\|\tilde{<}\bar{1}$, then $\bar{e} - \tilde{x}$ is invertible and $(\bar{e} - \tilde{x})^{-1} = \bar{e} + \sum_{n=1}^{\infty} \tilde{x}^n$.*

**Corollary 2.15.** ([19]) *Let $(G, A)$ be a soft Banach algebra. Let $\tilde{x}\tilde{\in}(G, A)$ and $\tilde{\mu}$ be a soft scaler such that $|\tilde{\mu}|\tilde{>}\|\tilde{x}\|$. Then $(\tilde{\mu}\bar{e} - \tilde{x})^{-1}$ exists and $(\tilde{\mu}\bar{e} - \tilde{x})^{-1} = \tilde{\mu}^{-1}\bar{e} + \sum_{n=1}^{\infty} \tilde{\mu}^{-n-1}\tilde{x}^n$.*

**Proposition 2.16.** ([19]) *Let $(G, A)$ be a soft Banach algebra. The soft set $\mathcal{S}$ generated by the set of all invertible soft elements of $(G, A)$ is a soft open subset in $(G, A)$.*

**Definition 2.17.** ([6]) *A soft linear functional $f$ is a soft linear operator such that $f : \mathcal{S}E(\check{X}) \to K$ where $\check{X}$ is a soft linear space and $K = \mathcal{R}(A)$ if $\check{X}$ is a real soft linear space and $K = \mathcal{C}(A)$ if $\check{X}$ is a complex soft linear space.*

We have the following definitions and theorem of soft linear functional $f$ in soft normed space $\check{X}$ ([5]).

**Definition 2.18.** *The soft linear functional $f$ is called bounded if there exists some positive soft real number $\tilde{M}$ such that for all $\tilde{x}\tilde{\in}\check{X}$, $\|f(\tilde{x})\|\tilde{\leq} \tilde{M}\|\tilde{x}\|$.*

**Definition 2.19.** *Let $f$ be a bounded soft linear functional. Then the norm of the functional $f$ denoted by $\|f\|$, is a soft real number defined as the following :*
*For each $\lambda \in A$, $\|f\|(\lambda)=inf\{t \in \mathrm{R}; \|f(\tilde{x})\|(\lambda) \leq t\|\tilde{x}\|(\lambda)$, for all $\tilde{x}\tilde{\in}\check{X}\}$.*

**Theorem 2.20.** *Let $f$ be a bounded soft linear functional. Then $\|f(\tilde{x})\|\tilde{\leq} \|f\|\|\tilde{x}\|$, for all $\tilde{x}\tilde{\in}\check{X}$.*

## 3 Main Results

Throughout this section $X$ is a Banach algebra over field C of complex numbers and $A$ is the parameter set.

**Definition 3.1.** *We say that a nonzero soft linear functional $\varphi : SE(\check{X}) \longrightarrow \mathcal{C}(A)$ is soft multiplicative on $\check{X}$, if $\varphi(\tilde{x}\tilde{y}) = \varphi(\tilde{x})\varphi(\tilde{x}), \forall \tilde{x}, \tilde{y}\tilde{\in}\check{X}$. Also a soft multiplicative linear functional $\varphi : SE(\check{X}) \longrightarrow \{\bar{\mu} : \bar{\mu}\tilde{\in}\mathcal{C}(A)\} \subset \mathcal{C}(A)$ which $\bar{\mu}(\lambda) = \mu$, $\forall\lambda \in A$ is said fixed soft multiplicative. We denote the space of all soft multiplicative linear functionals on $\check{X}$ by $\triangle(\check{X})$.*

**Definition 3.2.** *For $\bar{\varepsilon}\tilde{>}\bar{0}$ a nonzero soft linear functional $\varphi : SE(\check{X}) \longrightarrow \mathcal{C}(A)$ is called soft $\bar{\varepsilon}$-almost multiplicative on $\check{X}$, if $\|\varphi(\tilde{x}\tilde{y}) - \varphi(\tilde{x})\varphi(\tilde{x})\|\tilde{\leq} \bar{\varepsilon}\|\tilde{x}\|\|\tilde{y}\|, \forall \tilde{x}, \tilde{y}\tilde{\in}\check{X}$.*

**Example 3.3.** $I : \mathcal{C}(A) \longrightarrow \mathcal{C}(A)$ be such that $I(\tilde{r}) = \tilde{r}$, $\forall \tilde{r} \in \mathcal{C}(A)$. Then, $I$ is a soft multiplicative linear functional. Also, for any $\lambda_1 \in A$ the soft linear functional $\varphi_{\lambda_1} : \mathcal{C}(A) \longrightarrow \mathcal{C}(A)$ such that $[\varphi_{\lambda_1}(\tilde{r})](\lambda) = \tilde{r}(\lambda_1)$, $\forall \tilde{r} \in \mathcal{C}(A)$, is a fixed soft multiplicative linear functional. Because $[\varphi_{\lambda_1}(\tilde{r}\tilde{s})](\lambda) = (\tilde{r}\tilde{s})(\lambda_1) = \tilde{r}(\lambda_1)\tilde{s}(\lambda_1) = [\varphi_{\lambda_1}(\tilde{r})](\lambda)[\varphi_{\lambda_1}(\tilde{s})](\lambda) = [\varphi_{\lambda_1}(\tilde{r})\varphi_{\lambda_1}(\tilde{s})](\lambda)$, $\forall \tilde{r}, \tilde{s} \in \mathcal{C}(A)$, $\forall\lambda \in A$ then $\varphi_{\lambda_1}(\tilde{r}\tilde{s}) = \varphi_{\lambda_1}(\tilde{r})\varphi_{\lambda_1}(\tilde{s})$ for all $\tilde{r}, \tilde{s} \in \mathcal{C}(A)$. In addition, $[\varphi_{\lambda_1}(\bar{1})](\lambda) = \bar{1}(\lambda_1) = 1$, $\forall\lambda \in A$; i.e., $\varphi_{\lambda_1} \neq \bar{0}$.

**Example 3.4.** Let

$$\check{X} = \left\{ \left[ \begin{array}{cc} \tilde{m}_{11} & \bar{0} \\ \tilde{m}_{21} & \tilde{m}_{22} \end{array} \right] : \tilde{m}_{ij} \in \mathcal{C}(A) \right\}.$$

Then, $\check{X}$ is a soft sub Banach algebra of $M_2(\mathcal{C}(A))$. We define $\varphi : \check{X} \to \mathcal{C}(A)$ by $\varphi(\tilde{M}) = \tilde{m}_{11}$, for all $\tilde{M}\tilde{\in}\check{X}$ and $\psi : \check{X} \to \mathcal{C}(A)$ by

$\psi(\tilde{M}) = \tilde{m}_{21}$, for all $\tilde{M} \tilde{\in} \check{X}$. Then, $\varphi$ and $\psi$ are soft linear functionals, $\varphi$ is soft multiplicative but $\psi$ is not soft multiplicative. Because for all $\tilde{P}, \tilde{Q} \tilde{\in} \check{X}$, $\varphi(\tilde{P}\tilde{Q}) = \varphi(\tilde{P})\varphi(\tilde{Q}) = \tilde{p}_{11}\tilde{q}_{11}$, $\psi(\tilde{P}\tilde{Q}) = \tilde{p}_{21}\tilde{q}_{11} + \tilde{p}_{22}\tilde{q}_{21}$ and $\psi(\tilde{P})\psi(\tilde{Q}) = \tilde{p}_{21}\tilde{q}_{21}$.

**Note:** The set $\triangle(\check{X})$ is not a soft linear space. Because

(1). If $\varphi \in \triangle(\check{X})$ then $\bar{k}\varphi \notin \triangle(\check{X})$, $\forall\ \bar{k} \neq \bar{1}$. Because there is a soft element $\tilde{x}$ in $\check{X}$ such that $\varphi(\tilde{x}) = \bar{1}$. Thus, we have $(\bar{k}\varphi)(\tilde{x}^2) = \bar{k}(\varphi(\tilde{x}^2)) = \bar{k}\varphi(\tilde{x})\varphi(\tilde{x}) = \bar{k}$ and $(\bar{k}\varphi)(\tilde{x})(\bar{k}\varphi)(\tilde{x}) = \bar{k}^2$. But $\bar{k}^2 \neq \bar{k}$ for $\tilde{k} \tilde{\neq} \bar{1}, \tilde{k} \neq \bar{0}$. Then $\bar{k}\varphi \notin \triangle(\check{X})$, $\forall\ \bar{k} \neq \bar{1}$ in particular $-\varphi \notin \triangle(\check{X})$.

(2). If $\varphi, \psi \in \triangle(\check{X})$ then, $(\varphi + \psi) \notin \triangle(\check{X})$. For example, $\varphi + \varphi = \bar{2}\varphi$ and by part (1), $\bar{2}\varphi \notin \triangle(\check{X})$.

**Proposition 3.5.** *Every nonzero multiplicative linear functional on $X$ over* C *can be extended to a fixed soft multiplicative linear functional on $\check{X}$.*

**Proof.** Let $f$ be a multiplicative linear functional on $X$. Since $\check{X}$ is absolute soft Banach algebra, thus there exists a soft element $\tilde{x}_1$ in $\check{X}$ such that for some $\lambda_1 \in A$, $f(\tilde{x}_1(\lambda_1)) \neq 0$. We define $\varphi : SE(\check{X}) \longrightarrow \mathcal{C}(A)$ by $\varphi(\tilde{x}) = \bar{\mu}_{\tilde{x}}$, when $\mu_{\tilde{x}} = f(\tilde{x}(\lambda_1))$. Let $\tilde{x}$ and $\tilde{y}$ are arbitrary soft elements of $\check{X}$. Suppose that $\varphi(\tilde{x}) = \bar{\alpha}$, $\varphi(\tilde{y}) = \bar{\beta}$. Thus, we have $f((\tilde{x}\tilde{y})(\lambda_1)) = f(\tilde{x}(\lambda_1)(\tilde{y}(\lambda_1)) = f(\tilde{x}(\lambda_1))f(\tilde{y}(\lambda_1)) = \alpha\beta = \gamma$. Then $\varphi(\tilde{x}\tilde{y}) = \bar{\gamma}$ and $\gamma = \alpha\beta$. But $\bar{\gamma}(\lambda) = \gamma = \alpha\beta = (\bar{\alpha}\bar{\beta})(\lambda)$, $\forall \lambda \in A$. Therefore, $\varphi(\tilde{x}\tilde{y}) = \varphi(\tilde{x})\varphi(\tilde{y})$; i.e., $\varphi$ is a fixed soft multiplicative linear functional on $\check{X}$. □

**Proposition 3.6.** *Let $\varphi$ be a fixed soft multiplicative linear functional on $\check{X}$. Then $\varphi(\bar{e}) = \bar{1}$, when $\bar{e}$ is the unit element of $\check{X}$.*

**Proof.** Let $\varphi$ be a fixed soft multiplicative linear functional on $\check{X}$. By definition there is a soft element $\tilde{x} \neq \Theta$ in $\check{X}$ such that $\varphi(\tilde{x}) = \bar{r} \neq \bar{0}$ for some $r \in$ C. On the other hand, $\varphi(\tilde{x}) = \varphi(\tilde{x}\bar{e}) = \varphi(\tilde{x})\varphi(\bar{e})$, $\forall \tilde{x} \tilde{\in} \check{X}$. Then $\varphi(\bar{e}) = \bar{1}$. □

**Proposition 3.7.** *Let $\varphi$ be a soft multiplicative linear functional on $\check{X}$ and $\bar{e}$ be the unit element of $\check{X}$. Then $\varphi(\bar{e})(\lambda) \subseteq \{0, 1\}$, $\forall \lambda \in A$.*

**Proof.** Let $\varphi$ be a soft multiplicative linear functional on $\check{X}$. Thus, there is a soft element $\tilde{y} \neq \Theta$ in $\check{X}$ such that $\varphi(\tilde{y}) \neq \bar{0}$; i.e., there exist, a

$\lambda_1 \in A$ such that $[\varphi(\tilde{y})](\lambda_1) \neq 0$. Also $\varphi(\bar{e}) = \varphi(\bar{e}\bar{e}) = \varphi(\bar{e})\varphi(\bar{e})$. Hence, $[\varphi(\bar{e})](\lambda) = [\varphi(\bar{e})](\lambda)[\varphi(\bar{e})](\lambda)$, $\forall\lambda \in A$. Therefore, for each $\lambda \in A$, $[\varphi(\bar{e})](\lambda) = 1$ or $[\varphi(\bar{e})](\lambda) = 0$. □

**Corollary 3.8.** *Let $\varphi$ be a fixed soft multiplicative linear functional on $\check{X}$ with unit element $\bar{e}$ and let $\tilde{x}$ be an invertible element of $\check{X}$. Then $\varphi(\tilde{x}) \neq \bar{0}$ and we have $[\varphi(\tilde{x})]^{-1} = \varphi(\tilde{x}^{-1})$.*

**Proof.** Let $\varphi$ be a fixed soft multiplicative linear functional on $\check{X}$ and $\tilde{x}$ be an invertible element of $\check{X}$. We have $[\varphi(\tilde{x})](\lambda)[\varphi(\tilde{x}^{-1})](\lambda) = [\varphi(\tilde{x})\varphi(\tilde{x}^{-1})](\lambda) = [\varphi(\tilde{x}\tilde{x}^{-1})](\lambda) = [\varphi(\bar{e})](\lambda) = \bar{1}(\lambda) = 1$, $\forall\lambda \in A$. Then $\varphi(\tilde{x}) \neq \bar{0}$, and $[\varphi(\tilde{x})]^{-1} = \varphi(\tilde{x}^{-1})$. □

**Theorem 3.9.** *Let $\varphi$ be a fixed soft multiplicative linear functional on a commutative soft Banach algebra $\check{X}$ with unit element $\bar{e}$. Then $\|\varphi(\tilde{x})\| \leq \|\tilde{x}\|$, for all $\tilde{x} \tilde{\in} \check{X}$ and $\|\varphi\| = \bar{1}$.*

**Proof.** Let $\tilde{x}\tilde{\in} \check{X}$. If $\varphi(\tilde{x}) = \bar{0}$, then for each $\lambda \in A$ we have $\varphi(\tilde{x})(\lambda) = 0$, $\forall\lambda \in A$. Thus $\|\varphi(\tilde{x})\|(\lambda) = 0 \leq \|\tilde{x}\|(\lambda), \forall\lambda \in A$. Consequently, $\|\varphi(\tilde{x})\| \leq \|\tilde{x}\|$. Now, let $\varphi(\tilde{x}) = \bar{r} \neq \bar{0}$. We prove that $\|\varphi(\tilde{x})\| \leq \|\tilde{x}\|$. Suppose that $\|\varphi(\tilde{x})\| \tilde{>} \|\tilde{x}\|$, for some $\tilde{x} \tilde{\in} \check{X}$; i.e., $\|\varphi(\tilde{x})\|(\lambda) > \|\tilde{x}\|(\lambda)$ for all $\lambda \in A$. We set $\tilde{y}(\lambda) = \frac{\tilde{x}(\lambda)}{\varphi(\tilde{x})(\lambda)}$. Hence, $\|\tilde{y}\|(\lambda) = \frac{\|\tilde{x}\|(\lambda)}{\|\varphi(\tilde{x})\|(\lambda)} < 1 = \bar{1}(\lambda)$, for all $\lambda \in A$. Then $\|\tilde{y}\| \tilde{<} \bar{1}$. So, by Proposition 2.14, $\bar{e} - \tilde{y}$ is invertible. But for each $\lambda \in A$ we have $[\varphi(\bar{e} - \tilde{y})](\lambda) = [\varphi(\bar{e})](\lambda) - [\frac{\varphi(\tilde{x})}{\varphi(\tilde{x})}](\lambda) = \bar{1}(\lambda) - \bar{1}(\lambda) = 1 - 1 = 0$, so $\varphi(\bar{e} - \tilde{y}) = \bar{0}$. This is a contradiction. Consequently, $\|\varphi(\tilde{x})\| \leq \|\tilde{x}\|$, for all $\tilde{x} \tilde{\in} \check{X}$.
Now, from the last inequality and Theorem 2.20, we have $\|\varphi\| \leq \bar{1}$. In the other hand $\varphi(\bar{e}) = \bar{1}$, so by Definition 2.19, for each $\lambda \in A$, $\|\varphi\|(\lambda)=\inf\{t \in \mathrm{R} : \|\varphi(\tilde{x})\|(\lambda) \leq t\|\tilde{x}\|(\lambda)$ for all $\tilde{x}\tilde{\in}\check{X}\} \geq \inf\{t \in \mathrm{R} : \|\varphi(\bar{e})\|(\lambda) \leq t\|\bar{e}\|(\lambda)\}= \inf\{t \in \mathrm{R} : 1 \leq t\} = 1 = \bar{1}(\lambda)$. Thus, $\|\varphi\| \geq \bar{1}$. Therefore, we deduce that $\|\varphi\| = \bar{1}$. □

**Corollary 3.10.** *Let $\varphi$ be a fixed soft multiplicative linear functional on a commutative soft Banach algebra $\check{X}$, then $\varphi$ is continuous and bounded.*

**Proof.** It follows from Theorems 2.10 and 3.9. □

**Definition 3.11.** *Let $\tilde{x}$ be an element of a soft Banach algebra $(G, A)$ with unit element $\bar{e}$. Then we define the soft spectrum of $\tilde{x}$ by $Sp(\tilde{x}) = \{\tilde{\mu} \tilde{\in} \mathcal{C}(A) : (\tilde{\mu}\bar{e} - \tilde{x}) \tilde{\in}\ Sing(G, A)\}$ where $Sing(G, A)$ denotes the non invertible elements of $(G, A)$. We define the fixed soft spectrum of $\tilde{x}$ by $FSp(\tilde{x}) = \{\bar{\mu} \tilde{\in} \mathcal{C}(A) : (\bar{\mu}\bar{e} - \tilde{x}) \tilde{\in}\ Sing(G, A)\}$. Also, the fixed soft spectral radius of $\tilde{x}$ is denoted by $r(\tilde{x})$ and is defined by $r(\tilde{x}) = sup\{|\bar{\mu}| : \bar{\mu} \tilde{\in}\ FSp(\tilde{x})\}$. It is clearly that $FSp(\tilde{x}) \tilde{\subseteq}\ Sp(\tilde{x})$.*

**Example 3.12.** Let $\check{X}$ be the soft Banach algebra of $M_2(\mathcal{C}(A))$ and Let

$$\tilde{x} = \begin{bmatrix} \bar{0} & \tilde{r} \\ \tilde{r} & \bar{0} \end{bmatrix}.$$

We have $\tilde{x} = \bar{0}\bar{e} + \tilde{r}\tilde{y}$ where

$$\bar{e} = \begin{bmatrix} \bar{1} & \bar{0} \\ \bar{0} & \bar{1} \end{bmatrix} \text{and } \tilde{y} = \begin{bmatrix} \bar{0} & \bar{1} \\ \bar{1} & \bar{0} \end{bmatrix}.$$

We want to compute the spectrum of $\tilde{x}$. For $\tilde{\mu} \tilde{\in} \mathcal{C}(A)$ we have

$$\tilde{\mu}\bar{e} - \tilde{x} = \begin{bmatrix} \tilde{\mu} & -\tilde{r} \\ -\tilde{r} & \tilde{\mu} \end{bmatrix}.$$

Thus, $\tilde{\mu}\bar{e} - \tilde{x}$ is invertible in $M_2(\mathcal{C}(A))$ *if and only if* $\tilde{\mu}^2(\lambda) \neq \tilde{r}^2(\lambda), \forall \lambda \in A$. Hence, $\text{Sp}(\tilde{x}) = \{\tilde{\mu} \tilde{\in} \mathcal{C}(A) : \tilde{\mu}^2(\lambda) = \tilde{r}^2(\lambda)$, for some $\lambda \in A\}$. Then, $-\tilde{r}, \tilde{r} \tilde{\in} \text{Sp}(\tilde{x})$ and $\text{FSp}(\tilde{x}) = \{\bar{\mu} \tilde{\in} \mathcal{C}(A) : \mu = \pm\tilde{r}(\lambda)$, for some $\lambda \in A\}$.

**Example 3.13.** Let $\check{X}$ be the Banach algebra of $M_2(\mathcal{C}(A))$ and Let

$$\tilde{x} = \begin{bmatrix} \bar{1} & \bar{2} \\ \bar{8} & \bar{1} \end{bmatrix}.$$

For $\tilde{\mu} \tilde{\in} \mathcal{C}(A)$ we have

$$\tilde{\mu}\bar{e} - \tilde{x} = \begin{bmatrix} \tilde{\mu} - \bar{1} & -\bar{2} \\ -\bar{8} & \tilde{\mu} - \bar{1} \end{bmatrix}.$$

Thus, $\tilde{\mu}\bar{e} - \tilde{x}$ is invertible in $M_2(\mathcal{C}(A))$ *if and only if* $(\tilde{\mu} - \bar{1})^2(\lambda) \neq 16$, $\forall \lambda \in A$. Hence, $\text{Sp}(\tilde{x}) = \{\tilde{\mu} \tilde{\in} \mathcal{C}(A) : (\tilde{\mu} - \bar{1})(\lambda) = \pm 4$, for some $\lambda \in A\} = \{\tilde{\mu} \tilde{\in} \mathcal{C}(A) : \tilde{\mu}(\lambda) = -3$ or $5$, for some $\lambda \in A\}$. Then $-\bar{3}, \bar{5} \tilde{\in} \text{Sp}(\tilde{x})$ and $\text{FSp}(\tilde{x}) = \{-\bar{3}, \bar{5}\}$.

**Example 3.14.** Let $\check{X}$ be the Banach algebra $M_2(\mathcal{C}(A))$ and Let

$$\tilde{x} = \begin{bmatrix} \tilde{r} & \tilde{s} \\ \bar{0} & \tilde{r} \end{bmatrix}.$$

We have $\tilde{x} = \tilde{r}\bar{e} + \tilde{s}\tilde{y}$ where

$$\bar{e} = \begin{bmatrix} \bar{1} & \bar{0} \\ \bar{0} & \bar{1} \end{bmatrix} \text{ and } \tilde{y} = \begin{bmatrix} \bar{0} & \bar{1} \\ \bar{0} & \bar{0} \end{bmatrix}.$$

For $\tilde{\mu} \tilde{\in} \mathcal{C}(A)$ we have

$$\tilde{\mu}\bar{e} - \tilde{x} = \begin{bmatrix} \tilde{\mu} - \tilde{r} & -\tilde{s} \\ \bar{0} & \tilde{\mu} - \tilde{r} \end{bmatrix}.$$

Thus, $\tilde{\mu}\bar{e}-\tilde{x}$ is invertible in $M_2(\mathcal{C}(A))$ *if and only if* $\tilde{\mu}(\lambda) \neq \tilde{r}(\lambda), \forall\lambda \in A$. Hence, $\mathrm{Sp}(\tilde{x}) = \{\tilde{\mu} \in \mathcal{C}(A) : \tilde{\mu}(\lambda) = \tilde{r}(\lambda)$, for some $\lambda \in A\}$. Then, $\tilde{r} \tilde{\in} \mathrm{Sp}(\tilde{x})$ and $\mathrm{FSp}(\tilde{x}) = \{\bar{\mu} \tilde{\in} \mathcal{C}(A) : \mu = \tilde{r}(\lambda)$, for some $\lambda \in A\}$.

**Remark 3.15.** *Let $(G, A)$ be a soft Banach algebra of $X$ over* C *and let $\bar{e}$ be the identity element of $(G, A)$. By definition, $G(\lambda)$ is a sub Banach algebra of $X$, $\forall\lambda \in A$ and $\tilde{x}\bar{e} = \bar{e}\tilde{x} = \tilde{x}$, $\forall\tilde{x} \in (G, A)$. Hence, $(\tilde{x}\bar{e})(\lambda) = (\bar{e}\tilde{x})(\lambda) = \tilde{x}(\lambda)$, $\forall\tilde{x} \in (G, A)$, $\forall\lambda \in A$; i.e., $\tilde{x}(\lambda)\bar{e}(\lambda) = \bar{e}(\lambda)\tilde{x}(\lambda) = \tilde{x}(\lambda)$, $\forall\lambda \in A$. Hence, for all $\lambda \in A$, $\bar{e}(\lambda)$ is the identity of $G(\lambda)$. Thus, $\bar{e}(\lambda)$,$\forall\lambda \in A$ is the identity of $X$; i.e., $\bar{e}(\lambda) = e$, $\forall\lambda \in A$ which $e$ is the identity of $X$.*

**Proposition 3.16.** *Let $\tilde{x}$ be a soft element of a soft Banach algebra $(G, A)$. Then Sp$(\tilde{x})$ is a nonempty soft complex set.*

**Proof.** Let $\tilde{x}$ be a soft element of a soft Banach algebra $(G, A)$. Then, for each $\lambda$ in $A$, $z = \tilde{x}(\lambda)$ is an element of $(G(\lambda), \|.\|_\lambda)$. By Proposition 2.12, $G(\lambda)$ is a Banach algebra over C, $\forall\lambda \in A$ and we know that the spectrum of an element of Banach algebra over field C, is nonempty. Hence, $\mathrm{Sp}(z)$ is a nonempty subset of C. Thus, there is a complex number $\mu$ such that $\mu e - z$ is not invertible. But $\mu e - z = \bar{\mu}(\lambda)\bar{e}(\lambda) - \tilde{x}(\lambda) = (\bar{\mu}\bar{e} - \tilde{x})(\lambda)$, hence $\bar{\mu}\bar{e} - \tilde{x}$ is not invertible in $(G, A)$. Therefore, $\bar{\mu}\tilde{\in}$ $\mathrm{FSp}(\tilde{x})$; i.e., $\mathrm{FSp}(\tilde{x})$ is a nonempty soft complex set. Consequently, $\mathrm{Sp}(\tilde{x})$ is a nonempty soft complex set. □

**Proposition 3.17.** *Let $\tilde{x}$ be a soft element of soft Banach algebra $(G, A)$. Then $FSp(\tilde{x})$ is soft bounded and soft closed subset of $\mathcal{C}(A)$.*

**Proof.** Let $\bar{\mu} \in \mathrm{FSp}(\tilde{x})$, then by Corollary 2.15, $|\bar{\mu}| \leq \|\tilde{x}\|$. Hence, $\mathrm{FSp}(\tilde{x})$ is soft bounded subset of $\mathcal{C}(A)$. Now, we prove that $\rho(\tilde{x}) = \mathcal{C}(A)$ - $\mathrm{FSp}(\tilde{x})$ is soft open. Suppose that $\bar{\mu}\tilde{\in}\rho(\tilde{x})$. Then $(\bar{\mu}\bar{e} - \tilde{x})\tilde{\in}$ $\mathrm{Inv}(G, A)$. By Proposition 2.16, $\mathrm{Inv}(G, A)$ is soft open. So there exists $\tilde{\varepsilon}\tilde{>}\bar{0}$ such that $B(\bar{\mu}\bar{e}-\tilde{x}, \bar{\varepsilon})\tilde{\subseteq}$ $\mathrm{Inv}(G, A)$. Therefore, if $\|\tilde{y}\|\tilde{<}\bar{\varepsilon}$ then we have $(\bar{\mu}\bar{e} - \tilde{x} + \tilde{y}) \tilde{\in} B(\bar{\mu}\bar{e} - \tilde{x}, \bar{\varepsilon}) \tilde{\subseteq} \mathrm{Inv}(G, A)$. In particular, when $\bar{0}\tilde{<}\bar{\delta}\tilde{<}\bar{\varepsilon}$, $\bar{\mu}\bar{e} - \tilde{x} + \bar{\delta}\bar{e} = (\bar{\mu} + \bar{\delta})\bar{e} - \tilde{x}$ is invertible. Hence, we have $(\bar{\mu} + \bar{\delta})\tilde{\in}\rho(\tilde{x})$; i.e., $B(\bar{\mu}, \bar{\varepsilon})\tilde{\subseteq}\rho(\tilde{x})$. So $\rho(\tilde{x})$ is soft open. Therefore, $\mathrm{FSp}(\tilde{x})$ is soft closed subset of $\mathcal{C}(A)$. □

**Proposition 3.18.** *Let $(Y, A)$ be a soft bounded and soft closed subset of $\mathcal{C}(A)$. Then $(Y, A)$ is soft totally bounded.*

**Proof.** Let $(Y, A)$ be a soft bounded and soft closed subset of $\mathcal{C}(A)$. Let $\tilde{r}_n$ be a sequence of soft elements in $(Y, A)$. Then, for any $\lambda$ in $A$ the sequence $\tilde{r}_n(\lambda)$ is a bounded sequence in $(\mathrm{C}, \|.\|_\lambda)$. So, for all $\lambda \in A$ there exists a subsequence $\tilde{r}_{n_k}(\lambda)$ of $\tilde{r}_n(\lambda)$ such that is soft convergent to $\tilde{r}(\lambda)$. Hence, by Proposition 2.2, the subsequence $\tilde{r}_{n_k}$ of $\tilde{r}_n$ is soft convergent to $\tilde{r}$. Since $(Y, A)$ is soft closed subset of $\mathcal{C}(A)$ then, $\tilde{r} \in (Y, A)$. So, by Proposition 2.5, $(Y, A)$ is a soft totally bounded subset of $\mathcal{C}(A)$. □

**Theorem 3.19.** *Let $\tilde{x}$ be a soft element of a soft Banach algebra $(G, A)$. Then $FSp(\tilde{x})$ is a nonempty soft compact set.*

**Proof.** Let $\tilde{x}$ be a soft element of a soft Banach algebra $(G, A)$. By Proposition 3.16, $\mathrm{FSp}(\tilde{x})$ is nonempty soft complex set. By Proposition 3.17, $\mathrm{FSp}(\tilde{x})$ is soft bounded and soft closed subset of $\mathcal{C}(A)$. Then, from Proposition 3.18, $\mathrm{FSp}(\tilde{x})$ is soft totally bounded subset of $\mathcal{C}(A)$. On the other hand since every soft closed subset of soft complete space is soft complete, by Corollary 2.4, $\mathrm{FSp}(\tilde{x})$ is a soft complete subset of $\mathcal{C}(A)$. Hence, by Theorem 2.5 and 2.6, $\mathrm{FSp}(\tilde{x})$ is a soft compact subset of $\mathcal{C}(A)$. □

**Theorem 3.20.** *Let $\check{X}$ be a commutative soft Banach algebra and let $\tilde{x}\tilde{\in}\check{X}$. Then $FSp(\tilde{x}) = \{\varphi(\tilde{x}) : \varphi$ is a fixed soft multiplicative linear functional on $\check{X}\}$*

**Proof.** let $\varphi$ is a fixed soft multiplicative linear functional on $\check{X}$. Then $\varphi(\tilde{x})\bar{e} - \tilde{x}$ is not invertible. Because $[\varphi(\tilde{x})\bar{e} - \tilde{x}]\tilde{y} = \bar{e}$ implies that $[\varphi(\tilde{x})\varphi(\bar{e}) - \varphi(\tilde{x})]\varphi(\tilde{y}) = [\varphi(\tilde{x}) - \varphi(\tilde{x})]\varphi(\tilde{y}) = \bar{0} = \varphi(\bar{e}) = \bar{1}$, that is a contradiction. Therefore, $\varphi(\tilde{x})\tilde{\in}$ FSp$(\tilde{x})$.
Conversely, if $\bar{\mu}\tilde{\in}$ FSp$(\tilde{x})$ then $\bar{\mu}\bar{e} - \tilde{x}$ is not invertible in $\check{X}$. So, there exists a $\lambda_1 \in A$ such that $[\bar{\mu}\bar{e} - \tilde{x}](\lambda_1) = \bar{\mu}(\lambda_1)\bar{e}(\lambda_1) - \tilde{x}(\lambda_1) = \mu e - \tilde{x}(\lambda_1)$ is not invertible in $(X, \|.\|_{\lambda_1})$. Thus, $\mu \in$ Sp$(z)$ when $z = \tilde{x}(\lambda_1)$. So there exists a multiplicative linear functional $f$ on $X$ over C such that $\mu = f(z)$. By the same argument according to Proposition 3.5, we define $\varphi : SE(\check{G}) \longrightarrow \mathcal{C}(A)$ by $\varphi(\tilde{y}) = \bar{\mu}_{\tilde{y}}$, when $\mu_{\tilde{y}} = f(\tilde{y}(\lambda_1))$; i.e., $[\varphi(\tilde{y})](\lambda) = f(\tilde{y}(\lambda_1))$, $\forall \lambda \in A$. Then $\varphi$ is a fixed soft multiplicative linear functional on $\check{X}$ and $[\varphi(\tilde{x})](\lambda) = f(\tilde{x}(\lambda_1)) = f(z) = \mu$, $\forall \lambda \in A$. Therefore, $\varphi(\tilde{x}) = \bar{\mu}$, and so theorem is proved. □

**Proposition 3.21.** *Let $\tilde{x}$ be an element of a soft Banach algebra $\check{X}$ and $\tilde{\mu} \tilde{\in} Sp(\tilde{x})$. Then there exist $\bar{r} \tilde{\in} FSp(\tilde{x})$ and $\lambda$ in $A$ such that $\tilde{\mu}(\lambda) = \bar{r}(\lambda) = r$.*

**Proof.** Let $\tilde{x}$ be an element of a soft Banach algebra $\check{X}$ and $\tilde{\mu} \tilde{\in}$ Sp$(\tilde{x})$. So $\tilde{\mu}\bar{e} - \tilde{x}$ is not invertible element of $\check{X}$. Thus, there exists $\lambda \in A$ such that $\tilde{\mu}(\lambda)\bar{e}(\lambda) - \tilde{x}(\lambda)$ is not invertible in $X$. We set $r = \tilde{\mu}(\lambda)$, thus $\bar{r}(\lambda)\bar{e}(\lambda) - \tilde{x}(\lambda) = r\bar{e}(\lambda) - \tilde{x}(\lambda) = \tilde{\mu}(\lambda)\bar{e}(\lambda) - \tilde{x}(\lambda)$ is not invertible. Consequently, $\bar{r}\bar{e} - \tilde{x}$ is not invertible element of $\check{X}$ and so $\bar{r} \tilde{\in}$ FSp$(\tilde{x})$. The converse of Proposition 3.21 is very obvious. □

**Proposition 3.22.** *Let $M = \{\tilde{x}\tilde{\in}\check{X} : FSp(\tilde{x}) = \{\bar{0}\}\}$. Then $\tilde{x}\tilde{y} \tilde{\in} M$, $\forall \tilde{x}, \tilde{y} \tilde{\in} M$* if and only if *$\tilde{x} + \tilde{y} \tilde{\in} M$, $\forall \tilde{x}, \tilde{y} \tilde{\in} M$.*

**Proof.** Let for all $\tilde{x}, \tilde{y}\tilde{\in}M$, $\tilde{x}\tilde{y}\tilde{\in}M$. Suppose that $\tilde{x}$ and $\tilde{y}$ are arbitrary members of $M$ and $\bar{\mu} \neq \bar{0}$. We have to show that $\bar{\mu} \tilde{\notin}$ FSp$(\tilde{x} + \tilde{y})$. By hypothesis $\tilde{x}$ and $\tilde{y}$ are not invertible and for any $\bar{r} \neq \bar{0}$ in $\mathcal{C}(A)$, $\bar{r}\bar{e} - \tilde{x}$ and $\bar{r}\bar{e} - \tilde{y}$ are invertible. We set $\tilde{u} = \frac{\tilde{x}}{\bar{\mu}}$, $\tilde{v} = \frac{\tilde{y}}{\bar{\mu}}$. Then $\bar{\mu} \tilde{\in}$ FSp$(\tilde{x} + \tilde{y})$ *if and only if* $\bar{1} \tilde{\in}$ FSp$(\tilde{u} + \tilde{v})$. So, it is enough to show that $\bar{1}\tilde{\notin}$ FSp$(\tilde{u} + \tilde{v})$. We have

$$\begin{aligned} \tilde{1}\bar{e} - (\tilde{u} + \tilde{v}) &= (\bar{1}\bar{e} - \tilde{u})(\bar{1}\bar{e} - \tilde{v}) - \tilde{u}\tilde{v} \\ &= [(\bar{1}\bar{e} - \tilde{u}) - \tilde{u}\tilde{v}(\bar{1}\bar{e} - \tilde{v})^{-1}](\bar{1}\bar{e} - \tilde{v}) \\ &= (\bar{1}\bar{e} - \tilde{u})[\bar{1}\bar{e} - (\bar{1}\bar{e} - \tilde{u})^{-1}\tilde{u}\tilde{v}(\bar{1}\bar{e} - \tilde{v})^{-1}](\bar{1}\bar{e} - \tilde{v}) \end{aligned}$$

In addition, $\mathrm{FSp}(\bar{1}\bar{e}-\tilde{u}) = \bar{1}$- $\mathrm{FSp}(\tilde{u}) = \bar{1} - \{\bar{0}\} = \{\bar{1}\}$ and because $\bar{1}\bar{e}-\tilde{u}$ is invertible, hence we have $\mathrm{FSp}((\bar{1}\bar{e}-\tilde{u})^{-1}) = \{\bar{1}^{-1}\} = \{\bar{1}\}$. Also, $(\bar{1}\bar{e}-\tilde{u})^{-1}\tilde{u} = (\bar{1}\bar{e}-\tilde{u})^{-1}-\bar{1}\bar{e}$. Thus $\mathrm{FSp}((\bar{1}\bar{e}-\tilde{u})^{-1}\tilde{u}) = \mathrm{FSp}((\bar{1}\bar{e}-\tilde{u})^{-1}-\bar{1}\bar{e}) = \mathrm{FSp}((\bar{1}\bar{e}-\tilde{u})^{-1}) - \bar{1} = \{\bar{1}\} - \bar{1} = \{\bar{0}\}$. Hence, $\tilde{p} = (\bar{1}-\tilde{u})^{-1}\tilde{u}$ and $\tilde{q} = \tilde{v}(\bar{1}-\tilde{v})^{-1}$ are in $M$. Therefore, by hypothesis $\tilde{p}\tilde{q} \tilde{\in} M$. Consequently $\bar{1}\bar{e}-(\tilde{u}+\tilde{v})$ is a product of soft invertible elements. So, $\bar{1} \notin \mathrm{FSp}(\tilde{u}+\tilde{v})$. The converse of this statement similarly can be proved. □

**Conclusion:** In this paper we introduced the concept of soft multiplicative linear functionals in soft Banach algebras and deduced some of its properties. Then, we presented the concept of soft spectrum of an element in soft Banach algebras. We investigated some of their properties, and stated some new results about the concept of soft multiplicative linear functionals and soft spectrum conceptions.

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