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Multi-Period DEA-R Efficiency for Decision Making Units Using Network Structure

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Abstract. A P- stage network is studied in different time periods here which contains ratio data . Thus a method is given to evaluate the technical efficiency in each time period and overall efficiency after several desired time periods. Also, the efficiency of each stage of such a structure is evaluated in each time period and after several time periods. It has been shown that overall efficiency scores and the efficiency of each

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process obtained after several desired time periods are not less than to overall efficiency scores and the efficiency of each process in each time period. In addition, a unit becomes efficient after several periods of time if it is efficient in at least one period of time.

AMS Subject Classification: MSC code1; MSC code 2; more
Keywords and Phrases: data envelopment analysis (DEA), ratio data envelopment analysis (DEA-R), network, multi-period, efficiency

1 Introduction

DEA is one of the proper and efficient tools in evaluation of the decision-making units and it is non-parametric method. The primary method in DEA was proposed by Charnes, Cooper and Rhodes in 1978. They added mathematical programming to the Farrell non-parametric perspective that was proposed in 1957 to evaluate the efficiency of decision-making units with two inputs and one output called CCR model capable to measure efficiency with some inputs and outputs (Safari& Azar, 2004). In 1984, Banker, Charnes and Cooper proposed a new model with (BCC) by making shifts in the CCR model (Jahanshahlu et al., 2009). Network data envelopment analysis (NDEA) is a developed model of the data analysis envelopment that tries to consider the internal structures of the decision- making units. Fare and Grosskopf (1996) studied this issue for the first time and then many researchers studied network data envelopment analysis which has been used in various applications. Simple models only address the local optimization in a certain period in a specific and independent time period. Thus, the network optimization model is not suitable for the performance evaluation of complex supply networks with multiple levels. This model disregards the individual or joint relationships in the internal structures of the system and cannot assess the efficiency and performance in several successive and interdependent stages. To overcome this problem considering the efficiency in a long time, the researchers use the dynamic DEA with the transfer operation (Najari Alamuti et al.,2021). This model can measure the efficiency of a particular period based on long-term optimization (Tajik Yabr et al., 2022).
The functions of active organizations are like an interdependent chain. Thus, evaluating their performance during multiple periods is necessary

and provides better information for the managers. Thus, Nemoto and Goto(1999; 2003) introduced dynamic DEA (DDEA) models. These models consider the relationship of each unit with itself in successive periods and deliver the efficiency of each period, as well as the whole efficiency. However, these models consider the structure of units in each period a black box and disregard the internal structure. As we have discussed, researchers extended the network models to evaluate the efficiency of units in different processes to solve the black box problem of classical models. However, the network models are static, and the proposed DDEA models mainly consider DMU in any period in a single stage. Thus, a model is needed to consider the internal structure unit and time simultaneously, providing richer information on units' performances. The NDEA and DDEA models have been reviewed comprehensively by Kao (2014a) who proposed three suggestions for future studies(Kao, 2014b) one of which is to extend the dynamic models into network structures, i.e., designing DNDEA models. Besides Kao (2014a), other researchers emphasized the extension of these models(Fukuyama & Weber, 2010; Tone & Tsutsui, 2010, 2014).The efficiency of a set of units in a time period is evaluated by standard DEA just in terms of inputs and outputs variables. However, it is obvious that the efficiency in a time period t not only depends on the inputs of this period but also on inputs of one or some previous periods. This group of models is called multi-period DEA (Jablonsky et al., 2018). Gazari Neishaburi and his colleagues (2019) proposed a dynamic data envelopment analysis model that measures the process efficiency of a business actually. To determine the efficiency of the sub-processes, Chen et al (2010) used a DEA model with a network structure. Kao and Hwang (2013) proposed a model to measure the network efficiency of their model, the weight of the inputs and outputs are obtained so that the network efficiency is maximized under the condition that the efficiency of stages should not be more than one. However, Kao and Hwang (2013) model is incapable of determining the efficiency boundary and a pattern for inefficient units. Seiford and Zhu (1999) and Luo (2003) used a two-stage structure for the evaluation of banks. Cook et al (2010) studied the general issue of multi-stage networks. Tone et al (2014) proposed a network slacks-based measure (NSBM) for continuous network structures. Using this model, they could measure the process efficiency with overall efficiency. Tone et al (2014) introduced a new structure of multi-period data envelopment analysis named " multi-period data envelopment analysis with network structure ".Omrani and Soltanzadeh (2016) proposed a relational dynamic NDEA (DNDEA) model which measures the efficiencies of the system and its internal processes over time, simultaneously for evaluating the performance of a DMU with interrelated processes during specified multiple periods. Esmaeilzadeh and Kazemi Matin(2018) extended multi-period DEA models by considering more complex internal relations for the sub-processes of each decision-making unit, DMU. They presented novel multi-period network DEA models that were developed for performance evaluation of overall and specific time period efficiencies with parallel and series internal structures in the sub-processes for each time period. Esfidani et al. (2020) used a non-radial DEA model called the network slacks-based measure (NSBM) model to measure the efficiency of a system with a multi-period two-stage structure. Then they described the properties of the proposed model in detail. Moreover, they decomposed the overall efficiency of the system over a number of time periods as a weighted average of the efficiency in each period. Hosseinzadeh Lotfi et al. (2020) used R codes to solve DEA models with crisp and fuzzy data. R is a mathematical and subject-oriented programming language designed primarily for statistical calculations and data mining. The R programming language covers a wide range of linear and nonlinear programming, integer and quadratic models as well as statistical tests and time series analysis, with a high graphical capability. Moghaddas et al. (2020) evaluated revenue efficiency according to the piece linear theory in non-competitive situations and thus, they introduced a step-by-step pricing function that allows prices to change relative to the output. Then they proposed a novel and more accurate mathematical model for revenue efficiency. Therefore, they defined a dynamic weight function in the maximum revenue optimization model that no longer takes into account the fixed prices. Moghaddas et al. (2021) proposed an assessment method based on the network data proposed envelopment analysis to provide an efficient strategy for each step of a sustainable supply chain network. Their approach offers a robust design with decision-making units to avoid imposing additional costs on supply chains due to non-compliance with environmental and social issues. For doing so, they considered the inputs and outputs related to the concept of sustainability in the DEA network to select the most efficient strategy for sustainable supply chain design. By the proposed method decision-making units can select the appropriate strategy for each stage of the sustainable supply chain network maximizing the efficiency of the entire network. Moghaddas et al. (2022) developed combined scale returns of DEA models with- integer input and output data. They corrected the previous topic principles to introduce a minimal set of technical extrapolations and also formulated a pair of correct and incorrect linear programming models to assess the efficiency. In evaluating a decision unit, it is essential to take the view of the decision-maker into account on input and output weights to obtain correct results. However, the use of weight constraints in the DEA leads to some problems one of which is the use of the non-Archimedean constant for input and output weights in DEA models, which always leads to all non-zero weights, while some inputs may not play a role in producing output leading to inaccurate efficiency. Therefore,a model is introduced to work with the ratio data, input :output, instead of inputs and outputs separately, so that the input with no role in producing out will be omitted automatically. Consequently, Despic and Paradi (2007) introduced the ratio data envelopment analysis (DEA-R) models to evaluate the required efficiencies .Most of the weight constrains of DEA model can be changed into DEA-R model leading to an equivalent DEA-R model . The efficiency in DEA and DEA-R with weight constraints have been studied by Nazari et al (2014) studied. Wei et al. (2011)discussed the efficiency estimation in CCR models. Due to the assumption of limiting irrational and unnecessary weight, they used DEA-R models instead of traditional DEA models. Wei et al. (2011) also compared the optimal weights in DEA and DEA-R and they presented input-based DEA-R models and indicated that the efficiency calculated by their proposed model is greater than or equal to the efficiency obtained by the CCR model. To evaluate network efficiency , Gerami et al (2012) presented a DEA-R model for evaluating network efficiency and they showed that the efficiencies of each stage of the network and the overall efficiency of the network resulting from their model are greater than or equal to the corresponding values obtained from previous models. Also, in their proposed model, overall efficiency is obtained as the weighted average of the efficiencies of each stage. Mozaffari et al. (2017) have proposed a model for calculating the overall amount of efficiency in a two-stage network in DEA-R using a linear multi-objective programming structure. Having access to ratio data on 10 bank branches, Mozaffari et al. (2020) calculated the efficient hyperplanes for these branches using the method proposed in their paper. Overall, their evaluations revealed that the Royal Bank of Scotland (RBS) was not located on any hyperplanes. Ostovan et al.(2020) presented a number of models for calculating the average efficiency of two-stage networks using DEA and DEA-R with fuzzy data. Akbarian(2021) proposed two novel models namely, range directional DEA-R (RDD-R) and (weighted) Tchebychef norm DEA-R (TND-R) to calculate individual ratio efficiencies and overall ratio efficiency of two-stage DMUs. Kamyab et al.(2021) proposed a two-stage network incentives system for commercial banks. They used their proposed DEA-R-based CRA models to evaluate commercial banks in a two-stage case when the only ratios available are the assets-to-costs and income-to-assets vectors. Wanke et al.(2022) presented two-stage network models in the presence of stochastic ratio data. They obtained the relation between the efficiency scores obtained from the stochastic two -stage network DEA-ratio considering three different strategies. A multi-period production system is developed here which is based on the DEA-R approach for measuring the efficiency of a set of DMUs over a period of time. Thus ,a two-phase procedure is considered: The first phase, the efficiency of whole system is evaluated by using DEA-R model . Noting that some inputs may not play any role in producing some outputs due to its zero weight .Therefore, a model is developed based on the weight concepts in a multi-periodic system to overcome this problem; ,then after , the periodic efficiencies is evaluated for each separate time span in the second phase . Considering a subsystem corresponding to each time period, a network system with T subsystems is obtained .These subsystems are connected parallely where each one consists of P processes connected in series. Using the proposed multi-period DEA-R model and also the mathematical relationships in overall efficiencies of the overall system, subsystems and sub- processes, overall efficiency and the multi-period sub-processes efficiency of the system network are measured after several time periods and in any time period. This paper is organized as follows: an overview of the necessary concepts is introduced in section 2 . Section 3 involves the proposed model for determining the efficiency of a multi-period multistage network using the ratio data envelopment analysis (DEA-R) model. In section 4 ,an example is solved numerically by using the proposed method and then the results are analyzed . Conclusion is given in section 5.

2 Preliminaries

2.1 Network production systems

P-stage networks are evaluated via DEA models and internal relationships and intermediate products are considered. Networks are classified into various structures such as two-stage, series, parallel and a combination of series and parallel. In this section, DEA-R model for two-stage network structure has been proposed by Mozaffari et al. (2017) and then DEA-R model for multi-stage network structure in series has been introduced by Gerami and Mozaffari (2012).

2.1.1 Two-stage network DEA-R models

The evaluation of DMUs that have ratio data such as $\frac{Z_j}{X_j}$ and $\frac{Y_j}{Z_j}$ requires models that firstly have possibility of production and secondly, they are capable of calculation of the units' efficiency value. In this section, at first, the two-stage DEA-R models and then possibility of production in each stage have been suggested by Mozaffari and et al.(2017)(Fig.1)

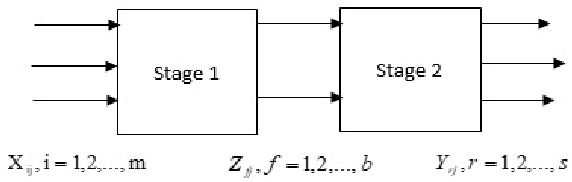

Figure 1: Two-stage network.

2.1.2 Efficiency in two-stage DEA- R

The evaluation of DMUs with ratio data such as $\frac{Z_j}{X_j}$ and $\frac{Y_j}{Z_j}$ requires models should enjoy possibility of production and also be able to calculate the value of efficiency of the units. In this section, at first, the two-stage DEA-R models and then possibility of production in each stage have been suggested by Mozaffari and et al.(2017)(Fig.1)

$$\begin{aligned} &\min E_1 = \beta_1 \\ s.t \quad &\sum_{t=1}^{m}\sum_{f=1}^{b} w_{if}\left(\frac{\frac{Z_{fi}}{X_{ij}}}{\frac{Z_{fo}}{X_{io}}}\right) \leq \beta_1, j = 1, ..., n \\ &\sum_{i=1}^{m}\sum_{f=1}^{b} w_{if} = 1 \qquad w_{if} \geq 0 i = 1, ..., m; f = 1, ..., b \end{aligned} \tag{1}$$

The evaluation of $DMU_o$by using output based $DEA-R$ envelop model is as follows :

$$\begin{aligned} &\max \alpha_1 \\ s.t \quad &\sum_{j=1}^{n} \lambda_j^1 \left(\frac{Z_j}{X_j}\right) \leq \alpha_1 \left(\frac{Z_o}{X_o}\right) \\ &\sum_{j=1}^{n} \lambda_j^1 = 1, \lambda_j^1 \geq 0, j = 1, ..., n \end{aligned} \tag{2}$$

The output based $DEA - R$ model for the second stage is as follows (Mozaffari et al., 2017):

$$\begin{aligned} & \min E_2 = \beta_2 \\ s.t \quad & \sum_{r=1}^{s}\sum_{f=1}^{b} v_{rf}\Big(\frac{\frac{Y_{rj}}{X_{fj}}}{\frac{Y_{ro}}{Z_{fo}}}\Big) \leq \beta_2, j = 1, ..., n \\ & \sum_{i=1}^{m}\sum_{f=1}^{b} v_{rf} = 1 \quad v_{rf} \geq 0 r = 1, ..., s; f = 1, ..., b \end{aligned} \tag{3}$$

To evaluate DMU_o in the second phase by using Output based envelopment model under CCR Mozaffari et al., 2017) suggested the

$$\begin{aligned} & \max \alpha_2 \\ s.t \quad & \sum_{j=1}^{n} \lambda_j^2 \Big(\frac{Y_j}{Z_j}\Big) \leq \alpha_2 \Big(\frac{Y_o}{Z_o}\Big) \\ & \sum_{j=1}^{n} \lambda_j^2 = 1, \lambda_j^2 \geq 0, j = 1, ..., n \end{aligned} \tag{4}$$

2.1.3 Two-stage network $DEA - R$ based on multi-objective linear programming (MOLP)

Mozaffari et al. (2017) suggested a two-objective linear programming model for measuring the overall efficiency of DMU_o with the ratio data defined as $\frac{Z_j}{X_j}$ and $\frac{Y_j}{Z_j}$ (CCR and BCC) as follows. Then, by combining the constraints of models (1) and (3), they introduced a two-objective linear model (5) for measuring the overall efficiency of the DEA-R two-stage network as follows :

$$
\begin{aligned}
& \min\{\gamma_1, \gamma_2\} \\
s.t \quad & \sum_{i=1}^{m}\sum_{f=1}^{b} w_{if}\Big(\frac{\frac{Z_{fj}}{X_{ij}}}{\frac{Z_{fo}}{X_{io}}}\Big) \leq \gamma_1, j = 1, ..., n \\
& \sum_{r=1}^{s}\sum_{f=1}^{b} v_{rf} = 1\Big(\frac{\frac{Y_{rj}}{Z_{fj}}}{\frac{Y_{ro}}{Z_{fo}}}\Big) \leq \gamma_2, j = 1, ..., n \\
& \sum_{r=1}^{s}\sum_{f=1}^{b} v_{rf} = 1, \ \sum_{i=1}^{m}\sum_{f=1}^{b} w_{if} = 1 \\
& v_{rf} \geq 0; w_{if} \geq 0 \quad r = 1, ..., s; f = 1, ..., b; \quad i = 1, ..., m
\end{aligned} \tag{5}
$$

An envelopment model for the evaluation of the overall efficiency of a two-stage network $DEA-R$ was proposed by Mozaffari et al. (2017) as follows:

$$
\begin{aligned}
& \max(\alpha_1 + \alpha_2) \\
& \sum_{j=1}^{n} \lambda_j^1 \Big(\frac{Z_j}{X_j}\Big) \geq \alpha_1 \Big(\frac{Z_o}{X_o}\Big) \\
& \sum_{j=1}^{n} \lambda_j^2 \Big(\frac{Y_j}{Z_j}\Big) \geq \alpha_2 \Big(\frac{Y_o}{Z_o}\Big) \\
& \sum_{j=1}^{n} \lambda_j^1 = p_1, \sum_{j=1}^{n} \lambda_j^2 = p_2 \\
& \lambda_j^1 \geq 0, \lambda_j^2 \geq 0, j = 1, ..., n
\end{aligned} \tag{6}
$$

Model (6) is a linear programming problem with the parameters p_1 and p_2 which determine the overall efficiency of the two-stage network. Variables λ_j^1 and λ_j^2 correspond to stages 1 and 2 respectively. If $\sum_{j=1}^{n} \lambda_j^2 = 0$ then only stage 1 of the network is considered. Similarly, if $\sum_{j=1}^{n} \lambda_j^1 = 0$ then only stage 2 is considered. If $\sum_{j=1}^{n} \lambda_j^2 = p_1$ and $\sum_{j=1}^{n} \lambda_j^2 = p_2$ where $p_1 + p_2 = 1$ and $p_1, p_2 > 0$ then the optimal Pareto solution (5) defines overall efficiency of DMU_o from two -stage models with ratio data.

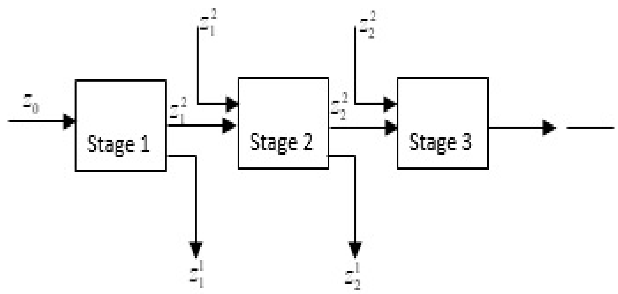

Figure 2: Network as series

2.1.4 . Measuring the network structure efficiency using DEA-R models

The efficiency of the network systems DEA-R is calculated In this section in terms of series as proposed by Gerami and Mozaffari (2012). A P - stage process is shown in Figure 2. Overall efficiency is denoted by θ^N and the efficiency of each process is denoted by θ_p^N. In the last phase P, all the outputs leave the system and we denote them by Z_{pr}^{j1}.

Table 1: Model variables

$w_{ir}^{(11)}$	Input variable ratio weight $(1,...,I_o)ith$ z_o^j to output variable of $rth(1,...,R_1)z_1^{j1}$
$w_{ik}^{(12)}$	Input variable ratio weight $(1,...,I_o)ith$ z_o^j to output variable of $kth(1,...,S_1)z_1^{j2}$
$w_{ir}^{(p1)}$	Input variable ratio weight $(1,...,I_p)ith$ z_{p-1}^{j3} to output variable of $rth(1,...,R_p)z_p^{j1}rth$
$w_{ik}^{(p2)}$	Input variable ratio weight $(1,...,S_{p-1})kth$ z_{p-1}^{j3} to output variable of $kth(1,...,S_p)z_p^{j2}rth$
$w_{kr}^{(p3)}$	Input variable ratio weight $(1,...,S_{p-1})kth$ to output variable of $rth(1,...,R_p)z_p^{j1}$
$w_{kk}^{(p4)}$	Input variable ratio weight $(1,...,S_{p-1})kth$ z_{p-1}^{j2} to output variable of $(1,...,S_p)kthz_p^{j2}$
$z_0^j = (z_{oi}^j)$	Input variable $(1,...,I_o)kth$ of the first unit stage $(1,...,n)jth$
$z_0^{j1} = (z_{1r}^{j1})$	The output vector $(1,...,R_1)rth$ for DMU_j which exits the first step and exits the system and does not enter the next step as input.
$z_{p-1}^{j3} = (z_{p-1i}^{j3})$	The output vector $(1,...,I_p)$ for DMU_j is in the stage $p(2,...,P)$ that enters the process at the beginning of this stage.
$z_p^{j1} = (z_{pi}^{j1})$	$r = 1,...,R_p$ is the output vector for DMU_j that exits the $p(2,...,P)$ stage and exits the process and does not enter as input in the stage $P+I$
$z_p^{j2} = (z_{pk}^{j2})$	$p = 2,...,P$ is the $(1,...,S_p)kth$ output vector for DMU_j that exits the pstage and enters as a part of input in the stage $P+I$
$z_{p-1}^{j2} = (z_{pk}^{j2})$	$p = 2,...,P$ is the input vector of $(1,...,S_{p-1}kth$ for DMU_j in the stage P that enters the process at the end of the stage $p-1$

Gerami and Mozaffari (2012) presented the $DEA-R$ model to calculate the efficiency of the DMU_o system with P network structure in series as follows:

$$
\begin{aligned}
&\theta^{NR} = \max \sum_{p=1}^{p} w_p \theta_p^{NR} \\
s.t. &\sum_{i=1}^{I_o} \sum_{r=1}^{R_1} W_{ir}^{(11)} \left(Z_{oi}^{j} / Z_{1r}^{j1} \Big/ Z_{0i}^{o} / Z_{1r}^{o1} \right) \\
&+ \sum_{i=1}^{I_o} \sum_{k=1}^{S_1} W_{ik}^{(12)} \left(Z_{oi}^{j} / Z_{1k}^{j2} \Big/ Z_{0i}^{o} / Z_{1k}^{o2} \right) \geq \theta_1^{NR} \\
&\sum_{i=1}^{I_p} \sum_{r=1}^{R_p} W_{ir}^{(p1)} \left(Z_{p-1i}^{j3} / Z_{pr}^{j1} \Big/ Z_{p-1i}^{o3} / Z_{pr}^{o1} \right) \\
&+ \sum_{i=1}^{I_p} \sum_{k=1}^{S_p} W_{ik}^{(p2)} \left(Z_{p-1i}^{j3} / Z_{pk}^{j2} \Big/ Z_{p-1i}^{o3} / Z_{pk}^{o2} \right) + \\
&\sum_{r=1}^{R_p} \sum_{k=1}^{S_{p-1}} W_{kr}^{(p3)} \left(Z_{p-1k}^{j2} / Z_{pr}^{j1} \Big/ Z_{p-1k}^{o2} / Z_{pr}^{o1} \right) \\
&+ \sum_{k=1}^{S_p} \sum_{k=1}^{S_{p-1}} W_{kk}^{(p4)} \left(Z_{p-1k}^{j2} / Z_{pk}^{j2} \Big/ Z_{p-1i}^{o2} / Z_{pk}^{o2} \right) \geq \theta_p^{NR} \\
&\sum_{i=1}^{I_o} \sum_{r=1}^{R_1} W_{ir}^{(11)} + \sum_{i=1}^{I_o} \sum_{k=1}^{S_1} W_{ik}^{(12)} = 1, j = 1, ..., n \quad p = 2, ..., P \\
&\sum_{i=1}^{I_p} \sum_{r=1}^{R_p} W_{ir}^{(p1)} + \sum_{i=1}^{I_p} \sum_{k=1}^{S_p} W_{ik}^{(p2)} + \\
&\sum_{r=1}^{R_p} \sum_{k=1}^{S_{p-1}} W_{kr}^{(p3)} + \sum_{k=1}^{S_p} \sum_{k=1}^{S_{p-1}} W_{kk}^{(p4)} = 1 \\
&W_{ir}^{(11)} \geq 0, W_{ik}^{(12)} \geq 0, W_{ir}^{(p1)}, W_{ik}^{(p2)} \geq 0, W_{kr}^{(p3)} \geq 0, \\
&W_{kk}^{(p4)} \geq 0, \theta_p^{NR} \geq 0, p = 1, ..., P, \sum_{p=1}^{p} W_p = 1 \qquad (7)
\end{aligned}
$$

The relative efficiency score of each weight vector is calculated first by the model (7) and then the smallest score will be calculated which is the efficiency score of this set of weights. Then, by adjusting the weights, the maximum efficiency score θ^{NR} is considered to be the overall efficiency score DMU_o. Since- the objective function is the weighted mean of the efficiencies of sub-phases efficiency in the network, then the values of $\theta_p^N R$ are maximized by this model for a selected set of weights leading to the overall maximum efficiency. A change in w_p causes a change in θ^{NR} and $\theta_p^N R$. Therefore, the overall efficiency and efficiencies of each process depend on the weights of the objective function.

3 Evaluating the Efficiency of Production Systems with a Relative Multi-Period Network Structure

Most of the multi-period DEA models proposed so far have considered production systems as a black box and the internal relationships within the system as well as the efficiency of each process are not considered . DEA models are also unable to calculate efficiency when the prices of inputs and outputs are unknown while the ratio of inputs to outputs or vice versa is known. Also, the previous models of the DEA-R two-stage network have been suggested without considering time. Therefore, a model is introduced in this section for determining the efficiency of multi-period decision units DEA-R (in L time period) with multi-network structure. Considering a subsystem corresponding to each period, we will have a network system with a parallel structure of L subsystems, each system consisting of P stages that are connected in series (Fig. 3).

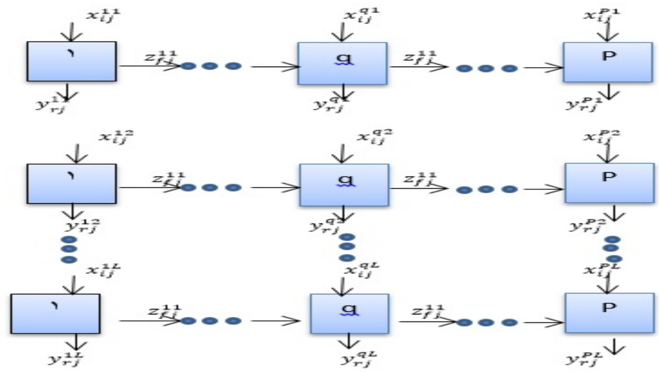

Figure 3: Parallel multi-period production network

3.1 Proposed model for determining the efficiency of $DEA-R$ of decision-making units with a multi-period network structure.

In this section, we consider a set of n DMU, which is observed in the L time period. Each DMU consists of P stages. The internal relations of the processes are the same for all DMU_s. In a network structure, each process works together with the others so that the entire decision-making unit achieves optimal efficiency. A multi-stage P network production model with external inputs x_{ij}^{1t}, x_{ij}^{2t} and x_{ij}^{pt} and final outputs y_{rj}^{1t}, y_{rj}^{2t} and y_{rj}^{pt} and the intermediate dimensions of z_{fj}^{1t} (first stage outputs and second stage inputs), z_{fj}^{2t} (second stage outputs and third stage inputs), and z_{fj}^{Pt} (P stage outputs) in the period t has been proposed under the following assumptions (Fig. 4).

A-The proposed model is the CCR input-oriented DEA-R.

B- The proposed model is a parametric linear model in P step network process. The purpose of this model is to reduce the inputs at each stage in order to evaluate the units with ratio data . In all network processes, we consider the constrain $\sum_{j=1}^{n} \lambda_j^k = p_k$ corresponding to each step t, provided that $\sum_{k=1}^{p} p_k$. Thus, since $\lambda_j^k \geq\geq 0$ if $\sum_{j=1}^{n} \lambda_j^k = 0$ then for every $1 \leq j \leq n$, $\lambda_j^k = 0$. In general, for p_k parameters we consider the following states: 1-If $p_k \in 0, 1$ and $\sum_{k=1}^{p} p_k = 1$, then the

proposed model can calculate the efficiency of each step. 2- If $p_k \in (0,1)$ and $\sum_{k=1}^{p} p_k = 1$, then the proposed model can calculate the overall efficiency of the network.
C- Suppose I_1, I_2 ,and I_p are sets of input indices in each P stage. Similarly, F_1, F_2, F_p and R_1, R_2, R_p are sets of intermediate size indices and final outputs.
D- In the proposed model, the parameters p_1, p_2, p_p correspond to the variables $\lambda_j^1, \lambda_j^2, \lambda_j^p$ respectively. Now since P is the network step corresponding to $\lambda_j^1, \lambda_j^2, \lambda_j^p$ so the parameters p_1, p_2, p_p play a very important role in calculating process efficiencies and overall efficiency.
E- The variables $\lambda_j^1, \lambda_j^2, \lambda_j^p$ correspond to processes 1, 2 , and P respectively.
F-Since the P-stage network process is input-driven, the purpose of the proposed model is to reduce input-to-output data that is reduced radially. The variables $\varphi_1, \varphi_2, \varphi_p$ are used to reduce the inputs in processes 1, 2, and P of the network with ratio data respectively.
Our proposed DEA-R model for calculating the overall efficiency of the system with DMU_o network structure after period L and also the time period t (actually subsystem efficiency) is as follows:

3.2 Proposed DEA-R model for overall efficiency of the system after the L time period

$$
\begin{aligned}
&\varphi_o = \min \sum_{p=1}^{p} \varphi_p \\
s.t. &\sum_{j=1}^{n} \lambda_j^{1t} \Big(\frac{x_{ij}^{1t}}{z_{fj}^{1t}}\Big) \leq \varphi_1 \Big(\frac{x_{io}^{1t}}{z_{fo}^{1t}}\Big) \; i \in I_1; f \in F_1; t = 1, ..., L \\
&\sum_{j=1}^{n} \lambda_j^{1t} \Big(\frac{x_{ij}^{1t}}{y_{rj}^{1t}}\Big) \leq \varphi_1 \Big(\frac{x_{io}^{1t}}{y_{ro}^{1t}}\Big) \; i \in I_1; r \in R_1; t = 1, ..., L \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{x_{ij}^{pt}}{z_{fj}^{pt}}\Big) \leq \varphi_p \Big(\frac{x_{io}^{pt}}{z_{fo}^{pt}}\Big) \; i \in I_p; f \in F_p; t = 1, ..., L; 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{x_{ij}^{pt}}{y_{rj}^{pt}}\Big) \leq \varphi_p \Big(\frac{x_{io}^{pt}}{y_{ro}^{pt}}\Big) \; i \in I_p; r \in R_p; t = 1, ..., L; 2 \leq p \leq P \qquad (8) \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{z_{fj}^{(p-1)t}}{y_{rj}^{pt}}\Big) \leq \varphi_p \Big(\frac{z_{fo}^{(p-1)t}}{y_{ro}^{pt}}\Big) \; r \in R_p; f \in F_(p-1); t = 1, ..., L; 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{z_{fj}^{(p-1)t}}{z_{fj}^{pt}}\Big) \leq \varphi_p \Big(\frac{z_{fo}^{(p-1)t}}{z_{fo}^{pt}}\Big) \; f \in F_{p-1)}; f \in F_p; 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{kt} = p_k, 1 \leq k \leq P \\
&\sum_{k=1}^{p} p_k = 1, \lambda_j^{kt} \geq 0 \; \forall 1 \leq j \leq n; 1 \leq k \leq p
\end{aligned}
$$

φ_1 in (8), indicates the first stage efficiency, φ_p , $2 \leq p \leq P$ indicates the efficiency of the stage p and φ_o shows the overall efficiency of the system after the L time period. The first constraint in (8) measures the radial decrease in the ratio of the value of external inputs to the value of intermediate sizes in the first stage. The second constraint in (8) also guarantees a radial decrease in the ratio of the value of external inputs to the value of final outputs in the first stage. The next four conditions in (8) are respectively the ratio of the value of external inputs to the value of intermediate sizes, the ratio of the value of external inputs to the value of final outputs, the ratio of the value of intermediate sizes to the final outputs and the ratio of the value of intermediate sizes of stage p to Step $p-1$ in step $2 \leq p \leq P$ shows.

3.3 Proposed DEA-R model for overall efficiency of the system in time period t

3.3.1 Proposed DEA-R model for overall efficiency of the system in time period t

" The overall efficiency of the system in time period t is the solution of the following problem

$$
\begin{aligned}
&\varphi_o^t = \min \sum_{p=1}^{p} \varphi_p \\
s.t. &\sum_{j=1}^{n} \lambda_j^{1t} \Big(\frac{x_{ij}^{1t}}{z_{fj}^{1t}}\Big) \leq \varphi_1 \Big(\frac{x_{io}^{1t}}{z_{fo}^{1t}}\Big) \ i \in I_1; f \in F_1 \\
&\sum_{j=1}^{n} \lambda_j^{1t} \Big(\frac{x_{ij}^{1t}}{y_{rj}^{1t}}\Big) \leq \varphi_1 \Big(\frac{x_{io}^{1t}}{y_{ro}^{1t}}\Big) \ i \in I_1; r \in R_1 \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{x_{ij}^{pt}}{z_{bj}^{pt}}\Big) \leq \varphi_p \Big(\frac{x_{io}^{pt}}{z_{bo}^{pt}}\Big) \ i \in I_p; b \in F_p; 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{x_{ij}^{pt}}{y_{rj}^{pt}}\Big) \leq \varphi_p \Big(\frac{x_{io}^{pt}}{y_{ro}^{pt}}\Big) \ r \in I_p; r \in R_p 2 \leq p \leq P \qquad (9) \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{z_{dj}^{(p-1)t}}{y_{rj}^{pt}}\Big) \leq \varphi_p \Big(\frac{z_{do}^{(p-1)t}}{y_{ro}^{(p-1)t}}\Big) \ r \in R_p; d \in F_{p-1} 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{pt} \Big(\frac{z_{dj}^{(p-1)t}}{z_{bj}^{pt}}\Big) \leq \varphi_p \Big(\frac{z_{do}^{(p-1)t}}{z_{do}^{(p-1)t}}\Big) \ d \in F_{p-1}; b \in F_P 2 \leq p \leq P \\
&\sum_{j=1}^{n} \lambda_j^{kt} = p_k, 1 \leq k \leq P \ , \sum_{k=1}^{p} p_k = 1, \lambda_j^{kt} \geq 0 \ \forall 1 \leq j \leq n, 1 \leq k \leq P
\end{aligned}
$$

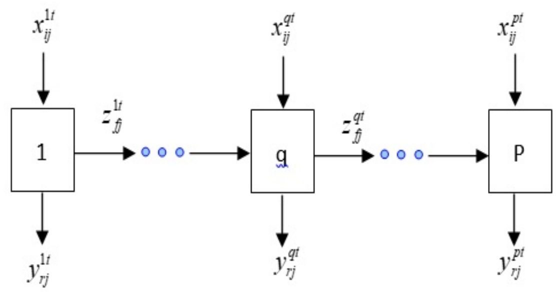

Figure 4: P stage network in time period t

φ_1 in (9) depicts the first stage efficiency and φ_p indicates the efficiency at stage P , $1 \leq p \leq P$ and φ_p shows the overall efficiency of the system in time period t.

Theorem 3.1. *models (8) and (9) are feasible.*

Proof. for $1 \leq p \leq P, p \neq K) \sum_{j=1}^{n} \lambda_j^{pt} = 0$ and $\sum_{j=1}^{n} \lambda_j^{kt} = 1$, In this case, $\lambda_j^{kt} = 0(j \neq 0); \lambda_o^{kt} = 1, \varphi_k = 1$ and $\varphi_p = 0(1 \leq p \leq P, p \neq k)$ is a feasible solution for model (8).
In general, $p_k = \frac{1}{p}$ for every $1 \leq p \leq P$, from the equality $\sum_{j=1}^{n} \lambda_j^{pt} = \frac{1}{p}$ for every $1 \leq p \leq P$, it can be concluded that $\lambda_j^{pt} = 0(j \neq 0)$ and $\lambda_o^{pt} = \frac{1}{p}$ and $\varphi \varphi_p \frac{1}{p}$ is a feasible solution for model (8). In addition, it can be concluded that the optimal value does not exceed one and is always greater than zero. Similar to model (8), model (9) is also feasible. □

Theorem 3.2. *the efficiency score of the entire network system after L time period is greater than or equal to the efficiency score of the entire system in the time period t.*

Proof. Since the number of constraints in (8) is more than that of (9), then the feasible region of model (8) is a subset of the feasible region of (9), since the objective functions of the two models are of the Minimum type, the smaller the feasible region is, the larger the objective function becomes, then the optimal value of model (8) which is the score of the whole system efficiency after L time period is greater than or equal to the optimal value of the model (9) or the same score of the whole system efficiency in the time period t. □

Corollary

If the multi-period system with DEA-R network structure is inefficient after L time period, then it is inefficient in all periods.

Proof. For the arbitrary unit under evaluation DMU_o and arbitrary time period t, we have $\varphi_o^t < 1$. To prove this, we use the absurd hypothesis. so if $\varphi_o^t \geq 1$, Because of $0 \leq \varphi_o^t \leq 1$,then $\varphi_o^t = 1$, according to theorem 2, it can be said that φ_o or the efficiency score of the entire network system after L time period is equal to one, which is contradictory Assuming the case. □

According to the above result, it can be said that a multi-period DEA-R system with a network structure after L time period is efficient if it is efficient in at least one of the periods.

4 Numerical Example

This example is qouted from Tohinia and Tohidi (2019). In this example we consider 10 $DMUs$. Each DMU has two stages, stage$I$ and stage II which have been observed in three time periods. $DMU_j, 1 \leq j \leq 10$, uses two external inputs x_{1j}^{1t} and x_{2j}^{1t} to produce two intermediate products z_{1j}^{1t} and z_{2j}^{1t} along with an external output y_{1j}^{1t}. In the second stage it uses two intermediate products and one external input x_{1j}^{2t} to produce two outputs y_{1j}^{2t} and y_{2j}^{2t} in time period t Figure (5).

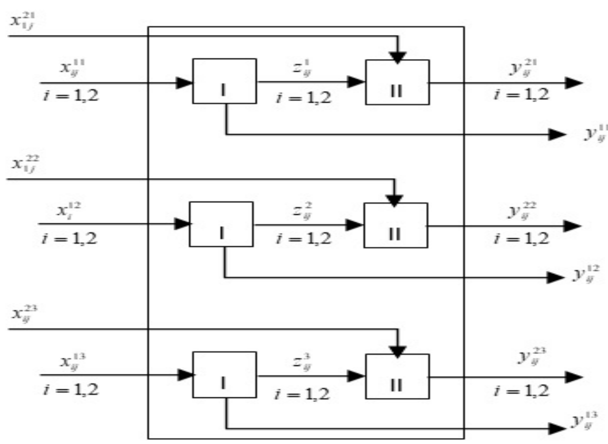

Figure 5: Multi-period network production system with $T = 3$ and $P = 2$

The data set is shown in three time periods ($t = 1, 2, 3$) in Tables 1, 2 and 3.

Table 2: the data set in the first time period

DMU	x_{1i}^{1t}	x_{2i}^{1t}	x_{1i}^{2t}	z_{1i}^{1t}	z_{2i}^{1t}	y_{1i}^{1t}	y_{1i}^{2t}	y_{2i}^{2t}
t=1								
A	4	2	7	8	1	9	12	15
B	7	1	9	5	3	11	8	9
C	5	3	6	10	2	8	17	13
D	8	1	1	10	6	3	15	16
E	7	1	7	7	5	8	14	10
F	9	5	8	3	1	11	9	16
G	10	3	6	6	3	10	12	9
H	6	4	9	4	4	7	10	11
I	3	1	5	8	2	9	8	8
J	8	2	5	5	5	15	10	12

Table 3: the data set in the first time period

DMU	x_{1i}^{1t}	x_{2i}^{1t}	x_{1i}^{2t}	z_{1i}^{1t}	z_{2i}^{1t}	y_{1i}^{1t}	y_{1i}^{2t}	y_{2i}^{2t}
t=2								
A	5.2	2	9.1	10.4	1.3	11.7	12	15
B	9.1	1	11.7	6.5	3.9	14.3	12	13.5
C	6.5	3	7.8	13	2.6	10.4	25.5	19.5
D	10.4	1	13	7.8	3.9	19.5	24	16.5
E	9.1	1	9.1	9.1	6.5	10.4	19	13
F	11.7	5	10.4	3.9	1.3	14.3	13	17.5
G	10	3	6	6	3	10	12	9
H	7.8	4	11.7	5.2	5.2	9.1	14	14
I	3.9	1	6.5	10.4	2.6	11.7	13	11
J	10.4	2	6.5	6.5	6.5	6.5	15	14.5

Table 4: the data set in the third time period

DMU t=3	x_{1i}^{1t}	x_{2i}^{1t}	x_{1i}^{2t}	z_{1i}^{1t}	z_{2i}^{1t}	y_{1i}^{1t}	y_{1i}^{2t}	y_{2i}^{2t}
A	7.8	3	13.65	11.44	1.43	12.87	13.2	16.5
B	13.65	1.5	17.55	9.75	5.85	21.45	18	20.25
C	9.75	4.5	11.7	19.5	3.9	15.6	38.25	29.25
D	15.6	1.5	19.5	30.42	5.85	29.25	36	24.75
E	13.65	1.5	15.5	17	7.5	15.5	25	20
F	17.55	7.5	14	7	2	16	14.5	22
G	19.5	4.5	11.2	10.5	4.9	17.5	18	14
H	11.7	6	15	9	8.5	12.6	19.2	17.5
I	5.85	1.5	10.5	15.5	3.5	13	16	17
J	15.6	3	10.5	9	8	24.3	17	16.5

After solving model (8) for each DMU The overall efficiency of the system aftervsolving (8) for each DMU for three time periods we obtained the result in table 4.

Table 5: Results of solving model (8) for $(p_1 = 0.5, p_2 = 0.5)$

$(p_1 = 0.5, p_2 = 0.5)$	Objective	φ_1	φ_2
A	0.875000	0.375000	0.500000
B	0.941388	0.499980	0.441408
C	0.877419	0.377419	0.500000
D	1.000000	0.500000	0.500000
E	0.984848	0.500000	0.484848
F	0.705128	0.205128	0.500000
G	0.708510	0.221939	0.486571
H	1.000000	0.500000	0.500000
I	0.863439	0.500000	0.363439
J	0.988939	0. 488939	0.500000

After solving (9) for each DMU, the overall efficiency for any time period are shown in tables 5 to 7 separately.

Table 6: Results of solving model (9) for $(p_1 = 0.5, p_2 = 0.5, t = 1)$

$(p_1 = 0.5, p_2 = 0.5)$ t=1	Objective	φ_1	φ_2
A	0.875000	0.375000	0.500000
B	0.790014	0.475105	0.314910
C	0.875000	0.375000	0.500000
D	1.000000	0.500000	0.500000
E	0.984848	0.500000	0.484848
F	0.703704	0.203704	0.500000
G	0.708510	0.221939	0.486571
H	0.909140	0.470842	0.438298
I	0.863439	0.500000	0.363439
J	0.988939	0. 488939	0.500000

Table 7: Results of solving model (9) for $(p_1 = 0.5, p_2 = 0.5, t = 2)$

$(p_1 = 0.5, p_2 = 0.5)$ t=2	Objective	φ_1	φ_2
A	0.854125	0.375000	0.479125
B	0.810123	0.475105	0.335018
C	0.875000	0.375000	0.500000
D	1.000000	0.500000	0.500000
E	0.984848	0.500000	0.484848
F	0.703704	0.203704	0.500000
G	0.632622	0.221939	0.410683
H	0.891495	0.470842	0.420654
I	0.842146	0.500000	0.342146
J	0.988939	0. 488939	0.500000

Table 8: Results of solving model (9) for $(p_1 = 0.5, p_2 = 0.5, t = 3)$

$(p_1 = 0.5, p_2 = 0.5)$ t=3	Objective	φ_1	φ_2
A	0.871250	0.371250	0.500000
B	0.941388	0.499980	0.441408
C	0.877419	0.377419	0.500000
D	0.813725	0.500000	0.313725
E	0.865500	0.500000	0.365500
F	0.705128	0.205128	0.500000
G	0.641030	0.218593	0.422437
H	1.000000	0.500000	0.500000
I	0.844713	0.500000	0.344713
J	0.934596	0.459541	0.475054

We investigate the relationship between overall efficiency of the system and the efficiency of each system process after 3 time periods and also the relationship between overall efficiency of the system and each system process in every time period by models (8) and (9). The second column of the table 4 shows the overall efficiency scores after three time periods and the third and fourth columns of the table 4 showes the efficiency scores of the first and second stages of network.Therfore , units D and H are efficient in overall network system. The contribution of both stages in the efficiency of units D and H is equal.The lowest efficiency score after three time periods is for unit F. The share of the first stage in the level of inefficiency of this unit is higher than the second stage. According to Tables 4, 5, 6 and 7, the efficiency scores of all units after three time periods are greater or equal to their efficiency scores in each of the time periods.The second column of the Table 5 summarizes overall efficiency of the system in the first time period and the third and fourth columns of the table 5 showes the efficiency scores of the first and second stages of network. According to this table Unit D is efficient and Unit F has the lowest efficiency score at this time period.The contribution of both stages in the efficiency of units D is equal. The share of the first stage in the level of inefficiency of unit F is higher than the second stage. The second column of the Table 6 shows overall efficiency of the network system in the second time period and the third and fourth columns of the table 6 showes the efficiency scores of the first and second stages of network. that unit D is efficient in the network system and unit G has the lowest efficiency score. The second column of the Table 7 shows overall efficiency of the network system in the third time period and the third and fourth columns of the table 7 showes the efficiency scores of the first and second stages of network. Unit H is efficient and G has the lowest efficiency score among the units of the network system in the third time period. By comparing Tables 4, 5, 6 and 7, we can see that the efficiency scores of each of the network stages after three time periods are greater or equal to their efficiency scores in each time period and the inefficient units of the whole system in table 4 are also inefficient in the rest of the tables. Also,According to Tables 4, 5, 6 and 7, it can be said that the efficiency scores obtained from model (8) are higher than or equal to the efficiency scores obtained from model (9). In fact, adding a time period variable increases the constraints of the secondary problem and it is possible that the optimal solution of the problem to be better and the value of the objective function, which is of the maximization type, is increased and since the primary problem optimal solution equals the secondary problem, so the obtained efficiency scores are increased. As it is clear from Table 4, the units D and H are efficient and the efficiency score of units D and H is at least one in one of the tables 5, 6 and 7. That is, if a unit is efficient with model (9), it will be efficient with model (8). So a unit becomes efficient after several periods of time if it is efficient in at least one period of time. In other words, if a unit is inefficient in the whole network system, then it is inefficient in all periods.

5 Conclusion

A model is required for measuring the efficiency of the system and its processes by considering time in order to evaluate the efficiency of a network system and its processes. In this paper, a model has been proposed for measuring overall efficiency of the system and its processes over several desired time periods. This model has three advantages: First, when the input and output data are unknown and only a proportion of them is known, we can also use this model to prevent false inefficiency and not use the non-Archimedean number ε. Second, in this model, the internal relationships among processes are considered. Finally, the proposed model focuses on changes over time period. It has been shown that overall efficiency scores and the efficiency of each process obtained from this model after several desired time periods are higher than or equal to overall efficiency scores and the efficiency of each process in each time period. Also, a unit becomes efficient after several periods of time if it is efficient in at least one period of time To explain the capability of the proposed model, the efficiency of 10 $DMUs$ has been calculated that each of them is consisting of two phases. The results achieved by model solving help us to identify network processes and periods that reduce system efficiency. In this way, significant results can be obtained by decomposing of a system into subsystems and sub processes. Since the data are not known precisely then it will be suitable to investigate this problem when data are given as fuzzy numbers or uncertain data using uncertainty analysis which is postponed as a future work.

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