Journal of Mathematical Extension
Vol. 17, No. 8, (2023) (6)1-25
URL: https://doi.org/10.30495/JME.2023.2474
ISSN: 1735-8299
Original Research Paper

Developing New Robust DEA Models to Identify the Returns to Damage Under Undesirable Congestion and Damages to Return Under Desirable Congestion Measured by DEA Environmental Assessment

M. Nassiri
Shahre Rey Branch, Islamic Azad University

M. Rostamy-Malkhalifih*
Tehran Branch, Islamic Azad University

H. Bagherzadeh Valami
Shahre Rey Branch, Islamic Azad University

Abstract. In recent years, the environmental issue has attracted wide spread concern from the international community, as gas waste, water waste, and solid wastes generated in the production process of factories. Recent studies on environmental management have forced commercial organizations to re-evaluate their roles and responsibilities for protecting the natural environment. This study focuses on the DEA environmental assessment via the concept of congestion. Recognizing the congestion of units is one of the most attractive issues in the literature of Data Envelopment Analysis (DEA), because the decision maker (DM) can use this concept to decide whether to increase or decrease the size of a Decision Making Unit (DMU). In the DEA literature, congestion is classified into

Received: June 2021; Accepted: December 2022
*Corresponding Author

Undesirable Congestion (UC) and Desirable Congestion (DC). In many real-world situations, we cannot determine the exact value for all data, hence, some parameters are inevitably reported as uncertain data, e.g. stochastic data, fuzzy data, interval data and so on. This study focuses on considering Returns to Damage (RTD) under UC and Damages to Return (DTR) under DC in the situation that the input and desirable and undesirable outputs are reported as interval data. For this purpose, some uncertain models under the different production possibility sets (PPS) are formulated and then we use the robust optimization technique to formulate the equivalent certain models. The potential of the proposed methods are illustrated by a numerical example.

Keywords and Phrases: Data Envelopment Analysis; Returns to Damage; Damages to Return; Undesirable Congestion; Desirable Congestion.

1 Introduction

Data envelopment Analysis (DEA) is a well-known mathematical programming method to assess the relative efficiency of units. See Charnes et al. (1978), Banker et al. (1984), F"a" re et al. (1985), Zhu (2002), Cooper et al. (2006) for more studies about the classical DEA models. Evaluating the performance of the DMUs in the presence of undesirable outputs may be difficult because, we should first decide about the way of treatment with these outputs. Thus, many researchers have been attracted to modelling the undesirable outputs in the DEA literature in the last two decades. There are some possible options to handle the undesirable outputs. We can ignore them from the production technology. The undesirable outputs can be treated as the regular inputs or as the normal outputs. Also, we can performe some necessary transformations to take the undesirable outputs into account. Halkos and Petrou (2019) reviewed the existing methods in the DEA literature to handle the undesirable outputs and showed that each method has some benefits and drawbacks which should be taken into account by the researcher. Zhou et al. (2019) proposed an exponential transformation of undesirable outputs into desirable outputs to measure the environmental efficiency by using all kinds of classic models. Toloo and Hančlová (2020) formulated two individual and summative selecting directional distance models and developed a pair of multiplier- and envelopment-based selecting approaches. Shi et al. (2021) proposed a slacks-based measure network data envelopment analysis (SBM-NDEA) model with undesirable outputs to evaluate the performance of production processes that have complex structure containing both series and parallel processes. For more studies about the undesirable outputs, see Yousefi et al. (2018), Zarbakhshnia and Jaghdani (2018), Mo et al. (2020), Pishgar-Komleh et al. (2020), Sun and Huang (2021), Streimikis and Saraji (2021) and Zhao et al. (2022).

The concept of congestion is an important subjects in the DEA literature. It is well-known that, if the decreasing in some inputs of a decision making unit (DMU) results in the increasing in some outputs of that DMU. The concept of congestion can help the decision maker (DM) in order to decide whether to increase or decrease the size of the unit under evaluation. This concept of congestion is called the undesirable congestion (UC). Recognizing UC has attracted the attention of many scholars, for more details, see Fare et al. (1986), Cooper et al. (2001), Tone and Sahoo (2004), Wei and Yan (2004), Sueyoshi and Sekitani (2009), Jahanshahloo and Khodabakhshi (2004), Wu et al. (2015), Wanke et al. (2019) and Khezri et al. (2021). Another type of congestion is called the desirable congestion (DC) which reports an existence of eco-technology innovation and the managerial challenges, used for mitigation of undesirable outputs. It should be noted that, because UC and DC influence the performance of energy firms, hence, the recognizing and separating between UC and DC are crucial from the viewpoint of operating energy sectors. In the environmental assessment literature, the main aim is to overcome global warming and climate change, and so, the DC is more important than UC. For more study, see Sueyoshi and Yuan (2016).

The conventional DEA model deals with precisely known data where inputs and outputs values are deterministic and exactly known. However, in many real-world situations, one may encounter uncertain data due to the different reasons, such as incomplete information, measurement errors or any other source of reason. Robust Optimization (RO) is a technique to model optimization problems with uncertain data which aims to determine an optimal solution which is the best for all or the most possible realizations of the uncertain parameters. For more details, see Ben-Tal and Nemirovski (1998, 1999, 2000) and Bertsimas and Sim

(2004). Some scholars incorporated the robust optimization technique into DEA, for more details, see Wang and Wei (2010), Sadjadi and Omrani (2008), Sadjadi and Omrani (2010), Sadjadi et al. (2011), Omrani (2013), Salahi et al. (2016) and Dehnokhalaji et al. (2022).
Given that the importance of UC and DC, this study focuses on proposing new methods to recognize the UC and DC in the situation that the inputs, desirable outputs and undesirable outputs are reported as interval data. For this purpose, we formulate some models under the different PPSs to evaluate the DMUs in the case of interval data and then the robust optimization technique is used to convert the proposed models into the certain models.
The rest of the paper is organized as follows: Section 2 reviews some basic definitions and preliminaries. Section 3 proposes the new method to recognize the undesirable congestion and the desirable congestion. Section 4 uses a data set to show the potentially of the proposed method. Section 5 concludes the paper.

2 Preliminaries and Basic Definitions

Suppose that there are n DMUs,$DMU_j, j = 1, ..., n.$ and each unit, e.g.DMU_j,uses m inputs to generate s outputs. It is assumed that,x_{ij} for all $i = 1, ..., m$,and y_{rj} for all $r = 1, ..., s$,are the i^{th} input and the r^{th} output, respectively. Assume that $DMU_o = (x_o, y_o)$ is the unit under evaluation.
The following PPS under the variable returns to scale (VRS), namely T_v, has been introduced by Banker et al. (1984):

$$T_v = \{(x, y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_{ij}, y \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, ..., n\}. \tag{1}$$

The output oriented BCC model proposed by Banker et al. (1984) for evaluating the efficiency score of units is as follows:

$$\begin{aligned}
&\psi = max \quad \rho + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)\\
&\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad i = 1, ..., m,\\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \rho_{ro} \quad i = 1, ..., m,\\
&\sum_{j=1}^{n} \lambda_j = 1,\\
&\qquad\qquad \lambda_j \geq 0 \;\; j = 1, ..., n,\\
&\qquad\qquad s_i^- \geq 0 \;\; i = 1, ..., m,\\
&\qquad\qquad s_r^+ \geq 0 \;\; r = 1, ..., s.
\end{aligned} \tag{2}$$

where ϵ is non-Archimedean.
Banker et al. (1984) presented the following definition:
Definition 1:. Suppose that $(\rho^*, s^{-*}, s^{+*}, \lambda^*)$ is an optimal solution for model (2). If $\rho^* = 1$ then DMU_o is called technically efficient. If $\psi^* = 1$, then DMU_o is called strongly efficient. Cooper et al. (2001) and Brocket et al. (2004) presented the classical definition of congestion as follows:
Definition 2:. The unit $DMU_o = (x_o, y_o)$ has congestion if the decreases (increases) in some inputs result in the increases (decreases) in some outputs without worsening (improving) other inputs or outputs. Tone and Sahoo (2004) defined the PPS accepting all assumptions to build T_v except one assumption, strong disposal. They considered weak disposal instead, which was defined as follows:
Definition 3: The PPS satisfies weak disposal assumption if for each $(\overline{x}, \overline{y})$ belonging to the PPS and vector (x, y) where $x = \overline{x}$ and $y \leq \overline{y}, (x, y)$ belongs to the PPS.
$p_{convex} = \{(x, y) \mid x = \sum_{j=1}^{n} \lambda_j x_{ij}, y \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, ...n\}$
They proposed the following model to estimate the efficiency score of DMU_o, with respect to P_{convex}:

$$\begin{aligned}
&\phi^* = max\ \phi + \epsilon(\sum_{r=1}^{s} s_r^+) \\
&\sum_{j=1}^{n} \lambda_j x_{ij} = x_{io}, \quad i = 1, ..., m, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{ro}, \quad r = 1, ..., s, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geq 0, \quad j = 1, ..., n, \\
&s_r^+ \geq 0, \quad r = 1, ..., s.
\end{aligned} \tag{3}$$

It is clear that the target unit for DMU_o, located on the strongly efficient frontier of P_{convex} is as follows:
$\widehat{x}_{io} = x_{io}, \quad i = 1, ..., m,$
$\widehat{y}_{ro} = \phi^* y_{ro} + s_r^{+*}, \quad r = 1, ..., s,$
Tone and Sahoo (2004), presented the following definitions for strongly efficient unit, strong congestion and weak congestion, respectively.
Definition 4: The unit $DMU_o = (x_o, y_o)$ is strongly efficient with respect to P_{convex}, if $\phi^* = 1$.
Definition 5: Suppose that $DMU_o = (x_o, y_o)$ is strongly efficient unit with respect to P_{convex}. DMU_o has strong congestion if there is $(\overline{x}_o, \overline{y}_o)\epsilon$ P_{convex} such that $\overline{x}_o = \alpha x_o (0 < \alpha < 1)$ and $\overline{y}_o \geq \beta y_o (\beta > 1)$.
Definition 6: Suppose that $DMU_o = (x_o, y_o)$ is strongly efficient unit with respect to P_{convex}. DMU_o has weak congestion if there is an activity in P_{convex} that uses less resources in some inputs to produce more products in some outputs.
There are several methods to recognize the congestion of units. The next section develops a method to recognize the desirable congestion (DC) and the undesirable congestion (UC) in the case of interval data.

3 Our Proposed Approach to Determine the Undesirable Congestion (UC)

Consider a system with n DMU_s, DMU_j,$j = 1, ..., n$. Suppose that each unit, e.g. DMU_j, uses an input vector $X_j = (x_{1j}, ..., x_{mj})$ and produces a vector of desirable outputs $G_j = (g_{1j}, ..., g_{sj})$ and a vector of undesirable outputs $B_j = (b_{1j}, ..., b_{hj})$. It is assumed that, $x_{ij} \in [x_{ij}^L, x_{ij}^U]$, for all $i = 1, ..., m$, and $g_{rj} \in [g_{rj}^L, g_{rj}^U]$, for all $r = 1, ..., s$, and $b_{kj} \in [b_{kj}^L, b_{kj}^U]$, for all $k = 1, ..., h$. Assume that the lower bound and the upper bound of the intervals have positive values.

3.1 A Possible Occurrence of Undesirable Congestion (UC)

This section proposes a model to recognize the undesirable congestion of units in the case of interval data. For this purpose, we formulate model (4) as follows:

$$
\begin{aligned}
& max\ \zeta \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \quad x_{ij}^L \le x_{ij} \le x_{ij}^U, i = 1, ..., m, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j g_{rj} \ge (1+\zeta) g_{ro}, \quad g_{rj}^L \le g_{rj} \le g_{rj}^U, r = 1, ..., s, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j b_{kj} \le (1-\zeta) b_{ko}, \quad b_{kj}^L \le b_{kj} \le b_{kj}^U, k = 1, ..., h, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \ge 0, \quad j = 1, ..., n,
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
& max\ \zeta \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij} + (\lambda_o - 1) x_{io} \le 0, \quad x_{ij}^L \le x_{ij} \le x_{ij}^U, i = 1, ..., m, j = 1, ..., n, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj} + (\lambda_o - 1 - \zeta) g_{ro} \ge 0, \quad g_{rj}^L \le g_{rj} \le g_{rj}^U, r = 1, ..., s, j = 1, ..., n, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj} (\lambda_o - 1 + \zeta) b_{ko} \le 0, \quad b_{kj}^L \le b_{kj} \le b_{kj}^U, k = 1, ..., h, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \ge 0, \quad j = 1, ..., n.
\end{aligned}
\tag{5}
$$

It is clear that $\lambda_o^*-1 \leq 0$ at the optimality of model (5). In the following, we prove that $\lambda_o^* - 1 + \zeta^* \leq 0$ at the optimality of model (5). Since, $0 \leq \zeta^* < 1$ and $0 \leq \lambda_o^* \leq 1$, we have: If $0 < \zeta^* < 1$, then DMU_o is an inefficient unit. Therefore, $\lambda_o^* = 0$ and so $\lambda_o^* - 1 + \zeta^* < 0$.
If $\zeta^* = 0$, then DMU_o is an efficient unit. Hence, $\lambda_o^* - 1 + \zeta^* \leq 0$. Hence, model (6) can be obtained as the robust counterpart of model (5) by using the method of Ben-Tal and Nemirovski (2000).Also, Model (6) can be converted into the model (7).

$$
\begin{aligned}
&max \;\; \zeta \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^U + (\lambda_o - 1)x_{io}^L \leq 0, \qquad i = 1, ..., m, \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^L + (\lambda_o - 1 - \zeta)g_{ro}^U \geq 0, \quad r = 1, ..., s, \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U (\lambda_o - 1 + \zeta)b_{ko}^L \leq 0, \quad k = 1, ..., h, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geq 0, \quad j = 1, ..., n.
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&max \;\; \zeta \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^U + \lambda_o x_{io}^L \leq x_{io}^L, \qquad i = 1, ..., m, \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^L + \lambda_o g_{ro}^U \geq (1 + \zeta)g_{ro}^U, \quad r = 1, ..., s, \\
&\sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U + \lambda_o b_{ko}^L \leq (1 - \zeta)b_{ko}^L, \quad k = 1, ..., h, \\
&\sum_{j=1}^{n} \lambda_j = 1, \qquad \lambda_j \geq 0, \quad j = 1, ..., n.
\end{aligned}
\tag{7}
$$

Model (4) is an uncertain model which aims to increase the desirable outputs and decrease the undesirable outputs by ζ. It is clear that $0 \leq \zeta < 1$, and DMU_o is efficient if the optimal value of model (4),$\zeta = 0$. We use the robust optimization technique to convert model (4) into a

certain model. For this purpose, we rewrite model (4) as follows and then the robust optimization technique is used to formulate the equivalent certain model.
We developed model (7) in order to identify the undesirable congestion of units. For this purpose, model (7) is considered under the weak disposability in the undesirable outputs. Hence, the third constrain of model (7) is considered as the equality constraint.
The data ranges for adjustment are determined by the upper and lower bounds on inputs and those of desirable and undesirable outputs. These upper and lower bounds are specified as follows:
$R_i^x = (m+s+h)^{-1}(max\{x_{ij}^U \mid j=1,...,n\} - min\{x_{ij}^L \mid j=1,...,n\})^{-1}$
$R_r^g = (m+s+h)^{-1}(max\{g_{rj}^U \mid j=1,...,n\} - min\{g_{rj}^L \mid j=1,...,n\})^{-1}$
$R_k^b = (m+s+h)^{-1}(max\{b_{kj}^U \mid j=1,...,n\} - min\{b_{kj}^L \mid j=1,...,n\})^{-1}$.
Next, we formulate model (8) to recognize the undesirable congestion of units in the case of interval data:

$$
\begin{aligned}
& max\zeta + \epsilon(\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g) \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^U + \lambda_o x_{io}^L + d_i^x = x_{io}^L, \qquad i=1,...,m, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^L + \lambda_o g_{ro}^U - d_r^g = (1+\zeta) g_{ro}^U, \quad r=1,...,s, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U + \lambda_o b_{ko}^L = (1-\zeta) b_{ko}^L, \qquad k=1,...,h, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad j=1,...,n.
\end{aligned}
\tag{8}
$$

puts and so these constraints are considered as the equality constraints. Model (8) maximizes the slack variables related to the input and desirable constraints.

Model (8) ignores the slack variables related to the undesirable out-

The dual of model (8) can be formulated as follows:

$$
\begin{array}{ll}
min \sum_{i=1}^{m} \upsilon_i x_{io}^L - \sum_{r=1}^{s} u_r g_{ro}^U + \sum_{k=1}^{h} w_k b_{ko}^L + \sigma & \\
\sum_{i=1}^{m} \upsilon_i x_{ij}^U - \sum_{r=1}^{s} u_r g_{rj}^L + \sum_{k=1}^{h} w_k b_{kj}^U + \sigma \geq 0 \quad j = 1, ..., n, j \neq 0 & \\
\sum_{i=1}^{m} \upsilon_i x_{io}^L - \sum_{r=1}^{s} u_r g_{ro}^U + \sum_{k=1}^{h} w_k b_{ko}^L + \sigma \geq 0 & \\
\sum_{r=1}^{s} u_r g_{ro}^U + \sum_{k=1}^{h} w_k b_{ko}^L = 1 & \\
u_r \geq \epsilon R_r^g, \quad r = 1, ..., s, & \\
\upsilon_i \geq \epsilon R_i^x, \quad k = 1, ..., h, & \\
w_k : URS. \quad k = 1, ..., h, & \\
\sigma : URS. &
\end{array}
\tag{9}
$$

The efficiency score of DMU_o is as follows:

$$
\begin{array}{l}
E_{UC} = 1 - (\zeta^* + \epsilon(\sum_{i=1}^{m} R_i^x d_i^{x*} + \sum_{r=1}^{s} R_r^g d_r^{g*})) = \\
1 - (\sum_{i=1}^{m} \upsilon_i^* x_{io}^L - \sum_{r=1}^{s} u_r^* g_{ro}^U + \sum_{k=1}^{h} w_k^* b_{ko}^L + \sigma)
\end{array}
\tag{10}
$$

Which shows a possible occurrence of UC. All variables in Eq. (10) are determined at the optimality of models (8) and (9).
After solving model (9), a possible occurrence of UC can be determined as follows:

a. If $w_k^* < 0$, for at least one $k = 1, ..., h$, then strong UC occurs on DMU_o.
b. If $w_k^* > 0$, for all $k = 1, ..., h$, then no UC occurs on DMU_o.
c. If $w_k^* = 0$, for at least one $k = 1, ..., h$, then weak UC occurs on DMU_o.
In the following, we identify the returns to damage (RTD) under UC. Suppose that the dual variables, obtained by model (7), are $v_i^*(i = 1, ..., m), u_r^*(r = 1, ..., s)$ and $w_k^*(k = 1, ..., h)$ and σ^* at the optimality of this model. Then, the supporting hyperplane on DMU_o can be

expressed as follows:

$$\sum_{r=1}^{s} u_r^* g_r^U = \sum_{i=1}^{m} \upsilon_i^* x_i^L + \sum_{k=1}^{h} w_k^* b_k^L + \sigma^* \tag{11}$$

which is characterized by

$$\sum_{i=1}^{m} \upsilon_i^* x_{ij}^U - \sum_{r=1}^{s} u_r^* g_{rj}^L + \sum_{k=1}^{h} w_k^* b_{kj}^U + \sigma^*, j \in R_o \tag{12}$$

where R_o is the reference set for DMU_o and $\sum_{r=1}^{s} u_r^* g_{rj}^L + \sum_{k=1}^{h} w_k^* b_{kj}^U = 1$.
Hence, the degree of RTD (DRTD) under UC on DMU_o is determined as follows:

$$DRTD = \frac{\sum_{k=1}^{h} w_k^* b_k^L}{\sum_{r=1}^{s} u_r^* g_r^U} = \frac{\sum_{k=1}^{h} w_k^* b_k^L}{\sum_{i=1}^{m} \upsilon_i^* x_i^L + \sum_{k=1}^{h} w_k^* b_k^L + \sigma^*} = \frac{1}{(1 + (\frac{\sum_{i=1}^{m} \upsilon_i^* x_i^L + \sigma^*}{(\sum_{k=1}^{h} w_k^* b_k^L)}))} \tag{13}$$

The type of RTD is classified as follows:

a. If there is an optimal solution for model (9) which satisfies $w_k^* > 0$ for all $k = 1, ..., h$, and $\sum_{i=1}^{m} \upsilon_i^* x_{io}^L + \sigma^* < 0$ then DMU_o has an increasing RTD.
b. If there is an optimal solution for model (9) which satisfies $w_k^* > 0$ for all $k = 1, ..., h$, and $\sum_{i=1}^{m} \upsilon_i^* x_{io}^L + \sigma^* = 0$ then DMU_o has the constant RTD.
c. If for each optimal solution for model (9),$w_k^* > 0$ for all $k = 1, ..., h$, and $\sum_{i=1}^{m} \upsilon_i^* x_{io}^L + \sigma^* > 0$ then DMU_o has the decreasing RTD.

d. If for each optimal solution for model (9),$w_k^* < 0$, for at least one $k = 1, ..., h$, athen DMU_o has the negative RTD.
e. For all other cases excluding the cases (a)-(e), DMU_o has no RTD.
In summary, the type of UC is identified by the sign of dual variables, i.e. w_k^*. The type of UC can be classified into the three categories. Meanwhile, the type of RTD is determined by not only the sign of w_k^*. But also the sign of $\sum_{i=1}^{m} \upsilon_i^* x_{io}^L + \sigma^*$.The type of RTD is classified into the five categorizes.

3.2 A Possible Occurrence of Desirable Congestion (DC)

This section proposes a model to recognize the desirable congestion of units in the case of interval data. For this purpose, we consider the following PPS and formulate model (15) to evaluate the units.

$$T_{DC} = \{(G, B) \mid X \leq \sum_{j=1}^{n} \lambda_j X_j, B \geq \sum_{j=1}^{n} \lambda_j B_j, G \leq \sum_{j=1}^{n} \lambda_j G_j, \sum_{j=1}^{n} \lambda_j = 1, \\ \lambda_j \geq 0, 1 \leq j \leq n\} \tag{14}$$

$$\begin{aligned}
& max \ \zeta \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \geq x_{io}, \qquad x_{ij}^L \leq x_{ij} \leq x_{ij}^U, i = 1, ..., m, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j g_{rj} \leq (1+\zeta) g_{ro}, \quad g_{rj}^L \leq g_{rj} \leq g_{rj}^U, r = 1, ..., s, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j b_{kj} \leq (1-\zeta) b_{ko}, \quad b_{kj}^L \leq b_{kj} \leq b_{kj}^U, k = 1, ..., h, j = 1, ..., n, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad j = 1, ..., n.
\end{aligned} \tag{15}$$

Model (15) is an uncertain model which aims to increase the desirable outputs and decrease the undesirable outputs by ζ. It is clear that $0 \leq \zeta \leq 1$. We use the robust optimization technique to convert model (15) into a certain model. For this purpose, we rewrite model (15) as follows and then the robust optimization technique is used to formulate the equivalent certain model.

$$
\begin{array}{ll}
max\ \ \zeta & \\
\sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij} + (\lambda_o - 1) \geq 0, & x_{ij}^L \leq x_{ij} \leq x_{ij}^U, i = 1, ..., m, j = 1, ..., n, \\
\sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj} + (\lambda_o - 1 - \zeta) g_{ro} \leq 0, & g_{rj}^L \leq g_{rj} \leq g_{rj}^U, r = 1, ..., s, j = 1, ..., n, \\
\sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj} + (\lambda_o - 1 + \zeta) b_{ko} \leq 0, & b_{kj}^L \leq b_{kj} \leq b_{kj}^U, k = 1, ..., h, j = 1, ..., n, \\
\sum_{j=1}^{n} \lambda_j = 1, & \\
\lambda_j \geq 0, \qquad j = 1, ..., n. &
\end{array}
\tag{16}
$$

It is clear that $\lambda_o^* - 1 \leq 0$ at the optimality of model (16). In the following that, we prove that $\lambda_o^* - 1 + \zeta \leq 0$ at the optimality of model (16). Since, $0 \leq \zeta^* < 1$ and $0 \leq \lambda_o^* \leq 1$, we have:

3. If $0 < \zeta^* < 1$, $then DMU_o$ is an inefficient unit. Therefore, $\lambda_o^* = 0$ and so $\lambda_o^* - 1 + \zeta^* < 0$.

4. If $\zeta^* = 0$, then DMU_o is an efficient unit. Hence, $\lambda_o^* - 1 + \zeta^* \leq 0$.

Hence, model (17) can be obtained as the robust counterpart of model (16) by using the method of Ben-Tal and Nemirovski (2000).

$$
\begin{array}{ll}
max \quad \zeta & \\
\sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^L + (\lambda_o - 1) x_{io}^U \geq 0, & i = 1, ..., m, \\
\sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^U + (\lambda_o - 1 - \zeta) g_{ro}^L \leq 0, & r = 1, ..., s, \\
\sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U + (\lambda_o - 1 + \zeta) b_{ko}^L \leq 0, & k = 1, ..., h, \\
\sum_{j=1}^{n} \lambda_j = 1, & \\
\lambda_j \geq 0, \quad j = 1, ..., n. &
\end{array}
\tag{17}
$$

Model (17) can be converted into the following model:

$$
\begin{aligned}
& max \quad \zeta \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^L + \lambda_o x_{io}^U \geq x_{io}^U, \qquad i=1,...,m, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^U + \lambda_o g_{ro}^L \leq (1+\zeta) g_{ro}^L, \quad r=1,...,s, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U + \lambda_o b_{ko}^L \leq (1-\zeta) b_{ko}^L, \quad k=1,...,h, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad j=1,...,n.
\end{aligned} \tag{18}
$$

In the following, we develop model (19) in order to identify the desirable congestion of units. For this purpose, model (18) is considered under the weak disposability in the desirable outputs. Hence, the second constrain of model (18) is considered as the equality constraint.

$$
\begin{aligned}
& max \quad \zeta + \epsilon (\sum_{i=1}^{m} R_i^x d_i^x \sum_{k=1}^{h} R_k^b d_k^b) \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j x_{ij}^L + \lambda_o x_{io}^U - d_i^x = x_{io}^U, \qquad i=1,...,m, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j g_{rj}^U + \lambda_o g_{ro}^L - \zeta g_{ro}^L = g_{ro}^L, \qquad r=1,...,s, \\
& \sum_{j=1 j\neq 0}^{n} \lambda_j b_{kj}^U + \lambda_o b_{ko}^L + d_k^b + \zeta b_{ko}^L = b_{ko}^L, \quad k=1,...,h, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& d_i^x \geq 0, \quad i=1,...,m, \\
& d_k^b \geq 0, \quad k=1,...,h, \\
& \lambda_j \geq 0, \quad j=1,...,n.
\end{aligned} \tag{19}
$$

Model (19) ignores the slack variables related to the desirable outputs such that these constraints are considered as the equality constraints.

The dual of model (19) can be formulated as follows:

$$
\begin{aligned}
& min -\sum_{i=1}^{m} \upsilon_i x_{io}^{U} - \sum_{r=1}^{s} u_r g_{ro}^{L} + \sum_{k=1}^{h} w_k b_{ko}^{L} + \sigma - \\
& \sum_{i=1}^{m} \upsilon_i x_{ij}^{L} - \sum_{r=1}^{s} u_r g_{rj}^{U} + \sum_{k=1}^{h} w_k b_{kj}^{U} + \sigma \geq 0, \quad j = 1, ..., n, j \neq 0 \\
& -\sum_{i=1}^{m} \upsilon_i x_{io}^{U} - \sum_{r=1}^{s} u_r g_{ro}^{L} + \sum_{k=1}^{h} w_k b_{ko}^{L} + \sigma \geq 0 \\
& \sum_{r=1}^{s} u_r g_{ro}^{L} + \sum_{k=1}^{h} w_k b_{ko}^{L} = 1 \\
& w_k \geq \epsilon R_k^b, \qquad k = 1, ..., h, \\
& \upsilon_i \geq \epsilon R_i^x, \qquad i = 1, ..., m, \\
& u_r : URS. \qquad r = 1, ..., s.
\end{aligned}
\tag{20}
$$

The efficiency score of DMU_o is as follows:

$$
\begin{aligned}
& E_{UC} = 1 - (\zeta^* + \epsilon(\sum_{i=1}^{m} R_i^x d_i^{x*} + \sum_{r=1}^{s} R_k^b d_k^{b*})) = \\
& 1 - (-\sum_{i=1}^{m} \upsilon_i^* x_{io}^{U} - \sum_{r=1}^{s} u_r^* g_{ro}^{L} + \sum_{k=1}^{h} w_k^* b_{ko}^{L} + \sigma^*)
\end{aligned}
\tag{21}
$$

Which shows a possible occurrence of DC. All variables in Eq. (21) are determined at the optimality of models (19) and (20).
After solving model (20), a possible occurrence of DC can be determined as follows:

d. If $u_r^* < 0$, for at least one $r = 1, ..., s$, then DMU_o has strong DC.
e. If $u_r^* > 0$, for all $r = 1, ..., s$ then DMU_o shows no DC.
f. If $u_r^* = 0$, for at least one $r = 1, ..., s$, then DMU_o has weak DC.
In the following, we present a method to measure the damage to return (DTR) under DC. Suppose that the dual variables, obtained by model (20), are $\upsilon_i^*(i = 1, ..., m)$, $u_r^*(r = 1, ..., s)$ and $w_k^*(k = 1, ..., h)$ and σ^*at the optimality of this model. Then, the supporting hyperplane on

DMU_o can be expressed as follows:

$$\sum_{k=1}^{h} w_k^* b_k^L = \sum_{i=1}^{m} \upsilon_i^* x_i^U + \sum_{r=1}^{s} u_r^* g_r^L - \sigma^* \tag{22}$$

which is characterized by

$$-\sum_{i=1}^{m} \upsilon_i^* x_{ij}^L - \sum_{r=1}^{s} u_r^* g_{rj}^U + \sum_{k=1}^{h} w_k^* b_{kj}^U + \sigma^*, \quad j \in R_o \tag{23}$$

where R_o is the reference set for DMU_o and $\sum_{r=1}^{s} u_r^* g_{rj}^L + \sum_{k=1}^{h} w_k^* b_{kj}^L = 1$. Hence, the degree of DTR (DDTR) under DC on DMU_o is determined as follows:

$$DDTR = \frac{\sum_{r=1}^{s} u_r^* g_r^L}{\sum_{k=1}^{h} w_k^* b_k^L} = \frac{\sum_{r=1}^{s} u_r^* g_r^L}{\sum_{i=1}^{m} \upsilon_i^* x_i^U + \sum_{r=1}^{s} u_r^* g_r^L - \sigma^*} = \frac{1}{(1 - (\frac{(\sigma^* - \sum_{i=1}^{m} \upsilon_i^* x_i^U)}{(\sum_{r=1}^{s} u_r^* g_r^L)}))} \tag{24}$$

The type of DTR is classified as follows:

f. If there is an optimal solution for model (20) which satisfies $u_r^* > 0$, for all $r = 1, ..., s$, and $\sigma^* - \sum_{i=1}^{m} \upsilon_i^* x_i^U > 0$, then DMU_o has an increasing DTR.

g. If there is an optimal solution for model (20) which satisfies $u_r^* > 0$, for all $r = 1, ..., s$, and $\sigma^* - \sum_{i=1}^{m} \upsilon_i^* x_i^U = 0$,then DMU_o has the constant DTR.

h. If there is an optimal solution for model (20) which satisfies $u_r^* > 0$,

for all $r = 1, ..., s$, and $\sigma^* - \sum_{i=1}^{m} \upsilon_i^* x_i^U < 0$,then DMU_o has the decreasing DTR.
i. If there is an optimal solution for model (20) which satisfies $u_r^* < 0$ for at least one $r = 1, ..., s$, then DMU_o has negative DTR.
j. For all other cases excluding the cases (a)-(i), DMU_o has no DTR.
In summary, the type of DC is identified by the sign of dual variables, i.e. u_r^*. The type of DC can be classified into the three categories. Meanwhile, the type of DTR is determined by not only the sign of u_r^*. But also the sign of $\sigma^* - \sum_{i=1}^{m} \upsilon_i^* x_i^U$. The type of DTR is classified into the five categorizes.
It should be noted that, in this study, the proposed DEA approaches assumes that all unified efficiency measures are uniquely determined on optimality. If the assumption is dropped, then the proposed approach needs to incorporate Strong Complementary Slackness Conditions (SC-SCs) into the formulations. See, for example, Sueyoshi and Goto (2017) on how to incorporate SCSCs. Also, see Sueyoshi and Yuan (2016) for more details.

4 Case Study

This section uses a data set, reported in Khalili-Damghani et al. (2015), to illustrate the potentially of the proposed methods. The data set considers 17 combined cycle power plants in Iran during a six year period, as the DMUs. There are six variables, fossil fuel is considered as input, electricity power is considered as the desirable output and gases such as CO_2, SO_2, SO_3 and NO_x are as the undesirable outputs. The data are summarized in Table 1.Now, we solve model (9) to identify the status of units for UC and RTD. The results are summarized in Table 2. In this table, columns 2, 3, 4 and 5 show the optimal value for the weights of the undesirable outputs and column 6 shows the value of σ. Column 7 shows the efficiency score of units, columns 8 and 9 show the UC status and RTD status of DMU_s. In the next step, we solve model (20) to identify the status of units for DC and DTR. The results are summarized in Table 3. In this table, column 2 shows the optimal value for

the weight of the desirable output and column 3 shows the value of σ. Column 4 shows the efficiency score of units, columns 5 and 9 show the DC status and DTR status of DMU_s, respectively.
Meanwhile, most combined cycle power plants in Iran, had a large potential to reduce their pollutions with eco-technology development because they had strong DC with negative DTR. However, there were two types of combined cycle power plants that have weak DC with no DTR, indicating the low level of potential for pollution mitigation. This shows that the obtained results by the proposed method can help the DM to decide about the size of the combined cycle power plants. Also, it is strongly hoped that the combined cycle power plants in Iran will be able to change the industrial structure to reduce the environmental pollution.

5 Conclusion

Environmental management is very important in the manufacturing sector due to the unavoidable generation of pollutants during the production process of industrial activities. Regarding the suitability of the DEA models for incorporating the production pollutants, called the undesirable factors, DEA has received great research attention recently. One of the most attractive issues in the literature of Data Envelopment Analysis (DEA) is to recognize the congestion of units, because the decision maker (DM) can use this concept to decide whether to increase or decrease the size of a Decision Making Unit (DMU). In general, congestion can be classified into Undesirable Congestion (UC) and Desirable Congestion (DC). In many real-world applications, the exact value for all data cannot be determined, hence, some parameters may be reported as uncertain data, such as the stochastic data, fuzzy data, interval data and so on. This study focused on considering Returns to Damage (RTD) under UC and Damages to Return (DTR) under DC in the situation that the input and desirable and undesirable outputs were reported as interval data. For this purpose, some uncertain models under the different production possibility sets (PPS) were formulated and then we used the robust optimization technique to formulate the equivalent certain models.

Table 1: Interval inputs and outputs of the power plants during the six-year study.

DMU	x_{1j}^L	x_{1j}^U	g_{1j}^L	g_{1j}^U	b_{1j}^L	b_{1j}^U
1	1002243	1534381	4663820	5948123	3.6	5
2	971509	1298112	4821296	5657392	3.7	4.4
3	1331457	1831098	7220851	7699512	5.3	5.8
4	766658	1117322	3781843	4628520	2.8	3.7
5	24213	1060942	356963	3184631	0.6	3.2
6	1045455	1283541	5339780	5975686	3.8	4.4
7	412442	758142	1925856	2631210	1.7	2.3
8	446094	1017339	1836793	4289004	1.8	3.6
9	1244520	1820737	4222796	7935571	4.3	8
10	1056182	1410680	5126256	6213138	3.4	4.4
11	311239	635257	1820209	2106015	1.6	1.9
12	204	796605	515	2128410	0	2.6
13	1234922	1303468	4500169	9886102	5.2	8.3
14	422191	905874	1770332	2761553	1.9	2.9
15	147683	2769634	5008772	1030008	5.5	8.5
16	161614	928637	1258570	2678996	1.9	4.5
17	1298688	1961314	4785753	5898717	5.3	5.7
DMU	b_{2j}^L	b_{2j}^U	b_{3j}^L	b_{3j}^U	b_{4j}^L	b_{4j}^U
1	1.8	6.6	2338	3015	0	0.1
2	2.1	4.7	2367	2727	0	0.1
3	2.8	6.7	3478	3631	0	0.1
4	1.7	4.8	1779	2250	0	0.1
5	0.4	2.2	318	2119	0	0
6	1.3	3	2545	2806	0	0
7	0.1	1.1	1052	1557	0	0
8	1.1	4.2	1089	2229	0	0.1
9	0.2	12.5	2806	4788	0	0.2
10	0.2	3	2262	2802	0	0
11	1	3.3	979	1091	0	0.1
12	0	5	1	1595	0	0.1
13	4.1	11.4	3209	4993	0.1	0.2
14	0.4	2.4	1222	1848	0	0
15	3	9.2	3546	5535	0	0.1
16	1.4	7.7	985	2661	0	0.1
17	0.5	4	3382	3828	0	0.1

Table 2: Type of UC on power plants

DMU	w_1^*	w_2^*	w_3^*	w_4^*
1	-6.0832	-8.5052	-5.9656	-0.4814
2	-2.3841	-1.3779	4.9027	9.7916
3	0.8787	0.2406	0.1085	0.4814
4	7.1350	1.4970	-1.6050	7.2246
5	-1.0119	-9.7433	-2.1147	1.9193
6	0	4.3239	-1.9678	2.0056
7	-3.9112	7.3420	-9.3795	4.9994
8	-1.7631	1.0110	-7.9729	1.7415
9	7.2990	-3.1707	0.6483	1.1642
10	-6.8771	-3.5683	2.1910	4.4664
11	0	3.3867	2.0956	1.7415
12	-8.8158	0.7800	-2.9453	2.8272
13	4.0265	-8.0555	-5.1171	-4.2690
14	-6.8177	-1.4641	8.9462	2.2667
15	-3.2600	-1.4009	5.0356	4.8144
16	-9.7301	-8.1600	6.1938	-1.5242
17	6.7831	-5.6518	-2.3287	-2.9935
DMU	σ	UEN	UC	RTD
1	18.5439	1.000	Strong	Negative
2	6.0633	1.000	Strong	Negative
3	0.2487	0.874	NO	Increasing
4	1.6415	1.000	Strong	Negative
5	2.4554	1.000	Strong	Negative
6	4.3867	1.000	Weak	No
7	0.6483	1.000	Strong	Negative
8	6.1614	0.935	Strong	Negative
9	7.3453	0.810	Strong	Negative
10	3.9753	0.759	Strong	Negative
11	2.5437	1.000	Weak	No
12	8.2234	1.000	Strong	Negative
13	-0.4814	1.000	Strong	Negative
14	7.4532	1.000	Strong	Negative
15	12.5450	1.000	Strong	Negative
16	7.9997	1.000	Strong	Negative
17	21.5690	1.000	Strong	Negative

Table 3: Type of DC on power plants

DMU	u_1^*	σ	UEN	DC	DTR
1	-8.7401	7.4647	1.000	Strong	Negative
2	-5.0532	8.2407	0.862	Strong	Negative
3	-5.3249	4.2157	1.000	Strong	Negative
4	-6.9406	-3.6323	0.694	Strong	Negative
5	0	-1.9005	1.000	Weak	No
6	-1.0745	0.1319	1.000	Strong	Negative
7	-2.2626	2.5519	1.000	Strong	Negative
8	-1.9364	6.0038	0.794	Strong	Negative
9	8.2157	6.9693	1.000	NO	Decreasing
10	-7.2371	2.9668	1.000	Strong	Negative
11	-2.1954	0.8889	1.000	Strong	Negative
12	-7.1983	7.4817	1.000	Strong	Negative
13	-3.0311	2.9416	1.000	Strong	Negative
14	6.8480	-2.3890	1.000	No	Decreasing
15	0	8.1680	1.000	Weak	No
16	-4.3400	-6.4704	1.000	Strong	Negative
17	1.4970	9.0019	1.000	No	Increasing

References

[1] Banker, R.D., Charnes, A., Cooper, W.W.'Some models for estimating technical and scale inefficiencies in DEA', *Management Science*, Vol. 30, pp.(1984), 1078-1092.

[2] Ben-Tal, A., Nemirovski, A. Robust convex optimization, *Mathematics of Operations Research*, 23 (4)(1998), 769-805.

[3] Ben-Tal, A., Nemirovski, A. Robust solutions of uncertain linear programs, *Operations Research Letters*, 25 (1) (1999), 1-13.

[4] Ben-Tal, A., Nemirovski, A. Robust solutions of linear programming problems contaiminated wit uncertain data, *Mathematical Programming*, 88 (3)(2000), 411-421.

[5] Bertsimas, D., Sim, M. The price of robustness, *Operations Research*, 52 (1)(2004), 35-53.

[6] Charnes, A., Cooper, W.W., Rhodes, E. 'Measuring the efficiency of decision making units', *European Journal of Operational Research*, Vol. 2, pp.(1978), 429-444.

[7] Cooper, W.W., Park, K.S. and Yu, G. 'IDEA (imprecise data envelopment analysis) with CMDs (column maximum decision making units)', *Journal of the Operational Research Society*, Vol. 52, No. 2, pp.(2001), 176-181

[8] Cooper, W. W., Gu, B., and Li, S. Comparisons and evaluations of alternative approaches to the treatment of congestion in DEA. *European Journal of Operational Research*, 132(1)(2001), 62-74.

[9] Cooper, W.W., Seiford, L.M. and Tone, K.'Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software', 2nd edition, Springer, New York(2006).

[10] Dehnokhalaji, A., Khezri, S., and Emrouznejad, A. A box-uncertainty in DEA: A robust performance measurement framework. *Expert Systems with Applications*, 187 (2022), 115855.

[11] Färe, R., Grosskopf, S. and Lovell, C.A.K. '*The measurement of efficiency of production*', Kluwer-Nijhoff, Boston(1985).

[12] Färe, R., Grosskopf, S., and Pasurka, C. Effects on relative efficiency in electric power generation due to environmental controls. *Resources and energy*, 8(2)(1986), 167-184.

[13] Halkos, G., and Petrou, K. N. Treating undesirable outputs in DEA: A critical review. *Economic Analysis and Policy*(2019), 62, 97-104.

[14] Jahanshahloo, G. R., and Khodabakhshi, M. Suitable combination of inputs for improving outputs in DEA with determining input congestion: Considering textile industry of China. *Applied mathematics and computation*, 151(1)(2004), 263-273.

[15] Khalili-Damghani, K., Tavana, M., and Haji-Saami, E. A data envelopment analysis model with interval data and undesirable output for combined cycle power plant performance assessment. *Expert Systems with Applications*, 42(2)(2015), 760-773.

[16] Khezri, S., Dehnokhalaji, A., and Lotfi, F. H. A full investigation of the directional congestion in data envelopment analysis. RAIRO-*operations research*, 55(2021), S571-S591

[17] Mo, R., Huang, H., and Yang, L. An interval efficiency measurement in DEA when considering undesirable outputs. *Complexity*, 2020.

[18] Omrani, H. Common weights data envelopment analysis with uncertain data: A robust optimization approach, *Computers Industrial Engineering*, 66 (4)(2013), 1163-1170.

[19] Pishgar-Komleh, S. H., Zylowski, T., Rozakis, S., and Kozyra, J. Efficiency under different methods for incorporating undesirable outputs in an LCA+ DEA framework: A case study of winter wheat production in Poland. *Journal of environmental management*, 260(2020), 110138.

[20] Sadjadi, S.J., Omrani, H. Data envelopment analysis with uncertain data: An application for Iranian electricity distribution companies, *Energy Policy*, 36 (11)(2008), 4247-4254.

[21] Sadjadi, S.J., Omrani, H. A bootstapped robust data envelopment analysis model for efficiency estimating of telecommunication companies in Iran, *Telecommunication Policy*, 34 (4)(2010), 221-232.

[22] Sadjadi, S.J., Omrani, H., Makui, A., Shahanaghi, K. An interactive robust data envelopment analysis model for determining alternative targets in Iranian electricity distribution companies, Expert *Systems with Applications*, 38 (8)(2011), 9830-9839.

[23] Salahi, M., Torabi, N. Amiri, A. An optimistic robust optimization approach to common set of weights in DEA, *Measurment*, 93(2016), 67-73.

[24] Shi, X., Emrouznejad, A., and Yu, W. Overall efficiency of operational process with undesirable outputs containing both series and parallel processes: A SBM network DEA model. *Expert Systems with Applications*, 178 (2021), 115062.

[25] Streimikis, J., and Saraji, M. K. Green productivity and undesirable outputs in agriculture: a systematic review of DEA approach and policy recommendations. *Economic Research-Ekonomska Istraživanja*(2021), 1-35.

[26] Sueyoshi, T., and Sekitani, K. DEA congestion and returns to scale under an occurrence of multiple optimal projections. *European Journal of Operational Research*, 194(2)(2009), 592-607.

[27] Sueyoshi, T., and Yuan, Y. Returns to damage under undesirable congestion and damages to return under desirable congestion measured by DEA environmental assessment with multiplier restriction: Economic and energy planning for social sustainability in China. *Energy Economics*(2016), 56, 288-309.

[28] Sun, S., and Huang, C. Energy structure evaluation and optimization in BRICS: A dynamic analysis based on a slack based measurement DEA with undesirable outputs. *Energy*, 216(2021), 119251.

[29] Toloo, M., and Hančlová, J. Multi-valued measures in DEA in the presence of undesirable outputs. *Omega*, 94(2020), 102041.

[30] Tone, K.,and Sahoo, B. K. Degree of scale economies and congestion: A unified DEA approach. *European journal of operational research*, 158(3)(2004), 755-772.

[31] Wang, K., Wei, F. Robust data envelopment analysis based MCDM with the consideration of uncertain data, *Journal of Systems Engineering and Electronics*, 21 (6)(2010), 981-989.

[32] Wanke, P., Barros, C. P., and Figueiredo, O. Efficiency and productive slacks in urban transportation modes: A two-stage SDEA-Beta Regression approach. *Utilities Policy*, 41(2016), 31-39.

[33] Wei, Q., and Yan, H. Congestion and returns to scale in data envelopment analysis. *European Journal of operational research*, 153(3)(2004), 641-660.

[34] Wu, F., Zhou, P., and Zhou, D. Q. Measuring energy congestion in Chinese industrial sectors: a slacks-based DEA approach. *Computational Economics*, 46(3)(2015), 479-494.

[35] Yousefi, S., Alizadeh, A., Hayati, J., and Baghery, M. HSE risk prioritization using robust DEA-FMEA approach with undesirable outputs: a study of automotive parts industry in Iran. *Safety science*(2018), 102, 144-158. DOI: 10.1016/j.ssci.2017.10.015

[36] Zarbakhshnia, N., and Jaghdani, T. J. Sustainable supplier evaluation and selection with a novel two-stage DEA model in the presence of uncontrollable inputs and undesirable outputs: a plastic case study. *The International Journal of Advanced Manufacturing Technology*(2018), 97(5), 2933-2945. DOI: 10.1007/s00170-018-2138-z

[37] Zhao, P., Zeng, L., Li, P., Lu, H., Hu, H., Li, C., ... and Qi, Y. China's transportation sector carbon dioxide emissions efficiency and its influencing factors based on the EBM DEA model with undesirable outputs and spatial Durbin model. *Energy*, 238(2022), 121934.

[38] Zhou, Z., Xu, G., Wang, C., and Wu, J. Modeling undesirable output with a DEA approach based on an exponential transformation: An application to measure the energy efficiency of Chinese industry. *Journal of Cleaner Production*, 236(2019), 117717.

[39] Zhu, J., '*Quantitative Models for Performance Evaluation and Benchmarking: Data Envelopment Analysis with Spreadsheets and DEA Excel Solver*', Kluwer Academic Publishers, Boston(2002).

Mahsa Nassiri
PhD student of Mathematics
Department of Mathematics, Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University
Tehran, Iran
E-mail: m.nasiri@gmail.com

Mohsen Rostamy-Malkhalifih
Associate Professor of Mathematics
Department of Mathematics, science and research Branch, Islamic Azad University
Tehran, Iran
E-mail: mohsen$_r$*ostamy@yahoo.com*

Hadi Bagherzadeh Valami
Associate Professor of Mathematics
Department of Mathematics, Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University
Tehran, Iran
E-mail: hadi$_b$*agherzadehvalami@yahoo.com*