Journal of Mathematical Extension
Vol. 17, No. 2, (2023) (9)1-17
URL: https://doi.org/10.30495/JME.2023.2461
ISSN: 1735-8299
Original Research Paper

Interval Congestion In Commercial Bank Branch

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Abstract. Congestion is one of the main problems in economic issues. In fact, congestion is a wasteful step in the production process, where outputs decrease as a result of increasing inputs. In the economy, congestion is important because its elimination reduces the cost and also increases the output. Therefore, there is a great benefit in identifying congestion and reducing it. Since it is difficult to compute congestion for $DMUs$ with interval data, owing to the computational complexity of the existing methods, we first present a new method for computing congestion and then obtain congestion in the case of interval data. It is well known that if DEA inputs and outputs are in the form of intervals, there will be an efficiency interval for each DMU. Since we assume interval data in this paper, we obtain an interval for the amount of congestion possible in each DMU and prove that the interval indicates the upper and lower bounds of congestion for each DMU.

Keywords and Phrases: Data Envelopment Analysis; Congestion; Linear Programming; Efficiency; Decision Maker Unit.

1 Introduction

Congestion is an important topic in data envelopment analysis and in economics. It was first introduced by Färe and Svensson [9] and Färe and

Received: July 2022 ; Accepted: December 2022

Grosskopf [10] provided a method for its computation. Later, Cooper et al.[5, 6] also worked on congestion and presented another method to compute it. Many researchers have studied congestion issues in recent decades. Cooper et al. [3, 4] developed their previous works to create new models. Noura et al. [13] presented a method based on input consumption in effective decision-making units. Many authors developed congestion concepts based on Noura's method including Adimi [1] who proposed congestion hyperplane to separate congestion units and other units, Navidi et al. [12] who explained congestion identification method without calculation, and Shadab et al. [15] illustrated the concept of anchor point and developed a corresponding congestion identification. Sueyoshi and Goto [16] and Fang [7] researched the congestion by considering undesirable outputs, and Kheirollahi et al. [11] developed a congestion identification method for stochastic DEA and fuzzy DEA. For a review of these methods and their strengths and weaknesses, the readers are referred to Flegg and Allen [8] and Ren et al. [14]. In the present paper, using the method of Noura et al.[13], we are going to enable the computation of congestion in $DMUs$ with interval data. The new Noura's method and the related theorems are presented in Section 2. We provide the proposed method for computing congestion in the case of interval data, together with the related theorems, in Section 3. Numerical examples are given in Section 4, and the results are discussed in Section 5.

2 Background

Definition 1 (Congestion): Congestion is present when the output can be increased by reducing one or more inputs without improving any other input or output. Conversely, congestion is present when some outputs are reduced by increasing one or more inputs without improving any other input or output [13].

To explain congestion, we outline Noura et al.[13] method in this section. Suppose we have n $DMUs$ with m inputs and s outputs, and that the vectors $x_j = (x_{1j}, ..., x_{mj})^T$ and $y_j = (y_{1j}, ..., y_{mj})^T$ denote the input and output values of DMU_j, j =1,...,n , respectively. First, we solve the output-oriented BCC [2] model which assumes VRS to obtain the efficiency of each DMU.

$$
\begin{aligned}
&\varphi_o^* = \text{Max} \quad \varphi + \xi\left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right)\\
&\text{s.t}\\
&\sum_{j=1}^{n} x_{ij}\lambda_j + s_{io}^- = x_{io} \quad i = 1, \cdots, m\\
&\sum_{j=1}^{n} y_{rj}\lambda_j - s_{ro}^+ = \varphi_o y_{ro} \quad r = 1, \cdots, s\\
&\sum_{j=1}^{n} \lambda_j = 1\\
&\left(\lambda_j, s_{io}^-, s_{ro}^+\right) \geq 0 \quad j = 1, \ldots, n, i = 1, \ldots, m, r = 1, \ldots, s
\end{aligned} \tag{1}
$$

We solve the above model for each DMU and obtain the optimal solution $(\varphi^*, \lambda^*, S^{+*}, S^{-*})$ of Model (1) for each DMU. Denoting the φ^* corresponding to DMUj by φ_j^* Set E is defined as follows:

$$E = \{j \mid \varphi_j^* = 1\} \tag{2}$$

Among the $DMUs$ in set E, there exists at least one DMU, say DMU_l, that has the highest consumption in its first input component as compared with the first input component of the rest of the $DMUs$ in set E. that is to say,

$$\exists \quad (l \in E) \quad s.t \quad forall j \ (j \in E) \Longrightarrow x_{1l} \geq x_{1j} \tag{3}$$

x_{1L} is denoted by x_1^* Then we find, among the $DMUs$ in set E, a DMU, say DMU_t, that has the highest consumption in its second input component as compared with that of the rest of the $DMUs$ in set E. In other words,
$\exists \ (t \in E) \ \ s.t \ \ \forall j \ (j \in E) \Longrightarrow x_{2t} \geq x_{2j}$
x_{2t} is denoted by x_2^*. Similarly, for all input components, $i = 1, ..., m$ we can find a DMU in set E whose i^{th} input consumption is higher than that of all other $DMUs$ in set E. We denote such input by x_i^* ,

$i = 1, ..., m$ Note that $x_1^*, x_2^*, ..., x_m^*$ need not necessarily be selected from a single DMU. Now, congestion is defined as follows.

Definition 2: Congestion is said to occur if only in an optimal solution $(\varphi^*, \lambda^*, S^{+*}, S^{-*})$ of (1) for DMU_o, at least one of the following two conditions is satisfied:

(i) $\varphi^* > 1$ and there is the least one $x_{io} > x_i^*$, $i = 1, ..., m$.
(ii) there exists at least one $s_r^{+^*} > 0$ $(r = 1, ..., s)$ and at least one $x_{io} > x_i^*$, $i = 1, ..., m$ the amount of congestion in the i^{th} input of DMU_o is denoted by $s_i^{c'}$ where $x_{io} > x_i^*$ and define it as:

$$s_i^{c'} = x_{io} - x_i^* \tag{4}$$

and congestion isn't present where $x_{io} \leq x_i^*$ and $s_i^{c'} = 0$. The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o. To demonstrate the validity of this method, see Noura et al.[13].

Illustrative example: For the purpose of clarification, consider seven hypothetical $DMUs, A, B, C, D, E, F$, and G, in Fig.1, each using one input, x, to produce one output, y.

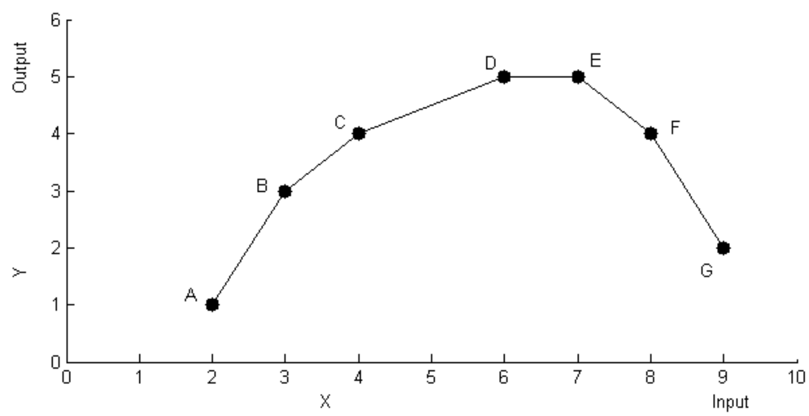

Figure 1: Numerical example. Source: Tone and Sahoo [17]

This example was solved by Cooper et al. [5], the results of which are provided in Table 1.

Table 1: Results from Cooper et al. [5] approach.

DMU	Input	output	φ^*	S^{-c^*}
A	2	1	1	0
B	3	3	1	0
C	4	4	1	0
D	6	5	1	0
E	7	5	1	0
F	8	4	1.25	1
G	9	2	2.5	2

Now we solve the same problem using our method. Considering the efficiency of $DMUs$, we have set $E = \{A, B, C, D, E\}$,and regarding relation(3)we have:
$X^* = x_E = 7 \geq x_j \quad \forall\, j \ (j \in A)$

As is known, the necessary condition for congestion is inefficiency. Thus, congestion is calculated as follows, using the proposed method.

$\varphi_F^* > 1 \quad , \quad x_F = 8 \quad S^{\acute{c}} = 8 - 7 = 1$
$\varphi_G^* > 1 \quad , \quad x_G = 9 \quad S^{\acute{c}} = 9 - 7 = 2$

As can be observed in table 2, the results of the proposed method and those from Cooper et al.'s [3] method are identical.

Table 2: Results from the new method

DMU	A	B	C	D	E	F	G
$S^{C'}$	0	0	0	0	0	1	2

3 Computation of interval congestion

Consider n $DMUs$ with m inputs and s outputs, with interval data, that is:

$x_{ij} \in [x_{ij}^l, x_{ij}^u] \quad i = 1, ..., m$
$y_{rj} \in [y_{rj}^l, y_{rj}^u] \quad r = 1, ..., s$

In order to measure congestion with interval data, we should first determine the efficiency interval of each DMU. To do so, we obtain the efficiency of each DMU in the most pessimistic and the most optimistic cases, using the following two models introduced by Wang, Greatbanks, and Yang [18]. In Model (5), which is the most pessimistic case in evaluating a DMU, we consider the unit under assessment with the highest inputs and the lowest outputs while the other $DMUs$ are assumed to have the lowest inputs and the highest outputs.

$$\begin{aligned} &\varphi_o^{*u} = \max \varphi \\ &\text{s.t} \\ &\sum_{\substack{j=1 \\ j\neq o}}^{n} x_{ij}^l \lambda_j + \lambda_o x_{io}^u + s_{io}^- = x_{io}^u \quad i = 1, \ldots, m \\ &\sum_{\substack{j=1 \\ j\neq o}}^{n} y_{rj}^u \lambda_j + \lambda_o y_{ro}^l + s_{ro}^+ = \varphi_o y_{ro}^l \quad r = 1, \ldots, s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\left(\lambda_j, s_{io}^-, s_{ro}^+\right) \quad j = 1, \ldots, n, i = 1, \ldots, m, \quad r = 1, \ldots, s. \end{aligned} \tag{5}$$

As for the most optimistic case, Model (6) assumes the unit under evaluation with the lowest inputs and the highest outputs and the other $DMUs$ with the highest inputs and the lowest outputs.

$$\begin{aligned} &\varphi_o^{*l} = \max \varphi \\ &s.t \\ &\textstyle\sum_{\substack{j=1 \\ j\neq o}}^{n} x_{ij}^u \lambda_j + \lambda_o x_{io}^l + s_{io}^- = x_{io}^l \quad i = 1, \ldots, m \\ &\textstyle\sum_{\substack{j=1 \\ j\neq o}}^{n} y_{rj}^l \lambda_j + \lambda_o y_{ro}^u + s_{ro}^+ = \varphi_o y_{ro}^u \quad r = 1, \ldots, s \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ &\left(\lambda_j, s_{io}^-, s_{ro}^+\right) \quad j = 1, \ldots, n, i = 1, \ldots, m, \quad r = 1, \ldots, s \end{aligned} \tag{6}$$

After determining the efficiency interval, we propose the Noura et al.13] method for computing congestion. With regard to the idea in this method, the highest input value for each component is specified to compute congestion among efficient $DMUs$. As was stated earlier, one need not consider a single DMU for selecting all the components. To this end, we define the set as follows.

$$E' = \{DMU_j \mid \varphi_j^{*l} = 1\} \tag{7}$$

E' is the largest efficient set that can possibly exist with the above data, i.e., it is the set of $DMUs$ that are efficient in the best case. Our aim is to determine a congestion interval (i.e., upper and lower bounds) such that the congestion value associated with any combination of values occurring in the input and output intervals of a DMU belongs to the interval obtained. Considering the fact that inefficiency is the necessary condition for congestion, there exists no congestion in $DMUs$ with $\varphi^{*L} = \varphi^{*u} = 1$ Furthermore, since the $DMUs$ in set E' are efficient in their best case, they do not exhibit congestion in this case. However, these $DMUs$ might be inefficient in their worst case. Thus, there exists the possibility of congestion in this case for these $DMUs$. In computing congestion in the most optimistic case possible, the lowest input consumption of a DMU is compared with the highest input consumption of the $DMUs$ belonging to set E' (i.e., those efficient in the best case). To this end, we find x_i^{*u} as follows:

$$\forall i \quad i = 1, ..., m \quad \exists\ t_i \quad s.t \quad x_{ti}^u = x_i^{*u} = Max\{x_{ij}^u \mid j \in E'\} \tag{8}$$

And in the most optimistic case possible, the highest input consumption of a DMU is compared with the lowest input consumption of the $DMUs$ belonging to set (i.e., those efficient in the best case). We obtain x_i^{l*} as follows:

$$\forall i, \quad i = 1, ..., m \quad \exists\ k_i \quad s.t \quad x_{k_i}^l = x_i^{*l} = Min\{x_{ij}^l \mid j \in E'\} \tag{9}$$

Now, we denote the lower bound of congestion in the i^{th} input of DMU_o by s_{io}^{cl} and define it as:

$$s_{io}^{cl} = x_{io}^l - x_i^{u*}, \quad i = 1, ..., m \tag{10}$$

If $s_{io}^{cl} \geq 0$ the amount of congestion is indicated; otherwise, congestion is zero in the best case. Moreover, we denote by $s_{io}^{cu}$the upper bound of congestion in the i^{th} input of $DMU_o$and define it as:

$$s_{io}^{cu} = x_{io}^{u} - x_i^{l*}, \quad i = 1, ..., m \tag{11}$$

If $s_{io}^{cu} \geq 0$ the amount of congestion is shown; otherwise, congestion is zero in the worst case.
Theorem: The interval $[s_o^{cl}, s_o^{cu}]$ indicates an upper and a lower bound for congestion present at DMU_o.
Proof: By contradiction, we assume that s_i^{-c} is the amount of congestion in the i^{th} input of DMU_o such that $s_{io}^{-c} < s_{io}^{cl}$ So, there exists an efficient $\overline{DMU}$ such that $\overline{DMU} \in E'$ and $s_{io}^{-c} = x_{io}^{l} - \overline{x}_i$. Regarding the assumption, we have the following relations:
$s_{io}^{-c} < s_{io}^{cl} \implies x_{io}^{l} - \overline{x}_i < x_{io}^{l} - x_i^{*u} \implies \overline{x}_i > x_i^{*u}$
Since $\overline{DMU} \in E'$ this is a contradiction to the definition of x_i^{*u}.
Similarly, for the upper bound, we suppose that s_i^{+c} is the amount of congestion in the i^{th} input of DMU_o such that $s_{io}^{+c} > s_{io}^{cu}$.
Then, there exists an efficient $\widetilde{DMU}$ such that $\widetilde{DMU} \in E'$ and $s_{io}^{+c} = x_{io}^{u} - \widetilde{x}_i$ from which the following can be concluded.

$$s_{io}^{+c} < s_{io}^{cu} \;\; x_{io}^{u} - \widetilde{x}_i > x_{io}^{u} - x_i^{*l} \quad Longrightarrow \;\; x_i^{*l} > \overline{x}_i$$

As $\widetilde{DMU} \in E'$ the above relations contradict the definition of x_i^{*l} Therefore, the interval $[s_o^{cl}, s_o^{cu}]$ indicates an upper and a lower bound for the congestion present at DMU_o.

4 Numerical Example

Example 1: Consider the interval data of Table 3 (six units with one input and one output).

Table 3: Interval Data

DMU_j	Input X	output Y	φ_l^*
A	[2,3]	[1,3]	1
B	[4,6]	[5,6]	1
C	[8,9]	[6,8]	1
D	[7,10]	[2,3]	1.72
E	[12,13]	[5,7]	1
F	[14,15]	[3,4]	1.38

Considering Model (6), set E' is defined as
$E' = \{A, B, C, E\}$
By relation (8), we have $X^{*u} = 13$,and by relation (9), $X^{*l} = 2$.
Finally, using relations (5) and (6), we compute the lowest possible amount of congestion (S^{cl}) and the highest possible amount of congestion (S^{cu}) , the results of which are presented in table 4.

Table 4: The Interval Congestion (IC).

DMU	A	B	C	D	E	F
IC	[0,1]	[0,4]	[0,7]	[0,8]	[0,11]	[1,13]

Example 2: This section considers an entire real data set of the Iranian banking industry in 1395.
Table 5, Table 6, and Table 7 show the data set for inputs and outputs respectively.This data is regarding 36 banks with 4 inputs and 5 which is the set of inputs including:Payable interest, Personnel, Non-performing loans, Number of branches and the set of outputs including: The total sum of four main deposits, Other deposits, loans granted, received interest, Fee.

Table 5: The set of interval inputs

DMU_j	IN_1^l	IN_1^u	IN_2^l	IN_2^u	IN_3^l	IN_3^u	IN_4^l	IN_4^U
1	7205.49	9613.37	37.04	37.65	0	84759	1	1
2	11555.5	15532.94	180.01	180.59	60699	61958	31	31
3	93796.62	126080.5	483.47	484.13	264789	276331	52	52
4	72343.26	96673.59	522.44	524.54	264789	203463	53	53
5	26138.65	36009.31	351.77	355.1	264789	83063	48	48
6	83466.53	126996.1	280.83	281.7	264789	514770	36	36
7	109503.1	148663.8	405.03	407.83	264789	99109	46	46
8	62495.56	84976.67	473.27	475.01	264789	312233	49	49
9	14952.54	19974.47	262.58	264.11	264789	100550	46	46
10	6368.1	8610.57	148.68	151.45	264789	29690	34	34
11	68143.25	91420.46	761.84	761.88	264789	56339	141	141
12	23356.27	31671.65	553.35	553.37	264789	63011	98	98
13	774.21	1033.36	28.62	28.67	264789	2294	6	6
14	7635.24	10211.72	181.39	181.94	264789	19690	45	45
15	8859.43	12098.68	186.67	187.25	264789	22074	48	48
16	52132.18	69644.45	704.89	707.47	264789	264966	144	144
17	4391.69	5904.57	136.97	136.97	264789	26135	33	33
18	5109.53	7579.14	48.37	48.78	264789	21150	14	14
19	21864.94	29790.28	430.06	435.85	264789	115841	89	89
20	4242.14	5715.69	104.75	105.05	264789	21621	28	28
21	5955.67	8842.88	153.26	153.35	264789	24258	34	34
22	6299.74	8538.53	144.66	147.42	264789	41735	30	30
23	13054.13	17588.21	428.25	431.41	264789	61578	97	97
24	20110.62	27252.82	465.78	466.47	264789	93704	80	80
25	3799.24	5096.07	137.24	138.57	264789	22765	30	30
26	2154.2	2896.07	65.98	66.45	264789	20220	18	18
27	4010.09	5350.62	127.33	127.44	264789	23367	27	27
28	6518.7	8779.18	113.79	114.59	264789	11205	26	26
29	2193.41	2956.67	166.52	166.52	264789	33618	36	36
30	8730.39	11379.31	255.99	256.65	264789	120629	57	57
31	10841.7	14582.94	307.33	307.67	264789	44077	69	69
32	40351.39	54564.38	1100.13	1103.05	264789	373142	195	195
33	8875.07	12683.31	180.26	180.44	264789	19440	38	38
34	6381.64	8621.95	154	154.08	264789	19242	35	35
35	8804.09	11987.37	218.32	218.49	264789	38563	43	43
36	20702.36	28061.32	532.5	534.96	264789	10288	93	93

Table 6: The set of interval outputs

DMU_j	OUT_1^l	OUT_1^u	OUT_2^l	OUT_2^u	OUT_3^l	OUT_3^u
1	3126798	3329887	263643	297174	1688579	1853365
2	1007068	1032209	45558	47213	584223	603535
3	5288816	5612194	482663	1029508	4605062	4915352
4	4942133	5055067	675226	751987	4246849	4451299
5	2333377	2352032	572648	587191	2605504	2658741
6	4058877	4086246	565479	572085	6837209	7229407
7	5944936	5963247	338858	350118	4249646	4310802
8	4431580	4856452	1558338	1619029	6617095	15404277
9	1076318	1086599	105165	130035	1026113	1036956
10	544571	561205	14932	17191	158314	160966
11	3616769	3668523	185694	207215	1178105	1233188
12	1377539	1395097	50862	61601	866787	885057
13	105914	111405	13968	20620	50994	51623
14	522050	531252	19809	23326	263764	264577
15	531538	545604	26046	50998	360624	388262
16	2784822	2816198	174751	228157	1832632	1845860
17	367709	375766	20619	21099	306978	316199
18	233929	235131	5727	6612	269196	275886
19	1864656	1883405	108135	114512	1061087	1110637
20	342251	409514	21650	23472	402252	408952
21	486802	489644	35165	41759	431241	459395
22	416258	425121	16530	18253	422150	423538
23	1031720	1048564	32392	34059	717839	742439
24	1503836	1518312	119584	125579	1188397	1246112
25	349075	365397	11092	13324	277316	283338
26	176940	177291	8075	9544	287681	296160
27	389967	397498	7174	7176	361642	380091
28	494267	497640	19702	21464	352431	368410
29	431522	437192	17788	23814	271441	280821
30	973594	1052715	54067	60694	672627	761748
31	809052	824138	26562	35747	743752	765280
32	2740688	2883002	144768	162295	2422523	2476927
33	659625	697401	17020	18101	1129728	1163837
34	428903	432605	15859	22450	242158	254390
35	713887	717260	34498	35503	583041	598293
36	1406476	1455764	60601	71063	1156873	1231176

Table 7: The set of interval outputs

DMU_j	OUT_4^l	OUT_4^u	OUT_5^l	OUT_5^u
1	116411.5	125740.3	4415.18	6957.33
2	92147.97	101954.3	1338.81	1933.16
3	410530.9	458971.4	9169	11675.5
4	1124745	1170466	3054.98	4165.45
5	270922.2	301877.3	8575.23	9823.13
6	1501993	1651658	7779.65	9597.59
7	282866.2	306716.7	5656.56	7749.21
8	525051.3	556634.3	13753.4	16761.5
9	136122.9	141755.8	1678.97	2037.73
10	24315.68	25769.91	604.85	760.47
11	195623	212174.7	3233.38	3878.89
12	134680.2	144835.5	1241.18	1542.24
13	3123.51	11145.17	143.72	330.08
14	30823.73	32961.04	673.87	823.72
15	45099.73	50012.51	849.6	1090.69
16	364781.4	396005.1	3825.39	4499.13
17	60829.67	66704.81	1564.88	1800.7
18	46033.25	47256.77	589.35	744.73
19	198796	214657.2	5127.03	5921.28
20	67906.38	73139.83	950.85	1119.49
21	55022.14	62111	1631.48	1933.55
22	96809.93	104604.2	834.09	959.48
23	104229.2	114658	2305.57	2599.56
24	169230.1	185292.4	3529.17	4478.81
25	48878.34	53301.03	1181.18	1296.07
26	62581.02	69356.54	675.57	786.42
27	79410.12	83297.25	918.24	1050.32
28	74544.62	78644.5	1196.42	1625.97
29	48070.58	52314.21	1653.17	1939.15
30	141401	158207.3	1922.09	2653.74
31	162685.8	173461.9	2052.31	2323.59
32	323326.9	349511.8	5928.62	6817.82
33	179966	194130.7	2202.09	2608.02
34	57194.69	60728.98	1197.48	1368.19
35	87768.63	98287.35	882.07	1038.17
36	182569.1	198089.6	2480.62	3013.47

By using Model (6), the most optimistic case of efficiency for the above DMU_s is obtained, as presented in Table 8:

Table 8: .The results of model (6)

DMU_j	1	2	3	4	5	6	7	8	9
φ_j^*	1.0000	1.5343	1.000	1.000	1.000	1.000	1.000	1.000	1.3438
DMU_j	10	11	12	13	14	15	16	17	18
φ_j^*	2.1399	1.000	1.6829	1.000	1.7030	1.5407	1.2515	1.000	1.2024
DMU_j	19	20	21	22	23	24	25	26	27
φ_j^*	1.000	1.000	1.000	1.000	1.4105	1.1384	1.0052	1.000	1.000
DMU_j	28	29	30	31	32	33	34	35	36
φ_j^*	1.000	1.000	1.000	1.000	1.0667	1.000	1.1037	1.000	1.000

Regarding the φ^{*l} for each DMU, set E' is defined as follows.

$$E' = \{1, 3, 4, 5, 6, 7, 8, 11, 13, 17, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 33, 36\}$$

Considering relations (8) and (9), we obtain x_i^{*l}, x_i^{*u} as follows.

$$x_1^{*l} = x_{1.13}^l = 774.21 \quad , x_2^{*l} = x_{2,13}^l = 28.62 \quad , x_3^{*l} = x_{3,1}^l = 0 \quad , x_4^{*l} = x_{4,1}^l = 1$$
$$x_1^{*u} = x_{1,7}^{u\prime} = 148663.81, x_2^{*u} = x_{2,11}^u = 761.88, x_3^{*} = x_{3,6}^u = 514770, x_4^{*u} = x_{4,11}^u = 141$$

And finally, we use relations (10) and (11) to determine the congestion interval for each DMU, as demonstrated in Table (9).

Table 9: The results of estimation of interval congestion

DMU_j	$[s_1^{cl}, s_1^{cu}]$	$[s_2^{cl}, s_2^{cu}]$	$[s_3^{cl}, s_3^{cu}]$	$[s_4^{cl}, S_4^{cu}]$
1	[0 , 8839.16]	[0 , 9.03]	[0 , 84759]	[0 , 0]
2	[0 , 14758.73]	[0 , 151.97]	[0 , 61958]	[0 , 30]
3	[0 , 125306.3]	[0 , 455.51]	[0 , 276331]	[0 , 51]
4	[0 , 95899.38]	[0 , 495.92]	[0 , 203463]	[0 , 52]
5	[0 , 35235.1]	[0 , 326.48]	[0 , 83063]	[0 , 47]
6	[0 , 126221.9]	[0 , 253.08]	[0 , 514770]	[0 , 35]
7	[0 , 147889.6]	[0 , 379.21]	[0 , 99109]	[0 , 45]
8	[0 , 84202.46]	[0 , 446.39]	[0 , 312233]	[0 , 48]
9	[0, 19200.26]	[0 , 235.49]	[0 , 100550]	[0 , 45]
10	[0 , 7836.36]	[0 , 122.83]	[0 , 29690]	[0 , 33]
11	[0, 90646.25]	[0 , 733.26]	[0 , 56339]	[0 , 140]
12	[0 , 30897.44]	[0 , 524.75]	[0 , 63011]	[0 , 97]
13	[0 , 259.15]	[0 , 0.05]	[0 , 2294]	[0 , 5]
14	[0 , 9437.51]	[0 , 153.32]	[0 , 19690]	[0 , 44]
15	[0 , 11324.47]	[0 , 158.63]	[0 , 22074]	[0 , 47]
16	[0 , 68870.24]	[0 , 678.85]	[0 , 264966]	[3 , 143]
17	[0 , 5130.36]	[0 , 108.35]	[0 , 26135]	[0 , 32]
18	[0 , 6804.93]	[0 , 20.16]	[0 , 21150]	[0 , 13]
19	[0 , 29016.07]	[0 , 407.23]	[0 , 115841]	[0 , 88]
20	[0 , 4941.48]	[0 , 76.43]	[0 , 21621]	[0 , 27]
21	[0 , 8068.67]	[0 , 124.73]	[0 , 24258]	[0 , 33]
22	[0 , 7764.32]	[0 , 118.8]	[0 , 41735]	[0 , 29]
23	[0 , 16814]	[0 , 402.79]	[0 , 61578]	[0 , 96]
24	[0 , 26478.61]	[0 , 437.85]	[0 , 93704]	[0 , 79]
25	[0 , 4321.86]	[0 , 109.95]	[0 , 22765]	[0 , 29]
26	[0 , 2121.86]	[0 , 37.83]	[0 , 20220]	[0 , 17]
27	[0 , 4576.41]	[0 , 98.82]	[0 , 23367]	[0 , 26]
28	[0 , 8004.97]	[0 , 85.97]	[0 , 11205]	[0 , 25]
29	[0 , 2182.46]	[0 , 137.9]	[0, 33618]	[0 , 35]
30	[0 , 10605.1]	[0 , 228.03]	[0 , 120629]	[0 , 56]
31	[0 , 13808.73]	[0 , 279.05]	[0 , 44077]	[0 , 68]
32	[0,53790.17]	[338.25 , 1074.43]	[0 , 373142]	[54 , 194]
33	[0 , 11909.1]	[0 , 151.82]	[0 , 19440]	[0 , 37]
34	[0 , 7847.74]	[0 , 125.46]	[0 , 19242]	[0 , 34]
35	[0 , 11213.16]	[0 , 189.87]	[0 , 38563]	[0 , 42]
36	[0 , 27287.11]	[0 , 506.34]	[0 , 10288]	[0 , 92]

5 Conclusion

In this paper, we presented a method to identify and measure congestion with interval data. In solving linear programming problems with interval data, the smaller the number of solved models and their complexity, the smaller the solution intervals and the more suitable for analysis. Since Nora et al.'s method has less computational burden than the existing methods for identifying congestion, it has been used to identify congestion in interval data. For example, to calculate congestion in Cooper et al., [4] two linear programming problems must be solved. But in Nora et al.'s method, instead of solving a model, the subtraction of two numbers are used to calculate the congestion. That is why we face less computational burden using this method in interval data. Ren et al. [14] is a useful resource for further information on any of the methods for measuring congestion and their strengths and weaknesses. We used the method to address congestion for obtaining the highest and lowest amount of congestion in the interval case. This method can also be employed for other non-deterministic data types, such as fuzzy and probabilistic data.

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