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Original Research Paper

Portion Reduction Procedure in the Two-Stage Network DEA

M. Zoriehhabib *
Sofian Branch, Islamic Azad University

M. Rostamy-Malkhalifeh
Science and Research Branch, Islamic Azad University

F. Hosseinzadeh Lotfi
Science and Research Branch, Islamic Azad University

Abstract. Data Envelopment Analysis (DEA) can be applied to a network structure systems. Considering two-stage network, this paper improves a leader-follower structure. Likewise, it is not possible to reduce the production of undesirable outputs without any cost. The motivation of this study is handling a fair treatment of the undesirable outputs in leader-follower structure. In the presence of the additional inputs in the second stage, this approach searches the proportional abatement in desirable and undesirable factors. Besides, it focuses on the less sacrifice of the desirable products. Moreover, the theoretical contributions of the proposed model have been illustrated in 13 poultry industry and 25 power plants in Iran.

AMS Subject Classification: MSC code1; MSC code 2; more
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*Corresponding Author

1 Introduction

Data Envelopment Analysis (DEA) as a non-parametric technique, was narrated by Charnes, Cooper, and Rhodes (CCR) [2]. DEA is able to evaluate the relative efficiency as well as contributing the developments, particularly in economic and management sciences. Many scholars are interested in modeling undesirable outputs (such as pollutant emissions and energy wasted, ...) of production activities. Frequently, the main aims of the managers are increasing desirable outputs, reducing undesirable ones, and controlling pollutant outputs of a set of decision-making units, (DMUs). (e.g., [19]).

As a role of thumb, setting the undesirable outputs in the optimization procedure knots to environmental issues. Recently, environmental protection and efficiency assessment have been discussed simultaneously. The researchers in [14] emphasize on the environmental protection and economic development. This study analyzes the relationship between national economic development and greenhouse gas emissions by dynamic DEA approach in European Union countries. The dynamic DEA can be applied to period specific efficiency measuring based on the long time optimization during the whole period. Therefore, this method can be applicable in particular cases. To evaluate the energy efficiency of China's transportation sector, authors in [22] have used two parallel subsystems (passenger and freight).This research faces a challenge while achieving a sustainable development goal in the transportation system precisely. It is determining the proportion of shared resource to confirm to reality. A noticeable contribution of DEA in the environment protection is presented in [7]. Authors in this article summarized the characteristics of air pollution protection development policies for 30 province-level areas in China. They explored economic and environmental performance perspective. This model is based on a single production system the internal without regarding differences of evaluation units.

Different assumptions are imposed to relax production computation. Particularly, dividing the single-stage system into sub-processes is developed as an important part of a real-life DEA-based models. Nowadays opening the black-box system and optimizing the multi-phase structure is expanded rapidly in many theoretical articles and applications. This is the main priority of the current paper that builds a network model

and flows in the interior structure. Some authors (e.g., [20 and 10]) have provided reviews about treating $DMUs$ in various models of the optimization problems by examining their structures as a network system. Traditional two-stage models normally disregarded undesirable intermediate products. But usually, managers declare their aim to reduce these products at each stage of the production process. Recently, some DEA approaches were taken into consideration, to estimate the two-phase feature at the presence of undesirable products by applying the definition of the weak-disposability. This concept was expressed originally by Shephard [18] and is powerfully alive in modeling efficiency evaluation (e.g., [9-5]). As yet, weak-disposability is considered not only for the production assessment process but also for the pollution abatement and cost function in the optimization problems. This parameter allows for the simultaneous contraction of desirable factors and waste outputs.
Another concept for relaxation of efficiency measurement is the game theory approach. This concept is presented in DEA-based studies since 2008 [13]. One of the articles that explores the leader-follower model, by applying the weak-disposability assumption, is [15]. In this article, undesirable outputs were assumed in the form of their additive inverse. Seminal work of [3] is performing a non-cooperative relationships on the efficiency of the Chinese industrial water system. This technology is characterized by a bargaining game framework. This issue have been employed for assessing the environmental efficiency score of $DMUs$ in different applications. Step by step, models are going to be developed and much modified, according to real-world requirements. Inspection of the efficiency measure appears while undesirable outputs are produced and external inputs are added to the second stage. However, giving the priority to decreasing of the undesirable products is the missing link that none of the previous papers are presented. As it is stated, scholars seldom prepare particular models for adopting less sacrifice of desirable factors. The main difference in the various network methods can be found in the structures that the preference of economic or environmental aims are applied to the model. The other difference is which technology has been adopted.
Imagine that a system desires to produce less undesirable factors while it wants to utilize less input factors. For measuring the efficiency of

DMU_o in such a system, it should be referred to a centralized model where the two-stage procedure is assumed as one stage process[1]. Then, both of the stages jointly specify one optimal plan to optimize the total efficiency of the whole system. However, it is clear that most of phenomena in the nature have a network structure. Leaving many technologies untouched, it is foccued on the two-stage approach that treated undesirable outputs by applying the weak-disposability assumption. In this respect, the authors in [14] modified a model that preserved fair evaluation of desirable and undesirable products in the single process based on the weak-disposability concept. According to that plan, a two-stage model is modified. Then, the optimized portion oriented measures of the first stage are applied as inputs to the second stage. In fact, these are intermediate measures that are employed as inputs to the second stage. Therefore, the current paper attempts to investigate a plan for a fair reduction of undesirable factors at both stages. The strategy of this research is that a good product of the system should be used by the system itself. At the same time, the sacrifice of the desirable outputs in both stages is presumed. Meanwhile, the environmental perspective of the efficiency assessment is. Both of these patterns happen in the model simultaneously. But a question arises: " How can we control less waste of the desirable factors?" Regarding to this plan, the suggested model applies leader-follower game theory to maintain each stage. Performance evaluation in the process is established due to the importance of fair treatment for waste products and desirable factors simultaneously. This proposed model not only provides a fair optimization at each stage, but also provides the optimization score for the whole system that was inspected in [24].

The rest of the paper is organized as follows: In the section below, we argue some basic DEA structures, a black box system, a general two-stage network structure, and a leader-follower structure briefly. A portion-oriented process extended into a two-stage system, as a proposed model, in the third section. Moreover, two individual examples are narrated in section four, as a significant contribution of this research by adopting the proposed model. A real case on a poultry farm in Guilan province, Iran, is highlighted. Also, we revisit the application of the proposed model in 25 power plant industries in Iran. Both of the empirical studies are discussed and compared with a general two-stage model. The results are presented by regarding the proposed model structure then they are compared with a general model, in order to make more clarification. Conclusion section extension appears in the last section.

2 Preliminaries and Background

This paper compares a centralized model and a proposed two-stage network structure. Both of proposed models are comprised with each other in two examples. According to the proposed network strategy, the undesirable products are considered at both stages. These products leave the system. However, reducing the undesirable outputs have more priority to the operation managers. On the other hand, the proposed method obeys the leader-follower design for the analysis and investigation of the two-stage systems. The proposed method deals with the network structure based on the leader-follower project and follows up reducing undesirable factors. The scenario of the centralized model and the two-stage model are comprised too. In the following essential basic concepts are discussed.

2.1 Black box system

Managers expect profit growth in the industry along with the protection of the environment nowadays. Industrialization makes experts conduct their process to be more productive and at the same time, they attempt to preserve the environment. It is clear that, the problem of pollutant emission of the industry or other waste in the environmental system is a major concern. Although, classical DEA models rely on the assumption that each DMU can improve its performance by increasing its current output level and decreasing its current input levels [4], the measures that are characterized by the property of being undesirable, are produced aside from the desired outputs and should not be ignored. Meanwhile, management scientists and economy experts have set modeling the undesirable outputs in detail because they regard it as one of the considerable importance. For the best of our knowledge, according to [5] weak-disposability technology was created and was applied to the production process of the suggested model. The Weak-disposable axiom assumes that desired factors and the generation of the waste outputs are not separated from each other and they are produced jointly. While desirable output is produced, the generation of undesired outputs cannot be embedded into zero. The null-joint feature of desirable and undesirable outputs can be an important challenge for scholars to give a share for both of them while planning to increase good products and decrease waste outputs. Applying this assumption convinces us to model undesirable outputs as outputs. Suppose that there is a set of $DMUs$. Each DMU is denoted by $DMU_j (j = 1, ..., N)$. Each unit has M inputs $x_j = (x_{1j}, ..., x_{Mj}) \geq 0$ to generate R desirable outputs $v_j = (v_{1j}, ..., v_{Rj}) \geq 0$ and K undesirable outputs $w_j = (w_{1j}, ..., w_{Kj}) \geq 0$ (Figure 1).

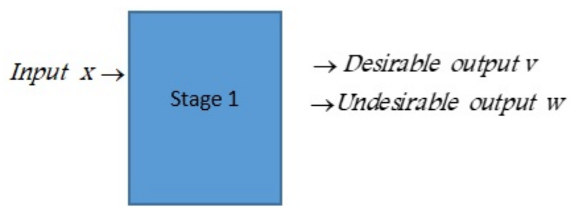

Figure 1: Black box system with undesirable output.

Furthermore, assume that $x_j \neq 0$, $v_j \neq 0$ and $w_j \neq 0$. The production technology can be represented by:

$$P(x) = \{(v, w)|x \; can \; produce(v, w), x \in R_M^+\}.$$

In fact, this production technology transfers the set of all feasible inputs into the set of all feasible desirable and undesirable outputs.

Definition 2.1. *Outputs (desirable and undesirable) are weakly disposable if* $0 \leq \theta \leq 1$ *implies that*

$$(\theta v, \theta w) \in P(x), x \in R_M^+([18]).$$

In other words, the reduction of waste factors cannot be achieved unless reducing the desirable outputs [21]. The contraction parameter θ corresponds to Shephard's definition of weak-disposability. This parameter allows for the simultaneous contraction of good and bad outputs.

Definition 2.2. *If* $(\theta v, \theta w) \in P(x)$ *and* $w = 0$ *then* $v = 0$ *(null-joint ness of the desirable and undesirable outputs)* [5]. This means that undesirable outputs can be obtained only if desirable outputs are produced. $P(x)$ can named as an environmental output set if both of the above definitions are taken into assumption. Moreover, note that the weak-disposability axiom is used widely and it gives precise results by applying to the network structure.

2.2 A general two-stage network structure

There is assumed a two-stage network process that the outputs from the first stage become inputs to the second stage. These intermediate factors play the role of outputs from the first stage and become inputs for the second stage at the same time. In some real cases, the second stage has its own inputs too. Suppose that there are N $DMUs$ that are denoted by $DMU_j(j = 1, ..., N)$. For the first stage of DMU_j the observed data on the vectors of inputs, desirable outputs and undesirable outputs are denoted by $x^{(1)}_{ij}(i = 1, ..., M)$, $v_{rj}(r = 1, ..., R)$ and $w_{kj}(k = 1, ..., K)$ respectively. The undesirable outputs w_j leaves the process. While, the output v_j can be used as the inputs for the second stage. The second stage employs (v_j) and an external input vector $x^{(2)}_{pj}(p = 1, ..., P)$ to generate both desirable and undesirable outputs. Vectors $y_{fj}(f = 1, ..., F)$ and $b_{hj}(h = 1, ..., H)$ are presenting final measure of the desirable and the undesirable output. Figure2 depicts the two-stage network structure.

As figure2 shows, establishing network system is a proper plan to reach the real-world objectives. Each stage is characterized by its own exogenous inputs and outputs, then a process offers intermediate flows in the system. According to this figure, undesirable intermediate output has left the first stage. Therefore, the second stage fed up with optimal desirable intermediate measure and external input to generate the final output y.

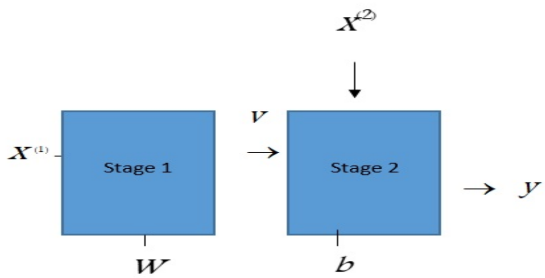

Figure 2: A general two-stage system.

The following two-stage centralized structure for evaluation of a specific DMU_o, is an example of a general system. For measuring the efficiency of DMU_o with abatement in undesirable factors and the reduction of inputs, the following linear programming problem can be solved. Furthermore, the two-stage procedure is assumed one stage process. Then, both of the stages jointly specify one optimal plan to optimize the total efficiency of the whole system. Assume that, $x^{(1)}_{io}$ is the ith $(i = 1, ..., M)$ input , v_{ro} is the rth $(r = 1, ..., R)$ desirable output and w_{ko} is kth $(k = 1, ..., K)$ undesirable output of the observed DMU_o in the first stage. Also, $x^{(2)}_{po}$ is the pth $(p = 1, ..., P)$ input, y_{fo} is the fth $(f = 1, ..., F)$ desirable output and b_{ho} is hth $(h = 1, ..., H)$ undesirable output of the observed DMU_o in the second stage.

$$\phi = min(1/2)\big[(\frac{1}{M+K})(\sum_{i=1}^{M} \alpha_i + \sum_{k=1}^{K} \theta_k) + (\frac{1}{P})\sum_{p=1}^{P} \delta_p\big]$$

$$s.t$$

$$stage\ 1$$

$$\sum_{j=1}^{N} \lambda_j x^{(1)}_{ij} \leq \alpha_i x^{(1)}_{io} \qquad i = 1, ..., M$$

$$\sum_{j=1}^{N} \lambda_j v_{rj} + s = v_{ro} \qquad r = 1, ..., R$$
$$\sum_{j=1}^{N} \lambda_j w_{kj} = \theta_k w_{ko} \qquad k = 1, ..., K$$
$$\lambda_j \geq 0 \qquad j = 1, ..., N$$
$$stage 2 \qquad\qquad\qquad Model\ 1$$
$$\sum_{j=1}^{N} \lambda'_j x^{(2)}_{pj} \leq \delta_p x^{(2)}_{po} \qquad p = 1, ..., P$$
$$\sum_{j=1}^{N} \lambda'_j y_{fj} \geq y_{fo} \qquad f = 1, ..., F$$
$$\sum_{j=1}^{N} \lambda'_j b_{hj} = b_{ho} \qquad h = 1, ..., H$$
$$\lambda'_j \geq 0, \quad j = 1, ..., N, \ \ s\ is\ free\ in\ sign$$

According to the centralized model, exogenous input vector $x^{(1)}_{ij}$ $(i = 1, ..., M) \geq 0$ is used in the first stage for DMU_j to produce the desirable output vector (intermediate products) $v_{rj}(r = 1, ..., R) \geq 0$ and the undesirable output vector $w_{kj}(k = 1, ..., K) \geq 0$. In the second stage, the desirable output vector of the first stage $v_{rj}(r = 1, ..., R) \geq 0$ and the exogenous inputs vector $x^{(2)}_{pj}(p = 1, ..., P) \geq 0$ are used to generate the final desirable outputs $y_{fi}(f = 1, ..., F) \geq 0$ and undesirable outputs $b_{hj}(h = 1, ..., H) \geq 0$. Also, undesirable outputs of the second stage leave the system. Likewise the abatement parameter α_i and δ_p are multiplied to input as contraction factors. Model 1 obtains the whole system efficiency as ϕ^*. The efficiency of each stage can be measured by the following equations according to [1].

$$E_1 = [(\frac{1}{M+F})(\sum_{i=1}^{M} \alpha_i + \sum_{f=1}^{F} \theta_f)] \quad (a) \ \ \& \ E_2 = [(\frac{1}{P}) \sum_{p=1}^{P} \delta_p] \quad (b)$$

Similar to the conventional DEA models, some efficiency scores obtained for stage 1 and stage 2 can be raised depending on how an inefficient unit improves its performance. The authors in [1] applied this summative model to the Green Hen poultry chain in Guilan province as a case study. The results of this article which are compared with our investigation model, are expressed in Table 3 and Table 4.

2.3 Leader-Follower game theory

In this section, the leader-follower approach is developed to analyze the mentioned general two-stage structure. In a non-cooperative game [12] (Stackelberg or leader-follower game), there is a preference for the leader and follower. In this case, the leader is more preferable than the follower. Suppose again that $x_{ij}^{(1)}(i = 1, ..., M) \geq 0$ are used to produce intermediate measures $v_{rj}(r = 1, ..., R) \geq 0$ and the undesirable outputs $w_{kj}(K = 1, ..., k) \geq 0$. In terms of weak-disposable technology, the following linear model evaluates the leader's efficiency score[24].

$$
\begin{aligned}
& e_1 = min\ \theta_k \\
& \quad s.t \\
& \quad \sum_{j=1}^{N} \lambda_j x_{ij}^{(1)} \leq \alpha_i x_{io}^{(1)} && i = 1, ..., M \\
& \quad \sum_{j=1}^{N} \lambda_j v_{rj} \geq v_{ro} && r = 1, ..., R && Model\ 2 \\
& \quad \sum_{j=1}^{N} \lambda_j w_{kj} = \theta_k w_{ko} && k = 1, ..., K \\
& \quad \sum_{j=1}^{N} \lambda_j = 1 \quad , \lambda_j \geq 0 && j = 1, ..., N \\
& \quad 0 \leq \theta_k \leq 1 && k = 1, ..., K
\end{aligned}
$$

The objective function minimizes the equal-proportional reduction factor for all undesirable outputs from preserving the current level of inputs and desirable outputs. Clearly, model 2 is a linear programming problem and it is always feasible and bounded. Having obtained the efficiency of the first stage, the efficiency of the second stage could be evaluated.

In addition to this estimate, the first stage efficiency is preserved unchanged in the second stage formulation. Following the weak-disposable technology for undesirable factors, the following programming is considered.
Based upon the leader-follower game theory for a two-stage process, the second stage only considers optimal solutions that maintain the first stage's efficiency statues. In this model the second stage treats the optimal output values of the first stage for DMU_o.To this end, the second stage treats the pair $(v, x^{(2)})$ subject to the restriction that the efficiency score of the first stage remains at optimality. It should be pointed out that a system is efficient if and only if the two-component processes remain efficient. In this approach, at last the two-stage process jointly determine one optimal plan to maximize the total efficiency of the whole system. The intermediate measures are bad outputs and they should be abated in both stages.

$$
\begin{aligned}
& e_2^o = min \theta'_h \\
& \quad s.t. \\
& \quad \sum_{j=1}^{N} \lambda'_j x_{pj}^{(2)} \leq x_{po}^{(2)} \qquad p = 1, ..., P \\
& \quad \sum_{j=1}^{N} \lambda'_j v_{rj}^{*} \leq v_{ro}^{*} \qquad r = 1, ..., R \\
& \quad \sum_{j=1}^{N} \lambda'_j y_{fj} \geq y_{fo} \qquad f = 1, ..., F \qquad Model\ 3 \\
& \quad \sum_{j=1}^{N} \lambda'_j b_{hj} = \theta'_h b_{ho} \qquad h = 1, ..., H \\
& \quad \sum_{j=1}^{N} \lambda'_j = 1 \quad , \lambda'_j \geq 0 \qquad j = 1, ..., N \\
& \quad 0 \leq \theta'_h \leq 1 \qquad h = 1, ..., H
\end{aligned}
$$

3 Portion-Oriented Approach in Two-stage Structures

In real-world situations, there are many cases that handle the proportional changes in both desirable and undesirable outputs. In this circumstance, desirable factors can be assumed as a reduction part at the same time. In other words, the advent of the waste output in the production procedure is unavoidable. By regarding economic desire, a little part of this plan can be retaliated by reducing the desirable factor at the same time. This phenomenon has distinguished in a single process system [24]. Here, the extension of the model is scrutinized for a general two-stage system. There is no discussion about DEA- based models in portion reduction of the undesirable data as yet. Actually the motivation of this research is giving the priority to the reduction of undesirable outputs as much as it is possible. The proposed model is quite different from previous ones. This method deals with the network structure based on the leader-follower project and follows up reduction of undesirable factors. Particularly, in some cases, undesirable products such as the inputs, intermediate measures, and outputs would be proportionally (but not in the same rate) reduced. According to Figurer 2 and based upon the previous symbolization, the first stage pertains to the proportional change in both desirable and undesirable factors. The ε-based portion-oriented proposed model [24] is displayed as follows:

$$
\begin{aligned}
& w_1 = min\Big[\sum_{k=1}^{K}\theta_k + \frac{\varepsilon}{R}\sum_{r=1}^{R}\frac{t_{ro}}{v_{ro}}\Big] \\
& \quad s.t \\
& \quad \sum_{j=1}^{N}\lambda_j x_{ij}^{(1)} \leq x_{io}^{(1)} \qquad i = 1, ..., M \\
& \quad \sum_{j=1}^{N}\lambda_j v_{rj} \geq v_{ro} - t_{ro} \qquad r = 1, ..., R \qquad Model\ 4 \\
& \quad \sum_{j=1}^{N}\lambda_j w_{kj} = \theta_k w_{ko} \qquad k = 1, ..., K
\end{aligned}
$$

$$\sum_{j=1}^{N} \lambda_j = 1$$
$$0 \leq \theta_k \leq 1 \qquad\qquad k = 1, ..., K$$
$$t_{ro} \geq 0,\ \lambda_j \geq 0 \quad v_{ro} \geq \varepsilon \qquad j = 1, ..., N$$

This model supports the idea of decreasing the undesirable outputs with incorporation of the weak-disposable assumption. The abatement factor θ_k in the third constraint of model 4 secures this axiom. Besides, it leads to the reduction on undesirable outputs. In the light of proportional change in the desirable and undesirable outputs constraints $\sum_{j=1}^{N} \lambda_j v_{rj} \geq v_{ro} - t_{ro}$ and $\sum_{j=1}^{N} \lambda_j constraint w_{kj} = \theta_k w_{ko}$ propose a ratio decreament for good and bad outputs respectively. By the way, the equality constraint $\sum_{j=1}^{N} \lambda_j w_{kj} = \theta_k w_{ko}$ captulates the weak disposable concept. Concurrently, this constraint refers to the reduction of the undesirable outputs. According to this formulation, the constraints $0 \leq \theta_k \leq 1$ is the requirements for dominance. In addition, the modified inequality $\sum_{j=1}^{N} \lambda_j v_{rj} \geq v_{ro} - t_{ro}$ intends to the rest of the reduction from the desirable outputs. The defined abatement factor θ_k differentiate the same factor that is determined to the desirable outputs. The variable return to scale (VRS) is authorized by the constraint $\sum_{j=1}^{N} \lambda_j = 1$. Also, the unknown intensity variables $\lambda_j,\ j = 1, .., N$ are for the algebraic representation and they connect inputs and outputs by a convex combination associated with each DMU.
The ratio $\dfrac{t_{ro}}{v_{ro}}$ represented in objective function by $\sum_{k=1}^{K} \theta_k + \dfrac{\varepsilon}{R} \sum_{r=1}^{R} \dfrac{t_{ro}}{v_{ro}}$ is the shared reduction of good output for the DMU under assessment. According to the objective function, undesirable factors have the first priority in the reduction plan. The phrase $\dfrac{\varepsilon}{R} \sum_{r=1}^{R} \dfrac{t_{ro}}{v_{ro}}$ expresses that the desirable factors have the second preference. On the other hand, the non-Archimedean ε gives the priority of the reduction to the undesirable output. The priority goal of Model 4 is contracting of the undesirable output as much as possible. Then, it searches among all desirable outputs which can give these undesirable solutions [24]. In the end, the model selects a solution that can offer less reduction of the desirable one. The optimal value of the objective function places between 0 and 1. However, a unit is called efficient in model 4 if and only if the optimal

value $w_1^* = 1$ is achieved.
It is easy to show that the improved leader stage, which is defined above, is efficient and bounded [24]. The intermediate vectors v_{rj} $(r = 1, ..., R)$ have multiple roles. These are assumed as output for the first stage then they become input for the second stage. To treat the proportional change in both desirable and undesirable factors, leader- follower game theory is addressed. To fit the issue, the first stage is regarded as the leader stage and the second stage is as a follower. With a glance at the leader-follower game theory, the portion-oriented model 4 is applied to evaluate the efficiency of the second stage. The second stage employs the intermediate measure v_j of and additional inputs $x_{pj}^{(2)}$ $(p = 1, ..., P)$ to generate desirable outputs y_{fj} $(f = 1, ..., F)$ and undesirable outputs b_{hj} $(h = 1, ..., H)$. Since the undesirable output has left the system in the first stage. On the other hand, the second stage is only fed up by the intermediate measure. As a rational sight, the intermediate factor v should be increased in the first stage and oppositely it should be decreased in the second stage. As the portion-oriented method states the maximum possible reduction of the undesirable outputs. At the same time, a proportional reduction of the desirable outputs are expected. As a result, the efficiency measurement of DMU_o in terms of proportional abatement potential in the undesirable factors is desired. Simultaneously, the desirable outputs keep the first stage's status unchanged. Then, the following model is solved:

$$w_2 = min[\sum_{h=1}^{H} \theta'_h + \frac{\varepsilon}{F} \sum_{f=1}^{F} \frac{t'_{fo}}{y_{fo}}]$$

$$s.t$$

$$\sum_{j=1}^{N} \lambda'_j (v_r^* - t_r^*) \leq v_{ro}^* \qquad r = 1, ..., R$$

$$\sum_{j=1}^{N} \lambda'_j x_{pj}^{(2)} \leq x_{po}^{(2)} \qquad p = 1, ..., P \qquad Model\ 5$$

$$\sum_{j=1}^{N} \lambda'_j b_{hj} = \theta'_h b_{ho} \qquad h = 1, ..., H$$

$$\begin{aligned}
&\sum_{j=1}^{N} \lambda'_j y_{fj} \geq y_{fo} - t'_{fo} \quad f = 1, ..., F \\
&\sum_{j=1}^{N} \lambda'_j = 1 \\
&0 \leq \lambda'_h \leq 1 \qquad\qquad h = 1, ..., H \\
&t'_{fo} \geq 0, \; y_{fo} \geq \varepsilon \\
&\lambda'_j \geq 0, \; j = 1, ..., N \qquad ,
\end{aligned}$$

The second stage considers optimal solutions of the first stage. To address this issue, the second stage treats the pair $(v, x^{(2)})$ subject to a restriction that the efficiency score of the first stage remains at optimality. Hence, the first constraint of model 5 employs the optimal values of the first stage. Third and forth constraints impose the proportional reduction of both desirable and undesirable outputs in the second stage. Referring to the previous arguments, undesirable outputs do not vanish completely. But the desirable outputs can handle some part of the reduction at the same time. This situation is held while the first step‘s status remains unchanged. Actually, this condition is held by applying the optimized desirable intermediates as $v_r - t^*_r$ to the second stage as the input factors.
Additional inputs are demonstrated in$\sum_{j=1}^{N} \lambda'_j x^{(2)}_{pj} \geq x^{(2)}_{po}$ that are allowed to the second phase. Besides, the unknown intensity variables $\lambda'_j, \; j = 1, ..., N$ are applied to the algebraic representation and they connect inputs and outputs by a convex combination associated with each DMU. The objective function of the model 5 is composed of two terms. The first phrase is, $\sum_{h=1}^{N} \theta'_h$ and aggregates all reductions of undesirable outputs in the second step. This phrase is followed by $\dfrac{\varepsilon}{F} \sum_{f=1}^{F} \dfrac{t'_{fo}}{y_{fo}}$ that shows the proportion and option plan of the reduction in terms of the desirable outputs. Again, non-Archimedean ε indicates that the desirable output reduction has the second preference. Obviously, this model is always feasible and $\lambda'^*_j = 1 \; (j = k), \lambda'^*_j = 0 \; (j \neq k), \theta^*_k = 1, \; t'^*_{ro} = 1, t'^*_{fo} = 0$ is one feasible solution for it. The overall efficiency can be defined as a simple average function of the stages' efficiencies. As a re-

sult, the arithmetic average of optimal values of model 4 and model 5 is employed. Since the overall efficiency undercovers the performance of the individual stage, indicating each stage efficiency rather than the overall efficiency, leaves no hidden feature in stages.
It should be pointed out that a system is efficient if and only if the two-component processes are efficient. This model estimates the efficiency by using non-proportional adjustment for any desirable and undesirable products, which accounts for mix effects in evaluating production systems. To highlight the practical implication of the proposed approach,the real case13 poultry farms and 25 power plants are checked out individually. Interestingly, even if models 1 and the proposed model have imposed weak-disposability and null-joint on their technologies, they do not present the same results.

4 Empirical Examples

In this section, the newly developed model has been illustrated through empirical analysis. To further illustrate the effectiveness of the suggested model two different case studies are selected. First, the leader-follower and centralized plans are applied to thirteen real case paultry farms. The results of the comparison are demonstrated. Then the same scenario is applied to the real case twenty-five Iranian power plants.

4.1 Poultry farm

Providing a suitable design to prevent the poultry farm from threatening the environment (especially by carcass) should be taken into account. Effective management practices should be applied to the poultry industry. Unfortunately, there are few investigations about this issue. To optimize the operation size of the poultry industry, environmental cncerns would be reduced. In this respect, it is prefered to discuss briefly about this industry as an investigation of our proposed model where in addition to the desirable outputs, (Feed Conversion Ratio and Produced Meat) there are also some undesirable outputs (Mortality and Condemn). These undesirable outputs are assumed to leave the whole system. In order to demonstrate the formulated proposed model and its real applicability, Model 1 is applied to a real data set derived from [1]. For conducting the two-stage approach, poultry farms are selected from Guilan Province, in Iran. Gous [8] was reported that there can be two main sub-processes in the poultry industry. The first step as sub-process one is 7 or 21 days of broiler that is assumed as the golden time as a first stage. The remaining time of the production period that contains" chicken until turning into broiler", is assumed as the second stage. Figure (3) records the distinction between these two stages. However, almost all DEA studies consider a DMU as a single process. A centralized network DEA approach is applied to poultry farms operations [1]. It recorded two steps for the efficiency evaluation system. Golden time as the first stage is important in the poultry industry. On the other hand portion reduction of outputs proposes a fair assessment. As a result, taking into account the leader-follower structure, besides the portion oriented model affects the poultry operation. This plan not only increases the realism of our model but also a fairer performance assessment can be achieved. Thus, by disregarding the importance of a special stage and fair reduction of outputs, DEA models tend to label as efficient in poultry farms. The obtained results of the proposed models show that the leader-follower two-stage process is overall-efficient if and only if both of the two stages are efficient.

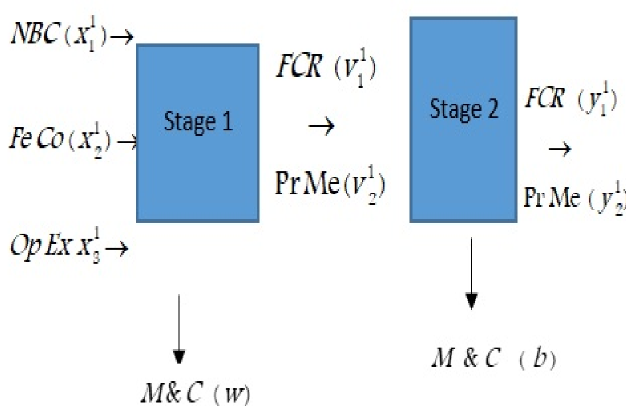

Figure 3: Poultry farm as a two-stage system.

Figure 3 fundamentally presents the case of a two-stage poultry structure likewise each sub-process in detail. Also, Table 1 displays inputs and outputs of thirteen poultry farms which are illustrated as $DMUs$, and the data set for thirteen poultry farms is revealed in Table 2.

Table 1: Inputs, outputs and intermediates (desirable and undesirable) with their abbreviations.

Stage	Input / Outout	Lable
		1.NBC X_{1j}^1 :New Born Chicks
	Input	2.Fe Co X_{2j}^1 : Feed Cost
		3.Op Ex X_{3j}^1 :Operational Expenses
STAGE 1 (Golden time)	Desirable Outout	1.FCR V_{1j}^1 : Feed Conversion Ratio
		2.Pr Me V_{2j}^1 : Produced Meat
	Undesirable Output	1.M 7& C W_{1j}^1 : Mortality and Condemn
		1.FCR V_{1j}^2 : Feed Conversion Ratio
		2.Pr Me V_{2j}^2 : Produced Meat
	Input	3.Op Ex X_{1j}^2 :Operational Expenses
		4.Fe Co X_{2j}^2 : Feed Cost
STAGE2 (Chicken to Broiler)		
	Desirable Output	FCR V_{1j}^2 : Feed Conversion Ratio
		2.Pr Me V_{2j}^2 : Produced Meat
	Undesirable Output	1.M & C W_{1j}^2 : Mortality and Condemn

Table 2: Observed data for thirteen poultry farms.

DMU	X_1^1	X_2^1	X_3^1	W	V_{1i}^1	V_{2i}^1	X_{1i}^2	X_{2i}^2	V_{1i}^2	V_{2i}^2	W_{1j}^2
	NBC	FeCo	OpEx	M & C	FCR	PrMe	FeCo	OpEx	FCR	PrMe	M & C
1	12700	148500	57370	467	1.69	6691.5	438500	97920	1.98	28582.2	173
2	14670	171740	63900	513	1.65	7871.3	491760	110160	1.93	32387.2	197
3	13300	154930	63220	1263	1.72	6921.3	435410	106150	2.00	58506.3	306
4	15000	182880	60590	421	1.71	8280.9	518560	126650	1.95	34075.0	79
5	12000	147490	57030	758	1.68	6340.5	415130	100700	1.98	26256.5	256

6	14000	105080	63640	1098	1.70	7134.8	449710	113700	1.97	29828.0	263
7	13000	168930	62020	646	1.75	7202.4	468450	110550	2.03	30158.7	144
8	14900	175430	71680	821	1.62	7475.9	532190	119100	2.04	33414.6	214
9	13500	169520	62300	518	1.71	7399.7	480800	106770	1.94	30439.0	246
10	12800	144130	60930	623	1.63	6356.4	433090	105240	2.03	28223.5	167
11	19800	235970	80960	1042	1.67	10373.2	685800	144430	2.01	44581.2	336
12	11000	133540	51340	385	1.68	5933.8	378100	80880	2.00	25683.4	89
13	12600	118870	57210	479	1.63	5933.8	440730	102420	1.88	28405.3	186

Table 3: Results of stage 1and stage 2 and overall efficiency scores of Model 1.

DMU	$E_{1stage1}$	$E_{2stage2}$	$\phi_{overall}$
DMU01	0.9480	0.9905	0.9692
DMU02	0.9421	1.0000	0.9710
DMU03	0.7974	1.0000	0.8987
DMU04	1.0000	1.0000	1.0000
DMU05	0.8495	1.0000	0.9248
DMU06	0.7962	0.9721	0.8841
DMU07	0.8902	0.9513	0.9207
DMU08	0.7956	1.0000	0.8987
DMU09	0.9282	0.9872	0.9577
DMU10	0.8480	1.0000	0.9240
DMU11	1.0000	1.0000	1.0000
DMU12	1.000	1.0000	1.0000
DMU13	0.9075	0.9880	0.9178

Average	0.8962	0.9917	
Standard deveation			0.0414

In order to carry out a new two-stage DEA approach to poultry farms, the results of Model 1 and the proposed model can be loaded on the data set, which is depicted in Table 3 and Table 4 respectively.

Table 4: Results of stage 1 and stage 2 and overall efficiency scores of the proposed approach.

DMU	$w_{1stage1}$	$w_{2stage2}$	$w_{overall}$
DMU01	0.8250	0.5240	0.6745
DMU02	0.7517	0.6901	0.7209
DMU03	0.3057	0.2772	0.2914
DMU04	0.9160	1.0000	0.9580
DMU05	0.5082	0.3478	0.4280
DMU06	1.0000	0.3191	0.6595
DMU07	0.5971	0.9977	0.7974
DMU08	0.4700	1.0000	0.7350
DMU09	0.7443	0.3364	0.5403
DMU10	0.6183	1.0000	0.8091
DMU11	0.3716	0.3134	0.3425
DMU12	1.000	1.0000	1.0000
DMU13	1.0000	0.8979	0.9489
Average	0.7006	0.6385	
Standard deveation			0.2300

By the way, the optimal overall efficiency value is represented in the last column of both tables. By explaining the efficiency scores of each stage and the overall efficiency score for both tables the following results can be expressed: According to column 4 of Table 4 and the same column of Table 3 it is clear that the proposed model gives lower efficiency scores. As a matter of fact, model 1 is overestimated (see figure 4). Comparing the average of efficiency scores of model 1 and the proposed model at both stages individually shows the following results $\bar{E}_1 = 0.8962$, $\bar{E}_2 = 0.9917$ and $\bar{w}_1 = 0.7006$, $\bar{w}_1 = 0.6385$. Subsequently model 1 still presents overestimated results.

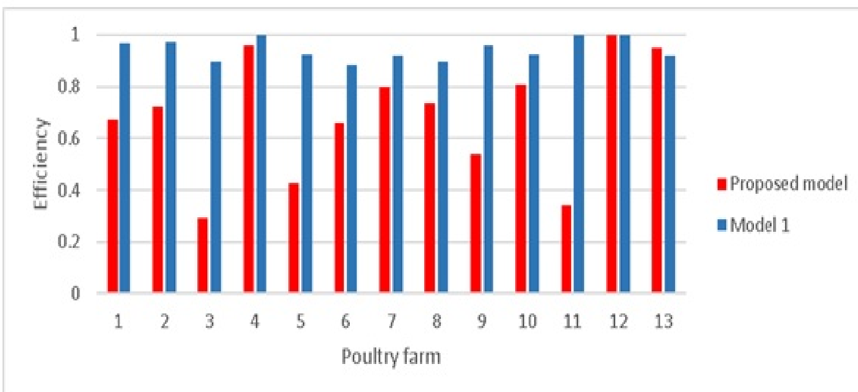

Figure 4: Comparing overall efficiency scores of proposed model and model 1.

Next, According to Table 3, the number of overall inefficient DMUs of model 1 is ten farms, and on the other hand Table 4 shows twelve overall inefficient farms (see figure 4). Third, by applying the proposed model, $DMU12$ is overall efficient, while other farms do not act well enough. $DMU12$ is efficient at both sages too. Therefore, other farms should concentrate on farm $DMU12$ as a benchmark. Benchmarks -that are presented in Table 3 -are farms $DMU4$, $DMU11$, and $DMU12$. Also, the number of efficient $DMUs$ that are suggested by the proposed model is less than efficient$DMUs$ that obtained in the model 1 at every stage as well as the overall efficiency score.

Furthermore, for some $DMUs$ like $DMU3$ in Table 4 there are many gaps to be efficient. An effective policy should be applied to each stage as well as the management's recommendation in these farms (see figure 4). The fifth point is that according to Table 3 and Table 4 $DMU12$ is a unique efficient farm at both stages of both models and an overall efficient farm at both models. The other significant matter is that a unit may be efficient at the first stage but it may be inefficient overall. For example, although $DMU13$ is efficient in the first stage of the proposed model, it is overall inefficient (see figure 5). The same scenario may happen in the second stage. $DMU4$ is efficient at the second stage but it is inefficient overall (see figure 6).

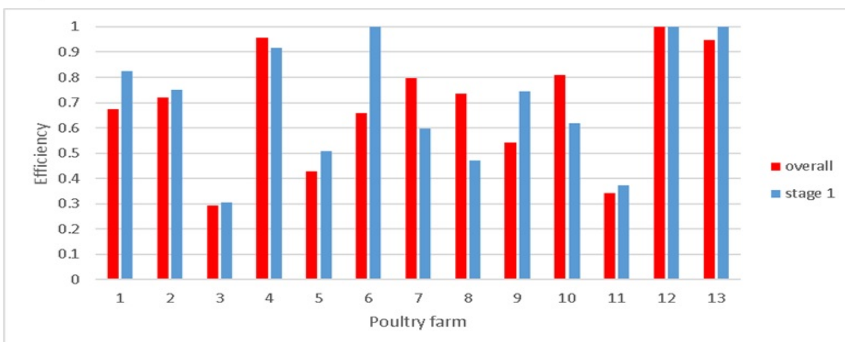

Figure 5: Results of efficiency scores of the first stage versus overall efficiency scores of the proposed model.

Seventh, amazingly, the standard deviation of the proposed model yields 0.2300, and the standard deviation of the model1 yields 0.0414. This result demonstrates that the nature of the proposed model acts better. Next, it can be mentioned that based on the concept of efficient and inefficient $DMUs$, inefficient $DMUs$ can be made efficient through supporting plans. For example, column four indicates that $DMU13$ and $DMU4$ can be easily shifted to efficiency score. Their efficiency rates are estimated near to one. $DMU4$ should improve its first stage according to the policy that $DMU12$ suggests (see figure 5). Also, the same plan should be done in the second stage for $DMU13$(see figure 6)

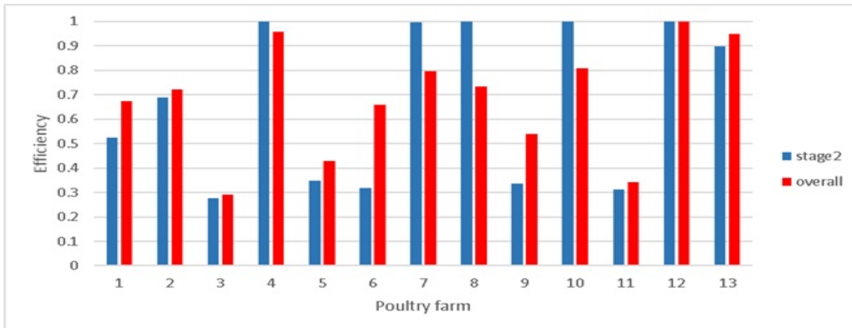

Figure 6: Results of efficiency scores of the second stage versus overall efficiency scores of the proposed model.

Actually, a real data-set that is drawn from the poultry was applied to the proposed model and general Model 1. The results involved optimized overall scores, each stage efficiency score, average, and standard deviation. The final results confirm the capability, validity, and the efficacy of the proposed models towards the general one.

4.2 Power plants

Another empirical example is provided which is based on the methodological framework of our proposed model. As efficiency measurement of the power plants has become a major topic, it has been explored by many researchers who investigate DEA (especially its economy and environmental sustainability concerns) [23]). Most of these DEA-based studies are provided with a single-stage system for efficiency assessment. For example, [6] revealed China's regional energy efficiency and carbon emission efficiency in a comprehensive survey of the empirical studies. They presented six widely-used single-stage DEA models. In order to assess the electricity companies the authors [16] applied the game theory concept to get more realistic results. They applied a single-stage model to 37 electricity distribution companies in order to examine their relative efficiencies in the competitive environment. In [11] researchers evaluated 38 electricity distribution units in Iran. They used a super-efficiency model to rank efficient units too. The major methodological

framework of this study was based on a stochastic DEA. Authors just analized the black box model.
The electric distribution company in Turkey is evaluated by a two-stage structure in [17]. Directional Distance Function approach is also integrated into the network DEA technique. The efficiency measurement of 20 electric distribution companies was assessed and the results were presented based on the profit efficiency measures. But, an environmental perspective of the efficiency evaluation is ignored in this research. This paper also provided the Malmquist Index by considering the variables which were related to the electricity distribution. By assuming the shortages, a combination of the portion-reduction and weak disposability is the motivation of this study in modeling the network DEA with undesirable intermediate measures. The proposed model is applied to 25 Iranian power plants to gain further insight. Table 5 reports the data set and Table 6 contains descriptive statistics of the studied data.

Table 5: inputs, outputs and intermediates (desirable and undesirable) with their abbreviations.

		I_1^1:Installed capacity (MVA)
	Input :	I_2^1 : Labour
		I_3^1: Natural Gas (Thousand cubicmeter)
SRAGE 1	Undersi rable Output :	
Generation		
		W_1^1 : Carbon dioxide (CO2) cubic meter
		W_2^1 : Ozone (O) cubic meter
		W_3^1 : nitrongen oxides (NOx) cubic meter
		Desirable Output
		V_1^1 : Netelectricity generation (MKWH)
		I_1^2 : Area (km^2)
	Input:	I_2^2 : Transmision staff
		I_3^2 :Operational expences
Transmission	Undesi rable Output:	b_1^2 : Lenght of cables substitution (km) STAGE 2
		b_2^2 : Waste in distribution (kw)
		Desirable Output y_1^2 : Net electricity transmission (MVH)

Table 6: Descriptive statistics of the studied data.

Variable	Min	Max	Average	Standard devition
Installed Capacity	33773	221254	111477.12	68054.8996
Labor	88	1157	345	307.1563
Natural Gas	91475	1322685	392301.25	343462.0395
Carbon dioxide	3398	44251	25761.8	20425.4162
Ozone	89	1002	398.64	270.940
Nitrogen oxides	46325	840645	180574.92	225311.9918
Net electricity generation	6547	42311	23947.668	34421.87338
Area	38251	1025947	221163.23	318969
Transmission staff	93	1061	324	293.214
Operational expenses	99347	144399	462490.32	384998.87
Waste in distribution	6859	9298	324974.28	351335.9545
Length of cables substitution	98	1198.646	495.0367	352.9649
Net electricity transmission	5750	58613	18391.64	11984.70356

The gathered information from educational and research institutions of the Ministry of Power of Iran. Since there are multiple production stages regarding the generation and transmission of electric power, the network DEA technique is used. The proposed model confirms realistic results by considering the weak disposability combined with proportional reduction and leader-follower structure. In this sample, the performance evaluation of the power plants is divided into two stages: Generation process and Transmission process. The important characteristics of this example and the main differences of our model with model 1 are the possibility of scarifying some parts of desirable outputs to satisfying the environmental purposes. To see how the combination of the proportional reduction of desirable and undesirable outputs with leader- follower structure influences the two-stage network structure, the proposed two-stage model is taken into consideration. First, the leader-follower (that is presented in section 2) is applieed to this data set and we assumed that the generation process is leader as a first stage and transmission is the label of the second stage. Moreover, in this example $\varepsilon = 0.01$ is applied. First, the results of the proposed model are illustrated in Table 7. The second column of this table reports the optimal scores of the first stage and the third column reveals the optimal results for the transmission stage as a second stage. By observing the second column of Table 7, when the generation process is leader, three power plants are efficient: $unit15$, $unit18$, and unit 19 (see figure 7). The second stage suggests five units as efficient power plants: $unit10$, $unit15$, $unit17$, $unit18$, and $unit21$ (see figure 8). However, only two units are overall efficient that is displayed in the fourth column of Table 7 ($unit15$, $unit18$). The point is that a DMU is efficient in a two-stage structure if and only if it is efficient at both stages like $unit15$ and $unit18$.

Table 7: Results of stage 1& stage 2& overall efficiency scores of the proposed model and overall efficiency scores of Model 1.

DMU	$W_{1stage1}$	$W_{2stage2}$	$W_{overall}$	ϕ^*
DMU01	0.1682	0.134	0.1511	1.0000
DMU02	0.2520	0.172	0.2120	1.0000
DMU03	0.4458	0.2455	0.3454	0.5964
DMU04	0.1614	0.1451	0.1532	1.0000
DMU05	0.2522	0.2263	0.2391	1.0000
DMU06	0.1703	0.1350	0.1526	0.7386
DMU07	0.4066	0.2691	0.3378	0.3925
DMU08	0.1645	0.1352	0.1497	0.5056
DMU09	0.3207	0.2068	0.2633	0.5670
DMU10	0.8113	1.000	0.9056	1.0000
DMU11	0.4388	0.3222	0.3804	0.7186
DMU12	0.1827	0.1587	0.1703	0.3156

DMU13	0.3592	0.2862	0.3226	0.8193
DMU14	0.5166	0.4340	0.4753	0.7916
DMU15	1.0000	1.000	1.000	0.7376
DMU16	0.4721	0.1712	0.3215	0.9356
DMU17	0.5894	1.000	0.7947	1.0000
DMU18	1.0000	1.000	1.000	1.0000
DMU19	1.0000	0.2212	0.6105	1.0000
DMU20	0.2015	0.1732	0.1872	1.0000
DMU21	0.5558	1.000	0.7779	1.0000
DMU22	0.1664	0.1361	0.1512	1.0000
DMU23	0.2930	0.2848	0.2885	0.9475
DMU24	0.5554	0.2413	0.3982	0.8867
DMU25	0.5704	0.2863	0.4282	1.0000
Average	0.4412	0.3752	0.4086	0.8381
Standard deveation			0.2769	0.2130

Second point is that, the average scores of the first and the second stages, are 0.4412 and 0.3751 respectively. It declares that units' activities are similar to each other at both stages (almost most cases) and there is not a significant difference between them.

Two last columns of Table 7 declare the overall efficiency scores of the proposed model and Model 1. It declares that the proposed model depicts just two efficient units but model 1 represents twelve efficient units. Indeed, Model 1 is overestimated and declares many benchmarks. Forth result, Table 7 shows that the average score of the proposed model is substantially lower than the average score of model 1.

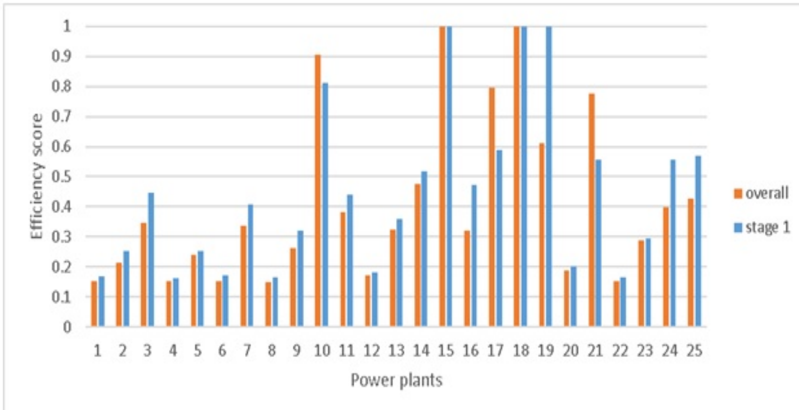

Figure 7: Comparing overall efficiency scores of the proposed model and the first stage of this model.

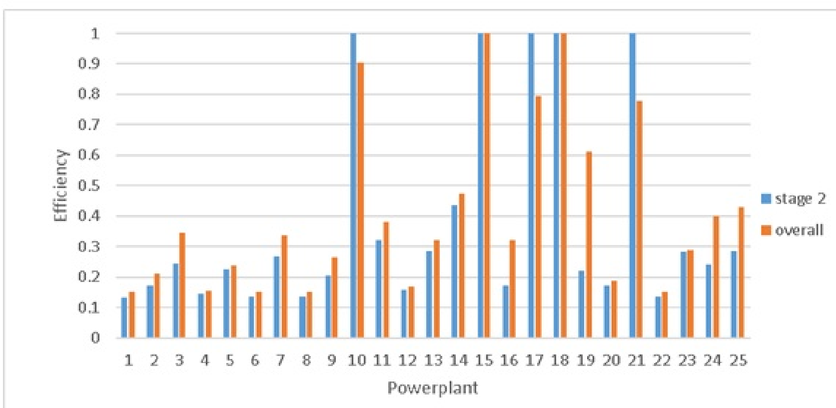

Figure 8: Comparing overall efficiency scores of the proposed model and the first stage of this model.

The average efficiency score of the proposed model is 0.4086 while in Model 1 the average efficiency score demonstrates 0.8381. This means that, in this example, the proposed approach will substantially reduce the efficiency scores and this has a strict viewpoint in assessing efficiency (see figure 9). Furthermore, the difference in the average efficiency scores records that the proposed model satisfies the reduction of the desirable and the undesirable outputs. Furthermore, its efficiency scores are strictly lower than Model 1.

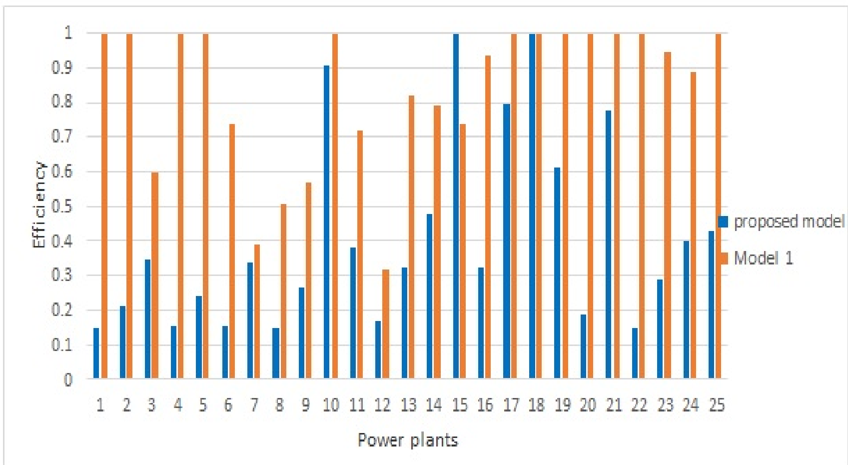

Figure 9: Comparing overall efficiency scores of proposed model and overall efficiency scores of model1

The overall efficiency scores in column 4 of Table 7 shows that $DMU10$ is a volunteer to be overall efficient. This unit obtains 0.9056 as its overall efficiency score and can easily move to get be efficient. On the other hand, $DMU14$ can be efficient by improving its first stage as well as improving the second stage. This unit should augment the shortfalls at both stages. By the way, the last row of Table 7 presents the standard deviation. The standard deviation of the suggested model is 0.2769, but Model 1 yields 0.2130. This is an outstanding challenge for the proposed model and shows the discriminatory power of its results against Model 1 because its productivity changes are distinguished better than Model 1. The proposed model encourages units to take the effect of the leader-follower structure as well as both faces of the reduction plan for undesirable and desirable outputs and gives lower efficiency scores.

5 Conclusion

In two-stage structure, usually a part from produced desirable outputs and consuming inputs, undesirable outputs are generated in the production process. The current paper develops a portion-oriented DEA-based model for performance evaluation. Commonly in a two-stage technology, a combination of the weak-disposability axiom and leader-follower plan with portion-reduction of the desirable and the undesirable outputs are ignored. According to this gadget, in the proposed model, the leader is first assessed by the portion oriented model. Then, the follower is evaluated by using the optimized objective function and the leader-optimized solutions of the intermediate measures. By giving the preference to the first stage, this plan tries to cover some pitfalls in the efficiency evaluation as well as optimizing the efficiency of both stages. Having used a two-stage structure as a real-life example, the current paper examines an optimization scheme where the overall efficiency is defined as a simple average of both stages. The proposed model encourages units to consider the influence of the leader-follower structure as well as both faces of the reduction plan for undesirable and desirable outputs. However, the superiority of the proposed model over a general two-stage network is discussed. However, the same scenario can be applied to Kuosmanen production technology. This is beyond the scope of this research and should be considered for further examination. Furthermore, this procedure can be assessed by using non-discretionary data at both stages according to the necessity. The consumption stage of power plants as the third stage is not examined as well as a two-stage process in this study, which are also possible interesting research topics in future researches.

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Monireh Zoriehhabib
Department of Mathematics
Assistant Professor of Mathematics
Sofian Branch, Islamic Azad University
Sofian, Iran
E-mail: zoriehhabib_m@iaut.ac.ir

Mohsen Rostamy-Malkhlifeh
Department of Mathematics
Associate Professor of Mathematics
Science and Research Branch, Islamic Azad University
Tehran, Iran
E-mail: mohsen_rostamy@yahoo.com

Farhad Hosseinzadeh Lotfi
Department of Mathematics
Professor of Mathematics
Science and Research Branch, Islamic Azad University
Tehran, Iran
E-mail: farhad@hosseinzadeh.ir