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Inference on the Ratio of Correlations of Two Independent Populations

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Abstract. The asymptotic distribution for the ratio of the sample correlations in two independent populations is established. The presented method can be used to derive the asymptotic confidence interval and hypothesis testing for the ratio of population correlations. The performance of the new interval is comparable with similar method. Then the simulation study is provided to compare our confidence interval with Fisher Z-transform and Cauchy-transform methods. The proposed confidence set has a good coverage probability with a larger power.

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1. Introduction

It is of interest to make inference about the ratio of correlations of two independent populations. This parameter is more applicable than the difference of correlations in some applications. The advantage of using ratio instead of difference lies in the fact that the difference of two small correlations is also small and has no meaningful description.

Raghunathan et al. (1996) discuss techniques for comparing correlated but non-overlapping correlations. Let us first make the distinction between overlapping and non-overlapping correlations. Suppose that one wish to compare the correlation between one pair of variables with that between a second, overlapping

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pair of variables (for example, when comparing the correlation between one IQ test(X) and grades(Y) with the correlation between a second IQ test (W) and grades (Y). Notice that variable Y is common to both correlations. You can use procedure explained in [5-7].

Now suppose that one wish to compare the correlation between one pair of variables and a second non-overlapping pair of variables. Assume that you have data on the same set of subjects for all four variables. Raghunathan et al. give several examples of research questions involving such non-overlapping correlations. One is comparing the correlation between two variables at two different points in time, for example, to learn whether, as children develop, their cognitive functions become less highly inter-correlated. Another of their examples involves comparing cross-lagged panel correlations, and their numerical example involves such correlations. To compare the correlation between one pair of variables in two independent populations, see the [9].Hittner et al.(2003) described their investigation of several methods for com-paring dependent correlations and found that all can be unsatisfactory, in terms of Type I errors, even with a sample size of 300.Baksh et al. (2006) extend the sequential approach to a comparison of the associations within two independent groups of paired continuous observations. Wilcox and Tian(2008) extended Hittner et al.'s research by considering a variety of alternative methods.

To test the null hypothesis that the correlation between X and Y in one population is the same as the correlation between X and Y in another population, commonly use the procedure developed by Fisher [3, 8]. First, transform each of the two correlation coefficients in this fashion:

$$r' = (0.5)\ln\left|\frac{1+r}{1-r}\right|.$$

Second, compute the test statistic

$$Z = \frac{r_1' - r_2'}{\sqrt{\frac{1}{m-3} + \frac{1}{n-3}}},$$

which is the standard difference of r_1'andr_2'. This statistic has asymptotic standard normal distribution.

As an alternative, one can use Cauchy-transform method. Consider the statistic

$$C = \sqrt{\frac{m-3}{n-3}}\frac{r_1'}{r_2'},$$

which is the standard ratio of r_1'andr_2'. This statistic has asymptotic standard Cauchy distribution.

Third, obtain p-value for the computed Z and C.
In this work, the asymptotic distribution for the ratio of sample correlations is presented. It will be applied to construct confidence interval and perform test statistics. This method is the most efficient way in comparison with Fisher Z-transformand Cauchy-transform methods.

2. Large Samples Inference

Let (X, Y) and (X^*, Y^*) be two independent random pairs with means (μ_X, μ_Y) and (μ_{X^*}, μ_{Y^*}), covariance matrices

$$\Sigma = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{pmatrix} \text{ and } \Sigma^* = \begin{pmatrix} \sigma_{X^*}^2 & \sigma_{X^*Y^*} \\ \sigma_{X^*Y^*} & \sigma_{Y^*}^2 \end{pmatrix},$$

and correlation coefficients ρ_{XY} and $\rho_{X^*Y^*}$, respectively. Also assume that (X, Y) and (X^*, Y^*) have the finite ij^{th} central product moments

$$\mu_{ij} = E(X - \mu_X)^i (Y - \mu_Y)^j,$$

and

$$\mu_{ij}^* = E(X^* - \mu_{X^*})^i (Y^* - \mu_{Y^*})^j,$$

for $(i, j) = (0, 4), (0, 3), (1, 3), (2, 2), (3, 0)$.
Let $(X_1, Y_1), \ldots, (X_m, Y_m)$ and $(X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)$ be two independent samples from $(X, Y)$and (X^*, Y^*), respectively. We are interested tomake inference about the parameter $\rho = \frac{\rho_{XY}}{\rho_{X^*Y^*}}$. Since $r_{XY} = \frac{\sum_{i=1}^{m} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{m} (x_i - \overline{x})^2} \sqrt{\sum_{i=1}^{m} (y_i - \overline{y})^2}}$ and $r_{X^*Y^*} = \frac{\sum_{i=1}^{n} \left(x_i^* - \overline{x^*}\right)(y_i^* - \overline{y^*})}{\sqrt{\sum_{i=1}^{n} \left(x_i^* - \overline{x^*}\right)^2} \sqrt{\sum_{i=1}^{n} (y_i^* - \overline{y^*})^2}}$ are consistent estimators for ρ_{XY} and $\rho_{X^*Y^*}$[1], respectively, it seemsthat $r = \frac{r_{XY}}{r_{X^*Y^*}}$ is a reasonable estimator for the parameter ρ.
There is no loss in assuming$m = n$. Also, since the correlationdoes not depend on locations, we may as well assume $(\mu_X, \mu_Y) = (\mu_X^*, \mu_Y^*) = \mathbf{0}$.

Lemma 2.1. *Under the above assumptions,*

$$\sqrt{n}\,(r_{XY} - \rho_{XY}) \xrightarrow{\mathcal{L}} N\left(0, \gamma^2\right), \quad as \quad n \to \infty,$$

where

$$\gamma^2 = \frac{1}{4}\rho_{XY}^2 \left[\frac{\mu_{40} - \sigma_X^4}{\sigma_X^4} + 2\frac{\mu_{22} - \sigma_X^2\sigma_Y^2}{\sigma_X^2\sigma_Y^2} + \frac{\mu_{04} - \sigma_Y^4}{\sigma_Y^4}\right]$$
$$-\rho_{XY}\left[\frac{\mu_{31} - \sigma_X^2\sigma_{XY}}{\sigma_X^3\sigma_Y} + \frac{\mu_{13} - \sigma_{XY}\sigma_Y^2}{\sigma_X\sigma_Y^3}\right] + \frac{\mu_{22} - \sigma_{XY}^2}{\sigma_X^2\sigma_Y^2}.$$

Proof. The outline of the proof is given in [1]. □

The asymptotic distribution for r is given in the following theorem.This theorem that is the main theorem of this article, will be needed to makeinference about the parameter ρ.

Theorem 2.2. *Under the assumptions of Lemma 1,*

$$\sqrt{n}\,(r-\rho) \xrightarrow{\mathcal{L}} N\left(0,\lambda^2\right), \quad as\ n\to\infty,$$

where

$$\lambda^2 = \frac{1}{4}\rho\left[\frac{\mu_{40}-\sigma_X^4}{\sigma_X^4} + 2\frac{\mu_{22}-\sigma_X^2\sigma_Y^2}{\sigma_X^2\sigma_Y^2} + \frac{\mu_{04}-\sigma_Y^4}{\sigma_Y^4}\right]$$

$$-\frac{\sqrt{\rho}}{\rho_2}\left[\frac{\mu_{31}-\sigma_X^2\sigma_{XY}}{\sigma_X^3\sigma_Y} + \frac{\mu_{13}-\sigma_{XY}\sigma_Y^2}{\sigma_X\sigma_Y^3}\right] + \frac{1}{{\rho_2}^2}\left[\frac{\mu_{22}-\sigma_{XY}^2}{\sigma_X^2\sigma_Y^2}\right]$$

$$+\frac{1}{4}\rho\left[\frac{\mu_{40}-\sigma_{X^*}^4}{\sigma_{X^*}^4} + 2\frac{\mu_{22}-\sigma_{X^*}^2\sigma_{Y^*}^2}{\sigma_{X^*}^2\sigma_{Y^*}^2} + \frac{\mu_{04}-\sigma_{Y^*}^4}{\sigma_{Y^*}^4}\right]$$

$$-\frac{\rho}{\rho_2}\left[\frac{\mu_{31}-\sigma_{X^*}^2\sigma_{X^*Y^*}}{\sigma_{X^*}^3\sigma_{Y^*}} + \frac{\mu_{13}-\sigma_{X^*Y^*}\sigma_{Y^*}^2}{\sigma_{X^*}\sigma_{Y^*}^3}\right]$$

$$+\frac{\rho}{{\rho_2}^2}\left[\frac{\mu_{22}-\sigma_{X^*Y^*}^2}{\sigma_{X^*}^2\sigma_{Y^*}^2}\right].$$

Proof. By using Lemma 1, we have,

$$\sqrt{n}\,(r_{XY}-\rho_{XY}) \xrightarrow{\mathcal{L}} N\left(0,{\gamma_1}^2\right), \quad as\ n\to\infty,$$

and

$$\sqrt{n}\,(r_{X^*Y^*}-\rho_{X^*Y^*}) \xrightarrow{\mathcal{L}} N\left(0,{\gamma_2}^2\right), \quad as\ n\to\infty,$$

where

$${\gamma_1}^2 = \frac{1}{4}\rho_{XY}^2\left[\frac{\mu_{40}-\sigma_X^4}{\sigma_X^4} + 2\frac{\mu_{22}-\sigma_X^2\sigma_Y^2}{\sigma_X^2\sigma_Y^2} + \frac{\mu_{04}-\sigma_Y^4}{\sigma_Y^4}\right]$$

$$-\rho_{XY}\left[\frac{\mu_{31}-\sigma_X^2\sigma_{XY}}{\sigma_X^3\sigma_Y} + \frac{\mu_{13}-\sigma_{XY}\sigma_Y^2}{\sigma_X\sigma_Y^3}\right] + \frac{\mu_{22}-\sigma_{XY}^2}{\sigma_X^2\sigma_Y^2},$$

and

$${\gamma_2}^2 = \frac{1}{4}\rho_{X^*Y^*}^2\left[\frac{\mu_{40}-\sigma_{X^*}^4}{\sigma_{X^*}^4} + 2\frac{\mu_{22}-\sigma_{X^*}^2\sigma_{Y^*}^2}{\sigma_{X^*}^2\sigma_{Y^*}^2} + \frac{\mu_{04}-\sigma_{Y^*}^4}{\sigma_{Y^*}^4}\right]$$

$$-\rho_{X^*Y^*}\left[\frac{\mu_{31}-\sigma_{X^*}^2\sigma_{X^*Y^*}}{\sigma_{X^*}^3\sigma_{Y^*}}+\frac{\mu_{13}-\sigma_{X^*Y^*}\sigma_{Y^*}^2}{\sigma_{X^*}\sigma_{Y^*}^3}\right]+\frac{\mu_{22}-\sigma_{X^*Y^*}^2}{\sigma_{X^*}^2\sigma_{Y^*}^2}.$$

Since the samples are independent, Slutsky's theorem gives

$$\sqrt{n}\left[\begin{pmatrix} r_{XY} \\ r_{X^*Y^*}\end{pmatrix}-\begin{pmatrix} \rho_{XY} \\ \rho_{X^*Y^*}\end{pmatrix}\right]\xrightarrow{\mathcal{L}}N\left(\mathbf{0},\begin{pmatrix} {\gamma_1}^2 & 0 \\ 0 & {\gamma_2}^2\end{pmatrix}\right).$$

Now define $f:\ \mathbb{R}^2\to\mathbb{R}$as $f\left(x_1,x_2\right)=\frac{x_1}{x_2}$. Then the gradient function with respect to $f$is $\nabla f\left(x_1,x_2\right)=\left(\frac{1}{x_2},-\frac{x_1}{{x_2}^2}\right)$.

Also $\nabla \mathrm{f}\left(\rho_{\mathrm{XY}},\rho_{\mathrm{X^*Y^*}}\right)\Sigma(\nabla \mathrm{f}\left(\rho_{\mathrm{XY}},\rho_{\mathrm{X^*Y^*}}\right))^{\mathrm{T}}{=}\lambda^2$.

Since ∇f is continuous in neighbourhoodof $\left(\rho_{XY},\rho_{X^*Y^*}\right)$, therefore, by Cramer's rule we have

$$\sqrt{n}\left(f\left(r_{XY},r_{X^*Y^*}\right)-f\left(\rho_{XY},\rho_{X^*Y^*}\right)\right)=\sqrt{n}\left(r-\rho\right)\xrightarrow{\mathcal{L}}N\left(0,\lambda^2\right),\ as\ n\to\infty.\ \square$$

So we have just proved

$$T_n=\sqrt{n}\left(\frac{r-\rho}{\lambda}\right)\xrightarrow{\mathcal{L}}N\left(0,1\right),\quad as\ n\to\infty. \tag{1}$$

This result can be used to construct an asymptotic confidence interval and hypothesis testing.

2.1 Asymptotic Confidence Interval

Note that the parameter λ in $T_n$given by (1) depends on the unknown parameter ρ, so it cannot be used as a pivotal quantity for the parameter ρ.

Theorem 2.1.1. *On the same hypothesis of Theorem 1,*

$$T_n^*=\sqrt{n}\left(\frac{r-\rho}{\widehat{\lambda}_n}\right)\xrightarrow{\mathcal{L}}N\left(0,1\right),\quad as\ n\to\infty, \tag{2}$$

where

$$\widehat{\lambda_n^2}=\frac{1}{4}\rho\left[\frac{m_{40}-S_X^4}{S_X^4}+2\frac{m_{22}-S_X^2S_Y^2}{S_X^2S_Y^2}+\frac{m_{04}-S_Y^4}{S_Y^4}\right]$$

$$-\frac{\sqrt{\rho}}{\rho_2}\left[\frac{m_{31}-\sigma_X^2\sigma_{XY}}{S_X^3S_Y}+\frac{m_{13}-\sigma_{XY}\sigma_Y^2}{S_XS_Y^3}\right]+\frac{1}{{\rho_2}^2}\left[\frac{m_{22}-S_{XY}^2}{S_X^2S_Y^2}\right]$$

$$+\frac{1}{4}\rho\left[\frac{m_{40}-S_{X^*}^4}{S_{X^*}^4}+2\frac{m_{22}-S_{X^*}^2S_{Y^*}^2}{S_{X^*}^2S_{Y^*}^2}+\frac{m_{04}-S_{Y^*}^4}{S_{Y^*}^4}\right]$$

$$-\frac{\rho}{\rho_2}\left[\frac{m_{31}-S_{X^*}^2S_{X^*Y^*}}{S_{X^*}^3S_{Y^*}}+\frac{m_{13}-S_{X^*Y^*}S_{Y^*}^2}{S_{X^*}S_{Y^*}^3}\right]+\frac{\rho}{{\rho_2}^2}\left[\frac{m_{22}-S_{X^*Y^*}^2}{S_{X^*}^2S_{Y^*}^2}\right],$$

and

$$m_{ij}=\frac{1}{n}\sum_{i=1}^{n}\left(X_k-\overline{X}\right)^i\left(Y_k-\overline{Y}\right)^j\ ;;\ {m_{ij}}^*=\frac{1}{n}\sum_{i=1}^{n}\left({X_k}^*-\overline{X^*}\right)^i\left({Y_k}^*-\overline{Y^*}\right)^j,$$

$(i,j)=\{(0,4),(1,3),\ (2,2),\ (1,3),\ (4,0)\}$, *are the ijth sample central product moments of* (X,Y)*and* (X^*,Y^*), *respectively.*

Proof. By the weak law of large numbers, we have
$m_{ij}\xrightarrow{p}\mu_{ij}\ ,\ {m_{ij}}^*\xrightarrow{p}{\mu_{ij}}^*\ ;;\ (i,j)=\{(0,4),(1,3),\ (2,2),\ (3,1),\ (4,0)\}$,
$S_{XY}\xrightarrow{p}\sigma_{XY}andS_{X^*Y^*}\xrightarrow{p}\sigma_{X^*Y^*}$, as $n\to\infty$.
Hence by Slutsky's theorem,$\widehat{\lambda}_n\xrightarrow{p}\lambda$, as $n\to\infty$.The proof is now completed by using Theorem 1.
Now, T_n^* can be used as a pivotal quantity to construct asymptotic confidence interval for ρ,

$$\left(r-\frac{\widehat{\lambda}_n}{\sqrt{n}}Z_{\alpha/2},r+\frac{\widehat{\lambda}_n}{\sqrt{n}}Z_{\alpha/2}\right).\quad\square\qquad(3)$$

2.2 Hypothesis Testing

Hypothesis testing about ρ is important in practice. For instance, the assumption $\rho=1$ is equivalent to the assumption $\rho_{XY}=\rho_{X^*Y^*}$. In general, to test$H_0:\ \rho=\rho_0$, the test statistic can be

$$T_0=\sqrt{n}\left(\frac{r-\rho_0}{\rho^*}\right),\qquad(4)$$

where

$${\rho^*}^2=\frac{1}{4}\rho_0\left[\frac{m_{40}-S_X^4}{S_X^4}+2\frac{m_{22}-S_X^2S_Y^2}{S_X^2S_Y^2}+\frac{m_{04}-S_Y^4}{S_Y^4}\right]$$

$$-\frac{\sqrt{\rho_0}}{\rho_2}\left[\frac{m_{31}-\sigma_X^2\sigma_{XY}}{S_X^3S_Y}+\frac{m_{13}-\sigma_{XY}\sigma_Y^2}{S_XS_Y^3}\right]+\frac{1}{{\rho_2}^2}\left[\frac{m_{22}-S_{XY}^2}{S_X^2S_Y^2}\right]$$

$$+\frac{1}{4}\rho_0\left[\frac{m_{40}-S_{X^*}^4}{S_{X^*}^4}+2\frac{m_{22}-S_{X^*}^2S_{Y^*}^2}{S_{X^*}^2S_{Y^*}^2}+\frac{m_{04}-S_{Y^*}^4}{S_{Y^*}^4}\right]$$

$$-\frac{\rho_0}{\rho_2}\left[\frac{m_{31}-S_{X^*}^2 S_{X^*Y^*}}{S_{X^*}^3 S_{Y^*}}+\frac{m_{13}-S_{X^*Y^*}S_{Y^*}^2}{S_{X^*}S_{Y^*}^3}\right]+\frac{\rho_0}{{\rho_2}^2}\left[\frac{m_{22}-S_{X^*Y^*}^2}{S_{X^*}^2 S_{Y^*}^2}\right].$$

By similar methodology applied in Theorem 2, it can be shown that under null hypothesis, T_0 has asymptotic standard normal distribution. Note that, in the case $n?m$, it is sufficient to replace n by $n^* = \min(m, n)$ in the above results.

3. Normal Populations

Assume that (X, Y) and (X^*, Y^*) have bivariate normal distributions. We find

$$\mu_{40} = 3\sigma_X^4,$$

$$\mu_{31} = 3\rho_{XY}\sigma_X^3\sigma_Y,$$
$$\mu_{22} = \left(1+2\rho_{XY}^2\right)\sigma_X^2\sigma_Y^2.$$

Hence,

$${\gamma_1}^2 = \left(1-\rho_{XY}^2\right)^2 \ and \ {\gamma_2}^2 = \left(1-\rho_{X^*Y^*}^2\right)^2.$$

So that for normal distributions

$$\sqrt{n}\,(r-\rho)\xrightarrow{\mathcal{L}}N\left(0,\lambda^{*2}\right),\quad as\ n\to\infty,$$

where

$$\lambda^{*2} = \frac{\left(1-\rho_{XY}^2\right)^2+\rho\left(1-\rho_{X^*Y^*}^2\right)^2}{\rho_{X^*Y^*}^2},$$

and hence

$$T_n^* = \sqrt{n}\left(\frac{r-\rho}{\widehat{\lambda}^*}\right)\xrightarrow{\mathcal{L}}N\,(0,1)\,,\quad as\ n\to\infty, \tag{5}$$

where

$$\widehat{\lambda}^{*2} = \frac{\left(1-r_{XY}^2\right)^2+r\left(1-r_{X^*Y^*}^2\right)^2}{r_{X^*Y^*}^2}.$$

This result can be used to construct an asymptotic confidence interval and hypothesis testing for the parameter ρ, in two normal populations, i.e.,

$$r \pm \frac{\widehat{\lambda}^*}{\sqrt{n}}Z_{\alpha/2}. \tag{6}$$

Also, to test$H_0 : \ \rho = \rho_0$, in two normal populations, we can use the test statistic

$$T_0 = \frac{r-\rho_0}{\lambda_0}, \tag{7}$$

where

$$\lambda_0{}^2 = \frac{\left(1 - r_{XY}^2\right)^2 + \rho_0\left(1 - r_{X^*Y^*}^2\right)^2}{r_{X^*Y^*}^2}.$$

We find that the test statistic T_0has asymptotic standard normal distribution.

4. Simulation

In this section, we provide a simulation study to compare our method with Fisher$Z$-transform and Cauchy-transform methods, i. e.,

$$Z = \frac{r_1' - r_2'}{\sqrt{\frac{1}{m-3} + \frac{1}{n-3}}},$$

and

$$C = \sqrt{\frac{m-3}{n-3}}\frac{r_1'}{r_2'}.$$

We simulate *50000* times of the presented confidence interval for two cases(normal and non-normal populations) with *m=n=50, 100, 200* and *500* for different values of $\rho_{XY}$and $\rho_{X^*Y^*}$.
The empirical coverage and mean lengths for normal and non-normal populations are summarized in the *Tables 1* and *3*, respectively.As can be seen, for two cases, in terms of the empirical probability coverage and the length of the interval, our method *(CI-1)* acts better than Fisher Z-transform *(CI-2)* and Cauchy transform *(CI-3)* methods.
The critical region which is constructed by inverting our confidence interval *(T-1)* has more power than the critical regions corresponding to the Fisher Z-transform *(T-2)* and Cauchy transform *(T-3)* methods. This subject can be seen by a simulation study for test $H_0 : \rho = 1$, the empirical powers of the tests for normal and non-normal populations are presented in *Tables 2* and *4*, respectively.
The power of the presented method is more than the Fisher Z-transform and Cauchy-transform methods and it can be stated that the test statistics (4) and (7) have a reasonable power in comparison with competing methods.

Table 1: The empirical probability and the length of the intervals for normal populations

n	CI	$\rho_{XY}=0.3$, $\rho_{X^*Y^*}=0.7$		$\rho_{XY}=0.8$, $\rho_{X^*Y^*}=0.7$		$\rho_{XY}=0.7$, $\rho_{X^*Y^*}=0.8$		$\rho_{XY}=0.7$, $\rho_{X^*Y^*}=0.3$	
		coverage	**length**	**coverage**	**length**	**coverage**	**length**	**coverage**	**length**
	CI-1	0.9501	0.3965	0.9496	0.5328	0.9503	1.0041	0.9487	2.3918
50	**CI-2**	0.9412	0.4114	0.9479	0.5378	0.9401	1.0966	0.9419	2.4947
	CI-3	0.9435	0.4125	0.9482	0.5400	0.9421	1.0928	0.9428	2.4702
	CI-1	0.9508	0.2797	0.9501	0.3729	0.9511	0.6799	0.9506	1.5978
100	**CI-2**	0.9456	0.2848	0.9468	0.3746	0.9462	0.7091	0.9456	1.6286
	CI-3	0.9462	0.2898	0.9465	0.3764	0.9470	0.7028	0.9475	1.6154
	CI-1	0.9499	0.1975	0.9500	0.2621	0.9501	0.4729	0.9501	1.1013
200	**CI-2**	0.9407	0.1993	0.9465	0.2627	0.9404	0.4827	0.9414	1.1114
	CI-3	0.9424	0.1991	0.9465	0.2620	0.9428	0.4832	0.9425	1.1120
	CI-1	0.9499	0.1248	0.9499	0.1653	0.9501	0.2958	0.9498	0.6859
500	**CI-2**	0.9402	0.1253	0.9402	0.1654	0.9404	0.2982	0.9411	0.6883
	CI-3	0.9456	0.1255	0.9456	0.1655	0.9421	0.3001	0.9426	0.6894

Table 2: The empirical powers of the tests for normal populations

n	***Critical Region***	$\rho_{XY}=0.3$, $\rho_{X^*Y^*}=0.7$	$\rho_{XY}=0.8$, $\rho_{X^*Y^*}=0.7$	$\rho_{XY}=0.7$, $\rho_{X^*Y^*}=0.8$	$\rho_{XY}=0.7$, $\rho_{X^*Y^*}=0.3$
	T-1	0.9930	0.9284	0.8610	0.9503
50	**T-2**	0.8802	0.8127	0.5880	0.8271
	T-3	0.9102	0.8402	0.6214	0.8512
	T-1	0.9994	0.9858	0.8833	0.9902
75	**T-2**	0.8986	0.8768	0.6825	0.8756
	T-3	0.9214	0.9124	0.7854	0.8997
	T-1	1	0.9969	0.9116	0.9980
100	**T-2**	0.9199	0.8947	0.7577	0.8948
	T-3	0.9266	0.9214	0.8103	0.9132
	T-1	1	0.9999	0.9348	0.9999
150	**T-2**	0.9341	0.9098	0.7065	0.9098
	T-3	0.9402	0.9235	0.8124	0.9232

Table 3: The empirical probability and the length of the intervals for non-normal populations

		$\rho_{XY}=0.3$ $\rho_{X^*Y^*}=0.7$		$\rho_{XY}=0.8$ $\rho_{X^*Y^*}=0.7$		$\rho_{XY}=0.7$ $\rho_{X^*Y^*}=0.8$		$\rho_{XY}=0.7$ $\rho_{X^*Y^*}=0.3$	
n	**CI**	**coverage**	**length**	**coverage**	**length**	**coverage**	**length**	**coverage**	**length**
50	**CI-1**	0.9502	0.3965	0.9501	0.5328	0.9499	1.0041	0.9497	2.3918
	CI-2	0.9412	0.4114	0.9409	0.5378	0.9486	1.0966	0.9479	2.4947
	CI-3	0.9421	0.4115	0.9428	0.5380	0.9492	1.0825	0.9480	2.4812
100	**CI-1**	0.9508	0.2797	0.9501	0.3729	0.9511	0.6799	0.9500	1.5978
	CI-2	0.9399	0.2848	0.9403	0.3746	0.9412	0.7091	0.9389	1.6286
	CI-3	0.9425	0.2901	0.9403	0.3765	0.9421	0.7100	0.9402	1.6245
200	**CI-1**	0.9499	0.1975	0.9500	0.2621	0.9498	0.4729	0.9501	1.1013
	CI-2	0.9307	0.1993	0.9401	0.2627	0.9304	0.4827	0.9414	1.1114
	CI-3	0.9422	0.1994	0.9416	0.26266	0.9356	0.4856	0.9402	1.2121
500	**CI-1**	0.9499	0.1248	0.9499	0.1653	0.9501	0.2958	0.9498	0.6859
	CI-2	0.9400	0.1253	0.9402	0.1654	0.9404	0.2982	0.9389	0.6883
	CI-3	0.9431	0.1249	0.9400	0.1664	0.9426	0.3012	0.9366	0.6880

Table 4: The empirical powers of the tests for non-normal populations

n	*Critical Region*	$\rho_{XY}=0.3$ $\rho_{X^*Y^*}=0.7$	$\rho_{XY}=0.8$ $\rho_{X^*Y^*}=0.7$	$\rho_{XY}=0.7$ $\rho_{X^*Y^*}=0.8$	$\rho_{XY}=0.7$ $\rho_{X^*Y^*}=0.3$
50	**T-1**	0.9930	0.9284	0.8610	0.9503
	T-2	0.8802	0.8127	0.5880	0.8271
	T-3	0.9002	0.8512	0.7645	0.8265
75	**T-1**	0.9994	0.9858	0.8833	0.9902
	T-2	0.8986	0.8768	0.6825	0.8756
	T-3	0.9102	0.8868	0.7825	0.8422
100	**T-1**	1	0.9969	0.9116	0.9980
	T-2	0.9199	0.8947	0.7577	0.8948
	T-3	0.9202	0.9012	0.8024	0.8611
150	**T-1**	1	0.9999	0.9348	0.9999
	T-2	0.9341	0.9098	0.7065	0.9098
	T-3	0.9402	0.9212	0.8426	0.9214

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References

[1] F. Baksh, G. Haars, S. Todd, P. A. H. Van Noord, and J. Whitehead, Comparing correlations of continuous observations from two independent populations using a sequential approach. *Statistics in Medicine,* 25 (2006), 4239-4310.

[2] Ferguson, S. Thomas, *A Course in Large Sample Theory,* Chapman & Hall, 1996.

[3] R. A. Fisher, On the probable error of a coefficient of correlation deduced from a small sample. *Metron*, 1 (1921), 3-32.

[4] J. B. Hittner, K. May, and N. C. Silver, A monte carlo evaluation of tests for comparing dependent correlations. *The Journal of General Psychology,* 130 (2003), 149-168.

[5] D. C. Howell, *Statistical Methods for Psychology*, 6th edition, Thomson Wadsworth, 2007.

[6] J. Lee, SAS Program for testing the difference between two correlated correlation coefficients. *Occup Environ Med.,* 51 (2) (1994), 141-141.

[7] X. Meng, R. Rosenthal, and D. B. Rubin, Comparing correlated correlation coefficients, *Psychological Bulletin*, 111 (1992), 172-175.

[8] A. Papoulis, *Probability and Statistics,* Prentence-Hall international editions, ISBN: 013711730, 2, 1990.

[9] T. E. Raghunathan, R. Rosenthal, and D. B. Rubin, Comparing correlated but non-overlapping correlations, *Psychological Methods*, 1 (1996), 178-183.

[10] R. R. Wilcox and T. Tian, Comparing dependent correlations, *The Journal of General Psychology,* 135 (1) (2008), 105-112.

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