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A New Method for Solving General Dual Fuzzy Linear Systems

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Abstract. According to fuzzy arithmetic, general dual fuzzy linear system (GDFLS) cannot be replaced by a fuzzy linear system (FLS). In this paper, we use new notation of fuzzy numbers and convert a GDFLS to two linear systems in crisp case, then we discuss complexity of the proposed method. Conditions for the existence of a unique fuzzy solution to $n \times n$ GDFLS are derived.

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1. Introduction

Fuzzy linear systems arise in many branches of science and technology such as economics, social sciences, telecommunications, image processing etc. Friedman *et al.* [14] introduced a model for solving a fuzzy linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. Also they studied duality in fuzzy linear systems $Ax = Bx + y$ where A and B are real $n \times n$ matrices, the unknown vector x is a vector consisting of n fuzzy numbers and the constant y is a vector consisting of n fuzzy numbers, in [15]. In [1, 3, 4, 9] the authors presented conjugate gradient and LU decomposition method for solving general FLS or symmetric FLS. Then, Allahviranloo has proposed iterative methods for solving a FLS [6, 7, 8]. In [5] Abbasbandy *et al.* investigated the existence of a minimal solution

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of general dual FLS of the form $Ax+f=Bx+c$, where A and B are real $m \times n$ matrices, the unknown vector x is a vector consisting of n fuzzy numbers and the constants f and c are vectors consisting of m fuzzy numbers. In [19, 20] Otadi *et al.* has proposed fuzzy neural networks for solving fully fuzzy linear systems.

Recently, the authors in [12, 13] proposed a method for solving a $n \times n$ FLS whose coefficients matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector by using the embedding method given in Cong-Xin and Min [10] and replace the original $n \times n$ fuzzy linear system by two $n \times n$ crisp linear systems. It is clear that in large systems, solving $n \times n$ linear system is better than solving $2n \times 2n$ linear system. Since perturbation analysis is very important in numerical methods. In [22, 23] presented the perturbation analysis for a class of a FLS which could be solved by an embedding method. Now, according to the presented method in this paper, we can investigate perturbation analysis in two $n \times n$ crisp linear systems.

2. Preliminaries

In [16, 17] a fuzzy number is defined as follows.

Definition 2.1. *A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \leqslant r \leqslant 1$, which satisfy the following requirements:*
i. $\underline{u}(r)$ is a bounded monotonically increasing, left continuous function on $(0,1]$ and right continuous at 0.
ii. $\overline{u}(r)$ is a bounded monotonically decreasing, left continuous function on $(0,1]$ and right continuous at 0.
iii. $\underline{u}(r) \leqslant \overline{u}(r), 0 \leqslant r \leqslant 1$.

Definition 2.2. [21] *For arbitrary fuzzy numbers $u = (\underline{u}, \overline{u})$ and $u = (\underline{v}, \overline{v})$ the quantity*

$$D(u,v) = \sup_{0 \leqslant r \leqslant 1} \{max[| \underline{u}(r) - \underline{v}(r) |, | \overline{u}(r) - \overline{v}(r) |]\}$$

is the Hausdorff distance between u and v.

The set of all these fuzzy numbers is denoted by E which is a complete metric space with Hausdorff distance. A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha,\ 0 \leqslant r \leqslant 1$.
For arbitrary fuzzy numbers $u = (\underline{u}(r), \overline{u}(r)),\ v = (\underline{v}(r), \overline{v}(r))$ and real number k, we may define the addition and the scalar multiplication of fuzzy numbers by using the extension principle as [17]

(i) $u = v$ if and only if $\underline{u}(r) = \underline{v}(r)$ and $\overline{u}(r) = \overline{v}(r)$,

(ii) $u + v = (\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r))$,

(iii) $ku = \begin{cases} (k\underline{u}, k\overline{u}), & k \geqslant 0, \\ (k\overline{u}, k\underline{u}), & k < 0. \end{cases}$

Remark 2.3. [2] *Let $u = (\underline{u}(r), \overline{u}(r)), 0 \leqslant r \leqslant 1$ be a fuzzy number, we take*

$$u^c(r) = \frac{\underline{u}(r) + \overline{u}(r)}{2},$$

$$u^h(r) = \frac{\overline{u}(r) - \underline{u}(r)}{2}.$$

It is clear that $u^h(r) \geqslant 0$, $\underline{u}(r) = u^c(r) - u^h(r)$ and $\overline{u}(r) = u^c(r) + u^h(r)$, also a fuzzy number $u \in E^1$ is said symmetric if $u^c(r)$ is independent of r for all $0 \leqslant r \leqslant 1$.

Remark 2.4. *Let $u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v}(r))$ and also k, s are arbitrary real numbers. If $w = ku + sv$ then*

$$\begin{aligned} & w^c(r) = ku^c(r) + sv^c(r), \\ & w^h(r) = |k|u^h(r) + |s|v^h(r). \end{aligned}$$

Definition 2.5. *The $n \times n$ linear system*

$$\begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_{11}x_1 + \cdots + b_{1n}x_n + y_1, \\ a_{21}x_1 + \cdots + a_{2n}x_n = b_{21}x_1 + \cdots + b_{2n}x_n + y_2, \\ \vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n = b_{n1}x_1 + \cdots + b_{nn}x_n + y_n, \end{cases} \tag{1}$$

where the given matrices of coefficients $A = (a_{ij}), 1 \leqslant i, j \leqslant n$ *and* $B = (b_{ij}), 1 \leqslant i, j \leqslant n$ *are real* $n \times n$ *matrices, the given* $y_i \in E$, $1 \leqslant i \leqslant n$, *with the unknowns* $x_j \in E$, $1 \leqslant j \leqslant n$ *is called a GDFLS. In the sequel, we will call the GDFLS (1) where* $b_{ij} = 0, 1 \leqslant i, j \leqslant n$, *a fuzzy linear system (FLS).*

Definition 2.6. [15] *A fuzzy number vector* $(x_1, x_2, ..., x_n)^t$ *given by*

$$x_j = (\underline{x}_j(r), \overline{x}_j(r)); \quad 1 \leqslant j \leqslant n, \ 0 \leqslant r \leqslant 1,$$

is called a solution of the GDFLS (1) if

$$\begin{cases} \sum_{j=1}^{n} \underline{a_{ij}x_j} = \sum_{j=1}^{n} \underline{b_{ij}x_j} + \underline{y}_i, \\ \\ \sum_{j=1}^{n} \overline{a_{ij}x_j} = \sum_{j=1}^{n} \overline{b_{ij}x_j} + \overline{y}_i. \end{cases}$$

If, for a particular i, $a_{ij}, b_{ij} > 0$, *for all* j, *we simply get*

$$\sum_{j=1}^{n} a_{ij}\underline{x}_j = \sum_{j=1}^{n} b_{ij}\underline{x}_j + \underline{y}_i, \qquad \sum_{j=1}^{n} a_{ij}\overline{x}_j = \sum_{j=1}^{n} b_{ij}\overline{x}_j + \overline{y}_i.$$

Finally, we conclude this section by a reviewing on the proposed method for solving FLS in [14].

Let $b_{ij} = 0$ for all i, j. Friedman *et al.* [14] wrote the FLS of Eq.(1) as follows:

$$SX = Y, \tag{2}$$

where s_{ij} are determined as follows:

$$\begin{array}{l} a_{ij} \geqslant 0 \Longrightarrow s_{ij} = a_{ij}, \ s_{i+m,j+n} = a_{ij}, \\ a_{ij} < 0 \Longrightarrow s_{i,j+n} = -a_{ij}, \ s_{i+m,j} = -a_{ij}, \end{array} \tag{3}$$

and any s_{ij} which is not determined by (3) is zero and

$$X = \begin{bmatrix} \underline{x}_1 \\ \vdots \\ \underline{x}_n \\ -\overline{x}_1 \\ \vdots \\ -\overline{x}_n \end{bmatrix}, \qquad Y = \begin{bmatrix} \underline{y}_1 \\ \vdots \\ \underline{y}_m \\ -\overline{y}_1 \\ \vdots \\ -\overline{y}_m \end{bmatrix}.$$

The structure of S implies that $s_{ij} \geqslant 0, 1 \leqslant i \leqslant 2m, 1 \leqslant j \leqslant 2n$ and that

$$S = \begin{pmatrix} H & C \\ C & H \end{pmatrix}, \tag{4}$$

where H contains the positive entries of A, and C contains the absolute values of the negative entries of A, i.e., $A = H - C$.

Theorem 2.7. [14] *The matrix S is nonsingular if and only if the matrices $A = H - C$ and $H + C$ are both nonsingular.*

Theorem 2.8. [14] *If S^{-1} exists it must have the same structure as S, i.e.*

$$S^{-1} = \begin{pmatrix} D & E \\ E & D \end{pmatrix}, \tag{5}$$

where

$$D = \frac{1}{2}[(H + C)^{-1} + (H - C)^{-1}], \qquad E = \frac{1}{2}[(H + C)^{-1} - (H - C)^{-1}].$$

We know that if S is nonsingular then

$$X = S^{-1}Y. \tag{6}$$

Recently, Ezzati [12] considered FLS and solved by using the embedding approach. Unfortunately he has not indicated conditions for the existence of a unique fuzzy solution to $n \times n$ linear system. Ezzati [12] wrote the linear system of Eq.(1) as follows:

$$A(\underline{x} + \overline{x}) = \underline{y} + \overline{y}, \tag{7}$$

where $h = (\underline{x} + \overline{x}) = (\underline{x}_1 + \overline{x}_1, \underline{x}_2 + \overline{x}_2, \ldots, \underline{x}_n + \overline{x}_n)^T$ and $\underline{y} + \overline{y} = (\underline{y}_1 + \overline{y}_1, \underline{y}_2 + \overline{y}_2, \ldots, \underline{y}_n + \overline{y}_n)^T$.

Theorem 2.9. [12] *Suppose the inverse of matrix A in Eq.(1) exists and $x = (x_1, x_2, \ldots, x_n)^T$ is a fuzzy solution of this equation. Then $\underline{x}(r) + \overline{x}(r)$ is the solution of the following system*

$$A(\underline{x}(r) + \overline{x}(r)) = \underline{y}(r) + \overline{y}(r). \tag{8}$$

We know that if A is a nonsingular real matrix then

$$h = A^{-1}(\underline{y}(r) + \overline{y}(r)).$$

Let matrices B and C have defined as Eq.(4). Now using matrix notation for Eq.(1), we get

$$\begin{cases} H\underline{x}(r) - C\overline{x}(r) = \underline{y}(r), \\ H\overline{x}(r) - C\underline{x}(r) = \overline{y}(r). \end{cases}$$

By substituting of $\overline{x}(r) = h - \underline{x}(r)$ and $\underline{x}(r) = h - \overline{x}(r)$ in the first and second equation of above system, respectively, we have

$$(H + C)\underline{x}(r) = \underline{y}(r) + Ch \tag{9}$$

and

$$(H + C)\overline{x}(r) = \overline{y}(r) + Ch.$$

If $H + C$ is nonsingular then

$$\underline{x}(r) = (H + C)^{-1}(\underline{y}(r) + Ch),$$

and

$$\overline{x}(r) = (H + C)^{-1}(\overline{y}(r) + Ch).$$

Therefore, we can solve FLS Eq.(1) by solving Eqs. (8)-(9).

3. The Model

In this section, we propose a new method for solving GDFLS.
Consider GDFLS Eq.(1). Usually, there is no inverse element for an arbitrary fuzzy number $u \in E^1$, i.e., there exists no element $v \in E^1$ such that

$$u + v = 0.$$

Actually, for all non-crisp fuzzy number $u \in E^1$ we have

$$u + (-u) \neq 0.$$

Therefore, the fuzzy linear equation system

$$Ax = Bx + y,$$

cannot be equivalently replaced by the fuzzy linear equation system

$$(A - B)x = y,$$

which had been investigated. By referring to remark 2 we have

$$\begin{cases} Ax^c(r) = Bx^c(r) + y^c(r), \\ A_1x^h(r) = B_1x^h(r) + y^h(r), \end{cases} \tag{10}$$

where $x^c(r) = (x_1^c(r), \ldots, x_n^c(r))^T$, $x^h(r) = (x_1^h(r), \ldots, x_n^h(r))^T$, $y^c(r) = (y_1^c(r), \ldots, y_n^c(r))^T$, $y^h(r) = (y_1^h(r), \ldots, y_n^h(r))^T$, A_1 and B_1 contains the absolute values of A and B, respectively. We known that if $A - B$ and $A_1 - B_1$ are nonsingular then

$$\begin{cases} x^c(r) = (A - B)^{-1}y^c(r), \\ x^h(r) = (A_1 - B_1)^{-1}y^h(r). \end{cases} \tag{11}$$

Therefore, we can solve GDFLS Eq. (1) by solving Eqs. (10) and we have

$$\begin{cases} \underline{x}(r) = x^c(r) - x^h(r), \\ \overline{x}(r) = x^c(r) + x^h(r). \end{cases} \tag{12}$$

Theorem 3.1. [11]. *A square crisp matrix is inverse-nonnegative if and only if it is the product of a permutation matrix by a diagonal matrix. And a square crisp matrix is inverse-nonnegative if and only if its entries are all zero except for a single positive entry in each row and column. The following result provides necessary conditions for the unique solution vector to be a fuzzy vector, given arbitrary input fuzzy vector y.*

Theorem 3.2. *The Eq. (1) have a unique fuzzy vector solution if $(A_1 - B_1)^{-1} \geqslant 0$, $(A_1 - B_1)^{-1} + (A - B)^{-1} \geqslant 0$ and $(A_1 - B_1)^{-1} - (A - B)^{-1} \geqslant 0$.*

Proof. Let $(A_1 - B_1)^{-1} \geqslant 0$, then

$$\underline{x}(r) = (A - B)^{-1}y^c(r) - (A_1 - B_1)^{-1}y^h(r), \tag{13}$$

$$\overline{x}(r) = (A-B)^{-1}y^c(r) + (A_1 - B_1)^{-1}y^h(r), \tag{14}$$

and by subtracting Eq. (13) from Eq. (14) we get

$$\overline{x}(r) - \underline{x}(r) = 2(A_1 - B_1)^{-1}y^h(r). \tag{15}$$

Thus, if y is arbitrary input vector which represents a fuzzy vector, i.e $\overline{y}(r) - \underline{y}(r) \geqslant 0$, then $y^h(r) \geqslant 0$, therefore a necessary condition $\overline{x}(r) - \underline{x}(r) \geqslant 0$. By using Eq. (13) and Eq. (14), we have

$$\underline{x}(r) = ((A-B)^{-1} + (A_1 - B_1)^{-1})\frac{\underline{y}(r)}{2} - ((A_1 - B_1)^{-1} - (A-B)^{-1})\frac{\overline{y}(r)}{2}, \tag{16}$$

$$\overline{x}(r) = ((A-B)^{-1} + (A_1 - B_1)^{-1})\frac{\overline{y}(r)}{2} - ((A_1 - B_1)^{-1} - (A-B)^{-1})\frac{\underline{y}(r)}{2}. \tag{17}$$

Since $\overline{y}(r)$ is monotonically decreasing and $\underline{y}(r)$ is monotonically increasing, the previous condition due to Eqs.(16)-(17) is also necessary for $\overline{x}(r)$ and $\underline{x}(r)$ to be monotonically decreasing and increasing, respectively. The bounded left continuity of $\overline{x}(r)$ and $\underline{x}(r)$ is obvious since they are linear combinations of $\overline{y}(r)$ and $\underline{y}(r)$. □

Consider FLS similar to Eq.(1), i.e. $b_{ij} = 0, 1 \leqslant i, j \leqslant n$. Therefor, we have the following theorem.

Theorem 3.3. *Assume that F_n, E_n and O_n are the number of multiplication operations that are required to calculate*

$$X = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n, -\overline{x}_1, -\overline{x}_2, \ldots, -\overline{x}_n)^T = S^{-1}Y$$

(the proposed method in Friedman et al. [14]),

$$X = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n, \overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n)^T$$

from Eqs. (8)-(9) (the proposed method in Ezzati [12]) and

$$X = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n, \overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n)^T$$

from Eqs. (10) and Eqs. (12), respectively. Then $O_n \leqslant E_n \leqslant F_n$ and $F_n - E_n = E_n - O_n = n^2$.

Proof. According to section 2, we have

$$S^{-1} = \begin{pmatrix} D & E \\ E & D \end{pmatrix},$$

where

$$D = \frac{1}{2}[(H+C)^{-1} + (H-C)^{-1}], \qquad E = \frac{1}{2}[(H+C)^{-1} - (H-C)^{-1}].$$

Therefore, for determining S^{-1}, we need to compute $(H+C)^{-1}$ and $(H-C)^{-1}$. Now, assume that M is $n \times n$ matrix and denote by $p_n(M)$ the number of multiplication operations that are required to calculate M^{-1}. It is clear that

$$p_{2n}(S) = p_n(H+C) + p_n(H-C) = 2p_n(A)$$

and hence

$$F_n = 2p_n(A) + 4n^2.$$

For computing $\underline{x} + \overline{x} = (\underline{x}_1 + \overline{x}_1, \underline{x}_2 + \overline{x}_2, \ldots, \underline{x}_n + \overline{x}_n)^T$ from Eq.(8) and $\underline{x} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n)^T$ from Eq.(9), the number of multiplication operations are $p_n(A) + n^2$ and $p_n(H+C) + 2n^2$, respectively. Clearly $p_n(H+C) = p_n(A)$, so

$$E_n = 2p_n(A) + 3n^2$$

and hence $E_n - F_n = n^2$. For computing $X = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n, \overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n)^T$, from Eqs. (10) and Eqs. (12), the number of multiplication operations are $p_n(A) + n^2$ and $p_n(A_1) + n^2$. Therefore

$$O_n = 2p_n(A) + 2n^2$$

and hence $F_n - E_n = E_n - O_n = n^2$. This proves theorem. $\square$

4. Numerical Examples

In this section we provide three examples illustrating the model in Section 3.

Example 4.1. An example of economic application
When some micro and macro problem are studied in the economic field, people often builds some equations or linear systems. And, when some parameters are not precise, fuzzy linear system appears. Here, we will give an economic application of general fuzzy linear systems [18].
The market price of a good and the quantity produced are determined by the equality between supply and demand. Suppose that demand and supply are linear functions of the price:

$$\begin{cases} q_d = a.p + b, \\ q_s = c.p + d, \end{cases}$$

where q_s is the quantity supplied, which is required to be equal to q_d, the quantity requested, p is the price and a, b, c and d are coefficients to be estimated, where the coefficients b and d are represented by fuzzy triangular numbers, a and c are crisp numbers. By imposing the equality between quantity supplied and requested, the following GDFLS should be solved:

$$\begin{cases} x_1 = -\frac{1}{2}x_2 + (18 + r, 20 - r), \\ x_1 = \frac{1}{2}x_2 + (-4 + r, -2 - r). \end{cases}$$

We obtain the exact solution by using Eqs. (10)

$$\begin{bmatrix} x_1^c(r) \\ x_2^c(r) \end{bmatrix} = \begin{bmatrix} 8 \\ 22 \end{bmatrix}$$

and

$$\begin{bmatrix} x_1^h(r) \\ x_2^h(r) \end{bmatrix} = \begin{bmatrix} 2 - 2r \\ 2 - 2r \end{bmatrix},$$

and hence $x_1(r) = (6 + 2r, 10 - 2r)$, $x_2(r) = (20 + 2r, 24 - 2r)$.

Example 4.2. Consider the 2×2 GDFLS

$$\begin{cases} 2x_1 - 2x_2 = x_1 - x_2 + (-1 + 2r, 4 - 3r), \\ 3x_1 + 4x_2 = 2x_1 + x_2 + (1 + 4r, 10 - 5r). \end{cases}$$

By using Eqs. (10), we have:

$$\begin{bmatrix} x_1^c(r) \\ x_2^c(r) \end{bmatrix} = \begin{bmatrix} \frac{5}{2} - \frac{r}{2} \\ 1 \end{bmatrix}$$

and

$$\begin{bmatrix} x_1^d(r) \\ x_2^d(r) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} - \frac{3}{2}r \\ 1 - r \end{bmatrix},$$

hence $x_1(r) = (1 + r, 4 - 2r)$ and $x_2(r) = (r, 2 - r)$. According to this fact that $\underline{x}_i \leqslant \overline{x}_i,\ i = 1, 1$, are monotonic decreasing functions then the solution $x_i(r) = (\underline{x}_i(r), \overline{x}_i(r)),\ i = 1, 2$, is a fuzzy solution.

Example 4.3. Consider the 3×3 GDFLS

$$\begin{cases} 5x_1 + x_2 - 2x_3 = x_1 - x_3 + (6r, 14 - 8r), \\ -x_1 + 4x_2 + x_3 = x_2 + (7 + 6r, 21 - 8r), \\ 2x_1 + 3x_2 + 5x_3 = 2x_2 + x_3 + (3 + 11r, 22 - 8r), \end{cases}$$

We obtain the exact solution by using Eqs. (10)

$$\begin{bmatrix} x_1^c(r) \\ x_2^c(r) \\ x_3^c(r) \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{9}{2} - \frac{r}{2} \\ \frac{3}{2} + \frac{r}{2} \end{bmatrix}$$

and

$$\begin{bmatrix} x_1^h(r) \\ x_2^h(r) \\ x_3^h(r) \end{bmatrix} = \begin{bmatrix} 1 - r \\ \frac{3}{2} - \frac{3}{2}r \\ \frac{3}{2} - \frac{3}{2}r \end{bmatrix},$$

hence $x_1(r) = (r, 2 - r)$, $x_2(r) = (3 + r, 6 - 2r)$ and $x_3(r) = (2r, 3 - r)$.

5. Conclusions

In this paper, we proposed a new model for solving a GDFLS with n fuzzy variables. The original fuzzy linear system with coefficient matrices A and B are replaced by two $n \times n$ crisp linear systems. Then, we discussed complexity of the proposed method. Also, we derived conditions for the existence of a unique fuzzy solution to $n \times n$ GDFLS.

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References

[1] S. Abbasbandy and M. Alavi, A new method for solving symmetric fuzzy linear systems, *Mathematics Scientific Journal, Islamic Azad University of Arak,* 1 (2005), 55-62.

[2] S. Abbasbandy, E. Babolian, and M. Alavi, Numerical method for solving linear Fredholm fuzzy integral equations of the second kind, *Chaos Solitons & Fractals,* 31 (2007), 138-146.

[3] S. Abbasbandy, A. Jafarian, and R. Ezzati, Conjugate gradient method for fuzzy symmetric positive definite system of linear equations, *Appl. Math. Comput.,* 171 (2005), 1184-1191.

[4] S. Abbasbandy, R. Ezzati, and A. Jafarian, LU decomposition method for solving fuzzy system of linear equations, *Appl. Math. Comput.,* 172 (2006), 633-643.

[5] S. Abbasbandy, M. Otadi, and M. Mosleh, Minimal solution of general dual fuzzy linear systems, *Chaos Solitons & Fractals,* 37 (2008), 1113-1124.

[6] T. Allahviranloo, Numerical methods for fuzzy system of linear equations, *Appl. Math. Comput.,* 155 (2004), 493-502.

[7] T. Allahviranloo, Successive over relaxation iterative method for fuzzy system of linear equations, *Appl. Math. Comput.,* 162 (2005), 189-196.

[8] T. Allahviranloo, The Adomian decomposition method for fuzzy system of linear equations, *Appl. Math. Comput.,* 163 (2005), 553-563.

[9] B. Asady, S. Abbasbandy, and M. Alavi, Fuzzy general linear systems, *Appl. Math. Comput.,* 169 (2005), 34-40.

[10] W. Cong-Xin and M. Ming, Embedding problem of fuzzy number space: PartI, *Fuzzy Sets and Systems,* 44 (1991), 33-38.

[11] R. Demarr, Nonnegative matrices with nonnegative inverses, *Proc. Am. Math. Soc.,* (1972), 307-308.

[12] R. Ezzati, Solving fuzzy linear systems, *Soft Computing,* 15 (2011), 193-197.

[13] M. A. Fariborzi Araghi and M. M. Hosseinzade, Solution of general dual fuzzy linear systems using ABS algorithm, *Applied Mathematical Sciences,* 6 (2012), 163-171.

[14] M. Friedman, Ma Ming and A. Kandel, Fuzzy linear systems, *Fuzzy Sets and Systems,* 96 (1998), 201-209.

[15] M. Friedman, M. Ming, and A. Kandel, Duality in fuzzy linear systems, *Fuzzy Sets and Systems,* 109 (2000), 55-58.

[16] O. Kaleva, Fuzzy differential equations, *Fuzzy Sets Syst.*, 24 (1987), 301-317.

[17] M. Ming, M. Friedman, and A. Kandel, A new fuzzy arithmetic, *Fuzzy Sets and Systems*, 108 (1999), 83-90.

[18] S. Muzzioli and H. Reynaerts, Fuzzy linear systems of the form $A_1x+b_1 = A_2x+b_2$, *Fuzzy Sets and Systems*, 157 (2006), 939951.

[19] M. Otadi and M. Mosleh, Simulation and evaluation of dual fully fuzzy linear systems by fuzzy neural network, *Applied Mathematical Modelling,* 35 (2011), 5026-5039.

[20] M. Otadi, M. Mosleh, and S. Abbasbandy, Numerical solution of fully fuzzy linear systems by fuzzy neural network, *Soft Computing,* 15 (2011), 1513-1522.

[21] M. L. Puri and D. Ralescu, Fuzzy random variables, *J. Math. Anal. Appl.,* 114 (1986), 409-422.

[22] K. Wang, G. Chen, and Y. Wei, Perturbation analysis for a class of fuzzy linear systems, *J. of Coimput. Appl. Math.,* 224 (2009), 54-65.

[23] Z. Tian, L. Hu, and D. Greenhalgh, Perturbation analysis of fuzzy linear systems, *Information Sciences*, 180 (2010), 4706-4713.

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