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Original Research Paper

On a Nonlinear Fractional-Order Model of COVID-19 Under AB-Fractional Derivative

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Abstract. In this paper, we present a BOX mathematical model for the release of COVID-19.We intend to generalize the model to fractional order derivative in Atangana-Baleanu sense and to show the existence of solution for the fractional model using Schaefer's fixed point theorem and for the uniqueness of solution we make use of Banach fixed point theorem. By using Shehu transform and Picard successive iterative procedure, we explore the iterative solutions and its stability for the considered fractional model. Given the beginning of a new wave of COVID-19 spread in Indonesia, we present a numerical simulation to study and predict the spread of the disease in this country.

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1 Introduction

In late 2019, a new virus from the corona virus, called the Novel Corona virus, was identified in Wuhan, China, and spread rapidly in several parts of China. It was initially reported that the virus had spread from the seafood market, now, it has been almost a year and a half since the virus was identified and its origin has not been determined. The SARS-CoV-2 quickly spread to other countries and spread widely, with the World Health Organization announcing a pandemic and informing all countries of rapid measures to control the spread of COVID-19, such as hand washing, mask use and social distancing. COVID-19, which is caused by SARSCOV-2, has different symptoms in different people. The most common symptoms are fever, lethargy, dry cough, and lung involvement, some other symptoms such as sore throat, body aches, digestive problems, and skin and heart problems, etc... can add to symptoms of the disease.At the beginning of the outbreak of COVID-19, it was announced that children and adolescents would not be infected, but by mutations in the SARSCOV-2 virus, new strains of the virus have identified that is more rapidly transmitted and infect children.Various vaccines have already been developed for COVID-19 and vaccination has begun in many countries, however, the spread of the disease continues and it is unclear whether vaccinated individuals will be resistant to the new strains of SARACOV-2, or no?

The theory of fractional calculus, especially differential equations of fractional order derivatives have a significance important in the modeling of many real world problems arising in science and engineering (see e.g. [2, 5, 6, 12, 18, 20, 27, 28, 32, 33, 35, 36] and references therein). Fractional derivative due to Riemann-Liouville and Caputo were used widely in the early literature. But due to the presence of singularities in their kernels, some new fractional derivatives were introduced which settled the arisen problem, for details, we refer [1, 3, 4, 10, 19, 21, 24, 29, 31, 37]. More precisely, to study the complex biological systems and diseases, fractional calculus played an important role as it provides better results than the integer order models (see e.g. [7, 14, 15, 16, 17, 22].

In the past, the integer-order derivative was used for modeling that

did not preserve the system's historical memory, but in recent decades, with the expansion of the use of the fractional order derivative that preserves the system's historical memory, researchers have begun to use the fractional order derivative instead of ordinary derivative. With the outbreak of COVID-19, mathematicians and biologists studied the spread of the disease. Depending on the behaviour of the SARSCOV-2 virus and how the disease spreads, various mathematical models have been written (see e.g. [8, 13, 23, 26, 34]).
Due to the nature of the prevalence of COVID-19, the number of infected people at any time depends on the number of infected people in previous times, and the disease transmission system has historical memory. So we use the fractional-order derivative, which preserves the historical memory of the system, to investigate the spread of COVID-19. On the other hand, because some fraction-order derivatives have a singularity in their kernels, we use the Atangana-Baleanu-Caputo fractional-order derivative whose kernel has no singularity. We also present a numerical simulation to better investigate the COVID- 19 transmission.
The structure of the paper will be as follows: In the next section, some basic definitions and concepts are recalled. The fractional order model is presented in Section 3 to investigate the release of COVID-19. The equilibrium points of the system and its stability conditions are determined in Section 4. The existence of the system solution is proved using the fixed point theory in Section 5. Using Shehu transform, a specific solution for the system is determined in Section 6, and the stability of the method in Section 7 is demonstrated. Numerical results and conclusions are presented in sections 8 and 9, respectively.

2 Preliminaries

In present section, we recall the some basic definitions of fractional calculus. We start with the definition of Caputo fractional-order derivative which can be found in many books (see, e.g., [18]).

Definition 2.1. For a differentiable function h, the Caputo derivative of order $\delta \in (0,1)$ is defined by

$$^{C}\mathfrak{D}^{\delta}h(t) = \frac{1}{\Gamma(1-\delta)}\int_0^t h^{'}(s)\frac{1}{(t-s)^{\delta}}ds.$$

Definition 2.2. [3] Let $h \in \mathcal{H}^1(0,1)$ and $\delta \in [0,1]$ then the Atangana-Baleanu-Caputo (ABC) fractional derivative is defined by

$$^{ABC}\mathfrak{D}^{\delta}h(t) = \frac{M(\delta)}{(1-\delta)} \int_0^t h^{'}(\omega)E_{\delta}[-\frac{\delta}{1-\delta}(t-\omega)^{\delta}]d\omega.$$

Definition 2.3. [3] The integral operator associated with ABC-fractional derivative is defined by

$$^{ABC}\mathfrak{I}^{\delta}h(t) = \frac{(1-\delta)}{M(\delta)}h(t) + \frac{\delta}{M(\delta)\Gamma(\delta)} \int_0^t h(\omega)(t-\omega)^{\delta-1}d\omega,$$

where $M(\delta)$ is the normalization function.

Definition 2.4. [25] For a function $\xi(t)$ in

$$A = \{\xi(t) : \text{there exist } \chi, t_1, t_2 > 0, |\xi(t)| < \chi exp\left(\frac{|t|}{t_i}\right), \text{ if } t \in (-1)^j \times [0,\infty)\},$$

the Shehu transform of $\xi(t) \in A$ is given by

$$S_h(\xi(t)) = \int\limits_0^{\infty} exp\left(-\frac{st}{u}\right)\xi(t)dt \qquad u \in (-t_1, t_2).$$

Lemma 2.5. *[9] Assume $h \in H^1(a,b)$, $b > a$, $\gamma \in (0,1)$ and $h(t) \in A$, the Shehu transform (S_h) of Atangana-Baleanu fractional derivative in Caputo sense is*

$$S_h(^{ABC}\mathfrak{D}^{\delta}h(t)) = \frac{M(\gamma)}{1-\gamma+\gamma\left(\frac{u}{s}\right)^{\gamma}}(S_h(h(t)) - \frac{u}{s}h(0)).$$

3 Fractional Model in Atangana-Baleanu Sense

In this section, we present a fractional-order mathematical model for the transmission of COVID-19, using the idea in the work of Chen *et al.* [11]. In this model, the people are divided into five compartments: susceptible people ($\mathfrak{S}_p$), exposed people ($\mathfrak{E}_p$), symptomatic infected people ($\mathfrak{I}_p$), asymptomatic infected people ($\mathfrak{A}_p$), and removed people ($\mathfrak{R}_p$) including recovered and dead people. Also, the virus repository from which the virus starts spreading is indicated by $\mathfrak{M}$. We consider the desired model as follows:

$$
\begin{aligned}
\frac{d\mathfrak{S}_p}{dt} &= \coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M},\\
\frac{d\mathfrak{E}_p}{dt} &= \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p - \Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p,\\
\frac{d\mathfrak{I}_p}{dt} &= (1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p, \qquad (1)\\
\frac{d\mathfrak{A}_p}{dt} &= \Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p,\\
\frac{d\mathfrak{R}_p}{dt} &= \tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p,\\
\frac{d\mathfrak{M}}{dt} &= \phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M},
\end{aligned}
$$

with the initial conditions

$$
\begin{aligned}
&\mathfrak{S}_p(0) = \mathfrak{S}_p(0) \geq 0, \mathfrak{E}_p(0) = \mathfrak{E}_p(0) \geq 0, \mathfrak{I}_p(0) = \mathfrak{I}_p(0) \geq 0,\\
&\mathfrak{A}_p(0) = \mathfrak{A}_p(0) \geq 0, \mathfrak{R}_p(0) = \mathfrak{R}_p(0) \geq 0, \mathfrak{M}(0) = \mathfrak{M}(0) \geq 0.
\end{aligned}
$$

The parameters of the model are: the birth rate $\coprod_p$, the death rate ω_p, the transmission rate from $\mathfrak{I}_p$ to $\mathfrak{S}_p$ as ζ_p, the transmission rate of $\mathfrak{A}_p$ to that of $\mathfrak{I}_p$ as Ψ, the transmission rate from $\mathfrak{M}$ to $\mathfrak{S}_p$ as ω_w, the proportion of asymptomatic infection rate of people Φ_p, the incubation period of people $\frac{1}{\eta_p}$, The latent period of people ϱ_p, the infectious period of symptomatic infection of people $\frac{1}{\tau_p}$, the infectious period of asymptomatic infection of people τ_{ap}, the shedding coefficients from $\mathfrak{I}_p$ to $\mathfrak{M}$ as ϕ_p, the shedding coefficients from $\mathfrak{A}_p$ to $\mathfrak{M}$ as ϖ_p,the lifetime of the virus in $\mathfrak{M}$ as $\frac{1}{\varphi}$.

We generalize the model (1) to a fractional order model using the Atangana-Baleanu derivative in Caputo sense as follows:

$$
\begin{aligned}
{}^{ABC}\mathfrak{D}^\delta\mathfrak{S}_p &= \coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M},\\
{}^{ABC}\mathfrak{D}^\delta\mathfrak{E}_p &= \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p - \Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p,\\
{}^{ABC}\mathfrak{D}^\delta\mathfrak{I}_p &= (1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p, \qquad (2)\\
{}^{ABC}\mathfrak{D}^\delta\mathfrak{A}_p &= \Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p,\\
{}^{ABC}\mathfrak{D}^\delta\mathfrak{R}_p &= \tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p,\\
{}^{ABC}\mathfrak{D}^\delta\mathfrak{M} &= \phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M},
\end{aligned}
$$

where δ denotes the the fractional order parameter and the model variables in (2) are nonnegative and the initial conditions are given by

$$\mathfrak{S}_p(0) = \mathfrak{S}_p(0) \geq 0, \mathfrak{E}_p(0) = \mathfrak{E}_p(0) \geq 0, \mathfrak{I}_p(0) = \mathfrak{I}_p(0) \geq 0,$$
$$\mathfrak{A}_p(0) = \mathfrak{A}_p(0) \geq 0, \mathfrak{R}_p(0) = \mathfrak{R}_p(0) \geq 0, \mathfrak{M}(0) = \mathfrak{M}(0) \geq 0.$$

4 Equilibrium Points

To determine the equilibrium points of the fractional order system (2), we solve the following equations

$$^{ABC}\mathfrak{D}^{\delta}\mathfrak{S}_p =^{ABC} \mathfrak{D}^{\delta}\mathfrak{E}_p =^{ABC} \mathfrak{D}^{\delta}\mathfrak{I}_p =^{ABC} \mathfrak{D}^{\delta}\mathfrak{A}_p =^{ABC} \mathfrak{D}^{\delta}\mathfrak{R}_p =^{ABC} \mathfrak{D}^{\delta}\mathfrak{M} = 0.$$

By solving the algebraic equations we obtain equilibrium points of system. The disease-free equilibrium point, the point where there is no disease, given by $E^0 = (\frac{\coprod_p}{\omega_p}, 0, 0, 0, 0, 0)$. In addition, if $R_0 > 1$,then the system (2) has a positive endemic equilibrium

$$E_1^* = (S_P^*, E_P^*, I_P^*, A_P^*, R_P^*, M^*),$$

$$S^* = \frac{xyz\varphi}{\tau_{ap}\Phi_p\varphi\psi\zeta_p y + \tau_{ap}\varphi_p\omega_w\Phi_p y - \omega_w\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \omega_w\phi_p\eta_p z + \varphi\eta_p\zeta_p z},$$

$$E^* = \frac{G^1_{a,p,w}}{x(\tau_{ap}\Phi_p\varphi\psi p y + \tau_{ap}\varphi_p\omega_w\Phi_p y - \omega_w\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \omega_w\phi_p\eta_p z + \varphi\eta_p\zeta_p z)}$$

with

$$G^1_{a,p,w} = \tau_{ap}\coprod_p \Phi_p\varphi\psi\zeta_p y + \tau_{ap}\coprod_p \varphi_p\omega_w\Phi_p y - \coprod_p \omega_w\Phi_p\phi_p\eta_p z$$
$$- \coprod_p \Phi_p\varphi\eta_p\zeta_p z + \coprod_p \omega_w\phi_p\eta_p z + \coprod_p \varphi\eta_p\zeta_p z - \varphi\omega_p xyz,$$

$$I^* = -\frac{G^2_{a,p,w}}{x(\tau_{ap}\Phi_p\varphi\psi\zeta_p y + \tau_{ap}\varphi_p\omega_w\Phi_p y - \omega_w\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \omega_w\phi_p\eta_p z + \varphi\eta_p\zeta_p z)y}$$

with

$$G^2_{a,p,w} = \eta_p(\tau_{ap}\coprod_p y(\Phi_p\varphi\psi\zeta_p + \varphi_p\omega_w\Phi_p)$$

$$-\coprod_p \eta_p(\omega_w\Phi_p\phi_p z - \Phi_p\varphi\zeta_p z + \zeta_p\phi_p z + \varphi\zeta_p z) - \varphi\omega_p xyz)(\Phi_p - 1),$$

$$A^* = \frac{G^3_{a,p,w}}{x(\tau_{ap}\Phi_p\varphi\psi\zeta_p y + \tau_{ap}\varphi_p\omega_w\Phi_p y - \omega_w\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \omega_w\phi_p\eta_p z + \varphi\eta_p\zeta_p z)z}$$

with

$$G^3_{a,p,w} = \Phi_p\tau_{ap}(\tau_{ap}\coprod_p \Phi_p\varphi\psi\zeta_p y + \tau_{ap}\coprod_p \varphi_p\omega_w\Phi_p y - \coprod_p \omega_w\Phi_p\phi_p\eta_p z$$

$$-\coprod_p \Phi_p\varphi\eta_p\zeta_p z + \coprod_p \omega_w\phi_p\eta_p z + \coprod_p \varphi\eta_p\zeta_p z - \varphi\omega_p xyz),$$

$$R^* = \frac{G^4_{a,p,w}}{x(\tau_{ap}\Phi_p\varphi\psi\zeta_p y + \tau_{ap}a\omega_w\Phi_p y - \omega_w\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \omega_w\phi_p\eta_p z + \varphi\eta_p\zeta_p z)\omega_p yz}$$

with

$$G^4_{a,p,w} = (\tau_{ap}\coprod_p \Phi_p y(\varphi\psi\zeta_p + \varphi_p\omega_w) + (-\coprod_p \Phi_p\eta_p z$$

$$+\coprod_p \eta_p z)(\omega_w\phi_p + \varphi\zeta_p) - \varphi\omega_p xyz)(\tau_{ap}\Phi_p y\Upsilon - \tau_p\eta_p z(\Phi_p - 1)),$$

$$W^* = \frac{G^5_{a,p,w}}{x(\tau_{ap}\Phi_p\varphi\psi\zeta_p y + \tau_{ap}\varphi_p\beta\Phi_p y - \beta\Phi_p\phi_p\eta_p z - \Phi_p\varphi\eta_p\zeta_p z + \zeta_p\phi_p\eta_p z + \varphi\eta_p\zeta_p z)\varphi yz}$$

with

$$G^5_{a,p,w} = (\tau_{ap}\coprod_p \Phi_p y(\varphi\psi\zeta_p + \varphi_p\omega_w) - (\coprod_p \Phi_p\eta_p z$$

$$-\coprod_p \eta_p z)(\omega_w\phi_p + \varphi\zeta_p) - \varphi\omega_p xyz)(\tau_{ap}\varphi_p\Phi_p y - \phi_p\eta_p z(\Phi_p - 1)),$$

where $y = (\tau_p + \omega_p)$, $x = (1 - \Phi_p)\eta_p + \Phi_p\varrho_p + \omega_p$, $z = \tau_{ap} + \omega_p$. Also, R_0 is the basic reproduction number and is obtained using the next generation method. To find R_0, we first consider the system as follows

$${}^{ABC}D^\delta\Psi(t) = F(\Psi(t)) - V(\Psi(t)),$$

where

$$F(\Psi(t)) = \begin{bmatrix} \zeta_p S(t)(I(t) + \psi A(t)) + \omega_w S(t)W(t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$V(\Psi(t)) = \begin{bmatrix} (1-\Phi_p)\eta_p E(t) + \Phi_p \varrho_p E(t) + \omega_p E(t) \\ -(1-\Phi_p)\eta_p E(t) + (\tau_p + \omega_p) I(t) \\ -\Phi_p \varrho_p E(t) + (\tau_{ap} + \omega_p) A(t) \\ -\phi_p I(t) - \varphi_p A(t) + \varepsilon W(t) \end{bmatrix}$$

At E^0, the Jacobian matrix for F and V are obtained as

$$J_F(E^0) = \begin{bmatrix} 0 & N_{3\times 3} \\ 0 & 0 \end{bmatrix}, N_{3\times 3} = \begin{bmatrix} \zeta_p S & \zeta_p \psi S & \omega_w S \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$J_v(E^0) = \begin{bmatrix} (1-\Phi_p)\eta_p + \Phi_p \varrho_p + \omega_p & 0 & 0 & 0 \\ -(1-\Phi_p)\eta_p & \tau_p + \omega_p & 0 & 0 \\ -\Phi_p \varrho_p & 0 & \tau_{ap} + \omega_p & 0 \\ 0 & -\phi_p & -\varphi_p & \varepsilon \end{bmatrix}$$

FV^{-1} is the next generation matrix for system (2), then the basic reproduction number is

$$\begin{aligned} R_0 &= \rho(FV^{-1}) \\ &= \frac{\zeta_p \coprod_p (1-\Phi_p)\eta_p(\omega_p + \tau_{ap}) + \zeta_p \psi \coprod_p \Phi_p \eta_p (\omega_p + \tau_p)}{\omega_p(\omega_p + \eta_p)(\omega_p + \tau_p)(\omega_p + \tau_{ap})} \\ &+ \frac{\omega_w \coprod_p \phi_p \eta_p (1-\Phi_p)(\omega_p + \tau_{ap}) + \omega_w \coprod_p \Phi_p \eta_p \varphi_p (\omega_p + \tau_p)}{\omega_p \varepsilon (\omega_p + \eta_p)(\omega_p + \tau_p)(\omega_p + \tau_{ap})}. \end{aligned}$$

This basic reproduction number R_0, is an epidemiologic metric used to describe the contagiousness or transmissibility of infectious agents.
In the next section we investigate the existence and uniqueness of the solution for system (2) by fixed point theory.

5 Existence of Solution

Using the initial conditions and fractional integral operator, we convert model (2) into integral equations

$$
\begin{aligned}
\mathfrak{S}_p(t)-\mathfrak{S}_p(0) &= {}^{ABC}\mathfrak{I}^{\delta}\left[\coprod_p -\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
\mathfrak{E}_p(t)-\mathfrak{E}_p(0) &= {}^{ABC}\mathfrak{I}^{\delta}\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Big],\\
\mathfrak{I}_p(t)-\mathfrak{I}_p(0) &= {}^{ABC}\mathfrak{I}^{\delta}[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p],\\
\mathfrak{A}_p(t)-\mathfrak{A}_p(0) &= {}^{ABC}\mathfrak{I}^{\delta}[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
\mathfrak{R}_p(t)-\mathfrak{R}_p(0) &= {}^{ABC}\mathfrak{I}^{\delta}[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
\mathfrak{M}(t)-\mathfrak{M}(0) &= {}^{ABC}\mathfrak{I}^{\delta}[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}].
\end{aligned}
\tag{3}
$$

For simplicity, we write the kernels

$$
\begin{aligned}
\mathfrak{F}_1(t,\mathfrak{S}_p(t)) &= \coprod_p -\omega_p\mathfrak{S}_p(t)-\zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t)+\Psi\mathfrak{A}_p(t))-\omega_w\mathfrak{S}_p(t)\mathfrak{M}(t),\\
\mathfrak{F}_2(t,\mathfrak{E}_p(t)) &= \zeta_p\mathfrak{S}_p(t)(\mathfrak{I}_p(t)+\Psi\mathfrak{A}_p(t))+\omega_w\mathfrak{S}_p(t)\mathfrak{M}(t)-(1-\Phi_p)\eta_p\mathfrak{E}_p(t)\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p(t)-\omega_p\mathfrak{E}_p(t),\\
\mathfrak{F}_3(t,\mathfrak{I}_p(t)) &= (1-\Phi_p)\eta_p\mathfrak{E}_p(t)-(\tau_p+\omega_p)\mathfrak{I}_p(t),\\
\mathfrak{F}_4(t,\mathfrak{A}_p(t)) &= \Phi_p\varrho_P\mathfrak{E}_p(t)-(\tau_{ap}+\omega_p)\mathfrak{A}_p(t),\\
\mathfrak{F}_5(t,\mathfrak{R}_p(t)) &= \tau_p\mathfrak{I}_p(t)+\tau_{ap}\mathfrak{A}_p(t)-\omega_p\mathfrak{R}_p(t),\\
\mathfrak{F}_6(t,\mathfrak{M}(t)) &= \phi_p\mathfrak{I}_p(t)+\varpi_p\mathfrak{A}_p(t)-\varphi\mathfrak{M}(t)
\end{aligned}
\tag{4}
$$

and the functions

$$
\Upsilon(\delta)=\frac{1-\delta}{M(\delta)},\qquad \Lambda(\delta)=\frac{\delta}{\Gamma(\delta)M(\delta)}.
\tag{5}
$$

Applying (4) and (5) in (3) and writing state variables in terms of kernels, we obtain

$$\begin{aligned}
\mathfrak{S}_p(t) &= \mathfrak{S}_p(0)+\Upsilon(\delta)\mathfrak{F}_1(t,\mathfrak{S}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_1(x,\mathfrak{S}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{E}_p(t) &= \mathfrak{E}_p(0)+\Upsilon(\delta)\mathfrak{F}_2(t,\mathfrak{E}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_2(x,\mathfrak{E}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{I}_p(t) &= \mathfrak{I}_p(0)+\Upsilon(\delta)\mathfrak{F}_3(t,\mathfrak{I}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_3(x,\mathfrak{I}_p(x))(t-x)^{\delta-1}dx, \quad (6)\\
\mathfrak{A}_p(t) &= \mathfrak{A}_p(0)+\Upsilon(\delta)\mathfrak{F}_4(t,\mathfrak{A}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_4(x,\mathfrak{A}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{R}_p(t) &= \mathfrak{R}_p(0)+\Upsilon(\delta)\mathfrak{F}_5(t,\mathfrak{R}_p(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_5(x,\mathfrak{R}_p(x))(t-x)^{\delta-1}dx,\\
\mathfrak{M}(t) &= \mathfrak{M}(0)+\Upsilon(\delta)\mathfrak{F}_6(t,\mathfrak{M}(t))+\Lambda(\delta)\int_0^t \mathfrak{F}_6(x,\mathfrak{M}(x))(t-x)^{\delta-1}dx.
\end{aligned}$$

The Picard iterations are given by

$$\begin{aligned}
\mathfrak{S}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_1(t,\mathfrak{S}_p^j(t))+\Lambda(\delta_1)\int_0^t \mathfrak{F}_1(x,\mathfrak{S}_p^j(x))(t-x)^{\delta-1}dx,\\
\mathfrak{E}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_2(t,\mathfrak{E}_p^j(t))+\Lambda(\delta_2)\int_0^t \mathfrak{F}_2(x,\mathfrak{E}_p^j(x))(t-x)^{\delta-1}dx,\\
\mathfrak{I}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_3(t,\mathfrak{I}_p^j(t))+\Lambda(\delta_3)\int_0^t \mathfrak{F}_3(x,\mathfrak{I}_p^j(x))(t-x)^{\delta-1}dx,\\
\mathfrak{A}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_4(t,\mathfrak{A}_p^j(t))+\Lambda(\delta_4)\int_0^t \mathfrak{F}_4(x,\mathfrak{A}_p^j(x))(t-x)^{\delta-1}dx, \quad (7)\\
\mathfrak{R}_p^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_5(t,\mathfrak{R}_p^j(t))+\Lambda(\delta_5)\int_0^t \mathfrak{F}_5(x,\mathfrak{R}_p^j(x))(t-x)^{\delta-1}dx,\\
\mathfrak{M}^{j+1}(t) &= \Upsilon(\delta)\mathfrak{F}_6(t,\mathfrak{M}^j(t))+\Lambda(\delta_6)\int_0^t \mathfrak{F}_6(x,\mathfrak{M}^j(x))(t-x)^{\delta-1}dx.
\end{aligned}$$

In order to show the existence and uniqueness of solution of the model (2), we make use of fixed point theory. First, we re-write the model (2) in the following way:

$$\begin{cases} {}^{ABC}\mathfrak{D}^{\delta}\zeta(t)=\mathfrak{F}(t,\zeta(t)),\\ \zeta(0)=\zeta_0, \quad 0<t<T<\infty. \end{cases} \tag{8}$$

The vector $\zeta(t) = (\mathfrak{S}_p, \mathfrak{E}_p, \mathfrak{I}_p, \mathfrak{A}_p, \mathfrak{R}_p, \mathfrak{M})$ and $\mathfrak{F}$ in (8) represent the state variables and a continuous vector function respectively defined as follows:

$$\mathfrak{F} = \begin{pmatrix} \mathfrak{F}_1 \\ \mathfrak{F}_2 \\ \mathfrak{F}_3 \\ \mathfrak{F}_4 \\ \mathfrak{F}_6 \\ \mathfrak{F}_6 \end{pmatrix} = \begin{pmatrix} \coprod_p -\omega_p \mathfrak{S}_p(t) - \zeta_p \mathfrak{S}_p(t)(\mathfrak{I}_p(t) + \Psi \mathfrak{A}_p(t)) - \omega_w \mathfrak{S}_p(t)\mathfrak{M}(t) \\ \zeta_p \mathfrak{S}_p(t)(\mathfrak{I}_p(t) + \Psi \mathfrak{A}_p(t)) + \omega_w \mathfrak{S}_p(t)\mathfrak{M}(t) - (1-\Phi_p)\eta_p \mathfrak{E}_p(t) \\ -\Phi_p \varrho_p \mathfrak{E}_p(t) - \omega_p \mathfrak{E}_p(t) \\ (1-\Phi_p)\eta_p \mathfrak{E}_p(t) - (\tau_p + \omega_p)\mathfrak{I}_p(t) \\ \Phi_p \varrho_P \mathfrak{E}_p(t) - (\tau_{ap} + \omega_p)\mathfrak{A}_p(t) \\ \tau_p \mathfrak{I}_p(t) + \tau_{ap}\mathfrak{A}_p(t) - \omega_p \mathfrak{R}_p(t) \\ \phi_p \mathfrak{I}_p(t) + \varpi_p \mathfrak{A}_p(t) - \varphi \mathfrak{M}(t) \end{pmatrix} \tag{9}$$

with initial conditions $\zeta_0(t) = (\mathfrak{S}_p(0), \mathfrak{E}_p(0), \mathfrak{I}_p(0), \mathfrak{A}_p(0), \mathfrak{R}_p(0), \mathfrak{M}(0))$. Corresponding to (8), the integral equation is give by

$$\zeta(t) = \zeta_0 + \Upsilon(\delta)\mathfrak{F}(t, \zeta(t)) + \Lambda(\delta) \int_0^t \mathfrak{F}(x, \zeta(x))(t-x)^{\delta-1} dx. \tag{10}$$

5.1 The Existence of Unique Solution

Consider $A = [0, T]$, $\mathcal{E} = \mathcal{C}(A, \mathbb{R}^6)$ and the Picard operator $\mathcal{P} : \mathcal{E} \to \mathcal{E}$ be given by

$$\mathcal{P}[\zeta(t)] = \zeta_0 + \Upsilon(\delta)\mathfrak{F}(t, \zeta(t)) + \Lambda(\delta) \int_0^t \mathfrak{F}(x, \zeta(x))(t-x)^{\delta-1} dx. \tag{11}$$

Together with the supremum norm $\|\cdot\|_{\mathcal{C}}$, on ζ is defined by

$$\|\zeta(t)\|_{\mathcal{C}} = \sup_{t \in A} \|\zeta(t)\|, \quad \zeta(t) \in \mathcal{E}, \tag{12}$$

$\mathcal{E}$ defines a Banach space. Assume the following

[$\mathcal{A}_1$] Let $\mathfrak{F} : A \times \mathbb{R}^6 \to \mathbb{R}^6$ is continuous.

[$\mathcal{A}_2$] There exists $C_{\mathfrak{F}} > 0$ such that

$$|\mathfrak{F}(t, \zeta) - \mathfrak{F}(x, \zeta')| \leq C_{\mathfrak{F}} |\zeta - \zeta'|$$

for all $\zeta, \zeta' \in \mathbb{R}^6, t \in A$.

[$\mathcal{A}_3$] There exist a constant $L > 0$ such that $|\mathfrak{F}(x,\zeta)| \leq L(1+|\zeta|)$ for each $x \in A$ and all $\zeta \in \mathbb{R}^6$.

We prove the existence of solution of (8) by Schaefer's fixed point theorem.

Theorem 5.1. *If [$\mathcal{A}_1$]-[$\mathcal{A}_3$] together with $1-\Upsilon(\delta)L > 0$ hold, then (8) has at least one solution.*

Proof. We first show that the operator $\mathcal{P}$ given in (11) is continuous. Consider a sequence (ζ_j) such that $\zeta_j \to \zeta$ in $\mathcal{E}$. Now

$$\begin{aligned}
&|\mathcal{P}\zeta_j(t)-\mathcal{P}\zeta(t)| = \Bigg|\Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx \\
&-\Upsilon(\delta)\mathfrak{F}(t,\zeta(t)) - \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\Bigg| \\
\leq\ & \Upsilon(\delta)\Big|(\mathfrak{F}(t,\zeta_j(t))-\mathfrak{F}(t,\zeta(t)))\Big| \\
&+\Lambda(\delta)\left|\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx - \int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\right| \\
\leq\ & \Upsilon(\delta)\Big|(\mathfrak{F}(t,\zeta_j(t))-\mathfrak{F}(t,\zeta(t)))\Big| + \Lambda(\delta)\int_0^t |\mathfrak{F}(x,\zeta_j(x))-\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
\leq\ & \Upsilon(\delta)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x))-\mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}} + \Lambda(\delta)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x))-\mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}}\frac{t^\delta}{\delta} \\
\leq\ & \left(\Upsilon(\delta)+\frac{\Lambda(\delta)T^\delta}{\delta}\right)C_{\mathfrak{F}}\|\mathfrak{F}(x,\zeta_j(x))-\mathfrak{F}(x,\zeta(x))\|_{\mathcal{C}}.
\end{aligned}$$

Continuity of $\mathfrak{F}$ implies the continuity of $\mathcal{P}$.

Now suppose that $W = \{\zeta \in \mathcal{E} : \|\zeta\| \leq c > 0\}$. We now show that $\mathcal{P}[W]$ is bounded, i.e. there exists $d > 0$ such that for every $\zeta \in W$,

$\|\mathcal{P}\zeta\| \leq d$. For any $t \in A$, we have

$$
\begin{aligned}
|\mathcal{P}\zeta(t)| = &\left|\zeta_0 + \Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t)) + \Lambda(\delta)\int_0^t \mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx\right| \\
\leq \quad & |\zeta_0| + \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))| + \Lambda(\delta)\left|\int_0^t \mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\right| \\
\leq \quad & |\zeta_0| + \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))| + \Lambda(\delta)\int_0^t |\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
\leq \quad & |\zeta_0| + \Upsilon(\delta)L(1+|\zeta|) + \Lambda(\delta)L\int_0^t (1+|\zeta(x)|)(t-x)^{\delta-1}dx \\
\leq \quad & |\zeta_0| + \Upsilon(\delta)L(1+\|\zeta\|) + \Lambda(\delta)L(1+\|\zeta\|)\frac{T^\delta}{\delta} \\
\leq \quad & |\zeta_0| + \Upsilon(\delta)L(1+c) + \Lambda(\delta)L(1+c)\frac{T^\delta}{\delta} \\
= \quad & |\zeta_0| + \left(\Upsilon(\delta) + \Lambda(\delta)\frac{T^\delta}{\delta}\right)L(1+c) = d,
\end{aligned}
$$

which implies $|\mathcal{P}\zeta(t)| \leq d$. For the equicontinuity of $\mathcal{P}$, let $t_1, t_2 \in A$ with $0 \leq t_1, t_2 \leq T$ and $\zeta \in W$. Utilizing $[\mathcal{A}_\ni]$, we have

$$
\begin{aligned}
&|\mathcal{P}\zeta(t_1) - \mathcal{P}\zeta(t_2)| = \left|\Upsilon(\delta)\mathfrak{F}(t_1,\zeta(t_1)) + \Lambda(\delta)\int_0^{t_1} \mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx\right. \\
&\left. - \Upsilon(\delta)\mathfrak{F}(t_2,\zeta(t_2)) - \Lambda(\delta)\int_0^{t_2} \mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right| \\
&\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big| \\
&+ \Lambda(\delta)\left|\int_0^{t_1} \mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx - \int_0^{t_2} \mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right| \\
&\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big| + \Lambda(\delta)\left|\int_0^{t_1} \mathfrak{F}(x,\zeta(x))(t_1-x)^{\delta-1}dx\right. \\
&\left. - \int_0^{t_1} \mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx - \int_{t_1}^{t_2} \mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right| \\
&\leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big| + \Lambda(\delta)\left|\int_0^{t_1} \mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1} - (t_2-x)^{\delta-1}]dx\right| \\
&+ \Lambda(\delta)\left|\int_{t_1}^{t_2} \mathfrak{F}(x,\zeta(x))(t_2-x)^{\delta-1}dx\right| \leq \Upsilon(\delta)\Big|(\mathfrak{F}(t_1,\zeta(t_1)) - \mathfrak{F}(t_2,\zeta(t_2)))\Big|
\end{aligned}
$$

$$+\Lambda(\delta)\left|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1}-(t_2-x)^{\delta-1}]dx\right|+\Lambda(\delta)L(1+|\zeta|)\int_{t_1}^{t_2}|(t_2-x)^{\delta-1}|dx$$
$$\leq\Upsilon(\delta)\left|(\mathfrak{F}(t_1,\zeta(t_1))-\mathfrak{F}(t_2,\zeta(t_2)))\right|+\Lambda(\delta)\left|\int_0^{t_1}\mathfrak{F}(x,\zeta(x))[(t_1-x)^{\delta-1}-(t_2-x)^{\delta-1}]dx\right|$$
$$+\Lambda(\delta)L(1+d)\frac{(t_2-t_1)^{\delta}}{\delta}.$$

As t_1 tends to t_2, continuity of $\mathfrak{F}$ tends R.H.S of above inequality to zero. Hence $\mathcal{P}$ is equicontinuous. Therefore, we conclude by Arzela-Ascoli Theorem that $\mathcal{P}$ is completely continuous.
Finally, we show that the set $Q(\mathcal{P})=\{\zeta\in\mathcal{E}:\zeta=\vartheta\mathcal{P}\zeta\}$ is bounded for some $\vartheta\in(0,1)$. For each $t\in A$, we have

$$\begin{aligned}
|\mathcal{P}\zeta(t)| &= \left|\zeta_0+\Upsilon(\delta)\mathfrak{F}(t,\zeta_j(t))+\Lambda(\delta)\int_0^t\mathfrak{F}(x,\zeta_j(x))(t-x)^{\delta-1}dx\right|\\
&\leq |\zeta_0|+\Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))|+\Lambda(\delta)\left|\int_0^t\mathfrak{F}(x,\zeta(x))(t-x)^{\delta-1}dx\right|\\
&\leq |\zeta_0|+\Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))|+\Lambda(\delta)\int_0^t|\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx\\
&\leq |\zeta_0|+\Upsilon(\delta)L(1+|\zeta(t)|)+\Lambda(\delta)L\int_0^t(1+|\zeta(x)|)(t-x)^{\delta-1}dx\\
&\leq |\zeta_0|+\Upsilon(\delta)L(1+|\zeta(t)|)+\Lambda(\delta)L\frac{T^{\delta}}{\delta}+\Lambda(\delta)L\int_0^t|\zeta(x)|(t-x)^{\delta-1}dx\\
&= |\zeta_0|+\Upsilon(\delta)L+\Upsilon(\delta)L|\zeta(t)|+\Lambda(\delta)L\frac{T^{\delta}}{\delta}+\Lambda(\delta)L\int_0^t|\zeta(x)|(t-x)^{\delta-1}dx.
\end{aligned}$$

Writing $S=|\zeta_0|+\Upsilon(\delta)L+\Lambda(\delta)L\frac{T^{\delta}}{\delta}$ and since $1-\Upsilon(\delta)L>0$, we can have

$$|\mathcal{P}\zeta(t)|\leq\frac{S}{1-\Upsilon(\delta)L}+\frac{\Lambda(\delta)L}{1-\Upsilon(\delta)L}\int_0^t|\zeta(x)|(t-x)^{\delta-1}dx,$$

utilizing Gronwall's inequality, we obtain

$$|\mathcal{P}\zeta(t)|\leq\frac{S}{1-\Upsilon(\delta)L}exp\left(\frac{\Lambda(\delta)LT^{\delta}}{(1-\Upsilon(\delta)L)\delta}\right).$$

Therefore $Q(\mathcal{P})$ is bounded. Consequently, by Schaefer's theorem $\mathcal{P}$ has a fixed point which is infact a solution of (8). □

We now show by using Banach contraction principle that solution of (8) is unique.

Theorem 5.2. *If $[\mathcal{A}_1]$-$[\mathcal{A}_2]$ together with $\left(\Upsilon(\delta)+\frac{\Lambda(\delta)T^\delta}{\delta}\right)C_{\mathfrak{F}}<1$ hold, then there exists a unique solution of (8).*

Proof. Considering the equation together with (8), we have

$$\zeta(t)=\mathcal{P}[\zeta(t)]. \tag{13}$$

The operator $\mathcal{P}$ given in (11), is well defined by $[\mathcal{A}_1]$. Now for all $\zeta,\zeta'\in\mathcal{E}$, we have

$$\begin{aligned}
& |\mathcal{P}[\zeta(t)]-\mathcal{P}[\zeta'(t)]| \\
= & \left|\Upsilon(\delta)(\mathfrak{F}(t,\zeta(t))-\mathfrak{F}(t,\zeta'(t)))+\Lambda(\delta)\int_0^t(\mathfrak{F}(x,\zeta(x))-\mathfrak{F}(x,\zeta'(x)))(t-x)^{\delta-1}dx\right| \\
\leq & \Upsilon(\delta)|\mathfrak{F}(t,\zeta(t))-\mathfrak{F}(t,\zeta'(t))|+\Lambda(\delta)\int_0^t|\mathfrak{F}(x,\zeta(x))-\mathfrak{F}(x,\zeta(x))|(t-x)^{\delta-1}dx \\
\leq & \Upsilon(\delta)C_{\mathfrak{F}}|\zeta(t)-\zeta'(t)|+\Lambda(\delta)C_{\mathfrak{F}}\int_0^t|\zeta(x)-\zeta'(x)|(t-x)^{\delta-1}dx \\
\leq & \Upsilon(\delta)C_{\mathfrak{F}}\|\zeta-\zeta'\|_{\mathcal{C}}+\Lambda(\delta)C_{\mathfrak{F}}\|\zeta-\zeta'\|_{\mathcal{C}}\int_0^t(t-x)^{\delta-1}dx \\
\leq & (\Upsilon(\delta)+\frac{\Lambda(\delta)T^\delta}{\delta})C_{\mathfrak{F}}\|\zeta-\zeta'\|_{\mathcal{C}} \\
= & \mathcal{A}\|\zeta-\zeta'\|_{\mathcal{C}},
\end{aligned}$$

where

$$\mathcal{A}=\left(\Upsilon(\delta)+\frac{\Lambda(\delta)T^\delta}{\delta}\right)C_{\mathfrak{F}}.$$

This implies

$$\|\mathcal{P}[\zeta(t)]-\mathcal{P}[\zeta'(t)]\|_{\mathcal{C}}\leq\mathcal{A}\|\zeta-\zeta'\|_{\mathcal{C}}, \tag{14}$$

Thus the defined operator $\mathcal{P}$ is a contraction, and hence Banach contraction principle guarantees that $\mathcal{P}$ has a unique fixed point which is the solution model (8). $\square$

6 Special Solution by Iterative Approach

We obtain iterative solution of the model (2). Apply Shehu transforms (S_h) on both sides of (2), we get

$$
\begin{aligned}
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{S}_p] &= S_h\left[\coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{E}_p] &= S_h\left[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p-\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\right],\\
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{I}_p] &= S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p], \qquad (15)\\
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{A}_p] &= S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{R}_p] &= S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
S_h[{}^{ABC}\mathfrak{D}^{\delta}\mathfrak{M}] &= S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}].
\end{aligned}
$$

Using definition of Shehu transforms of ABC-derivative, we get

$$
\begin{aligned}
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{S}_p)-\left(\frac{u}{s}\right)\mathfrak{S}_p(0)\right] &= S_h\left[\coprod_p -\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{E}_p)-\left(\frac{u}{s}\right)\mathfrak{E}_p(0)\right] &= S_h\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Big],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{I}_p)-\left(\frac{u}{s}\right)\mathfrak{I}_p(0)\right] &= S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{A}_p)-\left(\frac{u}{s}\right)\mathfrak{A}_p(0)\right] &= S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{R}_p)-\left(\frac{u}{s}\right)\mathfrak{R}_p(0)\right] &= S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
\frac{M(\delta)}{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}\left[S_h(\mathfrak{M}_p)-\left(\frac{u}{s}\right)\mathfrak{M}_p(0)\right] &= S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}],
\end{aligned}
$$

On rearranging

$$
\begin{aligned}
S_h(\mathfrak{S}_p) &= \left(\frac{u}{s}\right)\mathfrak{S}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p -\omega_p\mathfrak{S}_p-\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)-\omega_w\mathfrak{S}_p\mathfrak{M}\right],\\
S_h(\mathfrak{E}_p) &= \left(\frac{u}{s}\right)\mathfrak{E}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p+\Psi\mathfrak{A}_p)+\omega_w\mathfrak{S}_p\mathfrak{M}-(1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p-\omega_p\mathfrak{E}_p\Big],\\
S_h(\mathfrak{I}_p) &= \left(\frac{u}{s}\right)\mathfrak{I}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p-(\tau_p+\omega_p)\mathfrak{I}_p], \qquad (16)\\
S_h(\mathfrak{A}_p) &= \left(\frac{u}{s}\right)\mathfrak{A}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p-(\tau_{ap}+\omega_p)\mathfrak{A}_p],\\
S_h(\mathfrak{R}_p) &= \left(\frac{u}{s}\right)\mathfrak{R}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p+\tau_{ap}\mathfrak{A}_p-\omega_p\mathfrak{R}_p],\\
S_h(\mathfrak{M}_p) &= \left(\frac{u}{s}\right)\mathfrak{M}_p(0)+\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p+\varpi_p\mathfrak{A}_p-\varphi\mathfrak{M}],
\end{aligned}
$$

Operating S_h^{-1} on both sides of (16) and taking into account that $S_h^{-1}\left(\frac{u}{s}\right) = 1$, we get

$$\begin{aligned}
\mathfrak{S}_p &= \mathfrak{S}_p(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p - \zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) - \omega_w\mathfrak{S}_p\mathfrak{M}\right]\right\},\\
\mathfrak{E}_p &= \mathfrak{E}_p(0) + S_h^{-1}\Big\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\Big[\zeta_p\mathfrak{S}_p(\mathfrak{I}_p + \Psi\mathfrak{A}_p) + \omega_w\mathfrak{S}_p\mathfrak{M} - (1-\Phi_p)\eta_p\mathfrak{E}_p\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p - \omega_p\mathfrak{E}_p\Big]\Big\},\\
\mathfrak{I}_p &= \mathfrak{I}_p(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p - (\tau_p + \omega_p)\mathfrak{I}_p]\right\}, \qquad (17)\\
\mathfrak{A}_p &= \mathfrak{A}_p(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p - (\tau_{ap} + \omega_p)\mathfrak{A}_p]\right\},\\
\mathfrak{R}_p &= \mathfrak{R}_p(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\tau_p\mathfrak{I}_p + \tau_{ap}\mathfrak{A}_p - \omega_p\mathfrak{R}_p]\right\},\\
\mathfrak{M}_p &= \mathfrak{M}_p(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\phi_p\mathfrak{I}_p + \varpi_p\mathfrak{A}_p - \varphi\mathfrak{M}]\right\},
\end{aligned}$$

The recursive formula is given by

$$\begin{aligned}
\mathfrak{S}_p^{n+1} &= \mathfrak{S}_p^n(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^n - \zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p + \Psi\mathfrak{A}_p^n) - \omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\},\\
\mathfrak{E}_p^{n+1} &= \mathfrak{E}_p^n(0) + S_h^{-1}\Big\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h\Big[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) + \omega_w\mathfrak{S}_p^n\mathfrak{M}^n - (1-\Phi_p)\eta_p\mathfrak{E}_p^n\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p^n - \omega_p\mathfrak{E}_p^n\Big]\Big\},\\
\mathfrak{I}_p^{n+1} &= \mathfrak{I}_p^n(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n - (\tau_p + \omega_p)\mathfrak{I}_p^n]\right\}, \qquad (18)\\
\mathfrak{A}_p^{n+1} &= \mathfrak{A}_p^n(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p^n - (\tau_{ap} + \omega_p)\mathfrak{A}_p^n]\right\},\\
\mathfrak{R}_p^{n+1} &= \mathfrak{R}_p^n(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\tau_p\mathfrak{I}_p^n + \tau_{ap}\mathfrak{A}_p^n - \omega_p\mathfrak{R}_p^n]\right\},\\
\mathfrak{M}_p^{n+1} &= \mathfrak{M}_p^n(0) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\phi_p\mathfrak{I}_p^n + \varpi_p\mathfrak{A}_p^n - \varphi\mathfrak{M}^n]\right\},
\end{aligned}$$

The approximate solution of (18) is given by

$$\begin{aligned}
\mathfrak{S}_p &= \lim_{n\to\infty}\mathfrak{S}_p^n, \qquad \mathfrak{E}_p = \lim_{n\to\infty}\mathfrak{E}_p^n, \qquad \mathfrak{I}_p = \lim_{n\to\infty}\mathfrak{I}_p^n,\\
\mathfrak{A}_p &= \lim_{n\to\infty}\mathfrak{A}_p^n, \qquad \mathfrak{R}_p = \lim_{n\to\infty}\mathfrak{R}_p^n, \qquad \mathfrak{M}_p = \lim_{n\to\infty}\mathfrak{M}_p^n.
\end{aligned}$$

7 Stability Analysis and Iterative Solution

Consider a Banach space $\mathcal{X}$ together with norm $\|x\| = \max_{t\in[a,b]} |x(t)|$, $x \in \mathcal{X}$ and $\mathcal{F}$ a self map on $\mathcal{X}$. The recursive procedure is

$$S_{n+1} = h(\mathcal{F}, S_n). \tag{19}$$

The set of fixed points $Fix(\mathcal{F})$ of $\mathcal{F}$ is nonempty and S_n converges to a point of $Fix(\mathcal{F})$. Choose a sequence (f_n) in $\mathcal{X}$ and $e_n = \|f_{n+1} - h(\mathcal{F}, S_n)\|$. The recursive procedure (19) is $\mathcal{F}$-stable if $\lim_{n\to\infty} e_n = 0$. We suppose that the sequence (f_n) is bounded above, else it will diverge. Under these conditions, $S_{n+1} = \mathcal{F}S_n$ is Picard's iteration as described in [38], implies it is $\mathcal{F}$-stable.

Theorem 7.1. *Let $(\mathcal{X}, \|\cdot\|)$ be a Banach space and $\mathcal{F}$ be a self map on $\mathcal{X}$ satisfying*

$$\|\mathcal{F}_x - \mathcal{F}_y\| \le R\|x - \mathcal{F}_x\| + r\|x - y\| \tag{20}$$

for all $x, y \in \mathcal{X}$, where $R \ge 0$ and $0 \le r < 1$. Then $\mathcal{F}$ is Picard $\mathcal{F}$-stable.

Theorem 7.2. *A self map $\mathcal{F}$ given by*

$$\begin{aligned}
\mathcal{F}(\mathfrak{S}_p^n(t)) &= \mathfrak{S}_p^{n+1}(t)\\
&= \mathfrak{S}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\left[\coprod_p -\omega_p\mathfrak{S}_p^n - \zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) - \omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\}\\
\mathcal{F}(\mathfrak{E}_p^n(t)) &= \mathfrak{E}_p^{n+1}(t)\\
&= \mathfrak{E}_p^n(t) + S_h^{-1}\Big\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h\Big[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n + \Psi\mathfrak{A}_p^n) + \omega_w\mathfrak{S}_p^n\mathfrak{M}^n - (1-\Phi_p)\eta_p\mathfrak{E}_p^n\\
&\quad -\Phi_p\varrho_p\mathfrak{E}_p^n - \omega_p\mathfrak{E}_p^n\Big]\Big\}\\
\mathcal{F}(\mathfrak{I}_p^n(t)) &= \mathfrak{I}_p^{n+1}(t)\\
&= \mathfrak{I}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n - (\tau_p+\omega_p)\mathfrak{I}_p^n]\right\}\\
\mathcal{F}(\mathfrak{A}_p^n(t)) &= \mathfrak{A}_p^{n+1}(t)\\
&= \mathfrak{A}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\Phi_p\varrho_P\mathfrak{E}_p^n - (\tau_{ap}+\omega_p)\mathfrak{A}_p^n]\right\}\\
\mathcal{F}(\mathfrak{R}_p^n(t)) &= \mathfrak{R}_p^{n+1}(t)\\
&= \mathfrak{R}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\tau_p\mathfrak{I}_p^n + \tau_{ap}\mathfrak{A}_p^n - \omega_p\mathfrak{R}_p^n]\right\}\\
\mathcal{F}(\mathfrak{M}^n(t)) &= \mathfrak{M}^{n+1}(t)\\
&= \mathfrak{M}_p^n(t) + S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)} S_h[\phi_p\mathfrak{I}_p^n + \varpi_p\mathfrak{A}_p^n - \varphi\mathfrak{M}^n]\right\}
\end{aligned}$$

is $\mathcal{F}$-stable in $H^1(a,b)$ if the following conditions holds:

$$\begin{cases} 1-\omega_p f_1(\kappa)-\frac{\zeta_p}{\mathfrak{N}_p}(L_1+L_3'+\Psi L_1+\Psi L_4')f_2(\kappa)-\omega_w(L_1+L_6')\}f_3(\kappa)<1 \\ 1+\frac{\zeta_p}{\mathcal{N}_p}(L_1+L_3'+\psi L_1+\psi L_4')f_4(\kappa)+\omega_w(L_1+L_6')f_5(\kappa)-\{(1-\Phi_p)\eta_p+\Phi_p\varrho_p+\omega_p\}f_6(\kappa)<1 \\ 1+(1-\Phi_p)\eta_p f_7(\kappa)-(\tau_p+\omega_p)f_8(\kappa)<1 \\ 1+\Phi_p\varrho_P f_9(\kappa)-(\tau_{ap}+\omega_p)f_{10}(\kappa)<1 \\ 1+\tau_p f_{11}(\kappa)+\tau_{ap}f_{12}(\kappa)-\omega_p f_{13}(\kappa)<1 \\ 1+\phi_p f_{14}(\kappa)+\varpi_p f_{15}(\kappa)-\varphi_p(\kappa)f_{16}(\kappa)<1. \end{cases}$$

Proof. We first show that $\mathcal{F}$ has a fixed point. For $m,n\in\mathbb{N}$, we have

$$\begin{aligned}
&\mathcal{F}(\mathfrak{S}_p^n(t))-\mathcal{F}(\mathfrak{S}_p^m(t))\\
&=\mathfrak{S}_p^n(t)-\mathfrak{S}_p^m(t)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p^n-\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n)-\omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\}\\
&\quad-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p^m-\zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m+\Psi\mathfrak{A}_p^m)-\omega_w\mathfrak{S}_p^m\mathfrak{M}^m\right]\right\}\\
&\mathcal{F}(\mathfrak{E}_p^n(t))-\mathcal{F}(\mathfrak{E}_p^m(t))\\
&=\mathfrak{E}_p^n(t)-\mathfrak{E}_p^m(t)+S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h\Big[\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n)+\omega_w\mathfrak{S}_p^n\mathfrak{M}^n-(1-\Phi_p)\eta_p\mathfrak{E}_p^n\\
&-\Phi_p\varrho_p\mathfrak{E}_p^n-\omega_p\mathfrak{E}_p^n\Big]\Bigg\}\\
&\quad-S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h\Big[\zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m+\Psi\mathfrak{A}_p^m)+\omega_w\mathfrak{S}_p^m\mathfrak{M}^m-(1-\Phi_p)\eta_p\mathfrak{E}_p^m\\
&-\Phi_p\varrho_p\mathfrak{E}_p^m-\omega_p\mathfrak{E}_p^m\Big]\Bigg\}\\
&\mathcal{F}(\mathfrak{I}_p^n(t))-\mathcal{F}(\mathfrak{I}_p^m(t))\\
&=\mathfrak{I}_p^n(t)-\mathfrak{I}_p^m(t)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^n-(\tau_p+\omega_p)\mathfrak{I}_p^n]\right\}\\
&\quad-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h[(1-\Phi_p)\eta_p\mathfrak{E}_p^m-(\tau_p+\omega_p)\mathfrak{I}_p^m]\right\}\\
&\mathcal{F}(\mathfrak{A}_p^n(t))-\mathcal{F}(\mathfrak{A}_p^m(t))\\
&=\mathfrak{A}_p^n(t)-\mathfrak{A}_p^m(t)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p^n-(\tau_{ap}+\omega_p)\mathfrak{A}_p^n]\right\}\\
&\quad-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)}S_h[\Phi_p\varrho_P\mathfrak{E}_p^m-(\tau_{ap}+\omega_p)\mathfrak{A}_p^m]\right\}
\end{aligned}$$

$$
\begin{aligned}
&\mathcal{F}(\mathfrak{R}_p^n(t))-\mathcal{F}(\mathfrak{R}_p^m(t))\\
&=\mathfrak{R}_p^n(t)-\mathfrak{R}_p^m(t)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p^n+\tau_{ap}\mathfrak{A}_p^n-\omega_p\mathfrak{R}_p^n]\right\}\\
&-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\tau_p\mathfrak{I}_p^m+\tau_{ap}\mathfrak{A}_p^m-\omega_p\mathfrak{R}_p^m]\right\}\\
&\mathcal{F}(\mathfrak{M}^n(t))-\mathcal{F}(\mathfrak{M}^m(t))\\
&=\mathfrak{M}_p^n(t)-\mathfrak{M}_p^m(t)+S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p^n+\varpi_p\mathfrak{A}_p^n-\varphi\mathfrak{M}^n]\right\}\\
&-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h[\phi_p\mathfrak{I}_p^m+\varpi_p\mathfrak{A}_p^m-\varphi\mathfrak{M}^m]\right\}
\end{aligned}
$$

Taking norm, we have

$$
\begin{aligned}
&\|\mathcal{F}(\mathfrak{S}_p^n(t))-\mathcal{F}(\mathfrak{S}_p^m(t))\|\\
\leq\ &\|\mathfrak{S}_p^n(t)-\mathfrak{S}_p^m(t)\|+\left\|S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p^n-\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n+\Psi\mathfrak{A}_p^n)-\omega_w\mathfrak{S}_p^n\mathfrak{M}^n\right]\right\}\right.\\
&\left.-S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\left[\coprod_p-\omega_p\mathfrak{S}_p^m-\zeta_p\mathfrak{S}_p^m(\mathfrak{I}_p^m+\Psi\mathfrak{A}_p^m)-\omega_w\mathfrak{S}_p^m\mathfrak{M}^m\right]\right\}\right\|\\
\leq\ &\|\mathfrak{S}_p^n(t)-\mathfrak{S}_p^m(t)\|+S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Big[-\|\omega_p(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\zeta_p\mathfrak{S}_p^n(\mathfrak{I}_p^n-\mathfrak{I}_p^m)\|\\
&-\|\zeta_p\mathfrak{I}_p^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\Psi\zeta_p\mathcal{S}_p^n(\mathfrak{A}_p^n-\mathfrak{A}_p^m)\|-\|\Psi\zeta_p\mathcal{A}_p^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\omega_w\mathfrak{S}_p^n(\mathfrak{M}^n-\mathfrak{M}^m\|\\
&-\|\omega_w\mathfrak{M}^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m\|\Big]\Bigg\}. \qquad (21)
\end{aligned}
$$

Due to similar functioning of both solutions, we have

$$
\begin{aligned}
\|\mathfrak{S}_p^n(t)-\mathfrak{S}_p^m(t)\| &\cong \|\mathfrak{E}_p^n(t)-\mathfrak{E}_p^m(t)\|\\
&\cong \|\mathfrak{I}_p^n(t)-\mathfrak{I}_p^m(t)\|\\
&\cong \|\mathfrak{A}_p^n(t)-\mathfrak{A}_p^m(t)\| \qquad (22)\\
&\cong \|\mathfrak{R}_p^n(t)-\mathfrak{R}_p^m(t)\|\\
&\cong \|\mathfrak{M}^n(t)-\mathfrak{M}^m(t)\|.
\end{aligned}
$$

Replacing (22) in (21), we get

$$
\begin{aligned}
&\|\mathcal{F}(\mathfrak{S}_p^n(t))-\mathcal{F}(\mathfrak{S}_p^m(t))\|\\
\leq\ &\|\mathfrak{S}_p^n(t)-\mathfrak{S}_p^m(t)\|+S_h^{-1}\Bigg\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^{\delta}}{M(\delta)}S_h\Big[-\|\omega_p(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\zeta_p\mathfrak{S}_p^n(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|\\
&-\|\zeta_p\mathfrak{I}_p^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\Psi\zeta_p\mathfrak{S}_p^n(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\Psi\zeta_p\mathfrak{A}_p^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m)\|-\|\omega_w\mathfrak{S}_p^n(\mathfrak{S}^n-\mathfrak{S}^m\|\\
&-\|\omega_w\mathfrak{M}^m(\mathfrak{S}_p^n-\mathfrak{S}_p^m\|\Big]\Bigg\}. \qquad (23)
\end{aligned}
$$

The sequences $\mathfrak{S}_p^n, \mathfrak{I}_p^m, \mathfrak{A}_p^m, \mathfrak{M}^m$ are bounded being convergent, so there exist L_1, L_3', L_4', L_6' for all t such that

$$\|\mathfrak{S}_p^n\| < L_1, \|\mathfrak{I}_p^m\| < L_3', \|\mathfrak{A}_p^m\| < L_4', \|\mathfrak{M}^m\| < L_6'.$$

Together with this, (23) become

$$\begin{aligned}&\|\mathcal{F}(\mathfrak{S}_p^n(t)) - \mathcal{F}(\mathfrak{S}_p^m(t))\| \\ \leq\ & \{1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_2(\kappa) - \omega_w(L_1 + L_6')\}f_3(\kappa)\|\mathfrak{S}_p^n - \mathfrak{S}_p^m\| \end{aligned} \tag{24}$$

where f_i are the functions obtained by $S_h^{-1}\left\{\frac{1-\delta+\delta\left(\frac{u}{s}\right)^\delta}{M(\delta)} S_h[\cdot]\right\}$. In a similar fashion, we can have

$$\begin{aligned}\|\mathcal{F}(\mathfrak{E}_p^n(t)) - \mathcal{F}(\mathfrak{E}_p^m(t))\| \leq & \Big[1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_4(\kappa) + \omega_w(L_1 + L_6')f_5(\kappa) \\ & -\{(1-\Phi_p)\eta_p + \Phi_p\varrho_p + \omega_p\}f_6(\kappa)\Big]\|\mathfrak{E}_p^n - \mathfrak{E}_p^m\| \end{aligned} \tag{25}$$

$$\|\mathcal{F}(\mathfrak{I}_p^n(t)) - \mathcal{F}(\mathfrak{I}_p^m(t))\| \leq \{1 + (1-\Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p)f_8(\kappa)\}\|\mathfrak{I}_p^n - \mathfrak{I}_p^m\| \tag{26}$$

$$\|\mathcal{F}(\mathfrak{A}_p^n(t)) - \mathcal{F}(\mathfrak{A}_p^m(t))\| \leq \{1 + \Phi_p\varrho_p f_9(\kappa) - (\tau_{ap} + \omega_p)f_{10}(\kappa)\}\|\mathfrak{A}_p^n - \mathfrak{A}_p^m\| \tag{27}$$

$$\|\mathcal{F}(\mathfrak{R}_p^n(t)) - \mathcal{F}(\mathfrak{R}_p^m(t))\| \leq \{1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa)\}\|\mathfrak{R}_p^n - \mathfrak{R}_p^m\| \tag{28}$$

$$\|\mathcal{F}(\mathfrak{M}^n(t)) - \mathcal{F}(\mathfrak{M}^m(t))\| \leq \{1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa)f_{16}(\kappa)\}\|\mathfrak{M}^n - \mathfrak{M}^m\|, \tag{29}$$

where

$$\begin{cases} 1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_2(\kappa) - \omega_w(L_1 + L_6')\}f_3(\kappa) < 1 \\ 1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_4(\kappa) + \omega_w(L_1 + L_6')f_5(\kappa) - \{(1-\Phi_p)\eta_p + \Phi_p\varrho_p + \omega_p\}f_6(\kappa) < 1 \\ 1 + (1-\Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p)f_8(\kappa) < 1 \\ 1 + \Phi_p\varrho_P f_9(\kappa) - (\tau_{ap} + \omega_p)f_{10}(\kappa) < 1 \\ 1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa) < 1 \\ 1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa)f_{16}(\kappa) < 1. \end{cases}$$

Hence, $\mathcal{F}$ possesses a fixed point. Thus to prove that the assumptions of Theorem 7.1 are satisfied by $\mathcal{F}$, we assume inequalities (24)-(29) holds, denote $r = (0,0,0,0,0,0)$ and

$$R = \begin{cases} 1 - \omega_p f_1(\kappa) - \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_2(\kappa) - \omega_w(L_1 + L_6')\}f_3(\kappa) < 1 \\ 1 + \zeta_p(L_1 + L_3' + \Psi L_1 + \Psi L_4')f_4(\kappa) + \omega_w(L_1 + L_6')f_5(\kappa) - \{(1-\Phi_p)\eta_p + \Phi_p\varrho_p + \omega_p\}f_6(\kappa) < 1 \\ 1 + (1-\Phi_p)\eta_p f_7(\kappa) - (\tau_p + \omega_p)f_8(\kappa) < 1 \\ 1 + \Phi_p\varrho_P f_9(\kappa) - (\tau_{ap} + \omega_p)f_{10}(\kappa) < 1 \\ 1 + \tau_p f_{11}(\kappa) + \tau_{ap} f_{12}(\kappa) - \omega_p f_{13}(\kappa) < 1 \\ 1 + \phi_p f_{14}(\kappa) + \varpi_p f_{15}(\kappa) - \varphi_p(\kappa)f_{16}(\kappa) < 1. \end{cases}$$

Hence all the conditions of Theorem 7.1 are satisfied, therefor $\mathcal{F}$ is Picard $\mathcal{F}$-stable. $\square$

8 Numerical Results

In this section, using the three-step Adams method, we solve the system of equations (2) and present an approximate solution and perform a simulation for forecasting of transmission of COVID-19 in Iran.

8.1 Numerical Method

Using the Adams-Bashforth scheme, we present a numerical solution for the COVID-19 transmission model (2). Owolabi and Atangana introduced the three-step Adams-Bashforth scheme with the Atangana-Baleanu-Caputo fractional derivative [30], we use this method to find three step Adams-Bashforth scheme for fractional order system (2).
Consider the fractional differential equation with the Atangana-Baleanu-Caputo fractional derivative

$$^{ABC}D_t^{\delta} z(t) = f(t, z(t)), \quad 0 < \delta < 1. \tag{30}$$

Using the fundamental calculus theorem, we get

$$z(t) - z(0) = \frac{(1-\delta)}{AB(\delta)} f(t, z(t)) + \frac{\delta}{AB(\delta)\Gamma(\delta)} \int_0^t (t-\tau)^{\delta-1} f(\tau, z(\tau)) d\tau. \tag{31}$$

By discretize the time interval [0,t] in steps of h, we obtain the sequence $t_0 = 0, t_{m+1} = t_m + h, m = 0, 1, 2, ..., n-1, t_n = t$. By replacing $t = t_{m+1}$ and $t = t_m$ in above equation and computing the difference of the resulting equations, we obtain

$$z(t_{m+1}) - z(t_m) = \frac{(1-\delta)}{AB(\delta)}[f(t_m, x(t_m)) - f(t_{m-1}, x(t_{m-1}))] + \frac{\delta}{AB(\delta)\Gamma(\delta)}$$

$$\times \int_0^{t_{m+1}} (t_{m+1}-\tau)^{\delta-1} f(\tau, z(\tau)) d\tau - \frac{\delta}{AB(\delta)\Gamma(\delta)} \int_0^{t_m} (t_m-\tau)^{\delta-1} f(\tau, z(\tau)) d\tau.$$

By putting $t_m = mh$ and $t_{m+1} = (m+1)h$ and $z(t_m) = z_m,\ z(t_{m+1}) = z_{m+1}$, we simplify the last equation as follows

$$z_{m+1} = z_m + f(t_m, z_m)\{\frac{1-\delta}{AB(\delta)} - \frac{\delta}{AB(\delta)\Gamma(\delta)} h^{\delta} [\frac{2(m+1)^{\delta}}{\delta} - \frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)} h^{\delta} [\frac{m^{\delta}}{\delta} - \frac{m^{\delta+1}}{\delta+1}]\} + f(t_{m-1}, z_{m-1})\{\frac{\delta-1}{AB(\delta)} - \frac{\delta}{AB(\delta)\Gamma(\delta)} h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\}.$$

Using this method, we can obtain the solution of the system (2) as follows

$$\mathfrak{S}_{p,m+1}=\mathfrak{S}_{p,m}+f(t_m,\mathfrak{S}_{p,m})\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},\mathfrak{S}_{p,m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\},$$

$$\mathfrak{E}_{p,m+1}=\mathfrak{E}_{p,m}+f(t_m,\mathfrak{E}_{p,m})\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},\mathfrak{E}_{p,m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\},$$

$$\mathfrak{I}_{p,m+1}=\mathfrak{I}_{p,m}+f(t_m,\mathfrak{I}_{p,m})\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},\mathfrak{I}_{p,m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\},$$

$$\mathfrak{A}_{p,m+1}=\mathfrak{A}_{p,m}+f(t_m,\mathfrak{A}_{p,m})\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},\mathfrak{A}_{p,m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\},$$

$$\mathfrak{R}_{p,m+1}=\mathfrak{R}_{p,m}+f(t_m,\mathfrak{R}_{p,m})\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},\mathfrak{R}_{p,m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\},$$

$$M_{m+1}=M_m+f(t_m,M_m)\{\frac{1-\delta}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{2(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}]$$

$$-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}[\frac{m^{\delta}}{\delta}-\frac{m^{\delta+1}}{\delta+1}]\}+f(t_{m-1},M_{m-1})\{\frac{\delta-1}{AB(\delta)}-\frac{\delta}{AB(\delta)\Gamma(\delta)}h^{\delta}$$

$$\times[\frac{(m+1)^{\delta}}{\delta}-\frac{(m+1)^{\delta+1}}{\delta+1}+\frac{m^{\delta+1}}{AB(\delta)\Gamma(\delta)h}]\}.$$

8.2 Simulation

In Indonesia, a new wave of the release of Covid 19 has started and the number of patients is increasing, so we present a numerical simulation to predict the release of Covid 19 in this country. To this end we assumed $\rho = 0.99$, some parameters are estimated, and the rest parameters are fitted by the least curve fitting technique. According to report of WHO, The total population of the Indonesia in 25-June 2021 is $N = 276351443$, the birth rate for the Indonesia in 2020 was 17.45 births per 1000 people, and the death rate was 6.6 per 1000 people. Thus for every day, we have $\Lambda = \frac{n\times N}{365} = 13211.87$ and $m = \frac{0.0066}{365} = 0.0000180821$. Given that in Indonesia, have no connection with the reservoir , so we considered $\beta_w = 0, \mu = 0, \mu^{'} = 0$. For the fitting, we use the information provided by the World Health Organization for COVID-19. The fitted curve and the reported cases of COVID-19 in 2021 at the Indonesia from 18 May to 23 June 2021 are plotted in Figure (1), so that every part is 3 days. Using this method, we obtain the parameters as follows

$$\Lambda = 13211.87, m = 18.0821\times 10^{-6}, \beta_p = 2.4\times 10^{-6}, \kappa = 0.001, \delta = 0.05,$$

$$\omega = 1.1\times 10^{-4}, \omega^{'} = 3.4\times 10^{-4}, \gamma = 0.03, \gamma^{'} = 0.07, \epsilon = 0.01$$

In Figures (2)-(4), we plotted the results of the system of COVID-19 transmission (2). As you can see in Figure (2)-(4), the variables have different results in different amounts of δ but exhibit the same behavior. Figure (2) shows that two months after the virus is released, almost the entire population is at risk for the disease. Figure (3) shows that

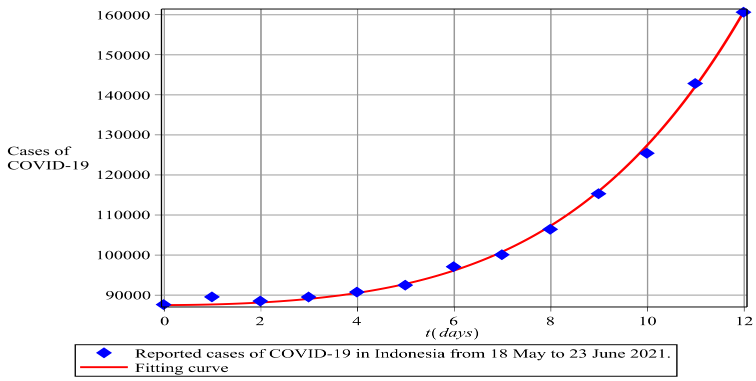

Figure 1: The fitted curve and the reported cases of COVID-19 in the Indonesia from 18 May to 23 June 2021.

the number of people with COVID-19 increases until 300 days. Also, the forecast is that the number of infected people could rise to 800,000. Figure (4) shows that the number of people who have recovered or died also increases over time and the number of virus in reservoir decreases.

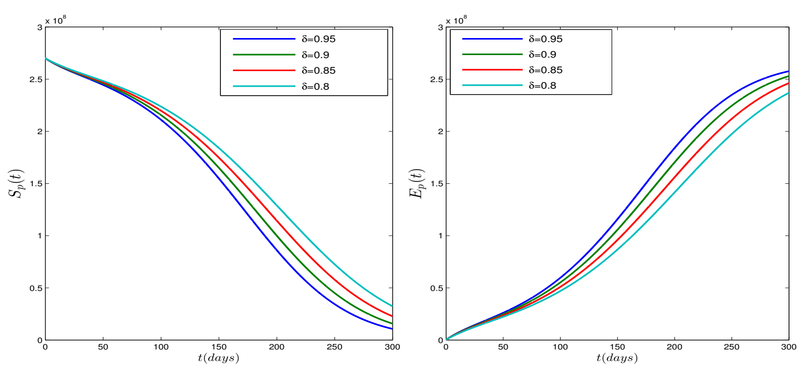

Figure 2: Plots of $\mathfrak{S}_p$ and $\mathfrak{E}_p$ for different values of $\delta = 0.95, 0.9, 0.85, 0.8$.

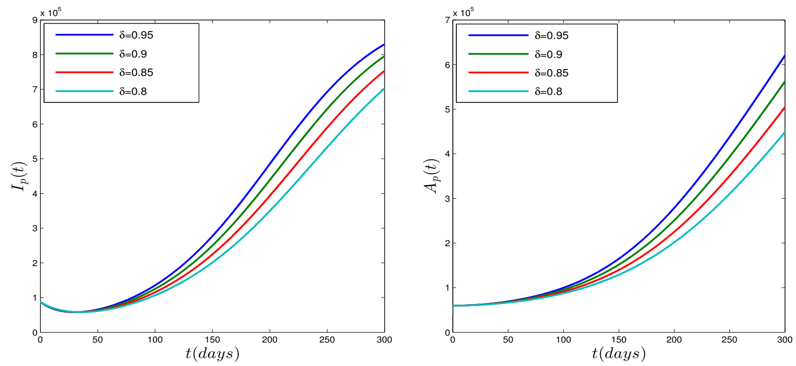

Figure 3: Plots of $\mathfrak{I}_p$ and $\mathfrak{A}_p$ for different values of $\delta = 0.95, 0.9, 0.85, 0.8$.

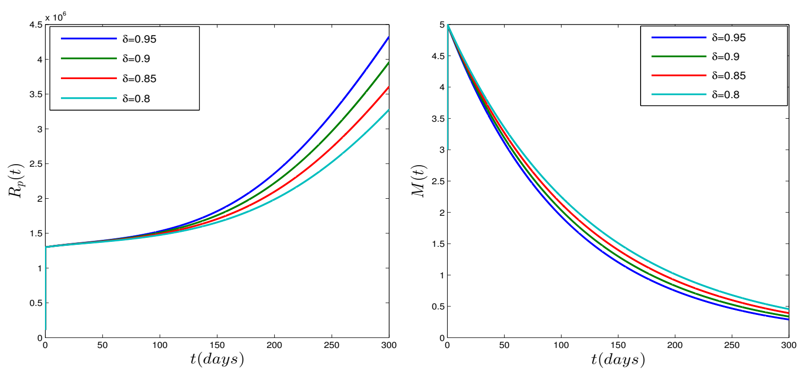

Figure 4: Plots of $\mathfrak{R}_p$ and $M(t)$ for different values of $\delta = 0.95, 0.9, 0.85, 0.8$.

9 Conclusion

In this paper, considering fractional order derivative due to Atangana and Baleanu we have studied mathematical model of COVID-19 transmission. We presented the existence and uniqueness of the related fractional differential equation of the model utilizing Schaefer's and Banach fixed point theorems respectively. Making use of Shehu transform and Picard iterative procedure, we presented iterative solutions and proved the stability of iterative method. Also, the equilibrium points of the system and its stability conditions are determined.The resulting differential system is solved using two-step Adams-Bashforth method, and we have obtained approximate solutions. A simulation of COVID-19 transmission based of real data in Indonesia is presented.

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