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A Novel Approach for Analyzing System Reliability Using Generalized Intuitionistic Fuzzy Pareto Lifetime Distribution

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Abstract. The present work concentrates on vagueness in the lifetime parameter and the generalized intuitionistic fuzzy set are extended to reliability characteristics. In order to satisfy this purpose, generalized intuitionistic fuzzy numbers are applied to evaluate the reliability of different systems. The reliability characteristics of systems using Pareto lifetime distribution are investigated where the lifetime scale parameter is assumed to be a generalized intuitionistic fuzzy number. In general, the generalized intuitionistic fuzzy reliability function, generalized intuitionistic fuzzy conditional reliability function, generalized intuitionistic fuzzy hazard function, generalized intuitionistic fuzzy mean time to failure and their cut sets are discussed. The whole mentioned reliability functions are discussed for generalized intuitionistic fuzzy Pareto lifetime systems. Furthermore, reliability analysis of the series and parallel systems are performed and numerical example is illustrated based on the proposed approach.

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1 Introduction

Reliability is an index to evaluate the performance of a system and product quality that represents efficiency and probability that the systems carry out the assigned tasks properly for a certain circumstances and time period. The quantitative reliability characteristics can be used as a decision-making system for lifetime data that needs to be calculated. The classical reliability methods are based on crisp or precise information on lifetime data and be inadequate to handle the uncertainty environments. However, in real situations, there are underlying systems whose information may not be necessarily crisp but rather uncertain which implies to the randomness, vagueness, ambiguity, and imprecision with different and specific characteristics. A reliability analysis shall often deal with the uncertainties associated with components, parameters, phenomena and underlying assumptions. Due to obtain realistic results, the estimation methods for reliability characteristics must be adjusted to the fuzzy lifetimes, which can capture the uncertainty or imprecision in the data. The fuzzy reliability of the system is the fuzzy probability that the system properly accomplishes its task without failing until attains the last state.

The fuzzy sets introduced by Zadeh [41] that considered as an appropriate tool to represent and manipulate imprecise decision-making problems. In a fuzzy set, any element can be defined by its membership function or grade. The membership function represents the possibility of occurrence of an object with a specified fuzzy set and possibility measure is an alternative to probability measures to account the uncertainty. For detailed studies on possibility theory, we cite [15] and [42].

Singer [38] presented a fuzzy set approach for fault tree and reliability analysis, afterwards fuzzy reliability has been introduced by [14]. Over the past decades, fuzzy set has received much attention to analyze the system reliability by many researchers ([18], [20], [11], [29], [22], [32], [34], [23], [30]).

Aliev and Kara [2] proposed a general procedure to construct the membership function of the fuzzy reliability using fuzzy failure rate. Liu et al. [28] expressed the fuzzy reliability analysis and mean time to failure of series, parallel, series-parallel, parallel-series and cold standby systems. Developing the system reliability evaluation problem, Baloui Jamkhaneh [6] considered the exponential lifetimes with fuzzy parameters. El-Damcese et al. [17] considered both series and parallel systems which included both different and identical components and performed the fuzzy reliability analysis and represented some characteristics based on fuzzy triangular membership functions. Baloui Jamkhaneh [7] assessed the fuzzy reliability with Weibull lifetime distribution and fuzzy triangular number and investigated fuzzy reliability and mean time to failure series and parallel system. Pak et al. [31] evaluated a Bayesian approach to estimate the parameter and fuzzy reliability function of Rayleigh distribution. The reliability of aero-engine blades with considering the fuzziness of the input variables and their limit states are investigated by [32] and based on the entropy equivalence method, the fuzzy variables were converted into stochastic variables and the fuzzy reliability index and failure probability obtained. Recently, Shafiq et al. [36] generalized fuzzy reliability characteristics estimation for the three-parameter Weibull, Pareto and Gamma lifetime distribution.

In real lifetime data, the definition of possibility of a membership degree may be encountered to hesitation or uncertainty, but fuzzy set includes only the degree of acceptance. An appropriate approach is to use intuitionistic fuzzy sets (IFS) defined by [5], which incorporated the degree of hesitation called hesitation margin and characterized by membership and non-membership functions. IFS is more appropriate to deal with the uncertainty and vagueness than fuzzy sets.

Kumar et al. [25] developed the fuzzy set to IFS and analyzed IFS reliability based on the profust reliability theory, where the failure rate of the system is represented by a time-dependent triangular intuitionistic fuzzy number. Also, the membership and non-membership functions of fuzzy reliability of both series and parallel systems are represented. Sharma et al. [37] computed the fuzzy reliability of the different system using intuitionistic fuzzy set and implemented the intuitionistic triangular fuzzy number and its arithmetic operations. Kumar and Singh [24] evaluated the intuitionistic fuzzy reliability of the network system and discussed the different fuzzy reliability of the various environmental systems using Weibull lifetime distribution. Bohra and Singh [12] presented fuzzy system reliability using intuitionistic fuzzy lifetime distribution. The intuitionistic fuzzy generalized probabilistic ordered weighted averaging operator is introduced by [43], which integrated of the probability and the ordered weighted averaging operator. Akbari and Hesamian [1] represented a method for constructing time-dependent reliability systems by intuitionistic fuzzy random variable with exact parameters.

Baloui Jamkhane and Nadarajah [10] introduced new generalized intuitionistic fuzzy sets (GIFS$_B$) and introduced some operators over GIFS$_B$, which has been extensively used in subsequent researches. Shabani and Baloui Jamkhaneh [35] introduced a new generalized intuitionistic fuzzy number (GIFN$_B$) based on the GIFS$_B$. Thereafter, Baloui Jamkhaneh [8] represented the values and ambiguities of the degree of membership and the degree of non-membership of GIFS$_B$ and Baloui Jamkhaneh [9] presented system reliability using generalized intuitionistic fuzzy exponential lifetime distribution based on GIFS$_B$. Ebrahimnejad and Baloui Jamkhaneh [16] considered system reliability of Rayleigh lifetime distribution with GIFN$_B$.

The power-law probability Pareto distribution, has been used often to modeling reliability and heavy tailed lifetime data which was first proposed as a model for the distribution of incomes at the extremities and city populations. The Pareto distribution has many applications in actuarial science, economics, life testing, hydrology, finance, physics and engineering.

The heavy-tailed distributions such as Pareto are applicable for modeling extreme loss, especially for the insecure types of insurance and for financial applications, prepare information about the potential for financial fiasco or financial ruin. Levy and Levy [27] used Pareto wealth distribution for investigation of the market efficiency and investment talent. Brazauskas and Serfling [13] consider Pareto distribution and investigated the performance of the generalized mean and trimmed mean robust estimators on real data.

The hazard rate function of Pareto distribution is decreasing, which implies that the survival function will decay more slowly to zero. For a thorough discussion on various properties and applications and different forms of the Pareto distribution, see [4] and [21]. Some relevant research on the Pareto distribution can be attained in [39], [40], [3], [19], [33] and [26].

The main purpose of the present paper is to extend the reliability characteristics by the generalized intuitionistic fuzzy set, which introduced by [10] that represent more accurate and flexible results. Motivated by these, we consider Pareto lifetime distribution to evaluate system reliability, which has the uncertainty in the lifetime parameter. The scale parameter of the Pareto distribution function is taken as a generalized intuitionistic fuzzy number. The vagueness in the reliability characteristics of the system are represented perfectly by fuzzification the parameter values into a GIFN_B for the system to perform its function properly and the generalized intuitionistic fuzzy reliability modeling is introduced via the generalized intuitionistic fuzzy probabilities. The fuzzy reliability, conditional reliability, hazard and mean time to failure functions are obtained via generalized intuitionistic fuzzy parameter. Fuzzy reliability of the series and parallel system has been evaluated separately, where the parameter of each component is taken as a GIFN_B.

This paper is organized as follows: In Section 2, we represent background and some basic concepts of GIFN_B. The generalized intuitionistic fuzzy probability is introduced in Section 3, where parameter is the GIFN_B. In Section 4, we obtain the generalized intuitionistic fuzzy reliability characteristics which include the reliability, conditional reliability, hazard and mean time to failure functions. Section 5, concentrate on generalized intuitionistic fuzzy reliability for both series and parallel system. Finally, in Section 6 the theoretical results are investigated based on numerical example.

2 Preliminaries

Intuitionistic fuzzy set is characterized by a membership function and a non-membership function which can take to account the hesitation of the membership degree while fuzzy set only focused on the degree of acceptance. Due to the flexibility of IFS, we concentrate on a generalization of intuitionistic fuzzy sets whose basic elements are generalization intuitionistic fuzzy numbers. In the following, we briefly review several definitions and terminologies regarding GIFN$_B$ which used throughout the paper.

Definition 2.1. ([10]) *Consider non-empty set X, a generalized intuitionistic fuzzy set (GIFS$_B$) A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, denote the degree of membership and degree of non-membership functions of x in A, respectively, and $0 \leq (\mu_A(x))^\delta + (\nu_A(x))^\delta \leq 1,\ \forall x \in X$ and $\delta = n$ or $\frac{1}{n},\ n = 1, 2, \ldots, N$.*

[10] showed that GIFS$_B$ is more applicable and flexible, since the Atanassov's intuitionistic fuzzy set, intuitionistic fuzzy sets of root type and second type can be considered as special cases of GIFS$_B$.

Definition 2.2. ([35]) *Consider GIFS$_B$ of the real line $\mathbb{R}$, a generalized L-R type intuitionistic fuzzy number (GIFN$_B$) A can be defined where the membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ are defined as follows*

$$\mu_A(x) = \begin{cases} f^L(x), & a \leq x \leq b \\ u, & b \leq x \leq c \\ f^R(x), & c \leq x \leq d \\ 0, & o.w \end{cases}, \qquad \nu_A(x) = \begin{cases} g^L(x), & a_1 \leq x \leq b \\ w, & b \leq x \leq c \\ g^R(x), & c \leq x \leq d_1 \\ 1, & o.w \end{cases},$$

with $a_1 \leq a \leq b \leq c \leq d \leq d_1$ and $0 \leq (\mu_A(x))^\delta + (\nu_A(x))^\delta \leq 1,\ \forall x \in X$. The left and the right basis functions $f^L(x), f^R(x), g^L(x)$ and $g^R(x)$ are continuous monotone membership and non-membership functions respectively, where $f^L(x), g^R(x)$ are increasing and $f^R(x), g^L(x)$ are decreasing functions.

A especial class of generalized L-R type intuitionistic fuzzy number

A defined as

$$\mu_A(x) = \begin{cases} (\frac{x-a}{b-a})^{\frac{1}{\delta}}, & a \le x \le b \\ 1, & b \le x \le c \\ (\frac{d-x}{d-c})^{\frac{1}{\delta}}, & c \le x \le d \\ 0, & \text{o.w} \end{cases}, \qquad \nu_A(x) = \begin{cases} (\frac{b-x}{b-a_1})^{\frac{1}{\delta}}, & a_1 \le x \le b \\ 0, & b \le x \le c \\ (\frac{x-c}{d_1-c})^{\frac{1}{\delta}}, & c \le x \le d_1 \\ 1, & \text{o.w} \end{cases}.$$

The GIFN$_B$ A is denoted as $A = (a_1, a, b, c, d, d_1, \delta)$.
The α-cut of a fuzzy set is the classical set that includes all the elements of the set whose membership grades are greater than or equal to the specified value of α. [8] introduced the (α_1, α_2)-cut of GIFN$_B$, that is briefly explained in Definition 2.3.

Definition 2.3. ([8]) *Consider fixed numbers $\alpha_1, \alpha_2 \in [0, 1]$ such that $0 \le \alpha_1^\delta + \alpha_2^\delta \le 1$, a set of (α_1, α_2)-cut generated by a GIFN$_B$ A is defined by*

$$A[\alpha_1, \alpha_2, \delta] = \{\langle x, \mu_A(x) \ge \alpha_1, \nu_A(x) \le \alpha_2\rangle : x \in X\}.$$

The α_1-cut set of a GIFN$_B$ A is a crisp subset of $\mathbb{R}$, which is defined as

$$A_\mu[\alpha_1, \delta] = \{\langle x, \mu_A(x) \ge \alpha_1\rangle : x \in X\} = \left[A_\mu^L(\alpha_1), A_\mu^U(\alpha_1)\right], \;\; 0 \le \alpha_1 \le 1,$$

$$A_\mu^L(\alpha_1) = a + (b-a)\,\alpha_1^\delta, \qquad A_\mu^U(\alpha_1) = d - (d-c)\,\alpha_1^\delta.$$

Similarity, the α_2-cut set of a GIFN$_B$ A is a crisp subset of $\mathbb{R}$, which is defined as

$$A_\nu[\alpha_2, \delta] = \{\langle x, \nu_A(x) \le \alpha_2\rangle : x \in X\} = \left[A_\nu^L(\alpha_2), A_\nu^U(\alpha_2)\right], \;\; 0 \le \alpha_2 \le 1,$$

$$A_\nu^L(\alpha_2) = b\left(1 - \alpha_2^\delta\right) + a_1\alpha_2^\delta, \qquad A_\nu^U(\alpha_2) = c\left(1 - \alpha_2^\delta\right) + d_1\alpha_2^\delta.$$

Therefore the (α_1, α_2)-cut set of a GIFN$_B$ is given by

$$A[\alpha_1, \alpha_2, \delta] = \left\{x, x \in \left[A_\mu^L(\alpha_1),\; A_\mu^U(\alpha_1)\right] \cap \left[A_\nu^L(\alpha_2),\; A_\nu^U(\alpha_2)\right]\right\}.$$

The GIFN$_B$ based on the α_1-cut and α_2-cut sets are shown as

$$A(\alpha_1, \alpha_2, \delta) = (A_\mu[\alpha_1, \delta], A_\nu[\alpha_2, \delta]).$$

Definition 2.4. *Let $[a,b]$ and $[c,d]$ be two α-cut sets, some relations and operations on α-cut sets are defined as bellows*

(i) $[a,b] \preccurlyeq [c,d] \Leftrightarrow a \leq c$ *and* $b \leq d$,

(ii) *If* $k > 0$, *then we have* $k \otimes [a,b] = [ka, kb]$ *and if* $k < 0$, *then* $k \otimes [a,b] = [kb, ka]$,

(iii) $k \oplus [a,b] = [k+a, k+b]$ *and* $k \ominus [a,b] = [k-b, k-a]$,

(iv) $[a,b] \oplus [c,d] = [a+c, b+d]$.

Definition 2.5. *Suppose $A(\alpha_1,\alpha_2,\delta)$ and $B(\alpha_1,\alpha_2,\delta)$ be two GIFN$_B$s, the following relations and operations on GIFN$_B$s are concluded*

(i) $A(\alpha_1,\alpha_2,\delta) \oplus B(\alpha_1,\alpha_2,\delta) = \Big(A_\mu[\alpha_1,\delta] \oplus B_\mu[\alpha_1,\delta]\,,\ A_\nu[\alpha_2,\delta] \oplus B_\nu[\alpha_2,\delta]\Big)$,

(ii) $k \otimes A(\alpha_1,\alpha_2,\delta) \oplus b = (k \otimes A_\mu[\alpha_1,\delta] \oplus b, k \otimes A_\nu[\alpha_2,\delta] \oplus b)$,

(iii) $b \ominus A(\alpha_1,\alpha_2,\delta) = (b \ominus A_\mu[\alpha_1,\delta], b \ominus A_\nu[\alpha_2,\delta])$,

(iv) $A(\alpha_1,\alpha_2,\delta) \preccurlyeq B(\alpha_1,\alpha_2,\delta)$, *if and only if* $A_\mu[\alpha_1,\delta] \preccurlyeq B_\mu[\alpha_1,\delta]$ *and* $A_\nu[\alpha_2,\delta] \preccurlyeq B_\nu[\alpha_2,\delta]$,

(v) $A(\alpha_1,\alpha_2,\delta) = B(\alpha_1,\alpha_2,\delta)$, *if and only if* $A_\mu[\alpha_1,\delta] = B_\mu[\alpha_1,\delta]$ *and* $A_\nu[\alpha_2,\delta] = B_\nu[\alpha_2,\delta]$.

3 Generalized Intuitionistic Fuzzy Probability

Some lifetime data might be imprecise and the parameter of the model must be represented in the form of fuzzy numbers. Thus, it is necessary to generalize classical probability definition from real numbers to fuzzy numbers. In this section, we introduce the fuzzy probability where its parameter is generalized intuitionistic fuzzy number.

Consider X as a continuous random variable from a density function $f(x,\tilde{\theta})$ where $\tilde{\theta}$ is a GIFN$_B$. Its corresponding crisp PDF is $f(x,\theta)$. Then a set of α_1-cut of membership function and α_2-cut set of non-membership function of generalized intuitionistic fuzzy probability density function is defined as

$$f_j(x,\tilde{\theta}) = \Big\{ f(x,\theta)|\theta \in \theta_j[\alpha_i,\delta] \Big\} = \Big[f_j^L(x)[\alpha_i], f_j^U(x)[\alpha_i] \Big],$$

for all $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1$ and $0 \leq \alpha_1^\delta + \alpha_2^\delta \leq 1$, where $f_j^L[\alpha_i] = \inf_{\theta\in\theta_j[\alpha_i,\delta]} f(x)$, $f_j^U[\alpha_i] = \sup_{\theta\in\theta_j[\alpha_i,\delta]} f(x)$ and $(i,j) = (1,\mu),(2,\nu)$.

Then α_1-cut set of membership and α_2-cut set of non-membership functions a generalized intuitionistic fuzzy probability (GIFP) of C is defined as

$$P_j(C)\,[\alpha_i,\delta] = \{\, P\,(C)|\,\theta \in \theta_j\,[\alpha_i,\delta]\} = \Big[P_j^L(C)[\alpha_i],\; P_j^U(C)[\alpha_i] \Big], \quad (i,j) = (1,\mu)\,,(2,\nu)\,,$$

for all $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1$, $0 \leq \alpha_1^\delta + \alpha_2^\delta \leq 1$ where $P\,(C) = \int_C f\,(x,\theta)\,dx$, $P_j^L(C)[\alpha_i] = \inf\limits_{\theta\in\theta_j[\alpha_i,\delta]} P(C)$, $P_j^U(C)[\alpha_i] = \sup\limits_{\theta\in\theta_j[\alpha_i,\delta]} P(C)$, $(i,j) = (1,\mu)\,,(2,\nu)\,.$

Consequently, we have

$$\tilde{P}\,(C) = P\,(C)\,(\alpha_1,\alpha_2,\delta) = (P_\mu\,(C)\,[\alpha_1,\delta]\,,\; P_\nu(C)\,[\alpha_2,\delta])\,,$$

that it is a generalized intuitionistic fuzzy number, and a set of (α_1,α_2)-cut of generalized intuitionistic fuzzy probability of C is defined

$$P\,(C)\,[\alpha_1,\alpha_2,\;\delta] = \{w, w \in P_\mu(C)\,[\alpha_1,\delta] \cap\; P_\nu(C)\,[\alpha_2,\delta]\}\,.$$

Corollary 3.1. *Consider the generalized intuitionistic fuzzy probability* $P\,(C)$*,*

(i) $P\,(C^c)\,(\alpha_1,\alpha_2,\delta) = 1 \ominus P(C)\,(\alpha_1,\alpha_2,\delta)$*,*

(ii) If $C_1 \subset C_2$ *then* $P\,(C_1)\,(\alpha_1,\alpha_2,\delta) \preccurlyeq P\,(C_2)\,(\alpha_1,\alpha_2,\delta)$*.*

Proof. (i) Due to the definition of generalized intuitionistic fuzzy probability, we have

$$\begin{aligned}P_j\left(C^c\right)\left[\alpha_i,\delta\right] &= \left\{1-P\left(C\right)\right|\theta\in\theta_j\left[\alpha_i,\delta\right]\} = \left[P_j^L\left(C^c\right)\left[\alpha_i\right],\ P_j^U\left(C^c\right)\left[\alpha_i\right]\right]\\ &= \left[\inf_{\theta\in\theta_j\left[\alpha_i,\delta\right]}\left(1-P\left(C\right)\right),\ \sup_{\theta\in\theta_j\left[\alpha_i,\delta\right]}\left(1-P\left(C\right)\right)\right]\\ &= \left[1-\sup_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C\right),1-\inf_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C\right)\right]\\ &= 1\ominus\left[P_j^L\left(C\right)\left[\alpha_i\right],P_j^U\left(C\right)\left[\alpha_i\right]\right],\ \ (i,j)=\left(1,\mu\right),\left(2,\nu\right),\end{aligned}$$

which is verified by Definition 2.5-v.
(ii) Since $P\left(C_1\right)\leq P(C_2)$, for $(i,j)=\left(1,\mu\right),\left(2,\nu\right)$, so

$$\begin{aligned}P_j\left(C_1\right)\left[\alpha_i,\delta\right] &= \left[\inf_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C_1\right),\ \sup_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C_1\right)\right]\\ &\preccurlyeq \left[\inf_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C_2\right),\ \sup_{\theta\in\theta_j\left[\alpha_i,\delta\right]}P\left(C_2\right)\right]=P_j\left(C_2\right)\left[\alpha_i,\delta\right],\end{aligned}$$

and using Definition 2.5-iv, the proof is completed. □

By determination of GIFP, we can focus on fuzzification of some statistical concept such as expectation and variance.
A set of α_1-cut of membership function and α_2-cut set of non-membership function of generalized intuitionistic fuzzy expectation $\tilde{E}(g\left(X\right))$ is defined

$$\begin{aligned}E_j(g\left(X\right))\left[\alpha_i,\delta\right] &= \left\{E(g\left(X\right))\right|\theta\in\theta_j\left[\alpha_i,\delta\right]\}\\ &= \left[E_j^L\left(g\left(X\right)\right)\left[\alpha_i\right],\ E_j^U(g\left(X\right))[\alpha_i]\right],\end{aligned}$$

for all $0\leq\alpha_1\leq 1,\ 0\leq\alpha_2\leq 1,\ 0\leq\alpha_1^\delta+\alpha_2^\delta\leq 1$, where $E(g\left(X\right))=\int_x g\left(x\right)f\left(x,\theta\right)dx$, $E_j^L\left(g\left(X\right)\right)\left[\alpha_i\right]=\inf_{\theta\in\theta_j\left[\alpha_i,\delta\right]}E(g\left(X\right))$, $E_j^U(g\left(X\right))[\alpha_i]=\sup_{\theta\in\theta_j\left[\alpha_i,\delta\right]}E(g\left(X\right))$, $(i,j)=\left(1,\mu\right),\left(2,\nu\right)$. Therefore, we have

$$E(g\left(X\right))\left(\alpha_1,\alpha_2,\delta\right)=\left(E_\mu(g\left(X\right))\left[\alpha_1,\delta\right],E_\upsilon(g\left(X\right))\left[\alpha_2,\delta\right]\right),$$

and a set of (α_1, α_2)-cut of generalized intuitionistic fuzzy expectation of $g(X)$ is defined

$$E(g(X))[\alpha_1, \alpha_2, \delta] = E_\mu(g(X))[\alpha_1, \delta] \cap \ E_v(g(X))[\alpha_2, \delta].$$

Remark 3.1. *Assumption $g(X) = X$ and $g(X) = (X - E(X))^2$ lead to generalized intuitionistic fuzzy expectation of $X(\tilde{\mu})$ and generalized intuitionistic fuzzy variance of $X(\tilde{\sigma}^2)$, respectively.*

Corollary 3.2. *Consider a, b, c as constant numbers, then*

(i) $\tilde{E}(c) = c$,

(ii) $\tilde{E}(ag(X) + b) = a \otimes \tilde{E}(g(X)) \oplus b$,

(iii) $\tilde{\sigma}^2(c) = 0$,

(iv) $\tilde{\sigma}^2(aX + b) = a^2 \otimes \tilde{\sigma}^2(X)$.

Proof. (i) and (iii) are obvious and the proofs are omitted. (ii) is obtained as follows

$$\begin{aligned}
\inf_{\theta \in \theta_j[\alpha_i, \delta]} E(ag(X) + b) &= a \inf_{\theta \in \theta_j[\alpha_i, \delta]} E(g(X)) + b, \\
\sup_{\theta \in \theta_j[\alpha_i, \delta]} E(ag(X) + b) &= a \sup_{\theta \in \theta_j[\alpha_i, \delta]} E(g(X)) + b, \quad (i, j) = (1, \mu), (2, \nu).
\end{aligned}$$

(iv) is concluded by

$$\begin{aligned}
\inf_{\theta \in \theta_j[\alpha_i, \delta]} \sigma^2(aX + b) &= a^2 \inf_{\theta \in \theta_j[\alpha_i, \delta]} \sigma^2(X), \\
\sup_{\theta \in \theta_j[\alpha_i, \delta]} \sigma^2(aX + b) &= a^2 \sup_{\theta \in \theta_j[\alpha_i, \delta]} \sigma^2(X), \quad (i, j) = (1, \mu), (2, \nu),
\end{aligned}$$

which completed the proof. □

4 Generalized Intuitionistic Fuzzy Reliability Characteristics

The fuzzy reliability is a novel concept, since fuzzy set can easily capture subjective, uncertain, and ambiguous information. Therefore, to obtain more flexible information regarding the reliability of the system, we propose the intuitionistic fuzzy approach for reliability parameter descriptions.

Consider X as a lifetime variable of a component with a density function $f(x,\tilde{\theta})$ where $\tilde{\theta}$ is a GIFN$_B$ and the generalized intuitionistic fuzzy reliability characteristic (GIFRC) denoted by $\tilde{g}\,(t)$. A set of α_1-cut of membership and α_2-cut set of non-membership functions of GIFRC of the component is denoted by $g_j(t)\,[\alpha_i,\delta]$ and represented as

$$g_j(t)\,[\alpha_i,\delta] = \{\,g\,(t)|\,\theta \in \theta_j\,[\alpha_i,\delta]\} = \left[g_j^L\,(t)\,[\alpha_i],\ g_j^U(t)[\alpha_i]\right], \quad (i,j) = (1,\mu)\,,(2,\nu),$$

for all $0 \le \alpha_1 \le 1,\ 0 \le \alpha_2 \le 1,\ 0 \le \alpha_1^\delta + \alpha_2^\delta \le 1$, where $g_j^L(t)[\alpha_i] = \inf_{\theta\in\theta_j[\alpha_i,\delta]} g(t),\ \ g_j^U\,(t)\,[\alpha_i] = \sup_{\theta\in\theta_j[\alpha_i,\delta]} g(t),\quad (i,j) = (1,\mu)\,,(2,\nu)$. The function $g(t)$ can be considered as reliability, conditional reliability, hazard rate, cumulative risk and reverse hazard functions. It can be shown that $g\,(\alpha_1,\alpha_2,\delta) = (g_\mu(t)\,[\alpha_1,\delta]\,,g_\nu(t)\,[\alpha_2,\delta])$ and a (α_1,α_2)-cut set of GIFRC is defined as bellow

$$g(t)\,[\alpha_1,\alpha_2,\delta] = \{w, w \in g_\mu(t)\,[\alpha_1,\delta] \cap g_\nu(t)\,[\alpha_2,\delta]\}\,.$$

In the following different reliability characteristics are discussed, comprehensively.

4.1 Generalized intuitionistic fuzzy reliability function

Reliability of the systems can be evaluated in different methods and techniques, but fuzzy reliability measures the uncertainty of the possible membership and non-membership grade of the components. In this section, a notion of the generalized intuitionistic fuzzy reliability (GIFR) function, denoted by $\tilde{S}\,(t)$, is constructed based on the lifetime GIFN$_B$ parameter, which is the generalized intuitionistic fuzzy probability that a unit survives beyond time t.

A set of α_1-cut of membership function and α_2-cut set of non-membership function of GIFR function of component is denoted by $S_j(t)\,[\alpha_i,\delta]$, are obtained as

$$S_j(t)\,[\alpha_i,\delta] = \{\,S\,(t)|\,\theta \in \theta_j\,[\alpha_i,\delta]\} = \left[S_j^L\,(t)\,[\alpha_i],\ S_j^U(t)[\alpha_i]\right], \quad (i,j) = (1,\mu)\,,(2,\nu),$$

for all $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1, 0 \leq \alpha_1^{\delta} + {\alpha_2}^{\delta} \leq 1$, where $S(t) = \int_t^{\infty} f(x, \theta)\, dx$, $S_j^L(t)[\alpha_i] = \inf_{\theta \in \theta_j[\alpha_i, \delta]} S(t)$, $S_j^U(t)[\alpha_i] = \sup_{\theta \in \theta_j[\alpha_i, \delta]} S(t)$, $(i, j) = (1, \mu), (2, \nu)$ and it can be shown as

$$S(\alpha_1, \alpha_2, \delta) = (S_\mu(t)[\alpha_1, \delta], S_\nu(t)[\alpha_2, \delta]).$$

A set of (α_1, α_2)-cut of generalized intuitionistic fuzzy reliability is defined

$$S(t)[\alpha_1, \alpha_2, \delta] = \{w, w \in S_\mu(t)[\alpha_1, \delta] \cap S_\nu(t)[\alpha_2, \delta]\},$$

where $S_j(t)[\alpha_i, \delta]$, $(i, j) = (1, \mu), (2, \nu)$ are two-variables functions in terms of α_i, $i = 1, 2$ and t. For t_0, $\tilde{S}(t_0)$ is a generalized intuitionistic fuzzy number.
The GIFR curves are like bands whose width depends on the ambiguity parameter. The more bandwidth implies less certainty and when the crisp parameter is considered, the lower and upper bounds will become equal, that leads to classic reliability analysis. The GIFR bands have the following properties

(i) $S_j(0)[\alpha_{i0}, \delta] = [1, 1]$, i.e. no one starts off dead,

(ii) $S_j(\infty)[\alpha_{i0}, \delta] = [0, 0]$, i.e. everyone dies eventually,

(iii) $S_j(t_1)[\alpha_{i0}, \delta] \Leftrightarrow S_j(t_2)[\alpha_{i0}, \delta] \Leftrightarrow t_1 \leq t_2$, i.e. bands of $S_j(t)[\alpha_{i0}, \delta]$ declines monotonically.

4.2 Generalized intuitionistic fuzzy conditional reliability function

In reliability analysis, one of the most important character is the conditional reliability, which is the probability that an item survives for a time t, knowing that it has already survived until time τ.
Here, we extend the conditional reliability function to the uncertain case by generalized intuitionistic fuzzy set. The generalized intuitionistic fuzzy conditional reliability (GIFCR) function of component is denoted by $\tilde{S}(t|\tau)$. A set of α_1-cut of membership function and α_2-cut set of

non-membership function of $\tilde{S}\left(t|\,\tau\right)$ is given by

$$S_j(t|\,\tau)\left[\alpha_i,\delta\right]=\left\{S(t|\,\tau)|\,\theta\in\theta_j[\alpha_i,\delta]\right\}=\left[S_j^L\left(t|\,\tau\right)[\alpha_i],\ S_j^U\left(t|\,\tau\right)[\alpha_i]\right],\quad (i,j)=(1,\mu)\,,(2,\nu)$$

for all $0\leq\alpha_1\leq1,\ 0\leq\alpha_2\leq1,\ 0\leq\alpha_1^\delta+\alpha_2^\delta\leq1$, where $S(t|\,\tau)=\frac{S(t+\tau)}{S(\tau)}$, $S_j^L(t|\,\tau)[\alpha_i]=\inf\limits_{\theta\in\theta_j[\alpha_i,\delta]}S(t|\,\tau),\ S_j^U\left(t|\,\tau\right)[\alpha_i]=\sup\limits_{\theta\in\theta_j[\alpha_i,\delta]}S(t|\,\tau),\quad(i,j)=(1,\mu)\,,(2,\nu)$ and we have $S\left(\alpha_1,\alpha_2,\delta\right)=\left(S_\mu(t|\,\tau)\left[\alpha_1,\delta\right],\ S_\nu(t|\,\tau)\left[\alpha_2,\delta\right]\right)$.
A set of (α_1,α_2)-cut of GIFCR function is defined as

$$S(t\,|\tau)\left[\alpha_1,\alpha_2,\delta\right]=\left\{w,w\in S_\mu(t\,|\tau)\left[\alpha_1,\delta\right]\cap S_\nu(t\,|\tau)\left[\alpha_2,\delta\right]\right\},$$

where $S_j(t|\,\tau)\left[\alpha_i,\delta\right],\ (i,j)=(1,\mu)\,,(2,\nu)$ are two-variables functions in terms of $\alpha_i,\ i=1,2$ and t.

4.3 Generalized intuitionistic fuzzy hazard function

Another fuzzy character of the lifetime distribution is the fuzzy hazard function, that is also known as the instantaneous failure rate at which a component will fail under the condition that it has already survived. We propose the concept of a generalized intuitionistic fuzzy hazard (GIFH) function of component, which is denoted by $\tilde{h}\left(t\right)$ and it means the probability of a device failing at a time interval Δt if it operates until t. A set of α_1-cut of membership function and α_2-cut set of non-membership function a GIFH function of the component is illustrated as follows

$$h_j(t)\left[\alpha_i,\delta\right]=\left\{h(t)|\theta\in\theta_j[\alpha_i,\delta]\right\}=\left[h_j^L\left(t\right)[\alpha_i],h_j^U(t)[\alpha_i]\right],\quad (i,j)=(1,\mu)\,,(2,\nu),$$

for all $0\leq\alpha_1\leq1,\ 0\leq\alpha_2\leq1,\ 0\leq\alpha_1^\delta+\alpha_2^\delta\leq1$, where $h_j^L(t)[\alpha_i]=\inf_{\theta\in\theta_j[\alpha_i,\delta]}h(t),\ \ h_j^U\left(t\right)[\alpha_i]=\sup_{\theta\in\theta_j[\alpha_i,\delta]}h(t),\quad(i,j)=(1,\mu)\,,(2,\nu)$, $h\left(t\right)=\frac{f(t)}{S(t)}$.
It can be shown that $h\left(\alpha_1,\alpha_2,\delta\right)=\left(h_\mu(t)\left[\alpha_1,\delta\right],h_\nu(t)\left[\alpha_2,\delta\right]\right)$ and a set of (α_1,α_2)-cut of GIFH function is defined by

$$h(t)\left[\alpha_1,\alpha_2,\delta\right]=\left\{w,w\in h_\mu(t)\left[\alpha_1,\delta\right]\cap h_\nu(t)\left[\alpha_2,\delta\right]\right\},$$

where $h_j(t)\left[\alpha_i,\delta\right],\ (i,j)=(1,\mu)\,,(2,\nu)$ are two-variables functions in terms of $\alpha_i,\ i=1,2$ and t.

Remark 4.1. *Same as the GIFR, for every especially* α_{10} *and* α_{20}*, the shapes of* $S_j(t|\tau)[\alpha_{i0}, \delta]$ *and* $h_j(t)[\alpha_{i0}, \delta]$*,* $(i, j) = (1, \mu), (2, \nu)$ *are like bands with upper and lower bounds and for every especially* t_0*,* $\tilde{S}(t_0|\tau)$ *and* $\tilde{h}(t_0)$ *are generalized intuitionistic fuzzy numbers.*

Remark 4.2. *If* $\delta = 1$*, then our method is named intuitionistic fuzzy reliability evaluation, in addition, if* $\alpha_1 = 1 - \alpha_2$*,* $a = a_1$ *and* $d = d_1$*, then it changes to fuzzy reliability evaluation, finally if assumption* $a = b = c = d$ *is added, it agree to classical reliability theory.*

4.4 Generalized intuitionistic fuzzy reliability analysis for Pareto lifetime distribution

Here, we consider Pareto lifetime distribution, which has the uncertainty in the scale parameter and the vagueness are represented by fuzzifying the parameter values into a GIFN_B.
In this section, the generalized intuitionistic fuzzy reliability, conditional reliability and hazard functions are discussed based on the Pareto distribution. Consider the Pareto lifetime random variable X with generalized intuitionistic fuzzy lifetime scale parameter $\tilde{\lambda} = (a_1, a, b, c, d, d_1, \delta)$, which has the following probability density function

$$f(x, \lambda) = \frac{\lambda k^{\lambda}}{x^{\lambda+1}}, \qquad x > k,\ k > 0.$$

The cut sets of GIFR function is obtained as follows

$$S_j(t)[\alpha_i, \delta] = \left\{ \left(\frac{k}{t}\right)^{\lambda} | \lambda \in \lambda_j[\alpha_i, \delta], \ \ (i, j) = (1, \mu), (2, \nu) \right\}.$$

Since $(\frac{k}{t})^{\lambda}$ is a monotonically decreasing function with respect to λ, the reliability bands are given by

$$\begin{aligned} S_{\mu}(t)[\alpha_1, \delta] &= \left[\left(\frac{k}{t}\right)^{d-(d-c)\alpha_1^{\delta}}, \left(\frac{k}{t}\right)^{a+(b-a)\alpha_1^{\delta}} \right], \\ S_{\nu}(t)[\alpha_2, \delta] &= \left[\left(\frac{k}{t}\right)^{c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}}, \left(\frac{k}{t}\right)^{b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}} \right]. \end{aligned}$$

For every especial t_0, membership function and non-membership function of $\tilde{S}\left(t_0\right)$ are represented as

$$\mu_{S(t_0)}\left(x\right)=\begin{cases}\left(\dfrac{d-\frac{\ln x}{\ln(\frac{k}{t_0})}}{d-c}\right)^{\frac{1}{\delta}}, & \left(\frac{k}{t_0}\right)^{d}\leq x\leq\left(\frac{k}{t_0}\right)^{c}\\ 1, & \left(\frac{k}{t_0}\right)^{c}\leq x\leq\left(\frac{k}{t_0}\right)^{b}\\ \left(\dfrac{\frac{\ln x}{\ln(\frac{k}{t_0})}-a}{b-a}\right)^{\frac{1}{\delta}}, & \left(\frac{k}{t_0}\right)^{b}\leq x\leq\left(\frac{k}{t_0}\right)^{a}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_{S(t_0)}(x)=\begin{cases}\left(\dfrac{\frac{\ln x}{\ln(\frac{k}{t_0})}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \left(\frac{k}{t_0}\right)^{d_1}\leq x\leq\left(\frac{k}{t_0}\right)^{c}\\ 0, & \left(\frac{k}{t_0}\right)^{c}\leq x\leq\left(\frac{k}{t_0}\right)^{b}\\ \left(\dfrac{b-\frac{\ln x}{\ln(\frac{k}{t_0})}}{b-a_1}\right)^{\frac{1}{\delta}}, & \left(\frac{k}{t_0}\right)^{b}\leq x\leq\left(\frac{k}{t_0}\right)^{a_1}\\ 1, & \text{o.w.}\end{cases}.$$

The cut sets of GIFCR function are shown by

$$S_j\left(t|\,\tau\right)\left[\alpha_i,\delta\right]=\left\{\left(\frac{\tau}{t+\tau}\right)^{\lambda}|\lambda\in\lambda_j[\alpha_i,\delta]\right\},\quad (i,j)=\left(1,\mu\right),\left(2,\nu\right).$$

Since $\left(\frac{\tau}{t+\tau}\right)^{\lambda}$ is a monotonically decreasing function with respect to λ, the conditional reliability bands are obtained as

$$\begin{aligned}S_\mu\left(t|\,\tau\right)\left[\alpha_1,\delta\right] &= \left[\left(\frac{\tau}{t+\tau}\right)^{d-(d-c)\alpha_1^{\delta}},\left(\frac{\tau}{t+\tau}\right)^{a+(b-a)\alpha_1^{\delta}}\right],\\ S_\nu\left(t|\,\tau\right)\left[\alpha_2,\delta\right] &= \left[\left(\frac{\tau}{t+\tau}\right)^{c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}},\left(\frac{\tau}{t+\tau}\right)^{b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}}\right].\end{aligned}$$

For every especial t_0, membership function and non-membership function of $\tilde{S}\left(t_0 \middle|\, \tau\right)$ are as follows

$$\mu_{S(\,t_0|\tau)}\left(x\right)=\begin{cases}\left(\dfrac{d-\frac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}}{d-c}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{d}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{c}\\ 1, & \left(\frac{\tau}{t_0+\tau}\right)^{c}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{b}\\ \left(\dfrac{\frac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}-a}{b-a}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{b}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{a}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_{S(\,t_0|\tau)}(x)=\begin{cases}\left(\dfrac{\frac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{d_1}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{c}\\ 0, & \left(\frac{\tau}{t_0+\tau}\right)^{c}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{b}\\ \left(\dfrac{b-\frac{\ln x}{\ln\left(\frac{\tau}{t_0+\tau}\right)}}{b-a_1}\right)^{\frac{1}{\delta}}, & \left(\frac{\tau}{t_0+\tau}\right)^{b}\leq x\leq\left(\frac{\tau}{t_0+\tau}\right)^{a_1}\\ 1, & \text{o.w.}\end{cases}.$$

Finally, the cut sets of GIFH function are demonstrated as

$$h_j\left(t\right)\left[\alpha_i,\delta\right]=\left\{\frac{\lambda}{t}\left|\lambda\in\lambda_j\left[\alpha_i,\delta\right]\right.\right\}=\left[h_j^L\left(t\right)\left[\alpha_i\right],h_j^U\left(t\right)\left[\alpha_i\right]\right],\quad \left(i,j\right)=\left(1,\mu\right),\left(2,\nu\right),$$

for all $0\leq\alpha_1\leq 1,\ 0\leq\alpha_2\leq 1,\ 0\leq\alpha_1^{\delta}+\alpha_2^{\delta}\leq 1$, where

$$\begin{aligned} h_j^L\left(t\right)\left[\alpha_i\right] &= \inf\left\{\frac{\lambda}{t}\left|\lambda\in\lambda_j\left[\alpha_i,\delta\right]\right.\right\},\\ h_j^U\left(t\right)\left[\alpha_i\right] &= \sup\left\{\frac{\lambda}{t}\left|\lambda\in\lambda_j\left[\alpha_i,\delta\right]\right.\right\},\qquad \left(i,j\right)=\left(1,\mu\right),\left(2,\nu\right).\end{aligned}$$

Therefore,

$$\begin{aligned} h_{\mu}(t)\left[\alpha_1,\delta\right] &= \left[\frac{a+(b-a)\,\alpha_1^{\delta}}{t},\frac{d-(d-c)\,\alpha_1^{\delta}}{t}\right],\\ h_{\nu}(t)\left[\alpha_2,\delta\right] &= \left[\frac{b\left(1-\alpha_2^{\delta}\right)+a_1\alpha_2^{\delta}}{t},\frac{c\left(1-\alpha_2^{\delta}\right)+d_1\alpha_2^{\delta}}{t}\right]. \end{aligned}$$

As can be seen, the GIFH function for generalized intuitionistic fuzzy Pareto distribution, is decreasing with respect to time. The membership function and non-membership function of $\tilde{h}\left(t_0\right)$ are as follows

$$\mu_{h(t_0)}(x)=\begin{cases} \left(\dfrac{t_0x-a}{b-a}\right)^{\frac{1}{\delta}}, & \frac{a}{t_0}\leq x\leq\frac{b}{t_0}\\ 1, & \frac{b}{t_0}\leq x\leq\frac{c}{t_0}\\ \left(\dfrac{d-t_0x}{d-c}\right)^{\frac{1}{\delta}}, & \frac{c}{t_0}\leq x\leq\frac{d}{t_0}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_{h(t_0)}(x)=\begin{cases} \left(\dfrac{b-t_0x}{b-a_1}\right)^{\frac{1}{\delta}}, & \frac{a_1}{t_0}\leq x\leq\frac{b}{t_0}\\ 0, & \frac{b}{t_0}\leq x\leq\frac{c}{t_0}\\ \left(\dfrac{t_0x-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \frac{c}{t_0}\leq x\leq\frac{d_1}{t_0}\\ 1, & \text{o.w.}\end{cases}.$$

Corollary 4.1. *If $\delta_1\leq\delta_2$ then we have*

(i) $S_{\mu}(t)\left[\alpha_1,\delta_1\right]\subset S_{\mu}(t)\left[\alpha_1,\delta_2\right]$ *and* $S_{\nu}(t)\left[\alpha_2,\delta_2\right]\subset S_{\nu}(t)\left[\alpha_2,\delta_1\right]$,

(ii) $S_{\mu}(t|\,\tau)\left[\alpha_1,\delta_1\right]\subset S_{\mu}(t|\,\tau)\left[\alpha_1,\delta_2\right]$ *and* $S_{\nu}(t|\,\tau)\left[\alpha_2,\delta_2\right]\subset S_{\nu}(t|\,\tau)\left[\alpha_2,\delta_1\right]$,

(iii) $h_{\mu}(t)\left[\alpha_1,\delta_1\right]\subset h_{\mu}(t)\left[\alpha_1,\delta_2\right]$ *and* $h_{\nu}(t)\left[\alpha_2,\delta_2\right]\subset h_{\nu}(t)\left[\alpha_2,\delta_1\right]$.

Corollary 4.2. *For every δ,*

$$S\left(t\right)\left[1,0,\delta\right]=\left[\left(\frac{k}{t}\right)^{b},\left(\frac{k}{t}\right)^{c}\right],\quad h\left(t\right)\left[1,0,\delta\right]=\left[\frac{b}{t},\frac{c}{t}\right],$$

$$S\left(t\right)\left[0,1,\delta\right]=\left[\left(\frac{k}{t}\right)^{a},\left(\frac{k}{t}\right)^{d}\right],\quad h\left(t\right)\left[0,1,\delta\right]=\left[\frac{a}{t},\frac{d}{t}\right].$$

Corollary 4.3. *Consider $g\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]$ as (α_1,α_2)-cut set of reliability characteristics (GIFR or GIFCR or GIFH), then we have*

(i) $g\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]=\begin{cases}\left[g_\nu^L(t_0)\left[\alpha_2\right],g_\nu^U(t_0)\left[\alpha_2\right]\right], & \eta<\min(z_1,z_2)\\ \left[g_\mu^L(t_0)\left[\alpha_1\right],g_\mu^U(t_0)\left[\alpha_1\right]\right], & \eta\geq\max(z_1,z_2)\end{cases},$

(ii) *if $\eta=1$ (i.e. $\alpha_2^\delta=1-\alpha_1^\delta$), then* $g\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]=\left[g_\mu^L(t_0)\left[\alpha_1\right],g_\mu^U(t_0)\left[\alpha_1\right]\right],$

(iii) *if $z_1=z_2=\eta$ then $g_\mu\left(t_0\right)\left[\alpha_1,\delta\right]=g_\nu\left(t_0\right)\left[\alpha_2,\delta\right]=g\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]$,*

where $\eta=\frac{\alpha_2^\delta}{1-\alpha_1^\delta}$, $z_1=\frac{b-a}{b-a_1}$ and $z_2=\frac{d-c}{d_1-c}$.

Corollary 4.4. *Consider the generalized intuitionistic fuzzy Pareto lifetime random variable, if $\mu_{g(t_0)}\left(x\right)=\nu_{g(t_0)}(x)$ and $z_1=z_2=z$, then we have*

(i) $S\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]=S_\mu\left(t_0\right)\left[\alpha_1,\delta\right]=S_\nu\left(t_0\right)\left[\alpha_2,\delta\right]=\left[\left(\frac{k}{t_0}\right)^\zeta,\left(\frac{k}{t_0}\right)^\xi\right],$

(ii) $h\left(t_0\right)\left[\alpha_1,\alpha_2,\delta\right]=h_\mu\left(t_0\right)\left[\alpha_1,\delta\right]=h_\nu\left(t_0\right)\left[\alpha_2,\delta\right]=\left[\frac{\zeta}{t_0},\ \frac{\xi}{t_0}\right],$

(iii) $S\left(t_0|\,\tau\right)\left[\alpha_1,\alpha_2,\delta\right]=S_\mu\left(t_0|\,\tau\right)\left[\alpha_1,\delta\right]=S_\nu\left(t_0|\,\tau\right)\left[\alpha_2,\delta\right]=\left[\left(\frac{\tau}{t_0+\tau}\right)^\zeta,\left(\frac{\tau}{t_0+\tau}\right)^\xi\right],$

(iv) $\alpha_1=\alpha_2=\left(\dfrac{z}{1+z}\right)^{\frac{1}{\delta}},$

Where $\zeta=\frac{d+cz}{1+z}$ and $\xi=\frac{a+bz}{1+z}$.

Theorem 4.1. *Suppose the lifetime variables T_1 and T_2 have generalized intuitionistic fuzzy density function $\tilde{f}_1(x,\theta)$ and $\tilde{f}_2(x,\theta)$, respectively. If for every $t>0$ relation $\tilde{h}_1(t)\leq\tilde{h}_2(t)$, is electric, it can be concluded that $\tilde{S}_1(t)\geq\tilde{S}_2(t)$.*

Proof. By using $\tilde{h}_1\left(t\right)\left(\alpha_1,\alpha_2,\delta\right)\preccurlyeq\tilde{h}_2\left(t\right)\left(\alpha_1,\alpha_2,\delta\right)$ it can be shown that

$$\left(h_{1\mu}\left(t\right)\left[\alpha_1,\delta\right],h_{1\nu}\left(t\right)\left[\alpha_2,\delta\right]\right)\preccurlyeq\left(h_{2\mu}(t)\left[\alpha_1,\delta\right],h_{2\nu}\left(t\right)\left[\alpha_2,\delta\right]\right),$$

that induces to

$$h_{1\mu}(t)[\alpha_1,\delta] \preccurlyeq h_{2\mu}(t)[\alpha_1,\delta], \quad h_{1\nu}(t)[\alpha_2,\delta] \preccurlyeq h_{2\nu}(t)[\alpha_2,\delta].$$

Therefore, for every $\gamma = L, U$ we have

$$h^{\gamma}_{1\mu}(t)[\alpha_1,\delta] \leq h^{\gamma}_{2\mu}(t)[\alpha_1,\delta], \quad h^{\gamma}_{1\nu}(t)[\alpha_2,\delta] \leq h^{\gamma}_{2\nu}(t)[\alpha_2,\delta],$$

consequently,

$$\int_0^t h^{\gamma}_{1\mu}(x)[\alpha_1,\delta]dx \leq \int_0^t h^{\gamma}_{2\mu}(x)[\alpha_1,\delta]dx,$$
$$\int_0^t h^{\gamma}_{1\nu}(x)[\alpha_2,\delta]dx \leq \int_0^t h^{\gamma}_{2\nu}(x)[\alpha_2,\delta]dx,$$

therefore, based on the definition of hazard rate function, we have

$$\int_0^t \frac{f^{\gamma}_{1\mu}(x)[\alpha_1,\delta]}{1-F(x)^{\gamma}_{1\mu}(x)[\alpha_1,\delta]}dx \leq \int_0^t \frac{f^{\gamma}_{2\mu}(x)[\alpha_1,\delta]}{1-F(x)^{\gamma}_{2\mu}(x)[\alpha_1,\delta]}dx,$$
$$\int_0^t \frac{f^{\gamma}_{1\nu}(x)[\alpha_2,\delta]}{1-F(x)^{\gamma}_{1\nu}(x)[\alpha_2,\delta]}dx \leq \int_0^t \frac{f^{\gamma}_{2\nu}(x)[\alpha_2,\delta]}{1-F(x)^{\gamma}_{2\nu}(x)[\alpha_2,\delta]}dx,$$

hence

$$-\ln\left(1-F(t)^{\gamma}_{1\mu}(x)[\alpha_1,\delta]\right) \leq -\ln\left(1-F(t)^{\gamma}_{2\mu}(x)[\alpha_1,\delta]\right),$$
$$-\ln\left(1-F(t)^{\gamma}_{1\nu}(x)[\alpha_2,\delta]\right) \leq -\ln\left(1-F(t)^{\gamma}_{2\nu}(x)[\alpha_2,\delta]\right),$$

then we have

$$\left(1-F(t)^{\gamma}_{1\mu}[\alpha_1,\delta]\right) \geq \left(1-F(t)^{\gamma}_{2\mu}[\alpha_1,\delta]\right),$$
$$(1-F(t)^{\gamma}_{1\nu}[\alpha_2,\delta]) \geq (1-F(t)^{\gamma}_{2\nu}[\alpha_2,\delta]),$$

therefore,

$$\left(S_{1\mu}(t)[\alpha_1,\delta], S_{1\nu}(t)[\alpha_2,\delta]\right) \succcurlyeq \left(S_2\mu(t)[\alpha_1,\delta], S_{2\nu}(t)[\alpha_2,\delta]\right)$$

and

$$S_1(t)(\alpha_1,\alpha_2,\delta) \succcurlyeq S_2(t)(\alpha_1,\alpha_2,\delta) \text{ and } \tilde{S}_1(t) \succcurlyeq \tilde{S}_2(t).$$

which complete the proof. $\square$

Theorem 4.2. *The decreasing condition on the $\tilde{S}(x|t)$ function is a necessary and sufficient condition for $f(x, \tilde{\theta})$ to belong to a class of distribution with an increasing failure rate (IFR).*
Proof. Suppose for every $t_1 < t_2$ we have

$$\tilde{S}(x|t_1) \succcurlyeq \tilde{S}(x|t_2) \text{ and } \tilde{S}(x|t_1)(\alpha_1, \alpha_2, \delta) \succcurlyeq \tilde{S}(x|t_2)(\alpha_1, \alpha_2, \delta),$$

we conclude that

$$\left(S_{1\mu}\left(x\,|t_1\,\right)[\alpha_1, \delta], S_{1\nu}\left(x\,|t_1\,\right)[\alpha_2, \delta]\,\right) \succcurlyeq \left(S_{2\mu}\left(x\,|t_2\,\right)[\alpha_1, \delta], S_{2\nu}\left(x\,|t_2\,\right)[\alpha_2, \delta]\,\right),$$

then

$$S_{1\mu}\left(x \mid t_1\right)[\alpha_1, \delta] \succcurlyeq S_{2\mu}\left(x \mid t_2\right)[\alpha_1, \delta],$$

and

$$S_{1\nu}\left(x \mid t_1\right)[\alpha_2, \delta] \succcurlyeq S_{2\nu}\left(x|t_2\right)[\alpha_2, \delta].$$

For every $\gamma = L, U$ we have

$$S_{1\mu}^{\gamma}\left(x|t_1\right)[\alpha_1, \delta] \geq S_{2\mu}^{\gamma}\left(x \mid t_2\right)[\alpha_1, \delta],$$

and

$$S_{1\nu}^{\gamma}\left(x \mid t_1\right)[\alpha_2, \delta] \geq S_{2\nu}^{\gamma}\left(x \mid t_2\right)[\alpha_2, \delta].$$

Therefore, S_{μ}^{γ} and S_{ν}^{γ} are decreasing functions and by using definition of GIFCR function, we have

$$\begin{aligned}
S_j^{\gamma}\left(x \mid t\right)[\alpha_i, \delta] &= \frac{S_j^{\gamma}(x+t)\,[\alpha_i, \delta]}{S_j^{\gamma}\left(t\right)[\alpha_i, \delta]}, \qquad (i,j) = (1, \mu), (2, \nu), \\
\frac{\partial S_j^{\gamma}\left(x \mid t\right)[\alpha_i, \delta]}{\partial t} &= \frac{-f_j^{\gamma}\left(x+t\right)[\alpha_i, \delta]\, S_j^{\gamma}\left(t\right)[\alpha_i, \delta]}{S_j^{\gamma}(t)\left[\alpha_i, \delta\right]^2} \\
&\quad + \frac{f_j^{\gamma}(t)\,[\alpha_i, \delta]\, S_j^{\gamma}(x+t)\,[\alpha_i, \delta]}{S_j^{\gamma}(t)\left[\alpha_i, \delta\right]^2}.
\end{aligned}$$

According to that S_j^{γ} is a decreasing function, so it result that

$$\frac{\partial S_j^{\gamma}\left(x \mid t\right)[\alpha_i, \delta]}{\partial t} \leq 0$$

and hence

$$f_j^{\gamma}\left(t\right)[\alpha_i, \delta]\, S_j^{\gamma}\left(x+t\right)[\alpha_i, \delta] \leq f_j^{\gamma}\left(x+t\right)[\alpha_i, \delta]\, S_j^{\gamma}\left(t\right)[\alpha_i, \delta],$$

so,

$$h_j^{\gamma}(t)[\alpha_i,\delta] \leq h_j^{\gamma}(x+t)[\alpha_i,\delta],$$

we conclude that

$$\begin{array}{lll} h_\mu^\gamma(t)[\alpha_i,\delta] & \leq & h_\mu^\gamma(x+t)[\alpha_i,\delta], \qquad h_\nu^\gamma(t)[\alpha_i,\delta] \leq h_\nu^\gamma(x+t)[\alpha_i,\delta], \\ h_\mu(t)[\alpha_1,\delta] & \preccurlyeq & h_\mu(x+t)[\alpha_1,\delta], \qquad h_\nu(t)[\alpha_2,\delta] \preccurlyeq h_\nu(x+t)[\alpha_2,\delta]. \end{array}$$

Finally, we have

$$h(t)(\alpha_1,\alpha_2,\delta) \preccurlyeq h(x+t)(\alpha_1,\alpha_2,\delta) \text{ and } \tilde{h}(t) \preccurlyeq \tilde{h}(x+t),$$

which complete the proof. □

Corollary 4.5. *The increasing condition on the $\tilde{S}(x|t)$ function is a necessary and sufficient condition for $f(x,\tilde{\theta})$ to belong to a class of distribution with an decreasing failure rate (DFR).*

4.5 Generalized intuitionistic fuzzy mean time to failure for Pareto distribution

Mean time to failure (MTTF) is a reliability measure that represents the length of time a non-repairable system can be expected to perform. MTTF can be used to evaluate the reliability and to improve maintenance and system management strategy. Generalized intuitionistic fuzzy mean time to failure (GIFMTTF) of component is the expected time to failure of fuzzy system and denoted by $\widetilde{MTTF}$. In this section the generalized intuitionistic fuzzy of MTTF function under the Pareto lifetime distribution is demonstrated.
GIFMTTF of any component with generalized intuitionistic fuzzy Pareto distribution defined as follows

$$GIFMTTF_j[\alpha_i] = \left\{ \int_0^\infty s(x)dx \,|\lambda \in \lambda_j[\alpha_i,\delta] \right\} = \left\{ \frac{k\lambda}{\lambda-1} \,|\lambda \in \lambda_j[\alpha_i,\delta] \right\}, \quad \lambda > 1, (i,j) = (1,\mu),(2,\nu)$$

then

$$\begin{aligned} \text{GIFMTTF}_j[\alpha_1] &= \left[\frac{k\left(d-(d-c)\alpha_1^\delta\right)}{\left(d-(d-c)\alpha_1^\delta\right)-1}, \frac{k\left(a+(b-a)\alpha_1^\delta\right)}{\left(a+(b-a)\alpha_1^\delta\right)-1} \right], \\ \text{GIFMTTF}_j[\alpha_2] &= \left[\frac{k\left(c\left(1-\alpha_2^\delta\right)+d_1\alpha_2^\delta\right)}{\left(c\left(1-\alpha_2^\delta\right)+d_1\alpha_2^\delta\right)-1}, \frac{k\left(b\left(1-\alpha_2^\delta\right)+a_1\alpha_2^\delta\right)}{\left(b\left(1-\alpha_2^\delta\right)+a_1\alpha_2^\delta\right)-1} \right], \end{aligned}$$

where membership function and non-membership function of GIFMTTF are as follows

$$\mu_G(x)=\begin{cases}\left(\dfrac{d-\frac{x}{x-k}}{d-c}\right)^{\frac{1}{\delta}}, & \frac{kd}{d-1}\leq x\leq\frac{kc}{c-1}\\ 1, & \frac{kc}{c-1}\leq x\leq\frac{kb}{b-1}\\ \left(\dfrac{\frac{x}{x-k}-a}{b-a}\right)^{\frac{1}{\delta}}, & \frac{kb}{b-1}\leq x\leq\frac{ka}{a-1}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_G(x)=\begin{cases}\left(\dfrac{\frac{x}{x-k}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & \frac{kd_1}{d_1-1}\leq x\leq\frac{kc}{c-1}\\ 0, & \frac{kc}{c-1}\leq x\leq\frac{kb}{b-1}\\ \left(\dfrac{b-\frac{x}{x-k}}{b-a_1}\right)^{\frac{1}{\delta}}, & \frac{kb}{b-1}\leq x\leq\frac{ka_1}{a_1-1}\\ 1, & \text{o.w.}\end{cases}.$$

5 GIFR Function of Series and Parallel System

In series system, the reliability of whole system depends on each component and system fails even if an individual component fails. In contrary, for parallel situation system works even if a single component works. In this section, we focus on the GIFR of series system and parallel system, such that failure of any component does not depend on any other component.

5.1 Series system

If n-components are connected in series, then the α_i-cut $(i=1,2)$ of GIFR with generalized intuitionistic fuzzy distribution is given by

$$\begin{aligned} S_j(t)\,[\alpha_i,\delta] &= \{\,P(Y_1>t)|\,\theta\in\theta_j[\alpha_i,\delta]\}=\{\,S(t)^n|\,\theta\in\theta_j[\alpha_i,\delta]\}\\ &= \left[S_j^L\,(t)\,[\alpha_i],S_j^U\,(t)[\alpha_i]\right], \qquad (i,j)=(1,\mu)\,,(2,\nu)\end{aligned}$$

for all $0\leq\alpha_1\leq1,\ 0\leq\alpha_2\leq1,\ 0\leq\alpha_1^\delta+\alpha_2^\delta\leq1$, where $S_j^L(t)[\alpha_i]=\inf_{\theta\in\theta_j[\alpha_i,\delta]}S(t)^n,\ S_j^U\,(t)\,[\alpha_i]=\sup_{\theta\in\theta_j[\alpha_i,\delta]}S(t)^n,\quad(i,j)=(1,\mu)\,,(2,\nu)$. The α_i-cut $(i=1,2)$ of GIFR with generalized intuitionistic fuzzy Pareto distribution is given by

$$
\begin{aligned}
S_j(t)[\alpha_i,\delta] &= \Big\{ (\frac{k}{t})^{n\lambda} | \lambda \in \lambda_j[\alpha_i,\delta] \Big\}, \quad i=1,2 \\
S_\mu(t)[\alpha_1,\delta] &= \Big[(\frac{k}{t})^{n(d-(d-c)\alpha_1^\delta)}, (\frac{k}{t})^{n(a+(b-a)\alpha_1^\delta)} \Big], \\
S_\nu(t)[\alpha_2,\delta] &= \Big[(\frac{k}{t})^{n(c(1-\alpha_2^\delta)+d_1\alpha_2^\delta)}, (\frac{k}{t})^{n(b(1-\alpha_2^\delta)+a_1\alpha_2^\delta)} \Big].
\end{aligned}
$$

For t_0, the membership function and non-membership function of GIFR $\tilde{S}(t_0)$ are obtained as follows

$$
\mu_{S(t_0)}(x) = \begin{cases}
\Big(\frac{d - \frac{\ln x}{n \ln(\frac{k}{t_0})}}{d-c} \Big)^{\frac{1}{\delta}}, & (\frac{k}{t_0})^{nd} \le x \le (\frac{k}{t_0})^{nc} \\
1, & (\frac{k}{t_0})^{nc} \le x \le (\frac{k}{t_0})^{nb} \\
\Big(\frac{\frac{\ln x}{n \ln(\frac{k}{t_0})} - a}{b-a} \Big)^{\frac{1}{\delta}}, & (\frac{k}{t_0})^{nb} \le x \le (\frac{k}{t_0})^{na} \\
0, & \text{o.w.}
\end{cases},
$$

$$
\nu_{S(t_0)}(x) = \begin{cases}
\Big(\frac{\frac{\ln x}{n \ln(\frac{k}{t_0})} - c}{d_1-c} \Big)^{\frac{1}{\delta}}, & (\frac{k}{t_0})^{nd_1} \le x \le (\frac{k}{t_0})^{nc} \\
0, & \Big(\frac{k}{t_0}\Big)^{nc} \le x \le \Big(\frac{k}{t_0}\Big)^{nb} \\
\Big(\frac{b - \frac{\ln x}{n \ln(\frac{k}{t_0})}}{b-a_1} \Big)^{\frac{1}{\delta}}, & (\frac{k}{t_0})^{nb} \le x \le (\frac{k}{t_0})^{na_1} \\
1, & \text{o.w.}
\end{cases}.
$$

5.2 Parallel system

If n-components are connected in parallel, then the α_i-cut $(i=1,2)$ of GIFR with generalized intuitionistic fuzzy distribution is given by

$$
\begin{aligned}
S_j(t)[\alpha_i,\delta] &= \{ P(Y_n > t) | \, \theta \in \theta_j[\alpha_i,\delta] \} = \{ 1 - (1 - S(t))^n | \, \theta \in \theta_j[\alpha_i,\delta] \} \\
&= \Big[S_j^L(t)[\alpha_i], S_j^U(t)[\alpha_i] \Big], \qquad (i,j) = (1,\mu), (2,\nu)
\end{aligned}
$$

for all $0 \le \alpha_1 \le 1,\ 0 \le \alpha_2 \le 1,\ 0 \le \alpha_1^\delta + \alpha_2^\delta \le 1$, where

$$
S_j^L(t)[\alpha_i] = \inf_{\theta \in \theta_j[\alpha_i,\delta]} (1 - (1 - S(t))^n), \quad S_j^U(t)[\alpha_i] = \sup_{\theta \in \theta_j[\alpha_i,\delta]} (1 - (1 - S(t))^n).
$$

The α_i-cut $(i = 1, 2)$ of GIFR with generalized intuitionistic fuzzy Pareto distribution is given by

$$S_j(t)\,[\alpha_i,\delta] = \Big\{1-\big(1-(\frac{k}{t})^{\lambda}\big)^n\,|\lambda\in\lambda_j[\alpha_i,\delta]\Big\},\quad i=1,2,$$

$$S_\mu\,(t)\,[\alpha_1,\delta] = \left[1-\Big(1-(\frac{k}{t})^{(d-(d-c)\alpha_1^\delta)}\Big)^n, 1-\Big(1-(\frac{k}{t})^{(a+(b-a)\alpha_1^\delta)}\Big)^n\right],$$

$$S_\nu\,(t)\,[\alpha_2,\delta] = \left[1-\Big(1-(\frac{k}{t})^{(c(1-\alpha_2^\delta)+d_1\alpha_2^\delta)}\Big)^n, 1-\Big(1-(\frac{k}{t})^{(b(1-\alpha_2^\delta)+a_1\alpha_2^\delta)}\Big)^n\right].$$

For t_0, the membership function and non-membership function of $\tilde{S}\,(t_0)$ are obtained as follows

$$\mu_{S(t_0)}(x) = \begin{cases} \left(\dfrac{d-\frac{\ln(1-(1-x)^{\frac{1}{n}})}{\ln(\frac{k}{t_0})}}{d-c}\right)^{\frac{1}{\delta}}, & 1-(1-(\frac{k}{t_0})^d)^n \le x \le 1-(1-(\frac{k}{t_0})^c)^n \\ 1, & 1-(1-(\frac{k}{t_0})^c)^n \le x \le 1-(1-(\frac{k}{t_0})^b)^n \\ \left(\dfrac{\frac{\ln(1-(1-x)^{\frac{1}{n}})}{\ln(\frac{k}{t_0})}-a}{b-a}\right)^{\frac{1}{\delta}}, & 1-(1-(\frac{k}{t_0})^b)^n \le x \le 1-(1-(\frac{k}{t_0})^a)^n \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{S(t_0)}(x) = \begin{cases} \left(\dfrac{\frac{\ln(1-(1-x)^{\frac{1}{n}})}{\ln(\frac{k}{t_0})}-c}{d_1-c}\right)^{\frac{1}{\delta}}, & 1-(1-(\frac{k}{t_0})^{d_1})^n \le x \le 1-(1-(\frac{k}{t_0})^c)^n \\ 0, & 1-(1-(\frac{k}{t_0})^c)^n \le x \le 1-(1-(\frac{k}{t_0})^b)^n \\ \left(\dfrac{b-\frac{\ln(1-(1-x)^{\frac{1}{n}})}{\ln(\frac{k}{t_0})}}{b-a_1}\right)^{\frac{1}{\delta}}, & 1-(1-(\frac{k}{t_0})^b)^n \le x \le 1-(1-(\frac{k}{t_0})^{a_1})^n \\ 1, & \text{o.w.} \end{cases}.$$

6 Numerical Example

Let lifetime of electronic component is modeled by a Pareto distribution with generalized intuitionistic fuzzy parameter $\tilde{\lambda} = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 2)$ and

$k=1$. Then cut sets of GIFP of $X \leq 2$ is obtained as follows

$$\begin{aligned}
P_j\left(X \leq 2\right)\left[\alpha_i, 2\right] &= \{1-\left(\frac{1}{2}\right)^{\lambda} | \lambda \in \lambda_j\left[\alpha_i, 2\right]\},\ i=1,2, \\
P_\mu\left(X \leq 2\right)\left[\alpha_1, 2\right] &= [1-\left(\frac{1}{2}\right)^{0.2+0.1\alpha_1^2}, 1-\left(\frac{1}{2}\right)^{0.5-0.1\alpha_1^2}], \\
P_\nu\left(X \leq 2\right)\left[\alpha_2, 2\right] &= [1-\left(\frac{1}{2}\right)^{0.3-0.2\alpha_2^2},\ 1-\left(\frac{1}{2}\right)^{0.4+0.2\alpha_2^2}].
\end{aligned}$$

The membership function and non-membership function of $\tilde{P}\left(X \leq 2\right)$ are given by

$$\mu_P(x) = \begin{cases} \left(\frac{\ln(1-x)}{0.1\ln 0.5} - 2\right)^{\frac{1}{2}}, & 1-\left(\frac{1}{2}\right)^{0.2} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.3} \\ 1, & 1-\left(\frac{1}{2}\right)^{0.3} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.4} \\ \left(5 - \frac{\ln(1-x)}{0.1\ln 0.5}\right)^{\frac{1}{2}}, & 1-\left(\frac{1}{2}\right)^{0.4} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.5} \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_P(x) = \begin{cases} \left(1.5 - \frac{\ln(1-x)}{0.2\ln 0.5}\right)^{\frac{1}{2}}, & 1-\left(\frac{1}{2}\right)^{0.1} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.3} \\ 0, & 1-\left(\frac{1}{2}\right)^{0.3} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.4} \\ \left(\frac{\ln(1-x)}{0.2\ln 0.5} - 2\right)^{\frac{1}{2}}, & 1-\left(\frac{1}{2}\right)^{0.4} \leq x \leq 1-\left(\frac{1}{2}\right)^{0.6} \\ 1, & \text{o.w.} \end{cases}.$$

The membership and non-membership functions of the generalized intuitionistic fuzzy probability are represented in Figure 1 and for different values of α_1 and α_2 the membership and non-membership bands of GIFP and the bands of GIFP are obtained in Table 1, respectively.

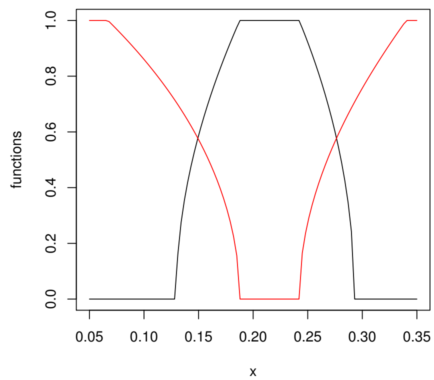

Figure 1: The black line (−) membership and red line (−) non-membership functions of GIFP.

Table 1: Different cut sets of GIFP function.

(α_1, α_2)	$P_\mu(x)[\alpha_1, 2]$	$P_\nu(x)[\alpha_2, 2]$	$P(x)[\alpha_1, \alpha_2, 2]$
(0,1)	[0.1294,0.2928]	[0.0669,0.3402]	[0.1294,0.2928]
(0.3,0.8)	[0.1384,0.2884]	[0.1123,0.3064]	[0.1384,0.2884]
(0.4,0.7)	[0.1390,0.2850]	[0.1305,0.2919]	[0.1390,0.2850]
(0.5,0.5)	[0.1444,0.2805]	[0.1591,0.2679]	[0.1591,0.2679]
(0.7,0.4)	[0.1585,0.2684]	[0.1695,0.2587]	[0.1695,0.2587]
(1,0)	[0.1877,0.2421]	[0.1877,0.2421]	[0.1877,0.2421]

With respect to Table 1, it can be concluded that with increasing α_1 and decreasing α_2, ambiguity decreases in membership and non-membership bands of GIFP and the bands of GIFP. The cut sets of GIFR are given by

$$\begin{aligned} S_\mu(t)\,[\alpha_1, 2] &= \Big[\big(\frac{1}{t}\big)^{0.5-0.1\alpha_1^2}, \big(\frac{1}{t}\big)^{0.2+0.1\alpha_1^2}\Big], \\ S_\nu(t)\,[\alpha_2, 2] &= \Big[\big(\frac{1}{t}\big)^{0.4+0.2\alpha_2^2}, \left(\frac{1}{t}\right)^{0.3-0.2\alpha_2^2}\Big]. \end{aligned}$$

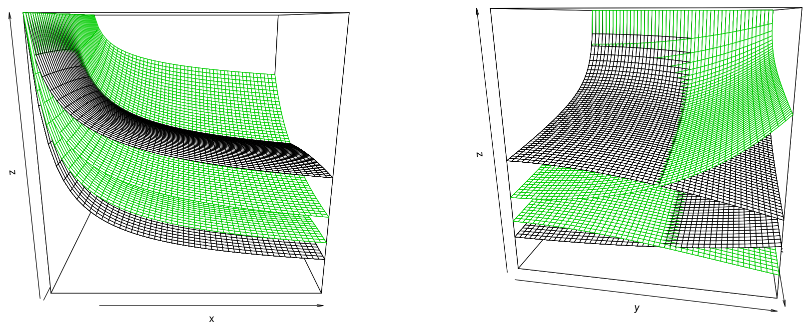

Figure 2: Surfaces of generalized intuitionistic fuzzy reliability.

Figure 2 shows surfaces of GIFR from different angles. The bands for $\alpha_1 = 0$ and $\alpha_2 = 1$ are given by

$$S_{\mu}(t)\,[0,2] = \Big[\big(\frac{1}{t}\big)^{0.5}, \big(\frac{1}{t}\big)^{0.2}\Big], \qquad S_{\nu}\,(t)\,[1,2] = \Big[\big(\frac{1}{t}\big)^{0.6}, \big(\frac{1}{t}\big)^{0.1}\Big].$$

The generalized intuitionistic fuzzy reliability bands for $\alpha_1 = 0$ and $\alpha_2 = 1$ are plotted in Figure 3. As can be seen, by increasing time t, the bandwidth of membership and non-membership functions are increased, which indicate increase in ambiguity.

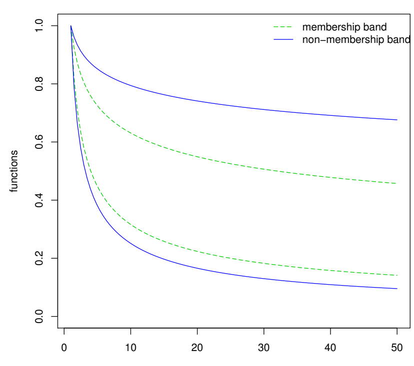

Figure 3: Generalized intuitionistic fuzzy reliability bands for $\alpha_1 = 0$ and $\alpha_2 = 1$.

If set $t = 2$, then cut sets of GIFR are computed as

$$S_\mu(2)\,[\alpha_1, 2] = \left[\Big(\frac{1}{2}\Big)^{0.5-0.1\alpha_1^2}, \Big(\frac{1}{2}\Big)^{0.2+0.1\alpha_1^2} \right],$$

$$S_\nu(2)\,[\alpha_2, 2] = \left[\Big(\frac{1}{2}\Big)^{0.4+0.2\alpha_2^2}, \Big(\frac{1}{2}\Big)^{0.3-0.2\alpha_2^2} \right].$$

The membership function and non-membership function of $\tilde{S}\,(2)$ are as follows

$$\mu_{S(2)}\,(x) = \begin{cases} \Big(5 + \dfrac{\ln x}{0.1 \ln 2}\Big)^{\frac{1}{2}}, & (\frac{1}{2})^{0.5} \le x \le (\frac{1}{2})^{0.4} \\ 1, & (\frac{1}{2})^{0.4} \le x \le (\frac{1}{2})^{0.3} \\ \Big(\dfrac{\ln x}{-0.1 \ln 2} - 2\Big)^{\frac{1}{2}}, & (\frac{1}{2})^{0.3} \le x \le (\frac{1}{2})^{0.2} \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{S(2)}(x) = \begin{cases} \Big(\dfrac{\ln x}{-0.2 \ln 2} - 2\Big)^{\frac{1}{2}}, & (\frac{1}{2})^{0.6} \le x \le (\frac{1}{2})^{0.4} \\ 0, & (\frac{1}{2})^{0.4} \le x \le (\frac{1}{2})^{0.3} \\ \Big(1.5 + \dfrac{\ln x}{0.2 \ln 2}\Big)^{\frac{1}{2}}, & (\frac{1}{2})^{0.3} \le x \le (\frac{1}{2})^{0.1} \\ 1, & \text{o.w.} \end{cases}.$$

In Figure 4, membership and non-membership functions of GIFR and in Table 2, the membership and non-membership bands of GIFR and bands of GIFR for different cuts α_1 and α_2 are prepared.

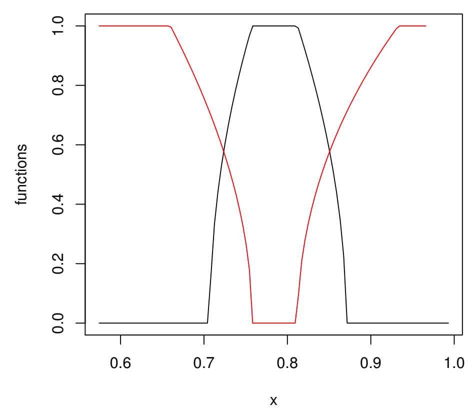

Figure 4: The black line (−) membership and red line (−) non-membership functions of GIFR.

Table 2: Different cut sets of GIFR function.

(α_1,α_2)	$S_\mu(t)[\alpha_1,2]$	$S_\nu(t)[\alpha_2,2]$	$S(t)[\alpha_1,\alpha_2,2]$
(0,1)	$[t^{-0.5},t^{-0.2}]$	$[t^{-0.6},t^{-0.1}]$	$[t^{-0.5},t^{-0.2}]$
(0.3,0.8)	$[t^{-0.491},t^{-0.209}]$	$[t^{-0.528},t^{-0.172}]$	$[t^{-0.491},t^{-0.209}]$
(0.4,0.7)	$[t^{-0.432},t^{-0.268}]$	$[t^{-0.498},t^{-0.201}]$	$[t^{-0.432},t^{-0.268}]$
(0.5,0.5)	$[t^{-0.475},t^{-0.225}]$	$[t^{-0.45},t^{-0.25}]$	$[t^{-0.45},t^{-0.25}]$
(0.7,0.4)	$[t^{-0.451},t^{-0.249}]$	$[t^{-0.432},t^{-0.268}]$	$[t^{-0.432},t^{-0.268}]$
(1,0)	$[t^{-0.4},t^{-0.3}]$	$[t^{-0.4},t^{-0.3}]$	$[t^{-0.4},t^{-0.3}]$

Based on Table 2, by increasing α_1 and decreasing α_2, the vagueness in membership and non-membership bands of GIFR and bands of GIFR are decreased.
The cut sets of GIFR are obtained in the following

$$
\begin{aligned}
S_\mu(t)[\alpha_1,\delta] &= \Big[\big(\frac{1}{t}\big)^{0.5-0.1\alpha_1^\delta},\big(\frac{1}{t}\big)^{0.2+0.1\alpha_1^\delta}\Big],\\
S_\nu(t)[\alpha_2,\delta] &= \Big[\big(\frac{1}{t}\big)^{0.4+0.2\alpha_2^\delta},\big(\frac{1}{t}\big)^{0.3-0.2\alpha_2^\delta}\Big],\\
S(t)[\alpha_1,\alpha_2,\delta] &= S_\mu(t)[\alpha_1,\delta]\cap S_\nu(t)[\alpha_2,\delta].
\end{aligned}
$$

The Reliability bands for the different values of δ and cut sets (α_1, α_2) are represented in Figure 5, which confirmed the result of Table 2, which by increasing α_1 and decreasing α_2, the uncertainty in reliability bands are reduced. Also, by increasing t, the uncertainty in GIFR is increased.

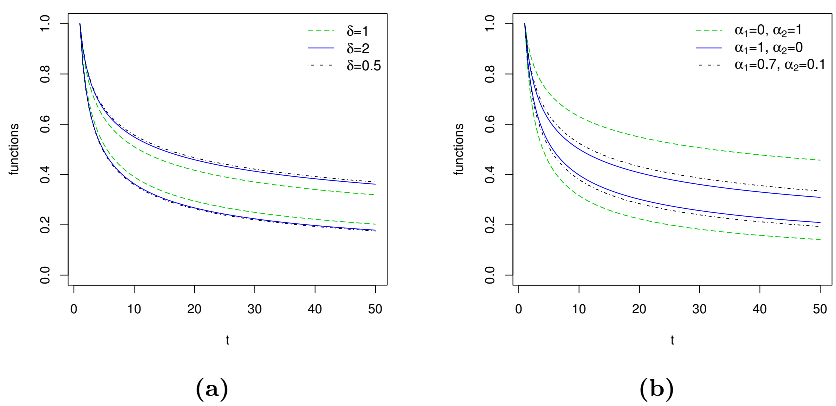

Figure 5: (a) Reliability bands of $S(t)[0.3, 0.2, \delta]$, (b) reliability bands of $S(t)[\alpha_1, \alpha_2, 1]$.

The α_i-cuts of GIFCR for $i = 1, 2$ are given by

$$S_\mu(t|\tau)[\alpha_1, 2] = \Big[\big(\frac{\tau}{t+\tau}\big)^{0.5-(0.1)\alpha_1^2}, \big(\frac{\tau}{t+\tau}\big)^{0.2+(0.1)\alpha_1^2}\Big],$$

$$S_\nu(t|\tau)[\alpha_2, 2] = \Big[\big(\frac{\tau}{t+\tau}\big)^{0.4\left(1-\alpha_2^2\right)+0.6\alpha_2^2}, \big(\frac{\tau}{t+\tau}\big)^{0.3\left(1-\alpha_2^2\right)+0.1\alpha_2^2}\Big].$$

Figure 6 shows surfaces of GIFCR from different angles. The GIFCR bands with $\tau = 1$ for $\alpha_1 = 0$ and $\alpha_2 = 1$ are expressed as

$$S_\mu(t|\tau)[0, 2] = \Big[\big(\frac{1}{t+1}\big)^{0.5}, \big(\frac{1}{t+1}\big)^{0.2}\Big],$$

$$S_\nu(t|\tau)[1, 2] = \Big[\big(\frac{1}{t+1}\big)^{0.6}, \big(\frac{1}{t+1}\big)^{0.1}\Big].$$

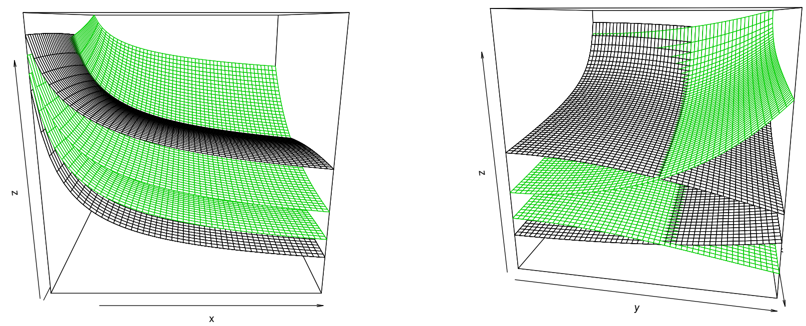

Figure 6: Surfaces of generalized intuitionistic fuzzy condition reliability with $\tau = 1$.

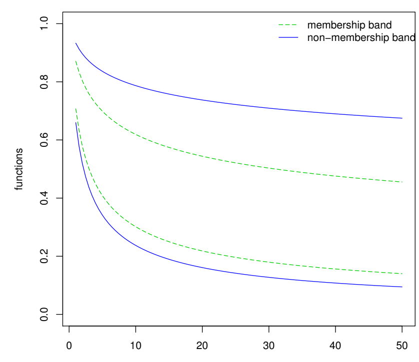

Figure 7: Generalized intuitionistic fuzzy condition reliability bands for $\alpha_1 = 0$ and $\alpha_2 = 1$.

The GIFCR bands for $\alpha_1 = 0$ and $\alpha_2 = 1$ are depicted in Figure 7, which indicate that increase in t leads to increase in the length of the band which means increase in uncertainty.

Let $t_0 = 2, \tau = 1$, the membership function and non-membership function of

$\tilde{S}\,(t_0|\,\tau)$ are obtained as follows

$$\mu_{S(\,t_0|\tau)}\,(x)=\begin{cases}\Big(\dfrac{\ln x}{0.1\ln 3}+5\Big)^{\frac{1}{2}}, & \big(\frac{1}{3}\big)^{0.5}\le x\le\big(\frac{1}{3}\big)^{0.4}\\ 1, & \big(\frac{1}{3}\big)^{0.4}\le x\le\big(\frac{1}{3}\big)^{0.3}\\ \Big(\dfrac{\ln x}{-0.1\ln 3}-2\Big)^{\frac{1}{2}}, & \big(\frac{1}{3}\big)^{0.3}\le x\le\big(\frac{1}{3}\big)^{0.2}\\ 0, & \text{o.w.}\end{cases},$$

$$\nu_{S(\,t_0|\tau)}(x)=\begin{cases}\Big(\dfrac{\ln x}{-0.2\ln 3}-2\Big)^{\frac{1}{2}}, & \big(\frac{1}{3}\big)^{0.6}\le x\le\big(\frac{1}{3}\big)^{0.4}\\ 0, & \big(\frac{1}{3}\big)^{0.4}\le x\le\big(\frac{1}{3}\big)^{0.3}\\ \Big(\dfrac{\ln x}{0.2\ln 3}+1.5\Big)^{\frac{1}{2}}, & \big(\frac{1}{3}\big)^{0.3}\le x\le\big(\frac{1}{3}\big)^{0.1}\\ 1, & \text{o.w.}\end{cases}.$$

The $(1,0)$-cut set of $\tilde{S}\,(t|\tau)$ are given by

$$S_\mu\,(t|\tau)\,[1,2]=\Big[\big(\frac{\tau}{\tau+t}\big)^{0.4},\big(\frac{\tau}{\tau+t}\big)^{0.3}\Big],$$

$$S_\nu\,(t|\tau)\,[0,2]=\Big[\big(\frac{\tau}{\tau+t}\big)^{0.4},\big(\frac{\tau}{\tau+t}\big)^{0.3}\Big],$$

$$S\,(t|\tau)\,[1,0,2]=S_\mu\,(t|\tau)\,[1,2]\cap S_\nu\,(t|\tau)\,[0,2]=\Big[\big(\frac{\tau}{\tau+t}\big)^{0.4},\big(\frac{\tau}{\tau+t}\big)^{0.3}\Big].$$

The membership function and non-membership function of GIFCR are represented in Figure 8 and the membership and non-membership bands of GIFCR and bands of GIFCR for different values of cus sets (α_1,α_2) are assembled in Table 3. Based on Table 3, the more accurate bands of membership and non-membership of GIFCR and bands of GIFCR are attained by the maximum value of α_1 and minimum of α_2.

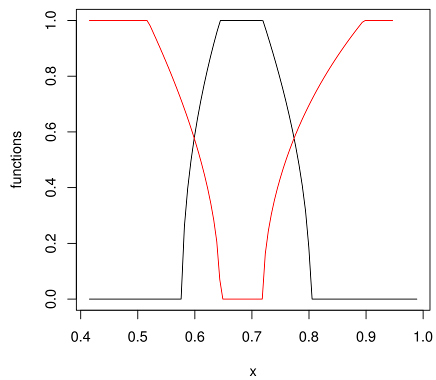

Figure 8: The black line (−) membership and red line (−) non-membership functions of GIFCR.

Table 3: Different cut sets of GIFCR function.

(α_1, α_2)	$S_\mu(t\|\tau)\,[\alpha_1, 2]$	$S_\nu(t\|\tau)\,[\alpha_2, 2]$	$S(t\mid\tau)[\alpha_1, \alpha_2, 2]$
(0,1)	$[(\frac{\tau}{t+\tau})^{0.5}, (\frac{\tau}{t+\tau})^{0.2}]$	$[(\frac{\tau}{t+\tau})^{0.6}, (\frac{\tau}{t+\tau})^{0.1}]$	$[(\frac{\tau}{t+\tau})^{0.5}, (\frac{\tau}{t+\tau})^{0.2}]$
(0.3,0.8)	$[(\frac{\tau}{t+\tau})^{0.491}, (\frac{\tau}{t+\tau})^{0.209}]$	$[(\frac{\tau}{t+\tau})^{0.528}, (\frac{\tau}{t+\tau})^{0.172}]$	$[(\frac{\tau}{t+\tau})^{0.491}, (\frac{\tau}{t+\tau})^{0.209}]$
(0.4,0.7)	$[(\frac{\tau}{t+\tau})^{0.484}, (\frac{\tau}{t+\tau})^{0.251}]$	$[(\frac{\tau}{t+\tau})^{0.498}, (\frac{\tau}{t+\tau})^{0.202}]$	$[(\frac{\tau}{t+\tau})^{0.484}, (\frac{\tau}{t+\tau})^{0.251}]$
(0.5,0.5)	$[(\frac{\tau}{t+\tau})^{0.475}, (\frac{\tau}{t+\tau})^{0.225}]$	$[(\frac{\tau}{t+\tau})^{0.450}, (\frac{\tau}{t+\tau})^{0.249}]$	$[(\frac{\tau}{t+\tau})^{0.450}, (\frac{\tau}{t+\tau})^{0.249}]$
(0.7,0.4)	$[(\frac{\tau}{t+\tau})^{0.451}, (\frac{\tau}{t+\tau})^{0.249}]$	$[(\frac{\tau}{t+\tau})^{0.432}, (\frac{\tau}{t+\tau})^{0.268}]$	$[(\frac{\tau}{t+\tau})^{0.432}, (\frac{\tau}{t+\tau})^{0.268}]$
(1,0)	$[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}]$	$[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}]$	$[(\frac{\tau}{t+\tau})^{0.4}, (\frac{\tau}{t+\tau})^{0.3}]$

The α_i-cuts of GIFH function for $i = 1, 2$ are given by

$$h_\mu\,(t)\,[\alpha_1, 2] = \Big[\frac{0.2 + 0.1\alpha_1^2}{t}, \frac{0.5 - 0.1\alpha_1^2}{t}\Big],$$

$$h_\nu\,(t)\,[\alpha_2, 2] = \Big[\frac{0.3 - 0.2\alpha_2^2}{t}, \frac{0.4 + 0.2\alpha_2^2}{t}\Big].$$

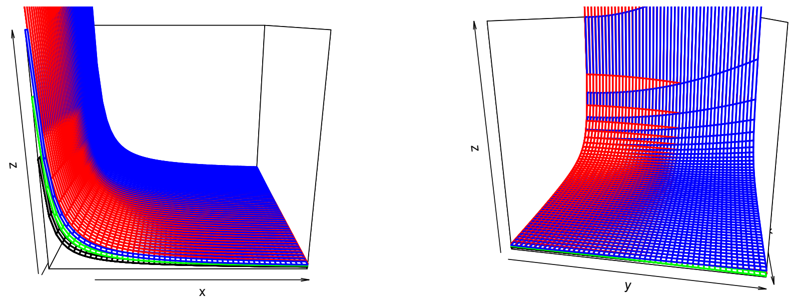

Figure 9: Surfaces of generalized intuitionistic fuzzy hazard.

Figure 9 shows the surfaces of GIFH function from different angles. The GIFH bands for $\alpha_1 = 0$ and $\alpha_2 = 1$ are computed as

$$h_\mu(t)[0,2] = \left[\frac{0.2}{t}, \frac{0.5}{t}\right], \qquad h_\nu(t)[1,2] = \left[\frac{0.1}{t}, \frac{0.6}{t}\right].$$

The GIFH bands of membership and non-membership functions for $\alpha_1 = 0$ and $\alpha_2 = 1$ are exhibited in Figure 10. Analogously, increasing t parameter cause the more accuracy in GIFH.

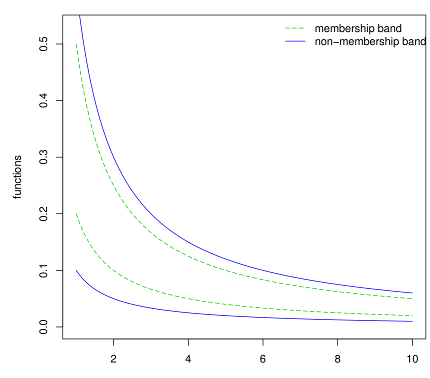

Figure 10: Generalized intuitionistic fuzzy hazard bands for $\alpha_1 = 0$ and $\alpha_2 = 1$.

The membership function and non-membership function of $\tilde{h}(2)$ are given as follows

$$\mu_{h(t_0)}(x) = \begin{cases} (20x-2)^{\frac{1}{2}}, & \frac{0.2}{2} \leq x \leq \frac{0.3}{2} \\ 1, & \frac{0.3}{2} \leq x \leq \frac{0.4}{2} \\ (5-20x)^{\frac{1}{2}}, & \frac{0.4}{2} \leq x \leq \frac{0.5}{2} \\ 0, & \text{o.w.} \end{cases},$$

$$\nu_{h(t_0)}(x) = \begin{cases} (1.5-10x)^{\frac{1}{2}}, & \frac{0.1}{2} \leq x \leq \frac{0.3}{2} \\ 0, & \frac{0.3}{2} \leq x \leq \frac{0.4}{2} \\ (10x-2)^{\frac{1}{2}}, & \frac{0.4}{2} \leq x \leq \frac{0.6}{2} \\ 1, & \text{o.w.} \end{cases}.$$

The membership and non-membership functions and bands of GIFH for different cut sets are displayed in Figure 11 and Table 4, respectively.

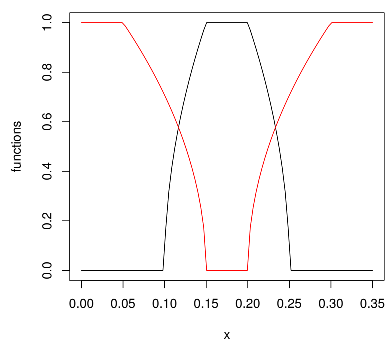

Figure 11: The black line (–) membership and red line (–) non-membership functions of GIFH.

Table 4: Different cut sets of GIFH function.

(α_1,α_2)	$h_\mu(t)[\alpha_1,2]$	$h_\nu(t)[\alpha_2,2]$	$h(t)[\alpha_1,\alpha_2,2]$
(0,1)	$[\frac{0.2}{t},\frac{0.5}{t}]$	$[\frac{0.1}{t},\frac{0.6}{t}]$	$[\frac{0.2}{t},\frac{0.5}{t}]$
(0.3,0.8)	$[\frac{0.209}{t},\frac{0.491}{t}]$	$[\frac{0.172}{t},\frac{0.528}{t}]$	$[\frac{0.209}{t},\frac{0.491}{t}]$
(0.4,0.7)	$[\frac{0.216}{t},\frac{0.484}{t}]$	$[\frac{0.2}{t},\frac{0.5}{t}]$	$[\frac{0.216}{t},\frac{0.484}{t}]$
(0.5,0.5)	$[\frac{0.225}{t},\frac{0.475}{t}]$	$[\frac{0.249}{t},\frac{0.450}{t}]$	$[\frac{0.249}{t},\frac{0.450}{t}]$
(0.7,0.4)	$[\frac{0.249}{t},\frac{0.451}{t}]$	$[\frac{0.268}{t},\frac{0.432}{t}]$	$[\frac{0.268}{t},\frac{0.432}{t}]$
(1,0)	$[\frac{0.3}{t},\frac{0.4}{t}]$	$[\frac{0.3}{t},\frac{0.4}{t}]$	$[\frac{0.3}{t},\frac{0.4}{t}]$

By Table 4 and the other tables we conclude that by increasing α_1 and decreasing α_2 ambiguity decreases in GIFR, GIFCR and GIFH bands. In addition, based on Figures 3, 7 and 10, as we expected, GIFR, GIFCR and GIFH are decreasing functions with respect to t.

Conclusion

In the present paper, the reliability characteristics is developed by the generalized intuitionistic fuzzy set and the GIFN$_B$ is extended to analyzing system reliability for different types of systems using Pareto lifetime distribution. The scale parameter of the Pareto lifetime distribution is considered as GIFN$_B$ and based on this assumption, various reliability functions are obtained. The reliability characteristics are represented through band, which attained their most precise bands for large value of cut set of the membership function and small value of cut set of the non-membership function. The numerical approach is extensively performed to examine the results. In this context, our study generalizes the various works of the literature.
Some of the topic of future research are as follows:

System reliability analysis using generalized intuitionistic fuzzy multi-parameters distributions.

Inference for the lifetime distributions based on generalized intuitionistic fuzzy data.

Compliance with Ethical Standards

Conflict of interest: The authors declare that they have no conflict of interest regarding the publication of this paper.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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