Journal of Mathematical Extension
Vol. 16, No. 9, (2022) (6)1-11
URL: https://doi.org/10.30495/JME.2022.1787
ISSN: 1735-8299
Original Research Paper

Compactness and Closed Maps in Topological Fuzzes

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Abstract. In this paper, we present the notion of weakly compact topological fuzzes and give some characterizations of them. In particular, a characterization of weakly compactness are given by the closedness of the projection fuzz maps. Also, we study some properties of proper fuzz maps as an important class of closed fuzz maps.

AMS Subject Classification: 54A40, 54D30.
Keywords and Phrases: Topological fuzz, Weakly compact, Proper fuzz map.

1 Introduction and Preliminaries

A completely distributive complete lattice is called a molecular lattice. In 1992, Wang introduced the concept of topological molecular lattices in terms of closed elements as a generalization of ordinary topological spaces, fuzzy topological spaces and L-fuzzy topological spaces in tools of molecules, remote neighborhoods and generalized order homomorphisms

Received: September 2020; Accepted: March 2021
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[9]. A fuzz is a pair $(F, ')$ consisting of a molecular lattice F and an order reversing involution $' : F \to F$, that is, $x \leq y$ if and only if $y' \leq x'$ and $x'' = x$ for all $x, y \in F$. A topological fuzz is a triple $(F, ', \tau)$ such that $(F, ')$ is a fuzz and $\tau \subseteq F$ is a topology, i.e., it is closed under finite meets, arbitrary joins and $0, 1 \in \tau$, where 0 and 1 are the smallest and the greatest elements of F, respectively. Every element of a topology τ is called open and every element of τ' is called closed, where $\tau' = \{a' \mid a \in \tau\}$.

For two molecular lattices F and G, and a mapping $f : F \to G$ which preserves arbitrary joins, suppose $\hat{f}$ denote the right adjoint of f, then $\hat{f} : G \to F$ is defined by $\hat{f}(y) = \bigvee\{x \in F \mid f(x) \leq y\}$ for every $y \in G$. A map $f : (F, ') \to (G, ')$ between fuzzes is called an order homomorphism or a **fuzz**-map in this paper if f preserves arbitrary joins and $\hat{f}$ preserves $'$ [10]. A **fuzz**-map $f : (F, ', \tau) \to (G, ', \mu)$ between topological fuzzes is said to be continuous if $b \in \mu$ implies $\hat{f}(b) \in \tau$.

The category of all fuzzes with **fuzz**-maps is denoted by **Fuzz**, and the category of all topological fuzzes with continuous **fuzz**-maps is denoted by **TopFuzz**. It is well known that these categories are both complete and cocomplete, and some categorical structures of them were introduced by many authors [5, 6, 8, 10, 11]. Topological fuzzes are more general frames for studding fuzzy topological spaces. Since the category **Top** of all topological spaces, as a full subcategory of **TopFuzz**, is reflective and coreflective [8], this points out the essential difference between the general topological spaces and topological fuzzes on the categorical level. For some categorical notions, readers are suggested to refer to [1].

The notion of compact topological fuzzes and some properties of them was introduced by B. Hutton [5]. In this paper, we present the notion of weakly compact topological fuzzes and give some characterizations of them.

In the following, we recall some definitions and properties of molecular lattices. We first recall the definition of extra order introduced by Li [7]. Extra-orders are useful tools to construct molecular lattices and function spaces in topological molecular lattices.

Definition 1.1. Let P be a poset and $\prec$ be a binary relation on P.

a) $\prec$ is called an extra order, if it satisfies the following conditions:

1. $x \prec y \Rightarrow x \leq y$,

2. $u \leq x \prec y \leq v \Rightarrow u \prec v$.

b) $\prec$ satisfies the interpolation property (short by INT), if $x \prec y$ implies that there exists $z \in P$ such that $x \prec z \prec y$.

Remark 1.2. [7] For a complete lattice L, an extra order $\lhd$ is defined by $a \lhd b$ if for every subset $S \subseteq L$, $b \leq \vee S$ implies $a \leq s$ for some $s \in S$. If L is a molecular lattice, then $\lhd$ satisfies the condition (INT).

Definition 1.3. [2] An extra order $\ll$ on a topological fuzz $(F, ', \tau)$ is defined by $a \ll b$ if for every subset $A \subseteq \tau$, $b \leq \vee A$ implies that there exists a finite subset D of A such that $a \leq \vee D$.

Definition 1.4. [9] An element a of a lattice L is called coprime, if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$, for every $b, c \in L$.

We denote by $CP(L)$ the set of all nonzero coprime elements L. Nonzero coprime elements are also called molecules.

Theorem 1.5. *[9] A complete lattice L is a molecular lattice if and only if $b = \bigvee \lhd(b) = \bigvee(\lhd(b) \cap CP(L))$ for every $b \in L$, where $\lhd(b) = \{a \in L \mid a \lhd b\}$.*

Remark 1.6. [11] The binary product of two topological fuzzes $(F, ', \tau)$ and $(G, ', \mu)$ is $(F \otimes G, ', \lambda)$, where $F \otimes G := \{D \subseteq F \times G \mid D = \bigcup_{(x,y) \in D} \lhd(x) \times \lhd(y)\}$, $D' = \bigcap_{(x,y) \in D}\{(\lhd(x') \times \lhd(1)) \cup (\lhd(1) \times \lhd(y'))\}$ for each $D \in F \otimes G$ and λ is generated by subbase $\{\hat{\pi}_1(x) \mid x \in \tau\} \cup \{\hat{\pi}_2(y) \mid y \in \mu\}$, such that the projection **fuzz**-maps π_1 and π_2 are defined by $\pi_1(D) = \bigvee\{x \in F \mid \exists y \in G, (x, y) \in D\}$ and $\pi_2(D) = \bigvee\{y \in G \mid \exists x \in F, (x, y) \in D\}$. The coprime elements of $F \otimes G$ are just $\{\lhd(x) \times \lhd(y) \mid x \in CP(F), y \in CP(G)\}$.

2 Compactness and Closed Maps

In this section, we present the notion of weakly compact topological fuzzes and give some characterizations of them. In particular, a characterization of weakly compactness are given by the closedness of the projection fuzz maps.

Definition 2.1. Let $(F, ', \tau)$ be a topological fuzz. A subset $A \subseteq \tau$ is called directed if for any finite set D of A there exists $a \in A$ such that $\vee D \leq a$.

Definition 2.2. [5] An element a of a topological fuzz F is called compact if $a \ll a$, and L is called compact if every its closed element is compact.

Definition 2.3. A topological fuzz F is called weakly compact, if the greatest element 1 is compact.

It is clear that each compact topological fuzz is weakly compact.

Definition 2.4. A **fuzz**-map $f : (F, ', \tau) \to (G, ', \mu)$ between topological fuzzes is said to be closed, if $f(a)$ is closed for every closed element $a \in F$.

Lemma 2.5. *Let $f : (F, ', \tau) \to (G, ', \mu)$ be a* **fuzz**-*map. The following statements are equivalent:*

1. *f is closed*
2. *$(f(a^{'}))^{'}$ is open for every $a \in \tau$.*
3. *The element $\bigvee\{m \in CP(G) | \hat{f}(m) \leq a\}$ is open for every $a \in \tau$.*

Proof. The equivalent of the parts 1 and 2 is clear. Let $m \in CP(G)$. Then we have $m \leq (f(a^{'}))^{'} \Leftrightarrow f(a^{'}) \leq m^{'} \Leftrightarrow a^{'} \leq \hat{f}(f(a^{'})) \leq (\hat{f}(m))^{'} \Leftrightarrow \hat{f}(m) \leq a$. Thus $(f(a^{'}))^{'} = \bigvee\{m \in CP(G) | \hat{f}(m) \leq a\}$. $\Box$

Remark 2.6. If L is a molecular lattice, then we have $CP(L) \cap \lhd(a) \subseteq CP(L) \cap \downarrow a$ for every $a \in L$, where $\downarrow a = \{b \in L | b \leq a\}$.

Definition 2.7. Let L be a molecular lattice. Then we say that L is admissible, if $CP(L) \cap \lhd(a) = CP(L) \cap \downarrow a$, for every $a \in L$.

Example 2.8. Let $L = \rho(X)$ be the lattice of the power set of an arbitrary set X. Then L is a fuzz, $CP(L) = \{\{x\} | x \in X\}$ and $\lhd(A) = \{\{a\} | a \in A\}$ for each $A \in L$. It is clear that L is admissible.

Example 2.9. If L is a finite molecular lattice (or a finite fuzz), then it is easy to show that L is admissible.

Example 2.10. Consider the lattice $L = [0, 1]$ with usual order. Define the involution $'$ on L by $a' = 1 - a$ for every $a \in L$. Then L is a fuzz and $CP(L) = (0, 1]$, $\lhd(a) = [0, a)$ for every nonzero element a. Thus $CP(L) \cap \lhd(a) = (0, a)$ and $CP(L) \cap \downarrow a = (0, a]$ for every nonzero element a, which shows that L is not admissible.

Theorem 2.11. *Let G be an admissible topological fuzz. Then the following statements are equivalent:*

1. *The projection* **fuzz***-map $\pi_1 : F \bigotimes G \to F$ is closed for every topological fuzz F.*
2. *The element $\overline{W} := \bigvee\{m \in CP(F) | \hat{\pi}_1(m) \leq W\}$ is open for every topological fuzz F and every open element W of $F \bigotimes G$.*
3. *G is weakly compact.*

Proof. $1 \Leftrightarrow 2$: This follows directly from Lemma 2.5.

$2 \Rightarrow 3$: Let $C = \{a_i | i \in I\}$ be a directed open cover of 1_G, i.e., $1_G = \bigvee_{i \in I} a_i$. We first construct a topological fuzz $F := \rho(\tau_G)$ the power set of τ_G, from G and C such that O is an open set in F if and only if the following conditions hold:

(1) If $a \in O$ and $a \leq b \in \tau_G$, then $b \in O$.

(2) If $\bigvee_{i \in I} a_i \in O$, then there exists $i \in I$ such that $a_i \in O$.

Using the fact that C is directed, such open sets are readily seen to form a topology and if $a \leqslant c$ for some $c \in C$, then $\uparrow a = \{b \in \tau_G | a \leq b\}$ is clearly open in F. Now, let

$$W = \bigcup\{\lhd(\{a\}) \times \lhd(p) | p \in CP(G), a \in \tau_F, p \leq a\}.$$

We show that W is an open set in $F \bigotimes G$. Let $a \in \tau_G$ and $p \in CP(G)$, such that $p \leq a$. We consider two cases. (1) : $p \leq a_{i_0}$ for some $i_0 \in I$, then $\uparrow (a \wedge a_{i_0})$ is an open set in F and $\lhd(\{a\}) \times \lhd(p) \leq \hat{\pi}_1(\uparrow (a \wedge a_{i_0})) \wedge \hat{\pi}_2(a \wedge a_{i_0}) \leq W$. (2) : $p \not\leq a_i$ for all $i \in I$. Since $p \in \lhd(a)$, it follows that $\bigvee_{i \in I} a_i \notin \uparrow a$. Thus $\uparrow a$ is an open set in F and $\lhd(\{a\}) \times \lhd(p) \leq \hat{\pi}_1(\uparrow a) \wedge \hat{\pi}_2(a) \leq W$. Finally, by the hypothesis, the set $\overline{W} = \bigcup\{\{a\} \in CP(F) | \lhd(\{a\}) \times \lhd(1_G) \leq W\}$ is open and clearly

$\{a\} \in \overline{W}$ if and only if $a \geq 1_G$. Hence $\{\bigvee_{i\in I} a_i\} \in \overline{W}$ and so there is $i_0 \in I$ such that $\{a_{i_0}\} \in \overline{W}$. Thus $1_G \leq a_i$, which shows that G is weakly compact.

$3 \Rightarrow 2$: Let W be an open element in $F \bigotimes G$ and $m \in CP(F)$ such that $\lhd(m) \times \lhd(1_G) \leq W$. Then there exist open elements $a_i \in \tau_F$ and $b_i \in \tau_G$ for an index set I, such that $W = \bigcup_{i\in I} \lhd(a_i) \times \lhd(b_i)$. For every $p \in CP(G) \cap \lhd(1_G)$, $(m, p) \in W$, so there exists $i_p \in I$ such that $m \lhd a_{i_p}$ and $p \lhd b_{i_p}$. Hence $1_G = \bigvee_{p\in\lhd(1_G)} b_{i_p}$, and by hypothesis $1_G = \bigvee_{p\in D} b_{i_p}$ for a finite set D of $\lhd(1_G)$. Now, if $a = \bigwedge_{p\in D} a_{i_p}$, then $a \in \tau_F$ and $\lhd(a) \times \lhd(1_G) = \lhd(\bigwedge_{p\in D} a_{i_p}) \times \lhd(\bigvee_{p\in D} b_{i_p}) \leq \bigcup_{p\in D} \lhd(a_{i_p}) \times \lhd(b_{i_p}) \leq W$. Thus $m \leq a \leq \overline{W}$, which shows that $\overline{W}$ is open. □

Definition 2.12. Let G be a subfuzz of topological fuzz F, then the collection $\tau_G = \{a \wedge 1_G | a \in \tau_F\}$ is a topology on G which is called subfuzz topology.

Theorem 2.13. *The following conditions are equivalent:*

1. *G is a weakly compact subfuzz of a topological fuzz H.*
2. *For every topological fuzz F and every open set W of $F \bigotimes G$, the element $\overline{W} = \bigvee\{a \in CP(F) | \lhd (a) \times \lhd(1_G) \leq W\}$ is open.*

Proof. The proof is similar to Theorem 2.11. □

3 Relative Compactness

In this section, we give the notion of relative compactness, which is a generalization of compactness and present some characterizations of it.

Definition 3.1. Let (G, τ) be a topological fuzz and $a, b \in G$. We say that a is relatively compact in b, if $a \ll b$.

Definition 3.2. Let (G, τ) be a topological fuzz and $a, b \in G$. We define the relation $\Subset$ on G by $a \Subset b$ if and only if $a \leq b^\circ$, where $b^\circ = \bigvee\{c \in \tau | c \leq b\}$ is the interior of b.

The following lemma is an immediate consequence of the definition of $\ll$.

Lemma 3.3. *Let (G, τ) be a topological fuzz and $a, b, c, d \in G$. Then the following statements hold.*

1. *If $a \ll b$, then $a \leq b$.*
2. *If $a \leq b \ll c \leq d$, then $a \ll d$.*
3. *$0 \ll a$ and if $a \ll c$ and $b \ll c$, then $a \vee b \ll c$.*

Theorem 3.4. *Let G be an admissible topological fuzz and $a, b \in G$. Then the following statements are equivalent:*

1. *a is relatively compact in b.*
2. *For each admissible topological fuzz F and every open element W of $F \bigotimes G$,*
   $\bigvee\{c \in CP(F) | \lhd(c) \times \lhd(b) \leq W)\} \Subset \bigvee\{c \in CP(F) | \lhd(c) \times \lhd(a) \leq W\}$.
3. *For every admissible topological fuzz F, every $c \in CP(F)$ and every open element W of $F \bigotimes G$, $\lhd(c) \times \lhd(b) \leq W \Rightarrow \lhd(v) \times \lhd(a) \leq W$ for some open element v of F such that $c \leq v$.*
4. *For every admissible topological fuzz F and all $A, B \in F \bigotimes G$, $A \Subset B \Rightarrow$*
   $$\{c \in CP(F) | \lhd(c) \times \lhd(b) \leq A\} \Subset \{c \in CP(F) | \lhd(c) \times \lhd(a) \leq B\}.$$

Proof. $2 \Leftrightarrow 3$: Follows directly from definition of interior.

$2 \Leftrightarrow 4$: Consider $W = B^\circ$ in one direction and $A = B = W$ in the other direction.

$1 \Rightarrow 3$: Let W be an open element of $F \bigotimes G$ and $c \in CP(F)$ such that $\lhd(c) \times \lhd(b) \leq W$. Then there exist open elements $a_i \in \tau_F$ and $b_i \in \tau_G$ for an index set I such that $W = \bigcup_{i \in I} \lhd(a_i) \times \lhd(b_i)$. For every $p \in CP(G) \cap \lhd(b)$, $(c, p) \in W$. So there exists $i_p \in I$ such that $c \lhd a_{i_p}$ and $p \lhd b_{i_p}$. Hence $b = \bigvee_{p \lhd b} b_{i_p}$ and by the hypothesis $a = \bigvee_{p \in D} b_{i_p}$ for a finite set D of $\lhd(b)$. Now, if $v = \bigwedge_{p \in D} a_{i_p}$, then $v \in \tau_F$, $c \leq v$ and $\lhd(v) \times \lhd(a) = \lhd(\bigwedge_{p \in D} a_{i_p}) \times \lhd(\bigvee_{p \in D} b_{i_p}) \leq \bigcup_{p \in D} \lhd(a_{i_p}) \times \lhd(b_{i_p}) \leq W$.

$3 \Rightarrow 1$: Let $K = \{b_i | i \in I\}$ be a directed open cover of b, i.e, $b = \bigvee_{i \in I} b_i$. We first construct a topological fuzz $F = \rho(\tau_G)$ from G and

K and also open set $W = \bigcup\{\lhd(m) \times \lhd(p) | p \in CP(G), m \in \tau_G, p \leq m\}$ as in the proof of Theorem 2.11. It is clear that $\lhd(\bigvee_{i\in I} b_i) \times \lhd(b) \leq W$. Finally, by the hypothesis, there exists open element v of F such that $\bigvee_{i\in I} a_i \leq v$. Thus $a_{i_0} \leq v$ for some $i_0 \in I$ and also $\lhd(a_{i_0}) \times \lhd(a) \leq \lhd(v) \times \lhd(a) \leq W$. Hence $a \leq a_{i_0}$, which shows that $a \ll b$. □

Corollary 3.5. *Let G be an admissible topological fuzz and $a \in G$. Then the following statements are equivalent:*

1. *a is compact, i.e, $a \ll a$.*
2. *For every admissible topological fuzz F and every open element W of $F \bigotimes G$, the element $\overline{W} = \bigvee\{c \in CP(F) \mid \lhd(c) \times \lhd(a) \leq W\}$ is open.*

Proof. By Theorem 3.4, we have $a \ll a$ if and only if $\bigvee\{c \in CP(F) \mid \lhd(c) \times \lhd(a) \leq W\} \subseteq (\bigvee\{c \in CP(F) \mid \lhd(c) \times \lhd(a) \leq W\})^{\circ}$. Thus $a \ll a$ if and only if $\overline{W}$ is open. □

4 Proper Fuzz-Maps

Recall that a continuous map $f : X \to Y$ in **Top** is called proper if the product map $id_Z \times f : Z \times X \to Z \times Y$ is closed, for every space Z, where $id_Z : Z \to Z$ is the identity map. This is equivalent to f being universally closed, in the sense that every pullback of f is a closed map. There are many characterizations of proper maps in **Top** [3, 4]. A useful characterization of them is in terms of compactness, that is, a continuous map $f : X \to Y$ is proper if and only if f is a closed map and the set $f^{-1}\{y\}$ is compact for every point $y \in Y$. In this section, we present some characterizations of proper maps in the full subcategory of admissible topological fuzzes of **TopFuzz**.

Definition 4.1. A **fuzz**-map $f : G \to H$ is called admissible proper, if the product map $id \otimes f : F \otimes G \to F \otimes H$ is closed for every admissible topological fuzz F, where $id : F \to F$ is the identity **fuzz**-map.

Theorem 4.2. *Let $f : G \to H$ be a continuous **fuzz**-map between admissible topological fuzzes. Then the following statements are equivalent:*

1. *f is admissible proper.*
2. *For every admissible topological fuzz F and every open element W of $F \bigotimes G$, the following set is open:* $\overline{W} = \bigvee\{\lhd(c) \times \lhd(p) \mid c \in CP(F), p \in CP(H), \lhd(c) \times \lhd(\hat{f}(p)) \leq W\}$.
3. *f is closed and $\hat{f}(a)$ is compact for every compact element a of H.*
4. *f is closed and $\hat{f}(p)$ is compact for every coprime element p of H.*

Proof. (1) $\Leftrightarrow$ (2): Since $\widehat{id \otimes f}(\lhd(c) \times \lhd(p)) = \hat{id} \otimes \hat{f}(\lhd(c) \times \lhd(p)) = \lhd(c) \times \lhd(\hat{f}(p))$, by Lemma 2.5, the result holds.

(1), (2) $\Rightarrow$ (3): Consider $F = \{0, 1\}$, then F is an admissible topological fuzz and $F \bigotimes L \cong L$ for every topological fuzz L. Thus every proper **fuzz**-map is closed. Now, let F be an arbitrary admissible topological fuzz and W be an open element of $F \bigotimes G$. By the hypothesis, the element $\overline{W} = \bigvee\{\lhd(c) \times \lhd(p) \mid c \in CP(F), p \in CP(H), \lhd(c) \times \lhd(\hat{f}(p)) \leq W\}$ is open. Hence by Corollary 3.5, the element $u = \bigvee\{m \in CP(F) | \lhd(m) \times \lhd(a) \leq \overline{W}\}$ is open, where a is a compact element of H. But $m \leq u \Leftrightarrow \lhd(m) \times \lhd(a) \leq \overline{W} \Leftrightarrow \lhd(m) \times \lhd(\hat{f}(a)) \leq W$. Since F and W are arbitrary, by Corollary 3.5, it follows that $\hat{f}(a)$ is compact.

(3) $\Rightarrow$ (4): Since H is an admissible topological fuzz, it follows that every coprime element of H is compact.

(4) $\Rightarrow$ (2): Let F be an arbitrary admissible topological fuzz and W be an open element of $F \bigotimes G$, $c \in CP(F)$ and $p \in CP(H)$ such that $\lhd(c) \times \lhd(\hat{f}(p)) \leq W$. Then there exist open elements $a_i \in \tau_F$ and $b_i \in \tau_G$ for an index set I such that $W = \bigcup_{i \in I} \lhd(a_i) \times \lhd(b_i)$. For every $x \in CP(G) \cap \lhd(\hat{f}(p))$, $(c, x) \in W$, so there exists $i_x \in I$ such that $c \lhd a_{i_x}$ and $x \lhd b_{i_x}$. Hence $\hat{f}(p) = \bigvee_{x \lhd \hat{f}(p)} b_{i_x}$ and by hypothesis, $\hat{f}(p) = \bigvee_{x \in D} b_{i_x}$ for a finite set D of $\lhd(\hat{f}(p))$. Now, if $a = \bigwedge_{x \in D} a_{i_x}$, then $a \in \tau_F$ and $c \leq a$. On the other hand, since f is closed, by Lemma 2.5, the element $k = \bigvee\{m \in CP(H) | \hat{f}(m) \leq u\}$ is open for every open element u of G. Let $u = \bigvee_{x \in D} b_{i_x}$. Then $p \leq k$ and also there exists an open element v of H such that $p \leq v \leq k$. Thus, $\lhd(a) \times \lhd(\hat{f}(v)) = \lhd(\bigwedge_{x \in D} a_{i_x}) \times \lhd(\bigvee_{x \in D} b_{i_x}) \leq \bigvee_{x \in D} \lhd(a_{i_x}) \times \lhd(b_{i_x}) \leq W$. Hence $\lhd(c) \times \lhd(p) \leq \lhd(a) \times \lhd(v) \leq \overline{W}$, which shows that $\overline{W}$ is open. $\square$

Acknowledgments
We are very grateful to the referee(s) for the careful reading of the paper and for the useful comments.

References

[1] J. Adamek, H. Herrlich and G. E. Strecker, *Abstract and Concrete Categories*, John Wiley and Sons Inc., New York, 1990.

[2] M. Akbarpour and GH. Mirhosseinkhani, Exponentiable objects in some categories of topological molecular lattices, *Hadronic Journal*, 40 (2017), 327–344.

[3] N. Bourbaki, *General Topology*, Volume 1. Addison-Wesley, London, 1988.

[4] M.M. Clementino and D. Hofmann, *On some special classes of continuous maps*, Open problems in topology II, Chapter 4, (2007), 367–376.

[5] B. Hutton, Products of fuzzy topological spaces, *Topology Appl.,* 11 (1980), 59–67.

[6] B. Hutton and I. Reilly, Separation axioms in fuzzy topological spaces, *Fuzzy Sets and Systems*, 3 (1980), 93–104.

[7] Y.M. Li, Exponentiable objects in the category of topological molecular lattices, *Fuzzy Sets and Systems*, 104 (1999), 407–414.

[8] Y.M. Li and Z.H. Li, Top is a reflective and coreflective subcategory of fuzzy topological spaces, *Fuzzy Sets and Systems*, 116 (2000), 429–432.

[9] G.J. Wang, Theory of topological molecular lattices, *Fuzzy Sets and Systems*, 47 (1992), 351–376.

[10] G.J. Wang, Order-homomorphisms on fuzzes, *Fuzzy Sets and Systems*, 12 (1984), 281–288.

[11] Z. Yang, The cartesian closedness of the category Fuzz and function spaces on topological fuzzes, *Fuzzy Sets and Systems*, 61 (1994), 341–351.

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