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Some New Fixed Point Results for Generalized Cyclic α-$f\pi\varpi$-Contractions in Metric-Like Spaces

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Abstract. In this article, we introduce generalized cyclic contractions in the context of metric-like spaces and prove some new fixed point results concerning these contractions. Our results, extend and unify several results in existing literature. We present suitable examples to make our findings worth mentioning.

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1 Introduction and Preliminaries

A vigorous research activity focuses on the research on fixed points for given mappings with certain contractive conditions in various abstract spaces. Banach contraction mapping has attracted the attention of many authors to generalize, extend and improve the metric fixed point theory. In this direction, the authors considered the extension of metric fixed point theory to different abstract spaces such as symmetric spaces, quasi-metric spaces, fuzzy metric spaces, partial metric spaces, probabilistic metric spaces, spaces with graph, (ordered) G-metric spaces (see, e.g. [1]-[30]). Also, one can observe the notions of cyclic contractions and cyclic contractive type mappings in several works. This line of research was initiated by Kirk, et al. [12]. For more results in this field, see ([7], [19]-[21] and [23]).

The notion of $\alpha-$admissible mappings has been introduced and applied by Samet et al. [24].

In this paper, we introduce the concept of cyclic-$\alpha-f\pi\varpi-$contractive mappings and their graphic version. In addition, we prove some fixed point results regarding this new notion.

In the following, we give a few auxiliary facts which will be used in our further considerations.

Definition 1.1. Let Υ be a self-mapping on a nonempty set Λ and $\alpha : \Lambda\times\Lambda \to [0,+\infty)$ be a function. Υ is called an $\alpha-$admissible mapping if $\alpha(\Upsilon\iota, \Upsilon\kappa) \geq 1$, for all $\iota, \kappa \in \Lambda$ with $\alpha(\iota, \kappa) \geq 1$.

The family Π of increasing functions $\pi : [0,+\infty) \to [0,+\infty)$ such that $\sum_{n=1}^{+\infty} \pi^n(t) < +\infty$ for each $t > 0$, where π^n is the n-th iterate of π, has been considered in [24].

Theorem 1.2. *[24] Let (Λ, d) be a complete metric space and Υ be an $\alpha-$admissible mapping. Assume that $\alpha(\iota,\kappa) d(\Upsilon\iota, \Upsilon\kappa) \leq \pi(d(\iota,\kappa))$, where $\pi \in \Pi$. Also, let:*

(i) $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$, for some $\iota_0 \in \Lambda$,

(ii) either Υ is continuous, or, for any sequence $\{\iota_n\}$ in Λ with $\alpha(\iota_n, \iota_{n+1}) \geq 1$ for all $n \in \mathbb{N}\cup\{0\}$ such that $\iota_n \to \iota$ as $n \to +\infty$, we have $\alpha(\iota_n, \iota) \geq 1$ for all $n \in \mathbb{N}\cup\{0\}$.

Then Υ possesses a fixed point.

Amini-Harandi [8] introduced any interesting generalization of the partial metric spaces, known as metric-like spaces. For more details on these spaces, we refer the reader to ([5, 6, 14, 23, 25, 26, 27]).

Definition 1.3. [8] A mapping $\hat{d} : \Lambda \times \Lambda \to [0, +\infty)$, where Λ is a nonempty set, is called a metric-like on Λ if for any $\iota, \kappa, z \in \Lambda$ one has:

($\hat{d}$ 1) $\hat{d}(\iota, \kappa) = 0$ implies $\iota = \kappa$;
($\hat{d}$ 2) $\hat{d}(\iota, \kappa) = \hat{d}(\kappa, \iota)$;
($\hat{d}$ 3) $\hat{d}(\iota, z) \leq \hat{d}(\iota, \kappa) + \hat{d}(\kappa, z)$.

The pair $\left(\Lambda, \hat{d}\right)$ is said to be a metric-like space. A metric-like $\hat{d}$ on Λ satisfies all conditions of a metric except that $\hat{d}(\iota, \iota)$ may be positive for some $\iota \in \Lambda$.

Definition 1.4. [8] Let $\left(\Lambda, \hat{d}\right)$ be a metric-like space, and let $\{\iota_n\}$ be a sequence of points in Λ. If $\lim_{n\to+\infty} \hat{d}(\iota, \iota_n) = \hat{d}(\iota, \iota)$, then ι is said to be the limit of the sequence $\{\iota_n\}$. In this case, we say that the sequence $\{\iota_n\}$ is convergent to ι which is denoted by $\iota_n \to \iota$ as $n \to +\infty$.

Definition 1.5. [8] Let $\left(\Lambda, \hat{d}\right)$ be a metric-like space, and let $\{\iota_n\}$ be a sequence of points in Λ. A sequence $\{\iota_n\}$ is called $\hat{d}-$Cauchy if and only if
$\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m)$ exists and is finite.

Definition 1.6. [8] A metric-like space $\left(\Lambda, \hat{d}\right)$ is said to be $\hat{d}-$complete if and only if for each $\hat{d}-$Cauchy sequence $\{\iota_n\}$ in Λ, there exists $\iota \in \Lambda$ such that

$$\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) = \hat{d}(\iota, \iota) = \lim_{n\to+\infty} \hat{d}(\iota_n, \iota).$$

Definition 1.7. ([5], [25]) A sequence $\{\iota_n\}$ in metric-like space $\left(\Lambda, \hat{d}\right)$ is called a 0-$\hat{d}$-Cauchy if and only if $\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) = 0$. A metric-like space $\left(\Lambda, \hat{d}\right)$ is said to be 0-$\hat{d}$-complete if and only if every 0-$\hat{d}$-Cauchy sequence $\{\iota_n\}$ in Λ converges to any element $\iota \in \Lambda$ such that

$$\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) = \hat{d}(\iota, \iota) = \lim_{n\to+\infty} \hat{d}(\iota_n, \iota) = 0.$$

Every partial metric space is a metric-like space, but the opposite is not true in general (see e.g., [8, 10, 25]).

Lemma 1.8. *[25] Let* $\left(\Lambda, \hat{d}\right)$ *be a metric-like space and* $\{\iota_n\}$ *be a sequence in* Λ *such that* $\iota_n \to \iota$ *as* $n \to +\infty$ *and* $\hat{d}(\iota, \iota) = 0$. *Then* $\lim_{n\to+\infty} \hat{d}(\iota_n, \kappa) = \hat{d}(\iota, \kappa)$ *for all* $\kappa \in \Lambda$.

Lemma 1.9. *[23] Let* $\left(\Lambda, \hat{d}\right)$ *be a metric-like space. Then,*

(a) if $\hat{d}(\iota, \kappa) = 0$, *then* $\hat{d}(\iota, \iota) = \hat{d}(\kappa, \kappa) = 0$;

(b) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $\lim_{n\to+\infty} \hat{d}(\iota_n, \iota_{n+1}) = 0$, *then* $\lim_{n\to+\infty} \hat{d}(\iota_n, \iota_n) = \lim_{n\to+\infty} \hat{d}(\iota_{n+1}, \iota_{n+1}) = 0$;

(c) if $\iota \neq \kappa$, *then* $\hat{d}(\iota, \kappa) > 0$;

(d) $\hat{d}(\iota, \iota) \leq \frac{2}{n} \sum_{i=1}^{n} \hat{d}(\iota, \iota_i)$ *holds for all* $\iota_i, \iota \in \Lambda$ *where* $1 \leq i \leq n$.

The concept of C-class functions which contains a large class of contractive conditions has been introduced in [2] and [5].

Definition 1.10. [2] A continuous function $f : [0, +\infty)^2 \to \mathbb{R}$ is called a C-class function if

(C1) $f(u, v) \leq u$;

(C2) $f(u, v) = u$ implies that either $u = 0$ or $v = 0$,

for all $u, v \in [0, +\infty)$.

We denote the class of all C-functions by $\mathcal{C}$.

Example 1.11. [2] Following examples show that the class $\mathcal{C}$ is nonempty:

1. $f(u, v) = u - v$.
2. $f(u, v) = ru$, for some $r \in (0, 1)$.
3. $f(u, v) = \frac{u}{(1+v)^k}$ for some $k \in (0, +\infty)$.
4. $f(u, v) = \frac{\log(t+u^{\alpha})}{1+v}$, for some $\alpha > 1$ and some $t < 1$.
5. $f(u, v) = u - \frac{v}{k+v}$, where $k \leq 1$.
6. $f(u, v) = u - g(u)$, where $g : [0, +\infty) \to [0, +\infty)$ is a continuous function such that $g(\iota) = 0$ if and only if $\iota = 0$.

Let $\Pi = \{\pi : [0, +\infty) \to [0, +\infty) : \pi \text{ is increasing and continuous}\}$ and

$\coprod = \{\varpi : [0, +\infty) \to [0, +\infty) : \varpi \text{ is increasing and lower semi-continuous}\}$.

Definition 1.12. A triple (π, ϖ, f) where $\pi \in \Pi$, $\varpi \in \coprod$ and $f \in \mathcal{C}$ is said to be monotone if

$$\iota \leq \kappa \text{ implies } f\left(\pi\left(\iota\right), \varpi\left(\iota\right)\right) \leq f\left(\pi\left(\kappa\right), \varpi\left(\kappa\right)\right),$$

for any $\iota, \kappa \in [0, +\infty)$.

Example 1.13. Let $f(u, v) = u - v$, $\varpi(\iota) = \sqrt{\iota}$ and

$$\pi(\iota) = \begin{cases} \sqrt{\iota}, & \text{if } 0 \leq \iota \leq 1, \\ \iota^2, & \text{if } \iota > 1. \end{cases}$$

Then (π, ϖ, f) is a monotone triple.

Lemma 1.14. *([5, 19, 25]) Let $\left(\Lambda, \hat{d}\right)$ be a metric-like space and let $\{\iota_n\}$ be a sequence in Λ such that $\lim_{n \to +\infty} \hat{d}(\iota_n, \iota_{n+1}) = 0$. If $\{\iota_n\}$ is not a 0-$\hat{d}$-Cauchy sequence in $\left(\Lambda, \hat{d}\right)$, then there exist $\varepsilon > 0$ and two sequences $\{m_k\}$ and $\{n_k\}$ of positive integers such that $n_k > m_k > k$ and the following sequences tend to ε^+ when $k \to +\infty$:*

$$\hat{d}\left(\iota_{m_k}, \iota_{n_k}\right), \ \hat{d}\left(\iota_{m_k}, \iota_{n_k+1}\right), \ \hat{d}\left(\iota_{m_k-1}, \iota_{n_k}\right), \ \hat{d}\left(\iota_{m_k-1}, \iota_{n_k+1}\right), \hat{d}\left(\iota_{m_k+1}, \iota_{n_k+1}\right).$$

2 Cyclic-$\alpha - f\pi\varpi-$Contractive Mappings

To start with, we enunciate some definitions and notations which are productive for the subsequent analysis.

(*) Let $\pi \in \Pi$, $\varpi \in \coprod$ and $f \in \mathcal{C}$ such that $\pi(v) - f(\pi(u), \varpi(u)) > 0$ for all $v > 0$ and $u = v$ or $u = 0$.

Note that condition (*) generalizes (1.2) of [13].

Following definition is a generalization of cyclic-type contractive mappings from [12]-[21] and [23].

Definition 2.1. Let $\left(\Lambda, \hat{d}\right)$ be a 0-$\hat{d}$-complete metric-like space, $p \in \mathbb{N}$ and $\Gamma_1, \Gamma_2, ..., \Gamma_p$, be $\hat{d}-$closed nonempty subsets of Λ. Let $\Delta = \bigcup_{i=1}^{p} \Gamma_i$ and $\alpha : \Delta \times \Delta \to [0, +\infty)$ be a mapping. $\Upsilon : \Delta \to \Delta$ is called a cyclic $\alpha - f\pi\varpi-$contractive mapping if

(1) $\Upsilon(\Gamma_j) \subseteq \Gamma_{j+1}$, for all $j = 1, 2, ..., p$, where $\Gamma_{p+1} = \Gamma_1$;

(2) for any $\iota \in \Gamma_i$ and $\kappa \in \Gamma_{i+1}$, $(i = 1, 2, ..., p)$, where $\Gamma_{p+1} = \Gamma_1$ and $\alpha(\iota, \Upsilon\iota)\, \alpha(\kappa, \Upsilon\kappa) \geq 1$ we have

$$\pi\left(\hat{d}(\Upsilon\iota, \Upsilon\kappa)\right) \leq f\left(\pi\left(M_{\hat{d}}(\iota, \kappa)\right), \varpi\left(M_{\hat{d}}(\iota, \kappa)\right)\right) \tag{1}$$

where $\pi \in \Pi$, $\varpi \in \coprod$ and $f \in \mathcal{C}$ such that the triple (π, ϖ, f) is monotone and

$$M_{\hat{d}}(\iota, \kappa) = \frac{a\hat{d}(\iota, \kappa) + b\hat{d}(\iota, \Upsilon\iota) + c\hat{d}(\kappa, \Upsilon\kappa) + e\frac{\hat{d}(\iota, \Upsilon\kappa) + \hat{d}(\kappa, \Upsilon\iota)}{2}}{m}, \tag{2}$$

with $a, b, c, e \geq 0$ and $m = a + b + c + 2e < 1$.

Remark 2.2. *If in Definition 2.1, $\Gamma_1 = \Gamma_2 = \cdots = \Gamma_p$, then we say Υ is an $\alpha - f\pi\varpi-$ contractive mapping. Further, if $\Upsilon : \Lambda \to \Lambda$ is an $\alpha - f\pi\varpi-$ contractive mapping, $\iota \in Fix(\Upsilon)$ and $\alpha(\iota, \iota) \geq 1$, then $\hat{d}(\iota, \iota) = 0$.*

Indeed, if $\iota \in Fix(\Upsilon), \alpha(\iota, \iota) \geq 1$ and $\hat{d}(\iota, \iota) > 0$, then $M_{\hat{d}}(\iota, \iota) \leq \hat{d}(\iota, \iota)$ and by (1), since the triple (π, ϖ, f) is monotone, we have

$$\pi\left(\hat{d}(\iota, \iota)\right) = \pi\left(\hat{d}(\Upsilon\iota, \Upsilon\iota)\right) \leq f\left(\pi\left(\hat{d}(\iota, \iota)\right), \varpi\left(\hat{d}(\iota, \iota)\right)\right).$$

So, $\pi\left(\hat{d}(\iota, \iota)\right) = 0$ or $\varpi\left(\hat{d}(\iota, \iota)\right) = 0$. Therefore, $\hat{d}(\iota, \iota) = 0$. This is a contradiction with $\hat{d}(\iota, \iota) > 0$. $\square$

Definition 2.3. [6] Let $\left(\Lambda, \hat{d}\right)$ be a metric-like space and let $\alpha : \Lambda \times \Lambda \to [0, +\infty)$ be a function. Then $\Upsilon : \Lambda \to \Lambda$ is said to be $\alpha-$continuous on $\left(\Lambda, \hat{d}\right)$, if

$$\iota_n \to \iota \text{ and } \alpha(\iota_n, \iota_{n+1}) \geq 1 \text{ for all } n \in \mathbb{N} \text{ implies } \Upsilon\iota_n \to \Upsilon\iota.$$

Example 2.4. Let $\Lambda = [0, +\infty)$ and $\hat{d}(\iota, \kappa) = \iota + \kappa$ be a metric-like on Λ. Assume that $\Upsilon : \Lambda \to \Lambda$ and $\alpha : \Lambda^2 \to [0, +\infty)$ are defined by

$$\Upsilon\iota = \begin{cases} \iota^4, & \text{if } \iota \in [0, 1] \\ \sin \pi\iota + 2, & \text{if } \iota \in (1, +\infty) \end{cases}, \quad \alpha(\iota, \kappa) = \begin{cases} 1, & \text{if } (\iota, \kappa) \in [0, 1]^2 \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, Υ is not continuous, but it is $\alpha-$continuous on $\left(\Lambda, \hat{d}\right)$. Indeed, if $\iota_n \to \iota$ as $n \to +\infty$ and $\alpha(\iota_n, \iota_{n+1}) \geq 1$, then $\iota_n \in [0,1]$ and so $\lim\limits_{n\to+\infty} \Upsilon\iota_n = \lim\limits_{n\to+\infty} \iota_n^4 = \iota^4 = \Upsilon\iota$.

Now, we formulate the following lemma which plays a significant role concerning the results invoking cyclic contractions. Its proof is similar to the corresponding result ([8], Lemma 1.8.) for partial metric spaces.

Definition 2.5. [18] Let Λ be a non-empty set, m a positive integer and $\Upsilon : \Lambda \to \Lambda$ an operator. By definition, $\Lambda = \bigcup\limits_{i=1}^{m} \Lambda_i$ is a cyclic representation of Λ with respect to Υ if

(1) Λ_i, $i = 1, ..., m$ are non-empty sets;

(2) $\Upsilon(\Lambda_1) \subseteq \Lambda_2$, ..., $\Upsilon(\Lambda_{m-1}) \subseteq \Lambda_m$, $\Upsilon(\Lambda_m) \subseteq \Lambda_1$.

Lemma 2.6. *Let* $\left(\Lambda = \bigcup\limits_{i=1}^{p} \Gamma_i, \hat{d}\right)$ *be a metric-like space and* $\Upsilon : \Lambda \to \Lambda$ *be a cyclic representation. Assume that*

$$\lim_{n\to+\infty} \hat{d}(\iota_n, \iota_{n+1}) = 0,$$

where $\iota_{n+1} = \Upsilon\iota_n$, *(we can suppose that* $\iota_1 \in \Gamma_1$*). If* $\{\iota_n\}$ *is not a 0-*$\hat{d}$*-Cauchy sequence, then there exist* $\varepsilon > 0$ *and two sequences* $\{n_k\}$ *and* $\{m_k\}$ *of positive integers such that* $n_k > m_k > k$ *and the following sequences tend to* ε^+ *when* $k \to +\infty$:

$$\hat{d}(\iota_{m_k - j_k}, \iota_{n_k}), \ \hat{d}(\iota_{m_k - j_k + 1}, \iota_{n_k}),$$
$$\hat{d}(\iota_{m_k - j_k}, \iota_{n_k + 1}), \ \hat{d}(\iota_{m_k - j_k + 1}, \iota_{n_k + 1}), ...$$

where $j_k \in \{1, 2, ..., p\}$ *is chosen so that* $n_k - m_k + j_k \equiv 1$, *for each* $k \in \mathbb{N}$. *Note that, if* $p = 1$, *Lemma 1.14 is a special case.*

3 Main Results

Now, we are ready to present our first new result of this section.

Theorem 3.1. *Let* $\left(\Lambda, \hat{d}\right)$ *be a 0-*$\hat{d}$*-complete metric-like space and* $\alpha : \Lambda \times \Lambda \to [0, +\infty)$ *be a mapping. Assume that* $\Upsilon : \Lambda \to \Lambda$ *is an* $\alpha - f\pi\varpi-$*contractive mapping satisfying the following assertions:*

(i) Υ *is an* $\alpha-$*admissible mapping,*

(ii) $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$ *for an element* ι_0 *in* Λ,

(iii) Υ *is* $\alpha-$*continuous, or;*

(iv) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $\alpha(\iota_n, \iota_{n+1}) \geq 1$ *for all* $n \in \mathbb{N}$ *and* $\iota_n \to \iota$ *as* $n \to +\infty$, *then* $\alpha(\iota, \Upsilon\iota) \geq 1$.

Then Υ *admits a fixed point in* Λ.

Moreover, if

(v) $\alpha(\iota, \iota) \geq 1$, *whenever* $\iota \in Fix(\Upsilon)$, *then* Υ *admits a unique fixed point.*

Proof. Define Picard's sequence $\iota_n = \Upsilon^n \iota_0$, where ι_0 is the given point for which $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$. Since Υ is an $\alpha-$admissible mapping, we get that $\alpha(\iota_1, \Upsilon\iota_1) = \alpha(\Upsilon\iota_0, \Upsilon(\Upsilon\iota_0)) \geq 1$. Again, from the same reason, it follows that $\alpha(\iota_2, \Upsilon\iota_2) = \alpha(\Upsilon\iota_1, \Upsilon(\Upsilon\iota_1)) \geq 1$. Continuing this process we have $\alpha(\iota_n, \Upsilon\iota_n) \geq 1$ for all $n \in \mathbb{N}$, and so, $\alpha(\iota_n, \Upsilon\iota_n)\alpha(\iota_{n-1}, \Upsilon\iota_{n-1}) \geq 1$ for all $n \in \mathbb{N}$. In the case when $\iota_{n-1} = \iota_n$ for some $n \in \mathbb{N}$, ι_{n-1} is a fixed point of Υ. Therefore, assume that $\iota_{n-1} \neq \iota_n$ for all $n \in \mathbb{N}$. Hence, by Lemma 1.9 (c), we have $\hat{d}(\iota_{n-1}, \iota_n) > 0$ for all $n \in \mathbb{N}$. Now, we show that the sequence $\hat{d}(\iota_n, \iota_{n+1})$ is increasing. Set

$$\Gamma = \frac{a\hat{d}(\iota_{n-1}, \iota_n) + b\hat{d}(\iota_{n-1}, \Upsilon\iota_{n-1}) + c\hat{d}(\iota_n, \Upsilon\iota_n) + e\frac{\hat{d}(\iota_{n-1}, \Upsilon\iota_n) + \hat{d}(\iota_n, \Upsilon\iota_{n-1})}{2}}{m}.$$

By (1) and (2) we get

$$\begin{aligned} \pi\left(\hat{d}(\iota_n, \iota_{n+1})\right) &= \pi\left(\hat{d}(\Upsilon\iota_{n-1}, \Upsilon\iota_n)\right) \\ &\leq f(\pi(\Gamma), \varpi(\Gamma)) \end{aligned} \tag{3}$$

On the other hand, from Lemma 3, part (d), we have $\hat{d}(\iota_n, \iota_n) \leq 2\hat{d}(\iota_{n-1}, \iota_n)$ and by $\left(\hat{d}3\right)$ we have $\hat{d}(\iota_{n-1}, \iota_{n+1}) \leq \hat{d}(\iota_{n-1}, \iota_n) + \hat{d}(\iota_n, \iota_{n+1})$. Therefore, from (3) we get,

$$\pi\left(\hat{d}(\iota_n, \iota_{n+1})\right) \leq f(\pi(B), \varpi(B)) \tag{4}$$

where

$$B = \frac{\left(a + b + \frac{3e}{2}\right)\hat{d}(\iota_{n-1}, \iota_n) + \left(c + \frac{e}{2}\right)\hat{d}(\iota_n, \iota_{n+1})}{m}.$$

From (4) it follows that

$$\hat{d}(\iota_n, \iota_{n+1}) \leq B,$$

that is,

$$m \cdot \hat{d}(\iota_n, \iota_{n+1}) \leq \left(a + b + \frac{3e}{2}\right) \hat{d}(\iota_{n-1}, \iota_n) + \left(c + \frac{e}{2}\right) \hat{d}(\iota_n, \iota_{n+1}),$$

or equivalently,

$$\hat{d}(\iota_n, \iota_{n+1}) \leq \hat{d}(\iota_{n-1}, \iota_n).$$

Therefore, there exists $\hat{d}^* \geq 0$ such that $\lim\limits_{n\to+\infty} \hat{d}(\iota_n, \iota_{n+1}) = \hat{d}^*$. To prove $\hat{d}^* = 0$, we use the method of reductio ad absurdum. For this purpose, we assume that $\hat{d}^* > 0$. By (4) together with the properties of π and ϖ we have

$$\pi\left(\hat{d}^*\right) \leq f\left(\pi\left(\hat{d}^*\right), \varpi\left(\hat{d}^*\right)\right),$$

so, $\pi\left(\hat{d}^*\right) = 0$, or, $\varpi\left(\hat{d}^*\right) = 0$. This is a contradiction. Hence,

$$\lim_{n\to+\infty} \hat{d}(\iota_n, \iota_{n+1}) = 0.$$

In order to prove that $\{\iota_n\}$ is a 0-$\hat{d}$–Cauchy sequence, let

$$\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) \neq 0.$$

Taking $\iota = \iota_{m_k}, \kappa = \iota_{n_k}$ in (6) and utilizing lemma 2.6, we have

$$\pi\left(\hat{d}(\iota_{m_k+1}, \iota_{n_k+1})\right) \leq f\left(\pi\left(M_{\hat{d}}(\iota_{m_k}, \iota_{n_k})\right), \varpi\left(M_{\hat{d}}(\iota_{m_k}, \iota_{n_k})\right)\right), \quad (5)$$

where

$$\begin{aligned} m \cdot M_{\hat{d}}(\iota_{m_k}, \iota_{n_k}) &= a\hat{d}(\iota_{m_k}, \iota_{n_k}) + b\hat{d}(\iota_{m_k}, \iota_{m_k+1}) + c\hat{d}(\iota_{n_k}, \iota_{n_k+1}) \\ &\quad + e\frac{\hat{d}(\iota_{m_k}, \iota_{n_k+1}) + \hat{d}(\iota_{m_k+1}, \iota_{n_k})}{2} \to \left(a + c + \frac{e}{2}\right)\varepsilon, \end{aligned}$$

as $k \to +\infty$. Now, taking the limit in (5) as $k \to +\infty$, we get the following:

$$\pi(\varepsilon) \leq f\left(\pi\left(\frac{1}{m}\left(\left(a+c+\frac{e}{2}\right)\varepsilon\right)\right), \varpi\left(\frac{1}{m}\left(\left(a+c+\frac{e}{2}\right)\varepsilon\right)\right)\right)$$
$$\leq f(\pi(\varepsilon), \varpi(\varepsilon)),$$

so, $\pi(\varepsilon) = 0$, or, $\varpi(\varepsilon) = 0$, that is, in both cases, we obtain that $\varepsilon = 0$. This is a contradiction. Hence, $\lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) = 0$. In other words, the sequence $\{\iota_n\}$ is a 0-$\hat{d}-$Cauchy sequence. Since $\left(\Lambda, \hat{d}\right)$ is 0-$\hat{d}-$complete, so there exists $\bar{\iota} \in \Lambda$ such that $\lim_{n\to+\infty} \iota_n = \bar{\iota}$. Equivalently, we have

$$\hat{d}(\bar{\iota}, \bar{\iota}) = \lim_{n\to+\infty} \hat{d}(\bar{\iota}, \iota_n) = \lim_{n,m\to+\infty} \hat{d}(\iota_n, \iota_m) = 0. \tag{6}$$

At first, assume that (iii) holds. That is, Υ is $\alpha-$continuous. Then $\bar{\iota} = \lim_{n\to+\infty} \iota_{n+1} = \lim_{n\to+\infty} \Upsilon\iota_n = \Upsilon\bar{\iota}$, i.e., $\bar{\iota}$ is a fixed point of Υ.

Secondly, assume that (iv) holds. Then from (iv) it follows that $\alpha(\bar{\iota}, \Upsilon\bar{\iota}) \geq 1$. Further, $\alpha(\bar{\iota}, \Upsilon\bar{\iota})\,\alpha(\iota_{n_k}, \Upsilon\iota_{n_k}) \geq 1$. In what follows, we prove that $\bar{\iota}$ is a fixed point of Υ. Since $\lim_{n\to+\infty} \iota_n = \bar{\iota}$, using the contractive condition (1), we obtain that

$$\pi\left(\hat{d}(\Upsilon\bar{\iota}, \Upsilon\iota_{n_k})\right) \leq f(\pi(\Gamma_k), \varpi(\Gamma_k)), \tag{7}$$

where

$$\Gamma_k = \frac{1}{m}\left\{a\hat{d}(\bar{\iota}, \iota_{n_k}) + b\hat{d}(\bar{\iota}, \Upsilon\bar{\iota}) + c\hat{d}(\iota_{n_k}, \Upsilon\iota_{n_k}) + e\frac{\hat{d}(\bar{\iota}, \Upsilon\iota_{n_k}) + \hat{d}(\iota_{n_k}, \Upsilon\bar{\iota})}{2}\right\}$$

and $m = a + b + c + 2e$. Now, taking the limit in (7) as $k \to +\infty$, using the (6) and the lower semi-continuous of the function ϖ, we obtain

$$\pi\left(\hat{d}(\bar{\iota}, \Upsilon\bar{\iota})\right) \leq f\left(\pi\left(\hat{d}(\bar{\iota}, \Upsilon\bar{\iota})\right), \varpi\left(\hat{d}(\bar{\iota}, \Upsilon\bar{\iota})\right)\right), \tag{8}$$

so, $\pi\left(\hat{d}(\bar{\iota}, \Upsilon\bar{\iota})\right) = 0$, or, $\varpi\left(\hat{d}(\bar{\iota}, \Upsilon\bar{\iota})\right) = 0$. Thus, $\hat{d}(\bar{\iota}, \Upsilon\bar{\iota}) = 0$ and, therefore, $\bar{\iota}$ is a fixed point of Υ.

For the uniqueness of the fixed point of the map Υ, suppose that $\bar{\iota}, \bar{\kappa}$ $(\bar{\iota} \neq \bar{\kappa})$ are two fixed points of Υ. Since $\bar{\iota}, \bar{\kappa} \in Fix(\Upsilon)$ we have, $\hat{d}(\bar{\iota}, \bar{\kappa}) > 0$, $\alpha(\bar{\iota}, \bar{\iota}) \geq 1$ and $\alpha(\bar{\kappa}, \bar{\kappa}) \geq 1$, (that is (v) holds). So, $\alpha(\bar{\iota}, \Upsilon\bar{\iota})\, \alpha(\bar{\kappa}, \Upsilon\bar{\kappa}) \geq 1$. Using (1) we have that $\hat{d}(\bar{\iota}, \bar{\iota}) = \hat{d}(\bar{\kappa}, \bar{\kappa}) = 0$. Further, it follows

$$\pi\left(\hat{d}(\Upsilon\bar{\iota}, \Upsilon\bar{\kappa})\right) \leq f\left(\pi\left(\Gamma_{\bar{\iota},\bar{\kappa}}\right), \varpi\left(\Gamma_{\bar{\iota},\bar{\kappa}}\right)\right), \tag{9}$$

where

$$\begin{aligned} \Gamma_{\bar{\iota},\bar{\kappa}} &= \frac{1}{m}\left\{a\hat{d}(\bar{\iota}, \bar{\kappa}) + b\hat{d}(\bar{\iota}, \Upsilon\bar{\iota}) + c\hat{d}(\bar{\kappa}, \Upsilon\bar{\kappa}) + e\frac{\hat{d}(\bar{\iota}, \Upsilon\bar{\kappa}) + \hat{d}(\bar{\kappa}, \Upsilon\bar{\iota})}{2}\right\} \\ &= \frac{1}{m}\left\{a\hat{d}(\bar{\iota}, \bar{\kappa}) + b\hat{d}(\bar{\iota}, \bar{\iota}) + c\hat{d}(\bar{\kappa}, \bar{\kappa}) + e\frac{\hat{d}(\bar{\iota}, \bar{\kappa}) + \hat{d}(\bar{\kappa}, \bar{\iota})}{2}\right\} \\ &= \frac{1}{m}\left\{a\hat{d}(\bar{\iota}, \bar{\kappa}) + b \cdot 0 + c \cdot 0 + e\frac{2\hat{d}(\bar{\iota}, \bar{\kappa})}{2}\right\} \\ &= \frac{1}{m}(a+e)\,\hat{d}(\bar{\iota}, \bar{\kappa}). \end{aligned} \tag{10}$$

Since $\frac{1}{m}(a+e) = \frac{a+e}{a+b+c+2e} < 1$, then from (9) and (10), it follows that

$$\pi\left(\hat{d}(\bar{\iota}, \bar{\kappa})\right) \leq f\left(\pi\left(\hat{d}(\bar{\iota}, \bar{\kappa})\right), \varpi\left(\hat{d}(\bar{\iota}, \bar{\kappa})\right)\right),$$

that is, $\hat{d}(\bar{\iota}, \bar{\kappa}) = 0$, which is a contradiction. This finishes the proof. □

We expound the following example to uphold our obtained result.

Example 3.2. Let $\Lambda = [0, +\infty)$ with the metric-like $\hat{d}(\iota, \kappa) = \max\{\iota, \kappa\}$ for all $\iota, \kappa \in \Lambda$ and $f(s,t) = \frac{s}{1+t}$. Let $\Upsilon : \Lambda \to \Lambda$ and $\alpha : \Lambda \times \Lambda \to [0, +\infty)$ be defined by

$$\Upsilon\iota = \begin{cases} \frac{1}{5}\iota^2, \text{ if } 0 \leq \iota < \frac{1}{3} \\ \frac{1-\iota^2}{6}, \text{ if } \frac{1}{3} \leq \iota \leq 1 \\ \frac{1}{6}\iota, \text{ if } 1 < \iota \leq 2 \\ 2\iota^2 + 1, \text{ if } \iota > 2, \end{cases} \quad \text{and } \alpha(\iota, \kappa) = \begin{cases} 5, \text{ if } (\iota, \kappa) \in [0, 2]^2 \\ 0, \text{ otherwise.} \end{cases}$$

Also, define $\pi, \varpi : [0, +\infty) \to [0, +\infty)$ by $\pi(t) = t$ and $\varpi(t) = 1$. Let $\alpha(\iota, \Upsilon\iota)\, \alpha(\kappa, \Upsilon\kappa) \geq 1$. Then $(\iota, \kappa) \in [0, 2]^2$. Now, we consider the following cases:

1. Let $0 \leq \iota, \kappa < \frac{1}{3}$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \frac{1}{5} \max\left\{\iota^2, \kappa^2\right\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

2. Let $\frac{1}{3} \leq \iota, \kappa \leq 1$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \frac{1}{2} \max\{1 - \iota, 1 - \kappa\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

3. Let $1 < \iota, \kappa \leq 2$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \frac{1}{6} \max\{\iota, \kappa\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

4. Let $0 \leq \iota < \frac{1}{3}$ and $\frac{1}{3} \leq \kappa \leq 1$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \max\left\{\frac{1}{5}\iota^2, \frac{1}{2}(1 - \kappa)\right\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

5. Let $0 \leq \iota < \frac{1}{3}$ and $\kappa > 1$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \max\left\{\frac{1}{5}\iota^2, \frac{1}{6}\kappa\right\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

6. Let $\frac{1}{3} \leq \iota \leq 1$ and $1 < \kappa \leq 2$. Then

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) = \max\left\{\frac{1}{2}(1 - \iota), \frac{1}{6}\kappa\right\} \leq \frac{1}{2} \max\{\iota, \kappa\} = \frac{1}{2}\hat{d}(\iota, \kappa).$$

So,

$$\begin{aligned}\pi\left(\hat{d}(\Upsilon\iota, \Upsilon\kappa)\right) &= \hat{d}(\Upsilon\iota, \Upsilon\kappa) \leq \frac{1}{2}\hat{d}(\iota, \kappa) \\ &\leq \frac{1}{2}M_{\hat{d}}(\iota, \kappa) = \frac{\pi\left(M_{\hat{d}}(\iota, \kappa)\right)}{1 + \varpi\left(M_{\hat{d}}(\iota, \kappa)\right)}.\end{aligned}$$

Since the conditions (i), (ii) and (iv) of Theorem 3.1 hold true, it follows that Υ is an $\alpha - f\pi\varpi-$contractive mapping, that is, Υ admits a fixed point.□

Our second new result of this paper is the following:

Theorem 3.3. *Let* $\left(\Lambda, \hat{d}\right)$ *be a 0-*$\hat{d}$*-complete metric-like space. Let* p *be a positive integer,* $\Gamma_1, \Gamma_2, ..., \Gamma_p$ *be nonempty* $\hat{d}-$*closed subsets of* Λ, $\Delta = \bigcup_{i=1}^{p} \Gamma_i$ *and* $\alpha : \Lambda \times \Lambda \to [0, +\infty)$ *be a mapping. Assume that* $\Upsilon : \Delta \to \Delta$ *is a cyclic* $\alpha - f\pi\varpi-$*contractive mapping satisfying the following assertions:*

(i) Υ *is an* $\alpha-$*admissible mapping;*

(ii) there exists an element ι_0 *in* Δ *such that* $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$;

(iii) Υ *is* $\alpha-$*continuous, or,*

(iv) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $\alpha(\iota_n, \iota_{n+1}) \geq 1$ *for all* $n \in \mathbb{N}$ *and* $\iota_n \to \iota$ *as* $n \to +\infty$, *then* $\alpha(\iota, \Upsilon\iota) \geq 1$.

Then Υ *admits a fixed point in* $\cap_{i=1}^{p}\Gamma_i$. *Moreover, if*

(v) $\alpha(\iota, \iota) \geq 1$, *whenever* $\iota \in Fix(\Upsilon)$,

Then Υ *admits a unique fixed point.*

Proof.

Let ι_0 be a point of Δ such that $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$. Then there exists some i_0 such that $\iota_0 \in \Gamma_{i_0}$. $\Upsilon(\Gamma_{i_0}) \subseteq \Gamma_{i_0+1}$ implies that $\Upsilon\iota_0 \in \Gamma_{i_0+1}$. Thus, there exists ι_1 in Γ_{i_0+1} such that $\Upsilon\iota_0 = \iota_1$. Similarly, $\Upsilon\iota_n = \iota_{n+1}$, where $\iota_n \in \Gamma_{i_n}$. Hence, for all $n \geq 0$, there exists $i_n \in \{1, 2, ..., p\}$ such that $\iota_n \in \Gamma_{i_n}$ and $\Upsilon\iota_n = \iota_{n+1}$. On the other hand, as Υ is an $\alpha-$admissible mapping, then we get $\alpha(\iota_1, \Upsilon\iota_1) = \alpha(\Upsilon\iota_0, \Upsilon(\Upsilon\iota_0)) \geq 1$. Again, since Υ is an $\alpha-$admissible mapping,

$$\alpha(\iota_2, \Upsilon\iota_2) = \alpha(\Upsilon\iota_1, \Upsilon(\Upsilon\iota_1)) \geq 1.$$

Continuing this process, we get $\alpha(\iota_n, \Upsilon\iota_n) \geq 1$ for all $n \in \mathbb{N}\bigcup\{0\}$ and so,

$$\alpha(\iota_n, \Upsilon\iota_n)\,\alpha(\iota_{n-1}, \Upsilon\iota_{n-1}) \geq 1 \text{ for all } n \in \mathbb{N}.$$

If $\iota_{n_0} = \iota_{n_0+1}$ for some $n_0 \in \mathbb{N}\bigcup\{0\}$, then it is clear that ι_{n_0} is a fixed point of Υ. Now, assume that $\iota_n \neq \iota_{n+1}$ for all n. Hence, by Lemma 3 (c) we have $\hat{d}(\iota_{n-1}, \iota_n) > 0$ for all n. Now, we show that the sequence $\left\{\hat{d}(\iota_n, \iota_{n+1})\right\}$ is increasing.

By (6) we get

$$\begin{aligned}\pi\left(\hat{d}(\iota_n, \iota_{n+1})\right) &= \pi\left(\hat{d}(\Upsilon\iota_{n-1}, \Upsilon\iota_n)\right) \\ &\leq f(\pi(\Gamma), \varpi(\Gamma)),\end{aligned}$$

where

$$\Gamma = \frac{a\hat{d}\left(\iota_{n-1}, \iota_n\right) + b\hat{d}\left(\iota_{n-1}, \iota_n\right) + c\hat{d}\left(\iota_n, \iota_{n+1}\right) + e\frac{a\hat{d}(\iota_{n-1},\iota_{n+1})+a\hat{d}(\iota_n,\iota_n)}{2}}{m},$$

and $m = a + b + c + 2e \in (0, 1)$.

Further, as in the proof of previous theorem we easily obtain that $\hat{d}\left(\iota_n, \iota_{n+1}\right) \leq \hat{d}\left(\iota_{n-1}, \iota_n\right)$ as well as $\hat{d}\left(\iota_n, \iota_{n+1}\right) \to 0$ as $n \to +\infty$. In order to prove that the sequence $\{\iota_n\}$ is a 0-$\hat{d}$−Cauchy, we will use Lemma 5. Therefore, putting $\iota = \iota_{m_k - j_k}, \kappa = \iota_{n_k}$ in (6) we get

$$\pi\left(\hat{d}\left(\iota_{m_k-j_k+1}, \iota_{n_k+1}\right)\right) \leq f\left(\pi\left(M_{\hat{d}}\left(\iota_{m_k-j_k}, \iota_{n_k}\right)\right), \varpi\left(M_{\hat{d}}\left(\iota_{m_k-j_k}, \iota_{n_k}\right)\right)\right), \tag{11}$$

where

$$\begin{aligned} m \cdot M_{\hat{d}}\left(\iota_{m_k-j_k}, \iota_{n_k}\right) &= a\hat{d}\left(\iota_{m_k-j_k}, \iota_{n_k}\right) + b\hat{d}\left(\iota_{m_k-j_k}, \iota_{m_k-j_k+1}\right) + \\ &+ c\hat{d}\left(\iota_{n_k}, \iota_{n_k+1}\right) + e\frac{\hat{d}\left(\iota_{m_k-j_k}, \iota_{n_k+1}\right) + \hat{d}\left(\iota_{m_k-j_k+1}, \iota_{n_k}\right)}{2} \\ &\to a\varepsilon + b \cdot 0 + c \cdot 0 + e\frac{2\varepsilon}{2} = (a + e)\,\varepsilon, \text{ as } \; k \to +\infty. \end{aligned}$$

Now, taking the limit in (11) as $k \to +\infty$, we obtain that

$$\pi\left(\varepsilon\right) \leq f\left(\pi\left(\frac{(a+e)\,\varepsilon}{m}\right), \varpi\left(\frac{(a+e)\,\varepsilon}{m}\right)\right) \leq f\left(\pi\left(\varepsilon\right), \varpi\left(\varepsilon\right)\right),$$

which implies that $\pi(\varepsilon) = 0$, or, $\varpi(\varepsilon) = 0$, that is, in both cases, we obtain that $\varepsilon = 0$. But, this is a contradiction. Hence, $\lim\limits_{n,m \to +\infty} \hat{d}\left(\iota_n, \iota_m\right) = 0$. In other words, the sequence $\{\iota_n\}$ is a 0-$\hat{d}$−Cauchy sequence. Since Δ is $\hat{d}$−closed in $\left(\Lambda, \hat{d}\right)$, this means that there exists a unique $\bar{\iota} \in \Delta$ such that

$$\hat{d}\left(\bar{\iota}, \bar{\iota}\right) = \lim_{n \to +\infty} \hat{d}\left(\iota_n, \bar{\iota}\right) = \lim_{n,m \to +\infty} \hat{d}\left(\iota_n, \iota_m\right) = 0.$$

Further, as $\Upsilon\left(\Gamma_i\right) \subseteq \Gamma_{i+1}, \Gamma_{p+1} = \Gamma_1$, it follows that the sequence $\{\iota_n\}$ has infinitely many terms in each Γ_i for $i \in \{1, 2, ..., p\}$. Hence, we have the subsequences $\{\iota_{n_i}\}$ of $\{\iota_n\}$ where $\{\iota_{n_i}\} \subseteq \Gamma_i, i = 1, 2, ..., p$. It is clear that each ι_{n_i} converges to $\bar{\iota}$. It follows that $\Gamma = \cap_{i=1}^{p} \Gamma_i \neq \emptyset$, because it contains at least the element $\bar{\iota}$. Obviously, $\left(\Gamma, \hat{d}\right)$ is a 0-$\hat{d}$-complete metric-like space and $\Upsilon : \Gamma \to \Gamma$. Also, the restriction $\Upsilon|_\Gamma$ of Υ on Γ satisfies all conditions of previous theorem . Hence, Υ admits a unique fixed point $\bar{\iota}$ in Γ. This completes the proof of Theorem 7. □

4 Fixed Point Results for Cyclic $f\pi\varpi-$Graphic Contractions

As in Jachymski [11], let $\left(\Lambda, \hat{d}\right)$ be a metric-like space and Ω be the diagonal of $\Lambda \times \Lambda$. Consider a directed graph G such that $V(G)$ consists of its vertices coincides with Λ, and $E(G)$ consists of its edges covers all loops, i.e., $E(G) \supseteq \Omega$. Let G has no parallel edges. So, we treat G to be the pair $(V(G), E(G))$. Moreover, we may consider G as a weighted graph (see [11]) by corresponding to each edge the distance between its vertices. For basic notations and further analysis on graph structure of fixed point theory, one may refer to the informative papers [11, 28, 29, 30].

Definition 4.1. [11] We say that a mapping $\Upsilon : \Lambda \to \Lambda$ is a Banach $G-$contraction or simply $G-$contraction if Υ preserves the edges of G, i.e.,

$$\text{for all } \iota, \kappa \in \Lambda \ \ ((\iota, \kappa) \in E(G) \ \text{ implies } \ (\Upsilon\iota, \Upsilon\kappa) \in E(G))$$

and Υ decreases the weights of the edges of G in the following way: there exists $\alpha \in (0, 1)$ such that

$$(\iota, \kappa) \in E(G) \ \text{ implies } d(\Upsilon\iota, \Upsilon\kappa) \leq \alpha d(\iota, \kappa),$$

for all $\iota, \kappa \in \Lambda$.

Definition 4.2. A mapping $\Upsilon : \Lambda \to \Lambda$ is called $G-$continuous, if for given $\iota \in \Lambda$ and for any sequence $\{k_n\}$ of positive integers, $\Upsilon^{k_n}\iota \to \kappa \in \Lambda$ as $n \to +\infty$, implies that $\Upsilon\left(\Upsilon^{k_n}\iota\right) \to \Upsilon\kappa$, as $n \to +\infty$.

Definition 4.3. [11] A mapping $\Upsilon : \Lambda \to \Lambda$ is called orbitally continuous, if for given $\iota \in \Lambda$ and for a sequence $\{\iota_n\} : \iota_n \to \iota$ as $n \to +\infty$ and $(\iota_n, \iota_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$ imply that $\Upsilon\iota_n \to \Upsilon\iota$.

Definition 4.4. A mapping $\Upsilon : \Lambda \to \Lambda$ is called orbitally $G-$continuous, if for given $\iota, \kappa \in \Lambda$ and for any sequence $\{k_n\}$ of positive integers, $\Upsilon^{k_n}\iota \to \kappa$ and $\left(\Upsilon^{k_n}\iota, \Upsilon^{k_{n+1}}\iota\right) \in E(G)$ for all $n \in \mathbb{N}$ imply that $\Upsilon\left(\Upsilon^{k_n}\iota\right) \to \Upsilon\kappa$, as $n \to +\infty$.

In the sequel, we define cyclic $f\pi\varpi-$graphic contractive mappings and prove the corresponding results in the framework of metric-like spaces. Our results extend, compliment, generalize, improve and enrich ones from the existing literature ([1, 11, 17, 23]).

Definition 4.5. Let $\left(\Lambda, \hat{d}\right)$ be a graphic metric-like space. Let p be a positive integer, $\Gamma_1, \Gamma_2, ..., \Gamma_p$ be nonempty $\hat{d}-$closed subsets of Λ and $\Delta = \bigcup\limits_{i=1}^{p} \Gamma_i$. We say that Υ is a cyclic $f\pi\varpi-$graphic contractive mapping if

(a) $\Upsilon(\Gamma_i) \subseteq \Gamma_{i+1},\ (i = 1, 2, ..., p)$, where $\Gamma_{p+1} = \Gamma_1$;
(b) For all $\iota, \kappa \in \Delta$ $((\iota, \kappa) \in E(G)$ implies that $(\Upsilon\iota, \Upsilon\kappa) \in E(G))$;
(c) For all $\iota \in \Gamma_i$ and $\kappa \in \Gamma_{i+1},\ (i = 1, 2, ..., p)$ where $\Gamma_{p+1} = \Gamma_1$ and $(\iota, \Upsilon\iota) \in E(G)$ and $(\kappa, \Upsilon\kappa) \in E(G)$ we have

$$\pi\left(\hat{d}(\Upsilon\iota, \Upsilon\kappa)\right) \le f\left(\pi\left(M_{\hat{d}}(\iota, \kappa)\right), \varpi\left(M_{d(\iota,\kappa)}\right)\right),$$

where $\pi \in \Pi$, $\varpi \in \coprod$ and

$$M_{\hat{d}}(\iota, \kappa) = \max\left\{\hat{d}(\iota, \kappa), \hat{d}(\iota, \Upsilon\iota), \hat{d}(\kappa, \Upsilon\kappa), \frac{\hat{d}(\iota, \Upsilon\kappa) + \hat{d}(\kappa, \Upsilon\iota)}{4}\right\}.$$

If we take $\Lambda = \Gamma_i,\ (i = 1, 2, ..., p)$, in the Definition 16, Υ is said to be an $f\pi\varpi-$graphic contractive mapping.

Theorem 4.6. *Let $\left(\Lambda, \hat{d}\right)$ be a complete graphic metric-like space . Let p be a positive integer, $\Gamma_1, \Gamma_2, ..., \Gamma_p$ be a nonempty $\hat{d}-$closed subsets of Λ, $\Delta = \bigcup\limits_{i=1}^{p} \Gamma_i$ and $\Upsilon : \Delta \to \Delta$ be a cyclic $f\pi\varpi-$graphic contractive mapping. Suppose that the following assertions hold:*

(i) $(\iota_0, \Upsilon\iota_0) \in E(G)$ for an $\iota_0 \in \Lambda$,

(ii) Υ *is orbitally* $G-$*continuous on* $\left(\Lambda, \hat{d}\right)$, *or,*

(iii) for any sequence $\{\iota_n\} \subset \Lambda$ *with* $\iota_n \to \iota$ *as* $n \to +\infty$ *and* $(\iota_{n+1}, \iota_n) \in E(G)$, *we have* $(\iota, \Upsilon\iota) \in E(G)$.

Then Υ *admits a fixed point say* $\bar{\iota} \in \Delta$ *and* $\bar{\iota} \in \cap_{i=1}^{p}\Gamma_i$.

Proof. Define $\alpha : \Delta \times \Delta \to [0, +\infty)$ by $\alpha(\iota, \kappa) = 1$, If $(\iota, \kappa) \in E(G)$, that is, $\alpha(\iota, \kappa) = 0$, otherwise. At first we prove that Υ is $\alpha-$admissible. Let $\alpha(\iota, \kappa) \geq 1$. Then $(\iota, \kappa) \in E(G)$. As Υ is a cyclic $f\pi\varpi-$graphic contractive mapping, we have $(\Upsilon\iota, \Upsilon\kappa) \in E(G)$, that is, $\alpha(\Upsilon\iota, \Upsilon\kappa) \geq 1$. So, Υ is an $\alpha-$admissible mapping. Let Υ be $G-$continuous on $\left(\Lambda, \hat{d}\right)$, that is,

$$\iota_n \to \iota \text{ as } n \to +\infty \text{ and } (\iota_n, \iota_{n+1}) \in E(G) \text{ for all } n \in \mathbb{N} \text{ imply that } \Upsilon\iota_n \to \Upsilon\iota.$$

So, Υ is $\alpha-$continuous on $\left(\Lambda, \hat{d}\right)$. From (i) there exists $\iota_0 \in \Lambda$ such that $(\iota_0, \Upsilon\iota_0) \in E(G)$. That is, $\alpha(\iota_0, \Upsilon\iota_0) \geq 1$.

Let $\iota \in \Gamma_i$ and $\kappa \in \Gamma_{i+1}$, where $\alpha(\iota, \Upsilon\iota)\,\alpha(\kappa, \Upsilon\kappa) \geq 1$. Then, $\iota \in \Gamma_i$ and $\kappa \in \Gamma_{i+1}$, where $(\iota, \Upsilon\iota) \in E(G)$ and $(\kappa, \Upsilon\kappa) \in E(G)$. Now, since Υ is a cyclic $f\pi\varpi-$graphic contractive mapping, then

$$\pi\left(\hat{d}(\Upsilon\iota, \Upsilon\kappa)\right) \leq f\left(\pi\left(M_{\hat{d}}(\iota, \kappa)\right), \varpi\left(M_{\hat{d}}(\iota, \kappa)\right)\right).$$

That is, Υ is a cyclic $\alpha - f\pi\varpi-$ contractive mapping. Let $\{\iota_n\} \subset \Lambda$ be a sequence such that $\iota_n \to \iota$ as $n \to +\infty$ and $\alpha(\iota_{n+1}, \iota_n) \geq 1$. Therefore, $(\iota_{n+1}, \iota_n) \in E(G)$ and then from (iii) we have $(\iota, \Upsilon\iota) \in E(G)$. That is, $\alpha(\iota, \Upsilon\iota) \geq 1$. Hence, all conditions of Theorem 3.1. are satisfied and Υ admits a fixed point, say $\bar{\iota} \in \Delta$ and $\bar{\iota} \in \cap_{i=1}^{p}\Gamma_i$. $\square$

If in Theorem 7, we take $\pi(t) = t$ and $f(s, t) = rs$ for some $r \in (0, 1)$, then we deduce the following corollary.

Corollary 4.7. *Let* $\left(\Lambda, \hat{d}\right)$ *be a graphic 0-*$\hat{d}$*-complete metric-like space* G. *Let* p *be a positive integer,* $\Gamma_1, \Gamma_2, ..., \Gamma_p$ *be nonempty* $\hat{d}-$*closed subsets of* Λ, $\Delta = \bigcup_{i=1}^{p} \Gamma_i$ *and* $\alpha : \Delta \times \Delta \to [0, +\infty)$ *be a mapping. Assume that* $\Upsilon : \Delta \to \Delta$ *be a mapping such that:*

(i) $\Upsilon(\Gamma_i) \subseteq \Gamma_{i+1}$, $(i = 1, 2, ..., p)$, *where* $\Gamma_{p+1} = \Gamma_1$;
(ii) $((\iota, \kappa) \in E(G)$ implies that $(\Upsilon\iota, \Upsilon\kappa) \in E(G))$, for all $\iota, \kappa \in \Delta$,
(iii) $(\iota_0, \Upsilon\iota_0) \in E(G)$, for an element ι_0 *in* Δ,
(iv) Υ *is orbitally* $G-$*continuous, or,*
(v) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $(\iota_n, \iota_{n+1}) \in E(G)$ *for all* $n \in \mathbb{N}$ *and* $\iota_n \to \iota$ *as* $n \to +\infty$, *then* $(\iota, \Upsilon\iota) \in E(G)$;
(vi) There exists $r \in (0, 1)$ *such that* $(\iota, \Upsilon\iota) \in E(G)$ *and* $(\kappa, \Upsilon\kappa) \in E(G)$ *implies that*

$$\hat{d}(\Upsilon\iota, \Upsilon\kappa) \leq r \max\left\{\hat{d}(\iota, \kappa), \hat{d}(\iota, \Upsilon\iota), \hat{d}(\kappa, \Upsilon\kappa), \frac{\hat{d}(\iota, \Upsilon\kappa) + \hat{d}(\kappa, \Upsilon\iota)}{4}\right\}$$

for any $\iota \in \Gamma_i$, $\kappa \in \Gamma_{i+1}$, $(i = 1, 2, ..., p)$, *where* $\Gamma_{p+1} = \Gamma_1$.
Then Υ *admits a fixed point, (say* z*) and* $z \in \cap_{i=1}^{p}\Gamma_i$.

If in Theorem 7, we take $\Gamma_i = \Lambda$, where $i = 1, 2, ..., p$, then we deduce the following result.

Theorem 4.8. *Let* $\left(\Lambda, \hat{d}\right)$ *be graphic a 0-*$\hat{d}-$*complete metric-like space. Assume that* $\Upsilon : \Lambda \to \Lambda$ *be an* $f\pi\varpi-$*graphic contractive mapping satisfying the following assertions:*

(i) $((\iota, \kappa) \in E(G)$ implies that $(\Upsilon\iota, \Upsilon\kappa) \in E(G))$, for all $\iota, \kappa \in \Lambda$,
(ii) $(\iota_0, \Upsilon\iota_0) \in E(G)$, *for an element* ι_0 *in* Λ,
(iii) Υ *is orbitally* $G-$*continuous, or,*
(iv) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $(\iota_n, \iota_{n+1}) \in E(G)$ *for all* $n \in \mathbb{N}$ *and* $\iota_n \to \iota$ *as* $n \to +\infty$, *then* $(\iota, \Upsilon\iota) \in E(G)$.
Then Υ *admits a fixed point in* Λ.

Corollary 4.9. *Let* $\left(\Lambda, \hat{d}\right)$ *be a graphic 0-*$\hat{d}-$*complete metric-like space* G. *Let* p *be a positive integer,* $\Gamma_1, \Gamma_2, ..., \Gamma_p$ *be nonempty* $\hat{d}-$*closed subsets of* Λ *and* $\Delta = \bigcup_{i=1}^{p} \Gamma_i$. *Assume that* $\Upsilon : \Delta \to \Delta$ *be a mapping such that:*

(i) $\Upsilon(\Gamma_i) \subseteq \Gamma_{i+1}$, $(i = 1, 2, ..., p)$, *where* $\Gamma_{p+1} = \Gamma_1$;
(ii) For all $\iota, \kappa \in \Delta$ $((\iota, \kappa) \in E(G)$ implies that $(\Upsilon\iota, \Upsilon\kappa) \in E(G))$;
(iii) $(\iota_0, \Upsilon\iota_0) \in E(G)$, *for an element* ι_0 *in* Δ,
(iv) Υ *is orbitally* $G-$*continuous, or,*

(v) if $\{\iota_n\}$ *is a sequence in* Λ *such that* $(\iota_n, \iota_{n+1}) \in E(G)$ *for all* $n \in \mathbb{N}$ *and* $\iota_n \to \iota$ *as* $n \to +\infty$, *then* $(\iota, \Upsilon\iota) \in E(G)$;

(vi) There exists $r \in (0,1)$ *such that* $(\iota, \Upsilon\iota) \in E(G)$ *and* $(\kappa, \Upsilon\kappa) \in E(G)$ *implies that*

$$\int_0^{\hat{d}(\Upsilon\iota, \Upsilon\kappa)} \rho(t)\, dt \le r \int_0^{\alpha} \rho(t)\, dt$$

for any $\iota \in \Gamma_i$, $\kappa \in \Gamma_{i+1}$, $(i = 1, 2, ..., p)$, *where* $\alpha = \max\left\{\hat{d}(\iota, \kappa), \hat{d}(\iota, \Upsilon\iota), \hat{d}(\kappa, \Upsilon\kappa), \frac{\hat{d}(\iota, \Upsilon\kappa) + \hat{d}(\kappa, \Upsilon\iota),}{4}\right\}$, $\Gamma_{p+1} = \Gamma_1$ *and* $\rho : [0, +\infty) \to [0, +\infty)$ *is a Lebesgue-integrable mapping satisfying* $\int_0^{\varepsilon} \rho(t)\, dt > 0$ *for all* $\varepsilon > 0$. *Then* Υ *admits a fixed point* $z \in \cap_{i=1}^{p} \Gamma_i$.

5 Application

The existence of solution for the following integral equation is the main purpose in this section.

$$\mu(\iota) = f\left(\iota, \int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa\right). \tag{12}$$

where $\iota \in [0, \infty)$.

By applying Theorem 3.1, we will establish the existence of solution for the integral equation (12).

Let $BC([0, \infty))$ be the space of all real, bounded and continuous functions on the interval $[0, \infty)$. We endow it with the metric like

$$d(\iota, \kappa) = \sup\{|\iota(t)| + |\kappa(t)| : t \in [0, \infty)\}.$$

Theorem 5.1. *Suppose that the following assumptions are satisfied:*

(i) $\rho, \varrho : [0, \infty) \longrightarrow [0, \infty)$ *are continuous functions so that*

$$\Lambda = \sup\{|\varrho(t)| : t \in [0, \infty)\} < 1,$$

(ii) *The function* $f : [0, \infty) \times \mathbb{R} \longrightarrow \mathbb{R}$ *is continuous so that*

$$\big|f\big(\iota, \mu\big)\big| \leq \big|\mu\big|,$$

for all $\iota \in [0, \infty)$ *and* $\mu \in \mathbb{R}, \mu \neq 0$*,*

(iii)

$$\Big|g(\iota, \kappa, \mu(\rho(\kappa)))\Big| \leq |\mu(\rho(\kappa))|$$

where $g : [0, \infty)^2 \times \mathbb{R} \longrightarrow \mathbb{R}$ *is continuous,*

(iv) $M = \max\{f(\iota, 0) : \iota \in [0, \infty)\} < \infty$ *and*
$G = \sup\Big\{\Big|g(\iota, \kappa, 0)\Big| : \iota \in [0, \infty)\Big\} < \infty.$

(v) $\pi(\lambda t) \leq \lambda[\pi(t)]$ *for all* $\lambda \in [0, 1)$ *and for all* $t \in [0, \infty)$*.*

Then the integral equation (12) *admits at least one solution in the space* $BC([0, \infty))$*.*

Proof. Let us consider the operator $\Upsilon : BC([0, \infty)) \longrightarrow BC([0, \infty))$ defined by

$$\Upsilon(\mu)(\iota) = f\big(\iota, \int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa).$$

In view of given assumptions, we infer that the function $\Upsilon(\mu)$ is continuous for arbitrarily$\mu \in BC([0, \infty))$. Now, we show that $\Upsilon(\mu)$ is bounded in $BC([0, \infty))$. As

$$\begin{aligned} |\Upsilon(\mu)(\iota)| &= \Big|f\big(\iota, \int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa)\Big| \\ &\leq \Big|f\big(\iota, \int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa) - f(\iota, 0)\Big| + \Big|f(\iota, 0)\Big|, \end{aligned}$$

we have

$$\begin{aligned} &\Big|f\big(\iota, \int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa) - f(\iota, 0)\Big| \\ &\leq \Big|\int_0^{\varrho(\iota)} g(\iota, \kappa, \mu(\rho(\kappa)))d\kappa\Big| \\ &\leq \Lambda\|\mu\| + \Lambda G. \end{aligned}$$

Thus,

$$\Big|f\big(\iota,\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu(\rho(\kappa)))d\kappa\big)-f(\iota,0)\Big|\leq \Lambda\|\mu\|+\Lambda G.$$

From the above calculations, we have

$$\|\Upsilon(\mu)(\iota)\|\leq \Lambda\|\mu\|+\Lambda G+M.$$

Due to the above inequality, the function Υ is bounded.

Now, we show that Υ satisfies all the conditions of Theorem 3.1. Let μ_1,μ_2 be some elements of $BC([0,\infty))$. Then we have

$$\begin{aligned}
&\pi\Big(\big|\Upsilon(\mu_1)(\iota)\big|+\big|\Upsilon(\mu_2)(\iota)\big|\Big)\\
&\leq \pi\Big(\Big|f\big(\iota,\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)))d\kappa\big)\Big|+\Big|f\big(\iota,\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_2(\rho(\kappa)))d\kappa\big)\Big|\Big)\\
&\leq \pi\Big(\big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)))d\kappa\big|+\big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_2(\rho(\kappa)))d\kappa\big|\Big)\\
&\leq \pi\Big(\int_0^{\varrho(\iota)} |\mu_1(\rho(\kappa))|d\kappa+\int_0^{\varrho(\iota)} |\mu_2(\rho(\kappa))\big|d\kappa\Big)\\
&\leq \pi\Big(\varrho(\iota)(d(\mu_1,\mu_2))\Big)\\
&\leq \pi\Big(\Lambda d(\mu_1,\mu_2)\Big)=\frac{\pi(d(\mu_1,\mu_2))}{1+\varpi(d(\mu_1,\mu_2))},
\end{aligned}$$

where $\pi,\varpi:[0,+\infty)\to[0,+\infty)$ are defined by $\pi(t)=t$ and $\varpi(t)=\frac{t-\Lambda t}{\Lambda t}$.

Thus, we obtain that

$$\pi(d(\Upsilon(\mu_1),\Upsilon(\mu_2)))\leq f(\pi(d(\mu_1,\mu_2)),\varpi(d(\mu_1,\mu_2))).$$

Using Theorem 3.1, we obtain that the operator Υ admits a fixed point. Thus, the functional integral equation (12) admits at least one solution in $BC([0,\infty))$. $\square$

Example 5.2. Let

$$\iota(t)=\tfrac{1}{8}\arctan\Big(\int_0^{\tanh t}\tfrac{\sinh\iota(s)+\arctan\iota(s)}{2e^t}ds\Big). \tag{13}$$

We observe that the integral equation (13) is a special case of (12) with $\rho(t) = t$ and $\varrho(t) = \tanh t$, where $t \in [0, 1]$. Also,

$$f(t, \iota) = \frac{\arctan(\iota)}{8},$$

and

$$g(t, s, \iota) = \frac{\sinh \iota + \arctan \iota}{2e^t}.$$

To solve this equation, we need to verify the conditions (i)-(iv) of Theorem 5.1.
Condition (i) is clearly evident. Take $\pi(t) = \cosh t$. Now,

$$\begin{aligned} |f(t, \iota)| &\leq \frac{|\arctan \iota|}{8} \\ &\leq \frac{|\iota|}{8} \leq |\iota|. \end{aligned}$$

So, we find that f satisfies condition (ii) of Theorem 5.1. Also,

$$M = \sup\{|f\big(t, 0\big)| : t \in [0, \infty)\} = 0.$$

Obviously, condition (iii) of Theorem 5.1 is valid, that is, g is continuous on $[0, \infty) \times [0, \infty) \times \mathbb{R}$, and

$$\begin{aligned} G &= \sup\left\{ \Big| \int_0^{\tanh t} \frac{1}{2e^t} ds \Big| : t \in [0, \infty) \right\} \\ &= \sup_{t \in [0,\infty)} \big(\frac{\tanh t}{4e^t}\big) \simeq 0.1. \end{aligned}$$

On the other hand,

$$\begin{aligned} |g(t, s, \iota(\rho(s)))| &= |\frac{\sinh \iota(s) + \arctan \iota(s)}{2e^t}| \\ &\leq |\iota(s)|. \end{aligned}$$

As a result, all the reservations of Theorem 5.1 are valid. So, there is at least one solution for integral equation (13) which is an element of $BC[0, \infty)$.

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