Journal of Mathematical Extension
Vol. 6, No. 2, (2012), 11-20

RNS-Approximately Nonlinear Additive Functional Equations

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Abstract. In this paper, we prove the generalized Hyers-Ulam-Rassias stability of a nonlinear additive functional equation in RNS.

AMS Subject Classification: 39B82; 39B52
Keywords and Phrases: Generalized Hyers-Ulam stability, random normed space, fixed point method

1. Introduction

A classical question in the theory of functional equations is the following question:
When is it true that a function which approximately satisfies a functional equation must be close to an exact solution of the equation?.
If the problem accepts a solution, we say that the equation is *stable*. The first stability problem concerning group homomorphisms was raised by Ulam ([29]) in 1940. In the next year, Hyers ([15]) gave a positive answer to the above question for additive groups under the assumption that the groups are Banach spaces. In 1978, Rassias ([23]) proved a generalization of Hyers's theorem for additive mappings. The result of Rassias has provided a lot of influence during the last three decades in the development of a generalization of the Hyers-Ulam stability concept. This new concept is known as generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability of functional equations. Furthermore, in 1994, a generalization of Rassias's theorem was obtained by Găvruta

Received: May 2011; Accepted: April 2012

([14]) by replacing the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem ([4]-[27]).
In the sequel, we adopt the usual terminology, notions and conventions of the theory of random normed spaces as in ([28]).
Throughout this paper, let Γ^+ denote the set of all probability distribution functions $F : \mathbb{R} \cup [-\infty, +\infty] \to [0, 1]$ such that F is left-continuous and nondecreasing on $\mathbb{R}$ and $F(0) = 0, F(+\infty) = 1$. It is clear that the set $D^+ = \{F \in \Gamma^+ : l^- F(-\infty) = 1\}$, where $l^- f(x) = \lim_{t \to x^-} f(t)$, is a subset of Γ^+. The set Γ^+ is partially ordered by the usual point-wise ordering of functions, that is, $F \leqslant G$ if and only if $F(t) \leqslant G(t)$ for all $t \in \mathbb{R}$. For any $a \geqslant 0$, the element $H_a(t)$ of D^+ is defined by

$$H_a(t) = \begin{cases} 0, & \text{if} \quad t \leqslant a, \\ 1, & \text{if} \quad t > a. \end{cases}$$

We can easily show that the maximal element in Γ^+ is the distribution function $H_0(t)$.

Definition 1.1. *A function $T : [0, 1]^2 \to [0, 1]$ is a continuous triangular norm (briefly, a t-norm) if T satisfies the following conditions:*
(a) T is commutative and associative; (b) T is continuous; (c) $T(x, 1) = x$ for all $x \in [0, 1]$; (d) $T(x, y) \leqslant T(z, w)$ whenever $x \leqslant z$ and $y \leqslant w$ for all $x, y, z, w \in [0, 1]$.
Three typical examples of continuous t-norms are as follows: $T_P(x, y) = xy, T_{max}(x, y) = \max\{a + b - 1, 0\}, T_M(x, y) = \min(a, b)$. Recall that, if T is a t-norm and $\{x_n\}$ is a sequence in $[0, 1]$, then $T_{i=1}^n x_i$ is defined recursively by $T_{i=1}^1 x_1 = x_1$ and $T_{i=1}^n x_i = T(T_{i=1}^{n-1} x_i, x_n)$ for all $n \geqslant 2$. $T_{i=n}^\infty x_i$ is defined by $T_{i=1}^\infty x_{n+i}$.

Definition 1.2. *A random normed space (briefly, RNS) is a triple (X, μ, T), where X is a vector space, T is a continuous t-norm and $\mu : X \to D^+$ is a mapping such that the following conditions hold:*
(a) $\mu_x(t) = H_0(t)$ for all $x \in X$ and $t > 0$ if and only if $x = 0$;
(b) $\mu_{\alpha x}(t) = \mu_x\left(\frac{t}{|\alpha|}\right)$ for all $\alpha \in \mathbb{R}$ with $\alpha \neq 0$, $x \in X$ and $t \geqslant 0$;

(c) $\mu_{x+y}(t+s) \geqslant T(\mu_x(t), \mu_y(s))$ *for all* $x, y \in X$ *and* $t, s \geqslant 0$. *Every normed space* $(X, \|\cdot\|)$ *defines a random normed space* (X, μ, T_M), *where* $\mu_u(t) = \frac{t}{t+\|u\|}$ *for all* $t > 0$ *and* T_M *is the minimum t-norm. This space* X *is called the induced random normed space. If the t-norm* T *is such that* $\sup_{0<a<1} T(a,a) = 1$, *then every RNS* (X, μ, T) *is a metrizable linear topological space with the topology* τ *(called the* μ*-topology or the* (ϵ, δ)*-topology, where* $\epsilon > 0$ *and* $\lambda \in (0,1)$*) induced by the base* $\{U(\epsilon, \lambda)\}$ *of neighborhoods of* θ, *where* $U(\epsilon, \lambda) = \{x \in X : \Psi_x(\epsilon) > 1 - \lambda\}$.

Definition 1.3. *Let* (X, μ, T) *be an RNS.*
(a) *A sequence* $\{x_n\}$ *in* X *is said to be convergent to a point* $x \in X$ *(write* $x_n \to x$ *as* $n \to \infty$*) if* $\lim_{n\to\infty} \mu_{x_n - x}(t) = 1$ *for all* $t > 0$.
(b) *A sequence* $\{x_n\}$ *in* X *is called a Cauchy sequence in* X *if* $\lim_{n\to\infty} \mu_{x_n - x_m}(t) = 1$ *for all* $t > 0$.
(c) *The RNS* (X, μ, T) *is said to be complete if every Cauchy sequence in* X *is convergent.*

Theorem 1.4. ([28]) *If* (X, μ, T) *is an RNS and* $\{x_n\}$ *is a sequence such that* $x_n \to x$, *then* $\lim_{n\to\infty} \mu_{x_n}(t) = \mu_x(t)$.

Definition 1.5. *Let* X *be a set. A function* $d : X \times X \to [0, \infty]$ *is called a generalized metric on* X *if* d *satisfies the following conditions:* (a) $d(x,y) = 0$ *if and only if* $x = y$ *for all* $x, y \in X$; (b) $d(x,y) = d(y,x)$ *for all* $x, y \in X$; (c) $d(x,z) \leqslant d(x,y) + d(y,z)$ *for all* $x, y, z \in X$.

Theorem 1.6. *Let (X,d) be a complete generalized metric space and* $J : X \to X$ *be a strictly contractive mapping with Lipschitz constant* $L < 1$. *Then, for all* $x \in X$, *either*

$$d(J^n x, J^{n+1} x) = \infty \quad , \tag{1}$$

for all nonnegative integers n *or there exists a positive integer* n_0 *such that*
(a) $d(J^n x, J^{n+1} x) < \infty$ *for all* $n_0 \geqslant n_0$;
(b) *the sequence* $\{J^n x\}$ *converges to a fixed point* y^* *of* J;
(c) y^* *is the unique fixed point of* J *in the set* $Y = \{y \in X : d(J^{n_0} x, y) < \infty\}$;
(d) $d(y, y^*) \leqslant \frac{1}{1-L} d(y, Jy)$ *for all* $y \in Y$.

In this paper, we prove the generalized Hyers-Ulam-Rassias stability of the following functional equation:

$$f(f(x)-f(y))+f(x)+f(y)=f(x+y)+f(x-y), \tag{2}$$

in RNS.

2. Main Results: RNS-Approximation of the Functional Equation (2)

In this section, using direct method, we prove the generalized Hyers-Ulam-Rassias stability of the functional equation (2) in random normed spaces.

Theorem 2.1. *Let X be a real linear space, $(Z,\mu',\min)$ be an RN-space and $\phi : X^2 \to Z$ be a function such that there exists $0<\alpha<\frac{1}{2}$ such that*

$$\mu'_{\phi(\frac{x}{2},\frac{y}{2})}(t) \geqslant \mu'_{\alpha\phi(x,y)}(t) \tag{3}$$

for all $x\in X$ and $t>0$ and $\lim_{n\to\infty}\mu'_{\phi(\frac{x}{2^n},\frac{y}{2^n})}\left(\frac{t}{2^n}\right)=1$ for all $x,y\in X$ and $t>0$. Let $(Y,\mu,\min)$ be a complete RN-space. If $f : X\to Y$ be a mapping such that

$$\mu_{f(f(x)-f(y))-f(x+y)-f(x-y)+f(x)+f(y)}(t) \geqslant \mu'_{\phi(x,y)}(t) \tag{4}$$

for all $x,y\in X$ and $t>0$. Then the limit $A(x)=\lim_{n\to\infty}2^n f\left(\frac{x}{2^n}\right)$ exists for all $x\in X$ and defines a unique additive mapping $A : X\to Y$ such that and

$$\mu_{f(x)-A(x)}(t) \geqslant \mu'_{\phi(x,x)}\left(\frac{(1-2\alpha)t}{\alpha}\right), \tag{5}$$

for all $x\in X$ and $t>0$.

Proof. Putting $y=x$ in (4), we see that

$$\mu_{f(2x)-2f(x)}(t) \geqslant \mu'_{\phi(x,x)}(t). \tag{6}$$

Replacing x by $\frac{x}{2}$ in (6), we obtain

$$\mu_{2f(\frac{x}{2})-f(x)}(t) \geqslant \mu'_{\phi(\frac{x}{2},\frac{x}{2})}(t) \quad , \tag{7}$$

for all $x \in X$. Replacing x by $\frac{x}{2^n}$ in (7) and using (3), we obtain

$$\mu_{2^{n+1}f(\frac{x}{2^{n+1}})-2^n f(\frac{x}{2^n})}(t) \geqslant \mu'_{\phi\left(\frac{x}{2^{n+1}},\frac{x}{2^{n+1}}\right)}\left(\frac{t}{2^n}\right) \geqslant \mu'_{\phi(x,x)}\left(\frac{t}{2^n\alpha^{n+1}}\right).$$

Since

$$2^n f\left(\frac{x}{2^n}\right) - f(x) = \sum_{k=0}^{n-1} 2^{k+1} f\left(\frac{x}{2^{k+1}}\right) - 2^k f\left(\frac{x}{2^k}\right),$$

and so

$$\begin{aligned}
\mu_{2^n f(\frac{x}{2^n})-f(x)}\left(\sum_{k=0}^{n-1} 2^k\alpha^{k+1}t\right) &= \mu_{\sum_{k=0}^{n-1} 2^{k+1}f(\frac{x}{2^{k+1}})-2^k f(\frac{x}{2^k})}\left(\sum_{k=0}^{n-1} 2^k\alpha^{k+1}t\right) \\
&\geqslant T_{k=0}^{n-1}\left(\mu_{2^{k+1}f(\frac{x}{2^{k+1}})-2^k f(\frac{x}{2^k})}(2^k\alpha^{k+1}t)\right) \\
&\geqslant T_{k=0}^{n-1}\left(\mu'_{\phi(x,x)}(t)\right) = \mu'_{\phi(x,x)}(t).
\end{aligned}$$

This implies that $\mu_{2^n f(\frac{x}{2^n})-f(x)}(t) \geqslant \mu'_{\phi(x,x)}\left(\frac{t}{\sum_{k=0}^{n-1} 2^k\alpha^{k+1}}\right)$. Replacing x by $\frac{x}{2^p}$ in the recent inequality, we obtain

$$\mu_{2^{n+p}f(\frac{x}{2^{n+p}})-2^p f(\frac{x}{2^p})}(t) \geqslant \mu'_{\phi(x,x)}\left(\frac{t}{\sum_{k=p}^{n+p-1} 2^k\alpha^{k+1}}\right). \tag{8}$$

Since $\lim_{p,n\to\infty} \mu'_{\phi(x,x)}\left(\frac{t}{\sum_{k=p}^{n+p-1} 2^k\alpha^{k+1}}\right) = 1$, it follows that $\left\{2^n f(\frac{x}{2^n})\right\}_{n=1}^{\infty}$ is a Cauchy sequence in a complete RN-space $(Y, \mu, \min)$ and so there exists a point $A(x) \in Y$ such that $\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = A(x)$. Fix $x \in X$ and put $p = 0$ in (8). Then we obtain

$$\mu_{2^n f(\frac{x}{2^n})-f(x)}(t) \geqslant \mu'_{\phi(x,x)}\left(\frac{t}{\sum_{k=0}^{n-1} 2^k\alpha^{k+1}}\right) \quad ,$$

and so, for any $\epsilon > 0$,

$$\begin{aligned} \mu_{A(x)\pm 2^n f(\frac{x}{2^n})-f(x)}(t+\epsilon) &\geqslant T\left(\mu_{A(x)-2^n f(\frac{x}{2^n})}(\epsilon), \mu_{2^n f(\frac{x}{2^n})-f(x)}(t)\right) \\ &\geqslant T\left(\mu_{A(x)-2^n f(\frac{x}{2^n})}(\epsilon), \mu'_{\phi(x,x)}\left(\frac{t}{\sum_{k=0}^{n-1} 2^k \alpha^{k+1}}\right)\right). \end{aligned} \tag{9}$$

Taking $n \to \infty$ in (9), we get

$$\mu_{A(x)-f(x)}(t+\epsilon) \geqslant \mu'_{\phi(x,x)}\left(\frac{(1-2\alpha)t}{\alpha}\right). \tag{10}$$

Since ϵ is arbitrary, by taking $\epsilon \to 0$ in (10), we get

$$\mu_{A(x)-f(x)}(t) \geqslant \mu'_{\phi(x,x)}\left(\frac{(1-2\alpha)t}{\alpha}\right).$$

Replacing x and y by $\frac{x}{2^n}$ and $\frac{y}{2^n}$ in (4), respectively, we get

$$\mu_{2^n\left[f\left(f\left(\frac{x}{2^n}\right)-f\left(\frac{y}{2^n}\right)\right)-f\left(\frac{x+y}{2^n}\right)-f\left(\frac{x-y}{2^n}\right)+f\left(\frac{x}{2^n}\right)+f\left(\frac{y}{2^n}\right)\right]}(t) \geqslant \mu'_{\phi(\frac{x}{2^n},\frac{y}{2^n})}\left(\frac{t}{2^n}\right)$$

for all $x, y \in X$ and $t > 0$. Since $\lim_{n\to\infty} \mu'_{\phi(\frac{x}{2^n},\frac{y}{2^n})}\left(\frac{t}{2^n}\right) = 1$, we conclude that A satisfies (2). On the other hand

$$2A\left(\frac{x}{2}\right) - A(x) = \lim_{n\to\infty} 2^{n+1} f\left(\frac{x}{2^{n+1}}\right) - \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = 0. \tag{11}$$

This implies that $A : X \to Y$ is an additive mapping. To prove the uniqueness of the additive mapping A, assume that there exists another additive mapping $B : X \to Y$ which satisfies (5). Then we have

$$\begin{aligned} \mu_{A(x)-B(x)}(t) &= \lim_{n\to\infty} \mu_{2^n A(\frac{x}{2^n})-2^n B(\frac{x}{2^n})}(t) \\ &\geqslant \lim_{n\to\infty} \min\left\{\mu_{2^n A(\frac{x}{2^n})-2^n f(\frac{x}{2^n})}\left(\frac{t}{2}\right), \mu_{2^n f(\frac{x}{2^n})-2^n B(\frac{x}{2^n})}\left(\frac{t}{2}\right)\right\} \\ &\geqslant \lim_{n\to\infty} \mu'_{\phi(\frac{x}{2^n},\frac{x}{2^n})}\left(\frac{(1-2\alpha)t}{2^{n+1}\alpha}\right) \geqslant \lim_{n\to\infty} \mu'_{\phi(x,x)}\left(\frac{(1-2\alpha)t}{2^{n+1}\alpha^n}\right). \end{aligned}$$

Since $\lim_{n\to\infty} \frac{(1-2\alpha)t}{2^{n+1}\alpha^n} = \infty$, we get $\lim_{n\to\infty} \mu'_{\phi(x,x)}\left(\frac{(1-2\alpha)t}{2^{n+1}\alpha^n}\right) = 1$.Therefore, it follows that $\mu_{A(x)-B(x)}(t) = 1$ for all $t > 0$ and so $A(x) = B(x)$. This completes the proof. $\square$

Corollary 2.2. *Let X be a real normed linear space, $(Z, \mu', \min)$ be an RN-space and $(Y, \mu, \min)$ be a complete RN-space. Let r be a positive real number with $r > 1$, $z_0 \in Z$ and $f : X \to Y$ be a mapping satisfying*

$$\mu_{f(f(x)-f(y))-f(x+y)-f(x-y)+f(x)+f(y)}(t) \geqslant \mu'_{(\|x\|^r+\|y\|^r)z_0}(t), \qquad (12)$$

for all $x, y \in X$ and $t > 0$. Then the limit $A(x) = \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$ exists for all $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that and

$$\mu_{f(x)-A(x)}(t) \geqslant \mu'_{\|x\|^p z_0}\left(\frac{(2^r-2)t}{2}\right),$$

for all $x \in X$ and $t > 0$.

Proof. Let $\alpha = 2^{-r}$ and $\phi : X^2 \to Z$ be a mapping defined by $\phi(x, y) = (\|x\|^r + \|y\|^r)z_0$. Then, from Theorem 2.1, the conclusion follows. □

Theorem 2.3. *Let X be a real linear space, $(Z, \mu', \min)$ be an RN-space and $\phi : X^2 \to Z$ be a function such that there exists $0 < \alpha < 2$ such that $\mu'_{\phi(2x,2y)}(t) \geqslant \mu'_{\alpha\phi(x,y)}(t)$ for all $x \in X$ and $t > 0$ and $\lim_{n\to\infty} \mu'_{\phi(2^n x,2^n y)}(2^n x) = 1$ for all $x, y \in X$ and $t > 0$. Let $(Y, \mu, \min)$ be a complete RN-space. If $f : X \to Y$ be a mapping satisfying (4). Then the limit $A(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for all $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that*

$$\mu_{f(x)-A(x)}(t) \geqslant \mu'_{\phi(x,x)}((2-\alpha)t), \qquad (13)$$

for all $x \in X$ and $t > 0$.

Proof. Putting $y = x$ in (4), we see that

$$\mu_{\frac{f(2x)}{2}-f(x)}(t) \geqslant \mu'_{\phi(x,x)}(2t). \qquad (14)$$

Replacing x by $2^n x$ in (14), we obtain that

$$\mu_{\frac{f(2^{n+1}x)}{2^{n+1}}-\frac{f(2^n x)}{2^n}}(t) \geqslant \mu'_{\phi(2^n x,2^n x)}(2^{n+1}t) \geqslant \mu_{\phi(x,x)}\left(\frac{2^{n+1}t}{\alpha^n}\right). \qquad (15)$$

The rest of the proof is similar to the proof of Theorem 2.1. □

Corollary 2.4. *Let X be a real normed linear space, $(Z, \mu', \min)$ be an RN-space and $(Y, \mu, \min)$ be a complete RN-space. Let r be a positive real number with $0 < r < 1$, $z_0 \in Z$ and $f : X \to Y$ be a mapping satisfying (12). Then the limit $A(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for all $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that*

$$\mu_{f(x)-A(x)}(t) \geqslant \mu'_{\|x\|^p z_0}\left(\frac{(2-2^r)t}{2}\right),$$

for all $x \in X$ and $t > 0$.

Proof. Let $\alpha = 2^r$ and $\phi : X^2 \to Z$ be a mapping defined by $\phi(x, y) = (\|x\|^r + \|y\|^r)z_0$. Then, from Theorem 2.2., the conclusion follows. $\square$

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